CONSTITUTIVE MODELLING OF HUMAN PERIVASCULAR ADIPOSE TISSUE

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Abstract. The mechanical properties of adipose tissue are studied rarely despite the fact that surrounding tissue plays an important role in arterial physiology and mechanobiology. This study deals with the determination of material parameters of human perivascular adipose tissue surrounding the abdominal aorta. The selected representative experimental curve from uniaxial tensile test was used to be fitted by Ogden, Gent, and Fung hyperelastic models assuming isotropic and incompressible material of perivascular tissue. The estimated material parameters are \( \mu = 0.001 \text{ MPa} \) and \( \alpha = 28 \), \( \mu = 0.026 \text{ MPa} \) and \( J_m = 0.067 \), \( \mu = 0.018 \text{ MPa} \) and \( b = 31.834 \), respectively. It was concluded that all applied mathematical models predicted nonlinear stress-strain relationship satisfactorily with coefficient of determination 0.995, 0.981, 0.994, respectively.

1 INTRODUCTION

One of the modifiable risk factors of mortality to cardiovascular diseases is obesity [1, 2]. The adipose tissue surrounds all blood vessels and organs and influences their mechanical state. This paper is focused on constitutive modelling of human perivascular adipose tissue that surrounds the abdominal aorta. The perivascular tissue should be taken into account in solving various types of boundary problems of abdominal aorta such as computational simulations of aneurysm or in vivo constitutive modeling.

The main function of fat tissue is the protection organs before trauma, and energy storage. The adipose tissue contains preadipocytes, adipocytes (fat cells), endothelial cells, monocytes/macrophages, nerve fibers and blood vessels [1-3].

The fat cells produce proteins such as adiponectin, leptin, adipokines and cytokines. Adiponectin significantly regulates metabolism of saccharides and lipids and increases the
transport of glucose and free fatty acids into muscle, liver and fat cells. Leptin is essential contribution to regulation of body weight [1-8]. Adipokines and cytokines are inflammatory proteins which may diffuse from surrounding adipose tissue into the arterial wall, where subsequently may cause endothelial dysfunction, hypercoagulability and proliferation of smooth muscle cells [1].

The aim of this study is to determine the material parameters of human perivascular adipose tissue surrounding abdominal aorta by fitting hyperelastic material models (Ogden, Gent, Fung) to uniaxial tensile test data.

2 MATERIALS AND METHODS

The human abdominal aorta specimen with perivascular adipose tissue was received from the Faculty Hospital Královské Vinohrady in Prague and was resected from cadaveric donor (male, 29 years old). Our study has been approved by the Ethics Committee of the Faculty Hospital Královské Vinohrady. The sample of perivascular tissue was manually separated from aortic wall using a scalpel and subsequently trimmed to approximately rectangular shape (Figure 1). Before the mechanical testing, reference sample dimensions (Table 1) were determined by image analysis of digital photographs (Nis-Elements, Nikon Instruments Inc., NY, USA). Axis of the adipose tissue specimen was parallel with axis of the abdominal aorta (Figure 2). The fat tissue sample was tested in air, at room temperature and 38 hours after removal from a deceased donor.

<table>
<thead>
<tr>
<th>Donor</th>
<th>Age [years]</th>
<th>Sex</th>
<th>Reference dimensions (mean ± SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M29</td>
<td>29</td>
<td>Male</td>
<td>7.998 ± 0.295</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.456 ± 0.456</td>
</tr>
</tbody>
</table>

Table 1: Reference dimensions of the perivascular tissue sample from male donor aged 29 years.

Figure 1: Sample of the human abdominal aorta and iliac arteries with surrounding perivascular adipose tissue.
2.1 Kinematics of uniaxial tensile test

Consider $x$ and $X$, respectively, denoting the position vector of material particle in some reference (undeformed) configuration $\beta_r$ and some deformed (current) configuration $\beta$. Motion of a continuum body is shown in Figure 2. The mapping from reference to current configuration is described by [9]

$$x = \chi(X,t),$$

where $\chi$ is the function describing the motion.

The deformation gradient tensor is described by equation [9]

$$F = \frac{\partial x}{\partial X}. \quad (2)$$

In case of incompressible material can be determinant of the deformation gradient expressed as [9]

$$\det F = \lambda_1 \lambda_2 \lambda_3 = 1, \quad (3)$$

where $\lambda$ are the principal stretches and index 1 indicates direction of the loading force, $\lambda_2$ and $\lambda_3$ are transversal stretches.
For an isotropic material during uniaxial tensile test can be written the following relationship [10]

\[ \lambda_2 = \lambda_3. \]  
(4)

From equation (3) and (4) implies [10]

\[ \lambda_2 = \frac{1}{\sqrt{\lambda_1}}. \]  
(5)

The deformation gradient when considering incompressibility and isotropy is [10]

\[ F = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda_1}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_1}} \end{pmatrix}. \]  
(6)

The right Cauchy-Green strain tensor is generated from the deformation gradient [9]

\[ C = F^T F, \]  
(7)

where \( T \) represents transpose of a second-order tensor.

### 2.2 Constitutive modelling

The strain energy density function for the hyperelastic Ogden [11], Gent [12], and Fung [13] model is expressed as

\[ W_\text{Ogden} = \frac{k}{\alpha} (\lambda_1^\alpha + \lambda_2^\alpha + \lambda_3^\alpha - 3), \]  
(8)

\[ W_\text{Gent} = \frac{-1}{2} \mu J_m \ln \left(1 - \frac{I_1 - 3}{J_m}\right), \]  
(9)

\[ W_\text{Fung} = \frac{1}{2} \mu (e^{b(I_1 - 3)} - 1), \]  
(10)

where \( \lambda_1, \lambda_2, \lambda_3 \) are the principal stretches and \( I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \) is the first invariant of the right Cauchy-Green strain tensor. \( \mu, \alpha, J_m, b \) are material parameters. \( \mu \) is positive stress-like material parameter corresponding at infinitesimal strain in shear modulus. \( \alpha, J_m, b \) are dimensionless material constants which indicate strain stiffening of material.

The Cauchy stress tensor for incompressible material is given by constitutive equation [11]

\[ \sigma = F \frac{\partial W}{\partial F} - pI, \]  
(11)
where $\mathbf{F}$ is the deformation gradient, $\mathbf{I}$ is the unit tensor. $p$ is the Lagrange multiplier which is interpreted as the hydrostatic pressure resulting from the incompressibility constrain $\sigma_{22} = 0$.

The Cauchy stress acting in the direction of axial force during uniaxial tensile loading for incompressible material can be express by equations (8) for Ogden, (9) for Gent, and (10) for Fung model.

$$
\sigma_{\text{mod}}^{\text{Ogden}} = \mu \lambda_1^2 - \mu \left( \frac{1}{\sqrt{\lambda_1}} \right)^\alpha,
$$

$$
\sigma_{\text{mod}}^{\text{Gent}} = \frac{\mu J_2 \left( \lambda_1^2 - 1 \right)}{J_2 + 3 \lambda_1^2 - 1},
$$

$$
\sigma_{\text{mod}}^{\text{Fung}} = \frac{\mu}{\lambda_1} \frac{h (\lambda_1 + 2)(\lambda_1 - 1)^2}{\lambda_1^3 - 1}.
$$

### 2.3 Uniaxial tensile test protocol

The multipurpose testing machine (Zwick/Roell, Ulm, Germany) was used for measurement of mechanical properties of human perivascular tissue (Figure 3a). Values of sample deformations were determined by built-in video extensometer by means of the distance of transverse marks on the sample surface (Figure 3b). Current force (U9B, ± 25 N, HBM, Darmstadt, Germany) and video extensometer data were recorded into the controlling computer. Four loading cycles were realized at constant velocity of the clamp 0.2 mm/s.

![Figure 3: (a) The multipurpose testing machine for uniaxial tensile test. (b) The tested sample of human perivascular tissue in the jaws of the multipurpose testing machine with transverse marks for the strain measurement.](image)
In the case of uniaxial tensile tests, incompressible and isotropic material is the Cauchy stress expressed as [10]

\[ \sigma_{\text{exp}} = \lambda_1 \frac{f}{A}. \]  \hfill (15)

2.4 Optimization

The material parameters \((\mu, \alpha, J_m, b)\) have been determined by means of nonlinear least squares regression method. The objective function \(Q\) was minimized in Maple (Maplesoft, Waterloo, Canada)

\[ Q^2 = \sum_{i=1}^{n} \left( \sigma_{\text{exp}_i} - \sigma_{\text{mod}_i} \right)^2, \]  \hfill (16)

where \(n\) is the number of observation points.

The quality of the regression was evaluated by means of the coefficient of determination

\[ \rho^2(\sigma) = 1 - \frac{\sum_{i=1}^{n} (\sigma_{\text{mod}_i} - \sigma_{\text{exp}_i})^2}{\sum_{i=1}^{n} (\sigma_{\text{exp}_i} - \text{Mean}(\sigma_{\text{exp}}))^2}. \]  \hfill (17)

3 RESULTS

The reference dimensions (Table 1) were determined as a mean value of six measurements of geometrically non-uniform sample (width, thickness and cross-section area).

The uniaxial tensile test data (the stress-stretch relationship) fitted by models (8), (9), (10) are displayed in Figure 4. The estimated material parameters, \(\alpha = 28\) and \(\mu = 0.001\) MPa, \(J_m = 0.067\) and \(\mu = 0.026\) MPa, \(b = 31.834\) and \(\mu = 0.018\) MPa, respectively, are summarized in Table 2. The goodness of fit of the nonlinear regression analysis is indicated by the coefficients of determination which are included in Table 2.

Table 2: Material parameters obtained from Ogden, Gent and Fung model and coefficient of determination.

<table>
<thead>
<tr>
<th>Hyperelastic model</th>
<th>Material parameter</th>
<th>Material parameter</th>
<th>Coefficient of determination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ogden</td>
<td>(\mu) [MPa]</td>
<td>(\alpha, J_m, b) [-]</td>
<td>0.995</td>
</tr>
<tr>
<td>Gent</td>
<td>0.001</td>
<td>28</td>
<td>0.995</td>
</tr>
<tr>
<td>Fung</td>
<td>0.026</td>
<td>0.067</td>
<td>0.981</td>
</tr>
<tr>
<td></td>
<td>0.018</td>
<td>31.834</td>
<td>0.994</td>
</tr>
</tbody>
</table>
4 DISCUSSION

This article is focused on the constitutive modelling of human perivascular adipose tissue. The sample of perivascular tissue from donor, 29 year old men (M29), was subjected to uniaxial tensile test at post mortem interval 38 hours (Figure 1). It has been observed nonlinear behavior with large deformations typical for biological materials (Figure 4). The experimental stress-stretch curve was used to fit three hyperelastic models (Ogden, Gent, Fung) assuming isotropic and incompressible material of the tissue (Figure 4). Material parameters (Table 2) of perivascular adipose tissue were obtained for each model applying a nonlinear regression analysis. We concluded that all applied material models predicted the uniaxial tensile test data satisfactorily.

To the best of authors’ knowledge, this is the first study which offers materials parameters for human perivascular adipose tissue surrounding abdominal aorta. In the literature, we can find studies dealing with mechanical properties of adipose tissue located in other areas of the body. Sommer et al. [14] found constitutive parameters from biaxial tensile tests of 8 samples \((c, k_1, k_2, \varphi, \kappa)\) and triaxial shear test with 6 specimens of subcutaneous adipose tissue \((c, k_1, k_2, \varphi, v, \kappa)\). Holzapfel-Gasser-Ogden hyperelastic model that includes incompressibility and anisotropy defined by two families of collagen fibers was used to determine the material parameters.
Material models are part of commercial FEM software, therefore the results of our study could be included in stress analysis of the human abdominal aorta with surrounding perivascular adipose tissue or simulation of aneurysm.

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REFERENCES