SHAKEDOWN ANALYSIS OF 3D FRAMES SUBJECTED TO COMPLEX STATICAL AND SEISMIC LOAD COMBINATIONS

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Abstract. The paper proposes a strategy for the treatment of load combinations making the shakedown analysis an affordable design tool for practical applications. The paper refers to 3D frames subjected to complex combinations of statical and dynamical loads defined according to Eurocode rules. The yield surface of the sections is defined by its support function values associated to presso-flexural mechanisms and it is approximated as Minkowski sum of ellipsoids. The detection of the significant vertexes of the elastic envelope is made possible by an efficient algorithm suitable for use in the case of response spectrum analyses.

1 Introduction

Structures, during their operational life, are subjected to a sequence of variable actions, including dead, anthropic and natural loads. Building codes fix the range and extension of load variability through combination formulas that usually involve a large number of load conditions. The loading scenario can become very complex in presence of seismic actions taken into account through nonlinear modal combination rules such as the well known SRSS or CQC rules.

The shakedown analysis [1, 2] plays an important role in reducing tip stresses coming from elastic solutions and allows reference to smoother stress fields, better suited for design purposes to improve both element and reinforcement design. For this reason, as well as shakedown analysis being particularly appropriate for steel structures due to their ductility behavior, it can also become an important design tool for reinforced concrete (RC) frames, especially when the re-bar design is obtained by fully automatic procedures.

The availability of high optimized IPM solvers, such as MCSEK [20], makes this approach
interesting. Alternative specialized formulations to evaluate the shakedown safety factors in a FE context of analysis, have also been proposed in [9, 10, 11, 21].

Despite its technical implications, nowadays shakedown analysis still seems confined to the research community instead of being a common tool in structural design. This is largely due to the difficulties in managing the large number of load conditions and the complex combination rules.

According to the König [2] theorem considering the set of vertexes of the load domain is obtained a convex polytope with \(2^n\) vertexes, \(p\) being the number of basic loads. In fact the stress envelope, is controlled by the admissibility of the stress fields corresponding to the load vertexes. However the number of vertexes becomes soon too large reducing both the efficiency and robustness of the solution process.

A suitable strategy is therefore needed for treating a large number of load conditions and complex combination rules. In particular at local level of analysis (i.e. finite element or Gauss point), exploiting the convexity of the yield function we can retain, the convex hull vertexes of the stress envelope, as their plastic admissibility implies that of the overall set. In this way an effective evaluation of the convex of the local stress envelope hull becomes crucial. General approaches approach becomes time and memory consuming when the number of loads grows, or even impossible when the load domain is not available in an explicit way, as in the case of the response spectrum seismic analysis using nonlinear combination rules such as SRSS or CQC.

For these reasons a computational strategy, called Selection Rule Algorithm (SRA), that directly evaluates the significant vertexes of the convex hull is proposed. The SRA is very efficient and suitable for dealing with both statical and seismic load combinations. In the Response Spectrum Seismic Analysis (RSSA) case it provides a simple and mechanically meaningful definition of the stress peak values to consider in the CQC or SRSS combination rules and provides an analytical evaluation of the stress envelope which coincides with the ellipsoid largely adopted in literature [22]. The SRA is general and suitable for an easy extension to other contexts of analysis.

The strategy is described for 3D frame analysis which is a simple, but technically relevant, structural context.

The yield function is usually evaluated, in a somewhat oversimplified form, considering only presso–flexural collapse mechanisms. Even with these simplifications, its accurate definition in a computational framework is not an easy task.

The approximation of the true nonlinear yield surfaces is done by a Minkowski Sum of Ellipsoids (MSE), proposed in [4] for limit analysis problems. The resultant Second Order Cone Programming problem is efficiently solved with the general purpose code Mosek.

Numerical examples, shows the efficiency and accuracy of the proposed SRA in the evaluation of the convex hull of the stress envelope.
2 The 3D beam model

The beam model used and its discrete finite element version, is briefly well described in [12, 13]. The construction of the 3D elastic domain for each beam section is obtained using the approach initially proposed in [5] and recently extended in [4] (see also [3]) where it is described by using the Minkowski sum of ellipsoids concept.

2.1 Support functions of the beam elastic domain

The beam section domain $\Omega$ is the union of the $n_d$ sub domains $\Omega_i$ in which the material is homogeneous and elastic-perfectly plastic. For each $\Omega_i$, the plastic admissibility condition is expressed in terms of normal stress only as $-\sigma_{ni} \leq \sigma_{ii} \leq \sigma_{ni}$ where $\sigma_{ni}$ is the ultimate normal stress in tension (positive) and $\sigma_{ni}$ in compression (negative). This corresponds to assuming infinitely resistant frame members with respect to shear effects as well as torsion. Letting $\mathbf{t}[s] = \{N_1, M_2, M_3\}$, the section yield function $\Phi[s, \mathbf{t}[s]]$ will be defined in a 3D space involving axial force $N_1$ and bending moments $M_2$ and $M_3$. Denoting the section elastic domain with $\mathbb{E}[s]$ we have

$$\mathbb{E}[s] = \{\mathbf{t}[s] : \Phi[s, \mathbf{t}[s]] \leq 0\}. \quad (1)$$

By this assumption, the plastic mechanisms will be defined by the versor

$$\hat{\mathbf{e}} = \{\epsilon_1, \chi_2, \chi_3\}, \quad ||\hat{\mathbf{e}}|| = 1 \quad (2)$$

We define

$$\pi_{\mathbb{E}}[\hat{\mathbf{e}}] = \max\{\hat{\mathbf{e}}^T \mathbf{t} : \mathbf{t} \in \mathbb{E}\} \quad (3)$$

the signed distance with respect to the origin of the hyperplane tangent to $\mathbb{E}$ and with normal $\hat{\mathbf{e}}$, i.e. it is the support function of $\mathbb{E}$.

2.2 The approximation of $\mathbb{E}$ using a Minkowski sum of ellipsoids

Once the support function values $\pi_{\mathbb{E}}[\mathbf{n}_k]$ for $N_p$ directions $\mathbf{n}_k := \hat{\mathbf{e}}_k$ have been evaluated, the approximation of $\mathbb{E}$ is obtained by means of a Minkowski sum of $N_\epsilon$ ellipsoids using the approach proposed in [4], which can be referred to for further details. In this way the elastic domain approximate as a Minkowski sum of ellipsoids

$$\mathbb{E}[s] \equiv \left\{ \mathbf{t}_s := \mathbf{c}_s + \sum_{l=1}^{N_\epsilon} \mathbf{t}_{sl} : \mathbf{t}_{sl} \in \mathcal{E}_l[C_l, 0] \right\} \quad (4)$$

where, from now on, the subscript $s$ denotes the quantities of a section.

Introducing the ellipsoid vector $\mathbf{t}_{s\epsilon} = \{\mathbf{t}_{s1}, \ldots, \mathbf{t}_{sN_\epsilon}\}$ the plastic admissibility of the stress $\mathbf{t}_s$ in terms of the its ellipsoid contributions becomes

$$\Phi[s, \mathbf{t}[s]] \leq 0 \iff \left\{ \begin{array}{l} \mathbf{t}_s = \mathbf{c}_s + \sum \mathbf{t}_{s\epsilon} \\ \Phi_{s\epsilon}[\mathbf{t}_{s\epsilon}] \leq 0 \end{array} \right\} \quad (5)$$
where \( \Phi_{\alpha\varepsilon}[t_{\varepsilon}] := \{ \phi_{s_1}[t_{s_1}], \ldots, \phi_{s_N}[t_{s_N}] \} \), the matrix \( \Sigma_\varepsilon \) is implicitly defined by Eq.(4) and, as usual, inequalities are intended component wise.

The finite element yield function \( \Phi_e[\beta_e] \) will be expressed in terms of those of its end sections as
\[
\Phi_e[\beta_e] := 
\begin{bmatrix}
\Phi[0, t[0]] \\
\Phi[\ell, t[\ell]]
\end{bmatrix}
\]  
where \( \Phi[0, t[0]] \) and \( \Phi[\ell, t[\ell]] \) are defined by Eq.(5) and the normal actions \( t[s] \), are obtained from \( \beta_e \) as \( t[s] = S[s] \beta_e \), with \( s = 0, \ell \).

From now on a vector column will be represented between curly brackets in the body of the text.

3 Shakedown analysis for static loads combinations

In this section the shakedown theorem is presented in a suitable format to deal with complex load conditions such as those considered in modern building codes.

3.1 The load and elastic stress envelopes for static actions

We assume here that the external actions \( p[t] \), varying with time \( t \), are expressed as a combination of \( p \) static load conditions \( \hat{p}_i \) belonging to an admissible load domain \( \mathbb{P} \). According to the Eurocode prescriptions \( \mathbb{P} \) is defined by \( p \) basic combinations, each obtained by assuming one of the load conditions \( \hat{p}_k \) as the leading one and \( \hat{p}_i, i \neq k \) as accompanying ones. All of these are affected by a scalar multiplier \( \alpha_i \) varying from \( \alpha_i^{\text{min}} \) to \( \alpha_i^{\text{max}} \) and by a combination factor \( 0 \leq \psi_{ki} \leq 1 \) (\( \psi_{kk} = 1 \)). The load domain is then defined by
\[
\mathbb{P} := \bigcup_{k=1}^{p} \mathbb{P}^{(k)}, \quad \mathbb{P}^{(k)} := \left\{ p = \sum_{i=1}^{p} \psi_{ki} \alpha_i \hat{p}_i : \alpha_i^{\text{min}} \leq \alpha_i \leq \alpha_i^{\text{max}} \right\}
\]  
where each combination \( \mathbb{P}_k \) is a convex polytope with \( 2^p \) vertexes. Denoting with \( \hat{t}[s] = \{ \hat{N}_1[s], \hat{M}_2[s], \hat{M}_3[s] \} \) the set of the elastic normal actions produced by each load \( \hat{p} \in \mathbb{P} \), we define the envelope of the elastic stress, from now on simply called stress envelope, \( S[s] \) associated to the generic beam section \( s \) as
\[
S[s] = \bigcup_{k=1}^{p} S^{(k)}[s], \quad S^{(k)}[s] := \left\{ \hat{t}[s] = \sum_{i=1}^{p} \psi_{ki} \alpha_i \hat{t}_i[s] : \alpha_i^{\text{min}} \leq \alpha_i \leq \alpha_i^{\text{max}} \right\}
\]  
\( \hat{t}_i[s] \) being the elastic solution for \( \hat{p}_i \). If the external loads increase by a load multiplier \( \lambda \), the stress envelope will become \( \lambda S[s] := \{ \lambda \hat{t}[s] : \hat{t}[s] \in S[s] \} \).

Note that both \( \mathbb{P} \) and \( S[s] \) are not convex in general and \( S[s] \) is locally defined for each section \( s \). Both domains are characterized by a number of vertexes exponentially growing with \( p \). However not all of them will be significant in the analysis, due to the convexity of the plastic constraints.
3.2 The plastic admissibility condition

Shakedown theorem \cite{1, 2, 11} requires the plastically admissibility of all the stresses contained in the amplified stress envelope $\lambda S[s]$, translated by $t_s := t[s]$. Due to the convexity of the yield function $\Phi[s(t_s) := \Phi[s, t[s]]]$, this is easily checked in terms of the plastic admissibility of the $N_s$ stress vertexes of the convex hull $S_H[s]$ of $S[s]$, even if $S[s]$ is not convex. This means that we can directly replace $S[s]$ with $S_H[s]$, in this case a convex polytope, without affecting the results. This provides a great simplification because the number of vertexes to consider is usually much lower than the total number of vertexes in $S[s]$. With this substitution, each $t_s \in S[s]$ can be expressed as a convex combination of the $N_s$ vertexes $t^\alpha_s$ of $S_H[s]$:

$$
\hat{t}_s = \sum_{\alpha=1}^{N_s} \xi^\alpha t^\alpha_s, \quad \xi^\alpha \geq 0, \quad \sum_{\alpha=1}^{N_s} \xi^\alpha = 1
$$

Equation (9)

where, from now on, a superscript will denote a vertex of $S_H[s]$.

The plastic admissibility condition for all stresses in $\lambda S[s] \oplus t_s$ reduces to that of its translated and amplified convex hull vertexes

$$
(\lambda \hat{t}^\alpha_s + t_s) \in E[s] \forall \alpha \in [1 \cdots N_s] \iff \Phi[s, \lambda \hat{t}^\alpha_s + t_s] \leq 0 \forall \alpha \in [1 \cdots N_s]
$$

Equation (10)

while the admissibility condition for the element becomes

$$
\begin{cases}
\Phi[0, \lambda \hat{t}^\alpha_0 + t_0] \leq 0, \forall \alpha \in [1 \cdots N_0] \\
\Phi[\ell, \lambda \hat{t}^\alpha_\ell + t_\ell] \leq 0, \forall \alpha \in [1 \cdots N_\ell]
\end{cases}
$$

Equation (11)

which, in the following, will be abbreviated to

$$
\Phi_e[\lambda, \beta] \leq 0
$$

Equation (12)

With the notation introduced before the discrete form of the Bleich–Melan static theorem \cite{1, 2, 11} expressing $t^\alpha_s = c_s + \sum_s t^\alpha_{s_0}$ as a Minkowski sum and remembering Eqs.(11) and (5) the shakedown theorem can be written as

$$
\begin{align*}
\text{maximize} & \quad \lambda \\
\text{subject to} & \quad Q^T/\beta = 0 \\
& \quad c_s + \sum_s t^\alpha_{s_0} = \lambda \hat{t}^\alpha_s + t_s \\
& \quad \Phi_{s\ell}[t^\alpha_{s_0}] \leq 0 \\
& \quad \forall e, \alpha, s = 0, \ell
\end{align*}
$$

Equation (13)

that reduces in the standard format for a single ellipsoid.
3.3 A selection rule algorithm in a general finite element framework

The evaluation of the convex hull of $S$ is not simple using standard strategies, even if the load domain is given in an explicit form as in Eq. (7).

To overcome these difficulties, we propose a slender and fast procedure directly aimed at providing a subset of the significant vertexes of $S_H$. These are obtained as the most restrictive with respect to a selected number of supporting hyperplanes of the section yield function $E[s]$. 

In fact, exploiting Eq. (3), the shakedown plastic admissibility condition in Eq. (10) becomes

$$ f[t, \lambda, \hat{\varepsilon}] := \hat{\varepsilon}^T (t + \lambda \hat{t}) - \pi_\varepsilon [\hat{\varepsilon}] \leq 0, \quad \forall \hat{t} \in S $$

(14)

for all unitary vectors $\hat{\varepsilon}$. Obviously, only vertexes $t^\alpha \in S_H$ satisfying the condition

$$ d[\hat{\varepsilon}] := \max \{ \hat{\varepsilon}^T \hat{t} \} $$

(15)

for some $\hat{\varepsilon}$ will be significant in Eq. (14), with $d[\hat{\varepsilon}]$ being the value of the support function of $S_H$ in the direction $\hat{\varepsilon}$. So condition (15) provides a simple rule for selecting the vertexes of $S_H$ within those of $S_H[s]$.

In practice only a finite number $N_h$ of relevant testing directions $\hat{\varepsilon}_h := n_h$ can be used to select the vertex $\hat{t}^h$. Note that the same vertex could be obtained from different directions and, in particular, the leading ones, are associated to a wider range of directions.

Recalling the definition (8) of $S$, Eq. (15) furnishes

$$ d_h := \max_{k=1 \ldots p} \{ d_{hk} := \sum_{j=1}^{p} \gamma_{kj} \hat{\varepsilon}_h^T \hat{t}_j \}, \quad \gamma_{kj} := \begin{cases} \psi_{kj} \alpha_{j}^{\text{min}} & \text{if } \hat{\varepsilon}_h^T \hat{t}_j < 0 \\ \psi_{kj} \alpha_{j}^{\text{max}} & \text{if } \hat{\varepsilon}_h^T \hat{t}_j \geq 0 \end{cases} $$

(16)

We obtain $\hat{t}^h = \sum_{j=1}^{p} \gamma_{kj} \hat{t}_j$ using the same set of $\gamma_{kj}$ that defines $d_h$.

The described selection rule algorithm (SRA) allows us to drastically reduce the number $N_v$ of vertexes for consideration, and therefore the number of constraints potentially active in the optimization problem (13), with a real improvement in terms of efficiency and robustness of the solution process. Moreover this can, in fact, be obtained without introducing significant approximations in the solution, even if using a relatively low number of testing directions. It seems appropriate to point out the potential of the method:

- The reduction ratio increases as $p$ increases, so in the case it is most needed.
- We do not introduce any approximation if the leading vertexes are kept in the selection. These are associated to a wide range of directions $\hat{\varepsilon}_h$, so they are selected also with few $N_h$.
- In practice it is quite easy to make appropriate choices for the test directions allowing accurate results to be obtained even for $N_h$ of the order of ten as will be shown in section 5.
It is important to mention here that the proposed selection algorithm is similar in same ways to methods based on a piecewise external linearization of the elastic domain [10, 5]. In fact both make use of support hyperplanes of normals $\mathbf{\epsilon}_h$. However, in the present case the support hyperplanes are only used to select the vertices of the convex hull $\mathcal{S}$ and the analysis always uses, the original yield function without any further assumptions which might imply additional approximations. Quadratic constraints are very easily managed by optimization algorithms so the linearization is not computationally convenient. This makes the proposed strategy a real improvement with respect to solutions based on the linearization of $\mathcal{E}$ in terms of both accuracy and computational efficiency.

The algorithm we propose is written in a format suitable for use in a generic finite element analysis context. In particular the plastic admissibility condition and the convex hull of the elastic stresses $\mathcal{S}_H$ need to be defined at the local level of analysis that is: i) the Gauss Point for standard compatible FE interpolations; ii) the finite element for more complex interpolations (see for example [18, 19]).

4 The SRA for modal response spectrum analysis

In the previous section we only considered static load conditions ruled by Eq. (7) but the proposed SRA can easily be generalized to more complex load cases and combination rules. In particular here it is extended to dynamical actions originating in modal response spectral analysis, because of its technical relevance in the context of seismic design.

4.1 The response spectrum seismic analysis

The RSSA is based on the natural vibration modes $\phi_m$ and the corresponding frequencies $\omega_m$ of the structure, both solutions of the eigenvalue problem (modal analysis)

$$\mathbf{K} \phi_i - \omega_i^2 \mathbf{M} \phi_i = 0, \quad \phi_i^T \mathbf{M} \phi_i = 1, \quad i = 1 \cdots m$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices respectively. Usually only few $m$ modes are considered, with $m$ in the order of tens.

The dynamical response associated to each mode $\phi_i$ is referred by the modal peak value of the seismic action $\ddot{\mathbf{p}}_i$ which is

$$\ddot{\mathbf{p}}_i = a_i \mathbf{M} \phi_i, \quad a_i := g_i S_a[T_m], \quad g_i = \mathbf{u}^T \mathbf{M} \phi_i$$

$\mathbf{u}$ being a unitary translation in the direction of the seismic action, and $S_a[T]$ is the so called design acceleration spectral value, depending on the period $T_m := 2\pi/\omega_m$ and on other characteristics of the structure (such as the seismic hazard of building site, foundation detail, structural ductility, required safety, etc.) which are prescribed by building codes or assigned at the design time.

As the main seismic direction is unknown in advance, the amplification factor $a_i$ is obtained by combining the effects in three possible directions $X, Y$ and $Z$ with a rule
such as the well known SRSS

\[
a_i = \pm \sqrt{a_{ix}^2 + a_{iy}^2 + a_{iz}^2}
\]  

(19)

where the single contributions \(a_{ix}, a_{iy}, \text{and } a_{iz}\) are obtained as in Eq. (18) by considering unitary translation in the \(X, Y, \text{and } Z\) directions.

The modal contributions \(\tilde{p}_i\) can be treated as static forces acting on the structures so we can derive associated stresses \(\tilde{t}_i[s]\) as the corresponding elastic solutions. However, the combination rule must take into account the random nature of the basic contributions and the mutual correlation between them. This is expressed by the so called Complete Quadratic Combination (CQC) which is obtained referring to the case of a scalar quantity \(c\) as combination of modal contributions \(c_i\) by

\[
c = \pm \sqrt{\sum_{i,j=1}^{N_m} \rho_{ij}c_ic_j}, \quad \rho_{ij} \in [0 \cdots 1], \quad \rho_{ii} = 1
\]  

(20)

\(\rho_{ij}\) being a factor expressing the correlation between modes \(i\) and \(j\). The same formula reduces to SRSS rule by assuming \(\rho_{ij} = \delta_{ij}\).

### 4.2 SRA for the seismic combination rules

Obviously we cannot use Eq. (20) to obtain the max and min peak values for each component \(\{N_1, M_2, M_3\}\) separately because they are all derived from the same load condition and so the stress must be treated as an overall multicomponent vector. However, the SRA can also be applied in this case as it is based on the support function which is a scalar value. In fact, for each test direction \(\hat{e}_h\) we can extend Eq. (16) by adding a further dynamical term to the expression for \(d_h\)

\[
\tilde{d}_h = \tilde{d}[\hat{e}_h] := \hat{e}_h^T \tilde{t}_h
\]  

(21)

where the stress \(\tilde{t}_h\) associated to \(\hat{e}_h\) is expressed as a linear combination of the modal contributions \(\tilde{t}_i\)

\[
\tilde{t}_h := \pm \sum_{i=1}^m \gamma_{hi} \tilde{t}_i
\]  

(22)

The combination factors \(\gamma_{hi}\) being implicitly defined by assuming \(c := \tilde{d}_h\) and \(c_i := \hat{e}_h^T \tilde{t}_i\) in (20)

\[
\gamma_{hi} = \frac{1}{d_h} \sum_{k=1}^m \rho_{ik} \left( \hat{e}_h^T \tilde{t}_k \right)
\]  

(23)

The stress \(\tilde{t}_h\), defined by Eq.(22) provides the dynamical contribution to be added to the static combination \(\tilde{t}_h\) directly, with the appropriate sign that makes \(\tilde{d}_h\) positive into \(d_h\).
5 Numerical results

In this section we present a test regarding the analysis of 3D steel frames. The aim is to show the effectiveness of our proposal and how some aspects affect the results and the feasibility of the analysis. In particular, we investigate the influence of the number of hyperplanes used in the SRA, with respect to both, accuracy and computational cost. Moreover the influence on accuracy using one or more ellipsoids in describing $E$ will also be tested.

5.1 The response spectrum seismic analysis

The simple frame of Fig. 1 is analyzed in the seismic case adopting an HEB320 steel section for all the beams. In the same figure the elastic domain approximation with one or three ellipsoids is reported.

The mass matrix is obtained considering the contribution of load $p_1$ and $p_2$ only. The same loads with a fixed value are included in the seismic combination. Only the 12 significant vibration modes are considered in the analysis. The same spectrum $S_a$ for the horizontal accelerations is adopted while the vertical one is assumed as $f_v S_a$ with $f_v = 0.4111$.

In Fig.2, the vertexes of the stress envelope of section A detected by the SRA for different values of $N_r$ are plotted and compared with the exact seismic ellipsoid, while the box represents the maximum value of each generalized stress components.

This simple test highlights a general behavior: the algorithm is capable of selecting the predominant vertexes even with the coarsest set of directions ($N_r = 1$) if the appropriate metric is used.

Finally in Tab.1 the values of the various multipliers are reported. Also in this case they are independent of $N_r$, the ellipsoids significant stresses being well detected, using...
only the 12 vertexes of the icosahedron \( N_r = 1 \). Furthermore the single ellipsoid approximation of \( E \), due to the use of a symmetric steel section, gives results comparable to those obtained by using 3 ellipsoids.

Finally, in Tab.2 we report the computational time and the memory allocation obtained using MCSEK for some values of \( N_r \).

### 6 CONCLUSIONS

A solution strategy for the shakedown analysis of 3D frames, subject to complex loading conditions as those proposed by the Eurocodes, has been presented. The formulation is suitable to treat a large number of different variable loads efficiently, including both statical and dynamical actions defined by the modal response spectrum analysis. Only

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presso-flxurh plastic mechanisms have been considered, while the yield surface of the section has been obtained by approximating the true values of the support function of the elastic domain as a Minkowski sum of ellipsoids. This simplification could be considered appropriate for the structural context at hand.

The main difficulty in managing different variable loads is related to the large number of plastic constraints. For static actions the number grows exponentially with the number of load conditions.

The proposed seleccon rule algorithm is able to reduce redundant constraints without affecting the accuracy.

The algorithm can be effectively used for the design of steel and reinforced concrete 3D framed structures. For the latter, it allows both a better dimensioning of sections and rebars. The analyses have been carried out using MOSEK, but SRA can be easily employed with any shakedown solver, and it seems particularly well suited for use within decomposition methods.

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