Real-Time Robust Loosely-Coupled GPS-aided PDR

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Master’s Degree Project
Stockholm, Sweden August 2015
Abstract

Fusion between inertial navigation systems (INS) and satellite-based systems like GPS are often used to enhance the overall position or navigation solution. The satellite-based systems are capable of correcting the drift errors from the inertial sensors in long-term measurements, but they have poor short-term solution and problems in indoors or harsh environments where the arrival of the satellite signals are quite challenging due to multipath or blockage of satellite signals among other errors. Therefore, the INS is also capable of helping the satellite system in dense urban environments or even in complete outages. This thesis proposes a GPS-aided foot-mounted pedestrian dead reckoning (PDR) system to have an improved overall positioning solution, in short-term and in long-term measurements. The positioning fusion algorithm is a loosely coupling integration between a GPS receiver and a PDR module through a Kalman filter. The thesis tests the performance of the coupling in two environments: in a clear sky environment and in an urban environment.
Sammanfattning

Acknowledgments

First of all, I would like to thank my supervisor John-Olof Nilsson for every moment dedicated to the master thesis, I really appreciated your advice in every step of the process.

I would also like to dedicate all the effort in this thesis to my partners Kike and Marcos. Everyday I spend with you was worth it, either studying, cooking, doing sports, traveling or partying. Our way back home from KTH, talking and helping each other was the main reason why the next morning I was motivated to continue. Knowing you both was enough reason to come to Stockholm. I hope we keep in touch in the future.

To my girlfriend Maria. Although one month ago you told that you hate acknowledgements, I need to dedicate you some words, because despite the distance you have always been by my side helping. Words can never express how grateful I am.

To my friends from Mallorca and Barcelona for these last six years. Because outside the hours dedicated to study and the University there are a lot of incredible moments, and you are part of them.

And finally and specially, to my family. To my parents, Àngels and Miquel, and to my sister Pilar. To all the happy moments and to all your worries during these years. I still remember when my father could not sleep because he thought I would not be able to do it. Now is done, and without your effort in my education and your affection during all these years this could not be possible. And I don’t say this a lot, but you are the joy of my life.

Thanks to all.
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Part I
Introduction

1 Positioning and Localization Systems

Since the beginning of mankind, knowing your or any position or location has been of special interest either for humans or animals. Through the years, positioning or localization has evolved from a survival matter like hunting, self-defense, cartography or mapping to an improvement for life quality, i.e. guidance, navigation, location-based services (LBS), indoor positioning, etc, even systems of localization have become of utmost importance in military systems, fire fighters and police officers, and are constantly under study in the related fields, e.g. battlefield positioning, tracking, targeting.

This introduction covers a brief explanation of the different systems of localization and positioning, starting probably with the most simple and used navigation system, i.e. the dead reckoning, subsequently a case of the dead reckoning is presented, the inertial navigation systems (INS). And finally, to end the introduction of one of the sides of the fusion, the pedestrian navigation. Later, the satellite-based systems are introduced, emphasizing the system that is used during the thesis, the Global Positioning System (GPS). This explanation is going to be useful to the reader for the comprehension of the later fusion between the GPS receiver and the pedestrian dead reckoning module.

1.1 Dead Reckoning, Inertial Navigation Systems and Pedestrian Navigation

Dead reckoning is known as the process of estimate the position of a certain object in a certain time $t_k$ with only the help of the previous position at time $t_{k-1}$ and the estimate of certain parameters as the velocity, acceleration, direction, drift, etc. Dead reckoning has been used for a long time in all kind of navigation, i.e. marine navigation, aircraft navigation, car navigation, pedestrian navigation, etc. Even animals use dead reckoning or path integration to estimate its own position. Although its short-term solution is good, the most important problem of the dead reckoning is that the error increases over time without boundaries. This is the main reason why stand-alone dead reckoning systems are nowadays obsolete and they have been more useful when they are fused or used as an aid with other navigation systems. Its output data is often combined or coupled with satellite-based navigation systems like GPS or GLONASS to increase the reliability of the overall position and velocity solution.
The inertial navigation systems are just a case of a dead reckoning system, which use inertial sensors like accelerometers, magnetometers, gyroscopes, etc., to calculate the acceleration, velocity, direction and thereby find the position of the object with the help of the previous position. Inertial navigation systems have a wide application market that include ships, military army (guided missiles), spacecraft, aircraft, cars, etc. An inertial navigation system is normally formed by an IMU (Inertial Measurement Unit), which outputs the measurements from the sensors, and a computer that can process the data given by the IMU to track the position. The IMU is often formed by accelerometers, which measure the linear acceleration or specific force on the object, and gyroscopes, which compute rotational changes in the direction of the object by calculating the angular velocity of the system. Normally, and IMU has 3 of these components, each one pointing to an orthogonal coordinate to create thereby a three coordinate body frame. Also, magnetometers and barometers can be included in a IMU to help to correct the drift in the solution. An Inertial Navigation System usually falls into two categories: gimbaled or strap down. Initial INS applications used gimbals to stable the platform and isolate the system from its own rotations. However, modern systems do not use these techniques but they attached the system rigidly to the body of the object, as we known as a strapped down system. This last kind of systems offers several benefits like lower cost, lower complexity, reduced size and better reliability compared to the gimbaled systems. The most important drawback is the sensor calibration and alignment [22].

As a case of dead reckoning, the inertial navigation systems suffer from the same problem, the accumulation of drift errors. Since small errors in the calculation of the linear acceleration and angular velocity appear, the integration of these parameters become to bigger errors in the velocity, which is integrated to get the position and even generates bigger errors. The errors of integrative nature, can accumulate and increase over time and therefore are a real problem of reliability for non-aided inertial navigation systems.
1.2 Global Navigation Satellite Systems

Since these systems don’t detect any other external signal or source of data to help to decrease the accumulated errors they need to be aid by other systems, either satellite-based systems or/and other algorithms as the Zero-Velocity Updates (ZUPTs). In our case, the fact of using a pedestrian navigation system gives us the opportunity to apply these ZUPTs, which are based in the fact that the gait of a pedestrian has a stance phase where the velocity has to be zero, and biases in the outputs of the sensors can be detected and corrected, so the drift from the system is significantly decreased. The main challenge resides in the detection of this stance epochs [28]. These systems are known as ZUPT-aided INS and are not limited to more aids, therefore they can also be combined with satellite-based systems for further improvement.

Moreover, the emergence in the last decade of Micro Electro Mechanical Systems (MEMS), which are miniaturized systems like MEMS gyroscopes and MEMS accelerometers, have increased the possibility of use IMU in every kind of systems and its integrations on more little systems that can be applied everywhere, e.g. in foot-mounted modules. Thus, pedestrian navigation is one that takes advantage of this.

One of the main uses of INS and mostly for satellite-based systems is to help people navigate as they travel around the world. Further, the increase of location-based services and social networks has catapulted the use of pedestrian positioning and navigation. With other aids like mapping, this navigation systems can be extended to track people and guiding them through his movements, as it has been done during several years in car or aircraft navigation. Normally, a user can be guided or located with the help of a GPS in its mobile, which not only uses satellite signals but radio tower signals. But unlike car or aircraft navigation, pedestrian movements are chiefly in indoor or urban environments where there is not a clear line of sight for the satellites. Therefore the solution is roughly good, with larger errors in position. That’s why in pedestrian navigation, the fusion of navigation systems is done too to improve the overall solution in a wide range of applications.

11.2 Global Navigation Satellite Systems

The term Global Navigation Satellite Systems (GNSS) refers to a constellation of satellites capable of transmitting signals used for localization and positioning worldwide. The time travel of these signals is computed to get the range between the satellites and the receiver and thereby find the position of the user. These signals are used in a wide range of applications as location, navigation, transport, agriculture, etc.

The origin of the satellite navigation can be found in military applications, were it was used as an improvement for armament, i.e. guided missiles, submarines, etc., or management and control of troops. Nowadays,
navigation systems are a must for every army and the study in this field is of utmost importance.

However, the improving of technology have cause an extension of this military use to a civilian use. Nowadays, a lot of applications like car navigation, pedestrian navigation, synchronization, emergency location systems, wildlife tracking, etc., that work with satellite systems are under constant study and present in our daily life.

![GPS satellite orbits and constellation](image)

**Figure 3:** GPS satellite orbits and constellation [15].

At present time, we can find two active satellite navigation systems, i.e. Global Positioning System (GPS), controlled by the United States Government and the Global Navigation Satellite System (GLONASS) operated by the Russian Aerospace Defense Forces. Others systems that are nowadays under development are the European Galileo, controlled by the European Union and the European Spatial Agency (ESA), the Beidou from China, the QZSS from Japan and the IRNSS from India. In this thesis, an off the shelf GPS receiver is used, so a brief explanation of this system is done below.

### 1.2.1 Global Satellite System

The NAVSTAR-GPS (Navigation System and Ranging - Global Positioning System), known as GPS, is a radio navigation system that uses signals to measure the ranges between the satellites and the receiver and so determine an accurate position. The system is operated by the United States Department of Defense and is the only satellite navigation system that is completely working. Until now, 69 Global Positioning System navigation satellites have been launched but not all of them are working. The current number of active satellite is 31, plus 3 or 4 residual satellites that can be activated if needed. The Air Force works to maintain at least 24 satellites available the 95% of the time. The satellites are identified by its Space Vehicle Number (SVN) and assigned a pseudorandom noise sequence (PRN) that the receiver is capable of decode and identify the satellite that has transmitted it.
The GPS system is divided in three segments: space, control and user. The space segment consists of the constellation of 32 satellites in orbit of 11:59 minutes with 6 planes of four satellites. Its attitude is approximately 20,000 km over the surface of the earth. The six orbits are almost circular with 55 degrees inclination and equally space in 60 degrees. This constellation distribution ensures that at every part of the world a GPS receiver is sure to be able to have four satellites in vision, which will result in a position fix. The control segment consists of a number of ground facilities that monitor and analyze the performance and transmissions of the GPS satellites and its transmitted signals. Nowadays, the control segment includes a master control station, an alternate master control station, 12 control and command antennas and 16 monitoring sites. The master control station, which is situated in Colorado (USA), controls and command the satellite an its constellation, ensuring its health and accuracy. The master control stations upload information to the satellites like the biases in its clock, which receives from the monitor stations. The monitor stations, as we have said, receive navigational, atmospheric and range data from the satellite and then redirect it to the master control station. Finally, the ground antennas are used to control and communicate with the satellites. The user segment consists of antennas and receivers. The GPS receivers are divided in three parts: radio frequency front end, the baseband and navigation. The radio frequency front end works on the amplification, filtering and shifting of the GPS signal from higher to lower frequency range. The baseband recognizes every satellite signal and determines parameters like the transit time and the carrier or code phase send by every satellite. Finally, the navigation part of the receiver uses the SV positions and the measured signal times from satellite to the receiver to compute the position, velocity and time estimate of the user [15].

2 Motivation

GPS signals have been used for pedestrian navigation almost two decades in outdoor positioning. But in indoor or harsh environments the GPS signals are not able to arrive due to blockage and this is where the inertial sensors are needed. On the other hand, dead reckoning and inertial navigation system lack a large time performance reliability due to sensors accumulative errors. Therefore, the GPS system characteristics are complemented by the INS and vice versa and that is why the fusion between this systems can minimize the position error estimate. The problem can be further minimized using a pedestrian navigation system, where a foot mounted pedestrian dead reckoning system is used, using zero velocity updates (ZUPTs). This thesis aims to minimize the heading error of a foot mounted pedestrian dead reckoning system with the aid of GPS signal loosely coupled with a Kalman filter.
The systems used are a Ublox LEA-6T GPS receiver and a foot-mounted pedestrian dead reckoning module designed by the KTH Signal Processing department. The performance of the system is tested in two different environments, a running track with a clear sky and continuous signal reception, and a building environment with a mix of blocked and clear signals zones. It should conclude that the drift of the IMU is improved significantly for a long period in environments where there is a clear signal reception, and that the performance in harsh environments enhance slightly although the loosely coupling is not the most suitable for these cases. This thesis has been done under the supervision of John-Olof Nilsson and examiner Peter Händel of the Signal Processing Department at KTH.

3 Thesis Outline

This thesis is divided in three parts with several sections in each part. In the first one, an introduction to the dead reckoning, inertial navigation systems, pedestrian navigation and satellite-based positioning systems, with special emphasis in the Global Positioning System, has been exposed. The remainder of the thesis is structured as follows. In part II, Systems Overview, the theoretical and mathematical basis for understanding the fusion between the two systems is explained. An introduction to the reference frames that both systems use and the transformations between them is presented. Further, the theory and related computational explanation of the mechanization equations of an INS and subsequently an explanation on the benefits of using ZUPTs-aided INS are presented followed by the typical INS errors. Next, a thorough explanation of the solution with pseudoranges for GPS is computed and followed with the most common GPS errors. Afterwards, the aspects of the Kalman filter for the data fusion and its different implementations in navigation are explained for a further comprehension of the reader. To end this part, in the Software and Hardware section, the GPS receiver and PDR module are presented with its own variables and parameters to personalize the later testing. In part III, the algorithm and theory presented above are applied to do the fusion, the process and the results are exposed and commented. Finally, a conclusion of the work done and of the thesis is presented with a future work text for next achievements.
Part II

Systems Overview

Up to now, an overall view of the thesis and the work coming next have been described. This part deals with the explanation of the theory that has been mostly studied during the realization of this master thesis. A description of the different reference frames used during the thesis is shown at the beginning jointly with the transformations between them, essential for the understanding of the later INS alignment and the fusion of the data between the two systems. Following on, the operation of the INS is explained from a computational point of view with the mechanization equations and typical errors. Next, the GPS system is fully explained, with the theory of how the pseudoranges and the satellite positions are measured and later how they are computed to fix the receiver position. An approach to the models of the most common GPS errors is also detailed. Once both systems are detailed, a generic view of the Kalman filter that fuses the data is presented with its characteristic equations and finally its versatility with navigation systems.

4 Geodetic Frames

There is no sense of finding a position or location if this is not applied to certain map or reference frame or, to put it differently, it is not possible to locate an object if its position is not related to another object or position. Navigation systems measures and data require to be computed in different frames to be related with different systems data and coordinates. Hence, the coordinate frames from the GPS system and the IMU have to be related through rotation matrices to be able to fuse its data. Although there is a myriad of coordinate frames that can be transformed from one to each other, in this thesis the main frames used are the Earth-Centered Earth-Fixed (ECEF) frame and the North-East-Down (NED) frame. The use of the ECEF frame is owing to the GPS and it is used jointly with the Latitude/Longitude/Height. The use of NED, which is a local tangent plane, is due to the foot-mounted IMU module. The IMU module navigation frame and the NED frame share the direction of the z-axis that points the same direction as the gravity, but not the x and y-axis. First we will need to convert the GPS measurements from ECEF coordinates to NED coordinates. Once we have converted the ECEF coordinates to NED we will have to relate this new NED coordinates to the navigation frame of the IMU system, because as we have explained before, the z-axis is the same but the x-axis and y-axis are not, so a rotation about the z-axis must be done to complete the coordinate frame conversion [7].
4.1 Earth-Centered Earth-Fixed Frame

To compute the conversion from one system to another the rotational matrices are needed. Three rotations have to be done, the first about the ECEF z-axis to align the ECEF y-axis to the NED East-axis. Then, we have to rotate about the new East-axis to align the ECEF z-axis to the NED Down-axis. Once we have the NED coordinates of the GPS receiver measurements, we will do a real-time alignment during the first steps between the NED frame and the IMU navigation frame. This final rotation calculates the angle difference between any of the two axis (x or y) of the frames and rotates about the z-axis until relates the axes from both reference frames. But before going further with the transformation matrices, a brief explanation of both frames is presented below.

4.1 Earth-Centered Earth-Fixed Frame

The Earth Centered Earth Fixed Frame, from now on ECEF, is a cartesian coordinate system. The points or positions of this coordinate system are represented in three axis, x, y and z. The point (0, 0, 0) is defined as the center of mass of the Earth, hence the name Earth Centered. The x-axis is aligned to the International Reference Meridian, consequently intersects the sphere of the Earth at Latitude 0° and Longitude 0°, and the Z-axis is aligned with the International Reference Pole (IRP). The y-axis completes the right handed coordinate system rule. The alignment of the z-axis can create confusion, since the Earth doesn’t rotate about the z-axis, unlike others systems like Earth Centered Inertial (ECI). The Earth-Fixed definition means that the ECEF coordinates and the Earth rotate together.

Figure 4: ECEF coordinates [8].
4.2 Local Geodetic or Tangent Plane Frame

A tangent plane or Local Geodetic frame is a plane that is often used in local navigation and it has its center in a certain point on the surface of the earth. Different tangent plane coordinates have been defined. In our case, we will use the NED (North, East, Down) frame, where we will fix the $x$-axis as the vector pointing to the north, the $z$-axis as the vector pointing to the center of the earth and the $y$-axis pointing to the east completing a right handed coordinate system. Another tangent coordinate system could be the ENU (North, East, Up), where the $z$-axis points to the opposite direction of the center of the Earth.

![Figure 5: Example of ECEF coordinates and a Local Tangent Plane][8]

4.3 Transforming ECEF to NED frame

To transform an ECEF coordinate to a NED coordinate first we will need to fix an initial ECEF coordinate that will be the origin of the NED plane and often the point of start of the movement of a user. Then, the vector from this initial position to any point of the NED plane coordinates is measured as

$$\Delta \hat{x}^e = [x, y, z]^e - [x_o, y_o, z_o]^e$$

where $(x_o, y_o, z_o)$ is the origin of the local tangent plane and $(x, y, z)$ any point on the plane.

Then, we have two rotational matrices to complete the transformation. As it has been explained above, the first matrix $R_1$ does a rotation about the ECEF $z$-axis to align the ECEF $y$-axis to the NED East-axis

$$R_1 = \begin{bmatrix}
\cos \phi & \sin \phi & 0 \\
-\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{bmatrix}$$
The second rotation is about the new y-axis to align the ECEF-z axis to the NED Down-axis

\[
R_2 = \begin{bmatrix}
  -\sin \lambda & 0 & \cos \lambda \\
  0 & 1 & 0 \\
  -\cos \lambda & 0 & -\sin \lambda
\end{bmatrix}
\]

So finally, combining the two rotations we get the rotational matrix \( R_{et} \) that converts a ECEF position to a NED position and is defined as

\[
R_{et} = R_2 R_1 = \begin{bmatrix}
  -\sin \phi \cos \lambda & -\sin \phi \sin \lambda & \cos \phi \\
  -\sin \lambda \cos \phi & \cos \lambda & 0 \\
  -\cos \phi \cos \lambda & -\cos \phi \sin \lambda & -\sin \phi
\end{bmatrix}
\]

where \( \phi \) and \( \lambda \) are the latitude and longitude from the initial position respectively.

Then, any (North, East, Down) position in a tangent plane can be expressed as

\[
\begin{bmatrix}
  N \\
  E \\
  D
\end{bmatrix} = R_{et}^T (\Delta \hat{x})^T
\]

Refer to [7], [8] for more information about the different coordinate frames and its transformations.

5 Inertial Navigation Systems

As it has been explained in the introduction section, an inertial navigation system is compounded by a number of sensors like accelerometers, gyroscopes and magnetometers, and a computer that is able to process the data from the sensors to compute the parameters that we want to know. To calculate the next position, the data from the sensors are used into a set of mechanization equations that estimate the position, orientation and velocity. In our case, the equations of motion will use the data to determine the state of an user, that includes its position and its heading. This section covers the explanation of the mechanization equations to understand how the state of the user is continuously computed. Then, a brief explanation of the benefits of using zero-velocity updates is presented. Finally, the main errors of the INS are described.

5.1 Equations of Motion

When the Foot-mounted INS system is initialized, its coordinate frame or body frame is aligned with the navigation frame, that in our case will be a
local tangent frame. But if the body frame starts moving and turning the body frame will be misaligned from initial navigation frame. That’s why in every step, a transformation is done and the measurements of the IMU are converted from the body frame to the navigation frame. Once the transformation of the measurements has been computed is when a dead reckoning interface can be applied. Remember that in a typical dead reckoning interface, the position is calculated as follows

\[ p_k = p_{k-1} + v_{k-1} t + \frac{1}{2} a_k t^2 \]

where \( p_k \) is the position, \( v_k \) is the velocity, \( a_k \) is the acceleration, \( t \) is the integration time and \( k \) is the time index.

In our system, every step updates the state of an user \( x_k \), compounded by its position \( p_k \), that is a three dimension vector \( p_k = \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} \) and its heading \( \theta_k \).

Hence, the state vector is defined as \( x_k = \begin{bmatrix} p_k \\ \theta_k \end{bmatrix} \)

Thereby, using dead reckoning the continuously state of an user can be expressed as

\[ x_k = x_{k-1} + R(\theta_{k-1})(u_k + w_k) \]

where \( k \) is the time index, \( u_k \) is the displacement of the position between \( k-1 \) and \( k \), \( u_k = \frac{dp_k}{d\theta_k} \), \( w_k \) is white gaussian noise with \( E\{w_k\} = 0 \) and covariance matrix \( Q_k \), and \( R \) is the rotation matrix about the \( z \)-axis that relates the body frame to the navigation frame

\[
R(\theta_k) = \begin{bmatrix}
\cos \theta_k & -\sin \theta_k & 0 & 0 \\
\sin \theta_k & \cos \theta_k & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Furthermore, the matrix \( P_k \) provides the covariance of the state \( x_k \)

\[
P_k = FP_{k-1}F^T + R(\theta_{k-1})dP_kR(\theta_{k-1})^T
\]

where \( F \) is the system matrix defined as

\[
F(\theta_{k-1}, dx_k, dy_k) = \begin{bmatrix}
1 & 0 & 0 & -\sin \theta_{k-1} dx_k - \cos \theta_{k-1} dy_k \\
0 & 1 & 0 & \cos \theta_{k-1} dx_k - \sin \theta_{k-1} dy_k \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Thus, the defined covariance matrix $P_k$ is summing up every step the errors of the user state (position and heading). For more information about the initialization and setup of the user position, refer to [23].

5.2 Zero Velocity Update

It is impossible to track pedestrian movement using only inertial sensors. A tiny drift in its gyroscopes can cause a growing error through time. The acceleration error is $9.8 \text{m/s}^2$ times the tilt error in radios [11]. Therefore, the double integration of the acceleration becomes a cubically error that grows with time of the user position. Therefore, the ZUPT is an important aid to the inertial systems and to constraint its errors. If we follow the step of a pedestrian, we can notice that every gait is a three phase movement: stance, forward swing and foot placement. The foot-mounted INS used in this thesis detect the stance phase and apply zero velocity updates to correct the error. Therefore, the main challenge here is to recognize the zero velocity or stance epochs. To detect this zero velocity moments is important due to this is the only moment were the information of the MEMS is previously known, therefore, any measurement different from zero on the MEMS in the stance phase can be assumed as errors. The interest of a foot-mounted IMU is that this zero velocity updates can be applied at the moment of the stance phase. As it has been said before, the use of ZUPT does not limit more aids for the INS, so satellite-based aids can be also used for more enhancement. For more information about zero velocity updates and its detection refer to [28].

5.3 Errors in INS

All the Inertial Navigation Systems suffer from drift error as we have explained above. Due to computing the positions in an integral manner, a small error in the acceleration causes an error in the calculation of the velocity (integral of acceleration), and thereby an error in the calculation of the position (integral of velocity). Furthermore, the calculation of the position is based on dead reckoning, so the next position will get the errors from the previous position and subsequently with the next positions. That causes an exponential error in the calculation of the parameters of an INS and is the main cause why nowadays the use of stand-alone INS systems are obsolete. INS are often aided by algorithms like ZUPTs and/or fused with other navigation systems, i.e. satellite-based navigation systems like GPS. In an INS, the error can come from a set of different sources. Four are the main ones:

**Measurement:** The data output by the sensors have measurement errors, i.e. bias, drift, random noise.
Processing: The error is caused because of the digital processing, e.g. quantization.

Alignment: When the sensors and the platform are not perfectly aligned it produces an error, i.e. in the frame rotation.

Environment: The modeling of the environment can cause errors, e.g. it is not possible to predict exactly the effective gravity vector.

6 Global Positioning System

Once a brief introduction of the GPS has been done in the first part, an accurate explanation of the system is approached in this section. Subsequently the position solution using pseudoranges is explained theoretically and mathematically. Finally, the main errors affecting the computation of the pseudoranges are also explained and modeled for a better comprehension.

As we have explained in the introduction, the GPS uses radio signals transmitted from the satellites to calculate the position of the user. These electromagnetic signals broadcasted by the satellites travel at the speed of light through the atmosphere until it arrive to the receiver. Every GPS spatial vehicle transmits continuously over two carrier frequencies known as L1 (1575.42 MHz) and L2 (1227.60 MHz). The L1, that is the one used by civilians and the one that interests to us, is modulated in quadrature by two code division multiple access (CDMA) signals: C/A and P(Y). The coarse/acquisition (C/A) code has a length of 1023 chips and 1.023 MHz chip rate, resulting in a code period of one millisecond. The military operators can degrade the accuracy of the C/A code intentionally and this is known as Selective Availability, capable of cause ranging errors of the order of 100m. There is a C/A PRN code for each satellite, and each of them are orthogonal to each other. So, a GPS receiver is capable of distinguish the signal between all the satellites by correlating internally the same codes with the arriving signals. So the range between the satellite and the receiver is the speed of light multiplied by the time that the signal lasts to arrive to the receiver. This would be certain if the signal doesn’t get delays. That is why the are called pseudoranges, because the traveling time is no the true time. The signals receive all kind of errors, first of all, the satellite time offset caused by a relativistic effect and biases from its atomic clocks used, then atmosphere delays from the ionosphere and the atmosphere and finally the receiver clock offset, that uses crystal oscillators which are inexpensive and quite less precise than the atomic ones. So if the delays caused by the atmosphere and the clock biases were zero, the true range between the
satellite and the receiver will be the traveling time of the arriving signal multiplied by the speed of light. With four equations (4 pseudoranges) and the knowledge of the satellite positions is possible to compute a position fix. Other more meticulous systems for the GPS solution have been used, e.g. the carrier phase. The carrier phase tracking is basically the same as the pseudoranges with the difference of the frequency. The main problem of the correlation between the PRN codes created by the satellites and the ones created internally by the GPS receiver is that the bits or cycles transmitted by the satellites are too wide so even when they are synchronized there is a lot of slop. So if instead of correlating the pseudorange code (1 MHz) we correlate the carrier phase (∼1GHz), so the pulses are going to be much narrower and therefore more accurate. This method counts the number of carrier cycles between the satellite and the receiver, so the challenge resides in counting these cycles because unlike the pseudoranges, the carrier phase is uniform so all the cycles are similar to the others.

6.1 Position Solution

In this section, the trilateration algorithm using pseudoranges to fix a receiver position is explained. Trilateration is a geometric method for finding a position using the distances between a certain set of points, that in our case are the pseudoranges and the position of the satellites, and is often used in satellite-based positioning and navigation systems. The process features two important steps, the ephemeris computation, where we find the position of the satellites at a certain time, and the pseudorange solution, where we compute the pseudoranges (not real ranges) to find the receiver position.

![Figure 6: GPS trilateration method for position solution.](image)
6.1.1 Ephemeris Computation

The user position accuracy is directly related to the accuracy of the satellites positions. To find the position of the satellites in orbit we need a set of parameters sent in the navigation message. With these parameters computed correctly to Kepler orbital equations, we can find the position of the satellites at any time. These parameters are known as ephemeris and are as follows [1]

\[ M_0 \]: Mean Anomaly at Reference Time

\[ \Delta n \]: Mean Motion Difference From Computed Value

\[ e \]: Eccentricity

\[ \sqrt{A} \]: Square Root of the Semi-Major Axis

\[ \omega_0 \]: Longitude of Ascending Node of Orbit Plane at Weekly Epoch

\[ i_0 \]: Inclination Angle at Reference Time

\[ w \]: Argument of Perigee

\[ \dot{\omega} \]: Rate of Right Ascension

IDOT: Rate of Inclination Angle

\[ c_{ac}\]: Amplitude of the Cosine Harmonic Correction Term to the Argument of Latitude

\[ c_{as}\]: Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude

\[ c_{rc}\]: Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius

\[ c_{rs}\]: Amplitude of the Sine Harmonic Correction Term to the Orbit Radius

\[ c_{ic}\]: Amplitude of the Cosine Harmonic Correction Term to the Angle of Inclination

\[ c_{is}\]: Amplitude of the Sine Harmonic Correction Term to the Angle of Inclination

\[ t_{oe}\]: Reference Time Ephemeris
Once the ephemeris is downloaded to the receiver, it can be used for days, although it is recommended to download the ephemeris data every four hours, which is the time that the parameters are updated. The ephemeris parameters are applied to the orbital body equations published by Johannes Kepler to know the position of a satellite at a certain time \( t \), which is the only variable that changes (not the Ephemeris parameters) in every calculation that we want to do during these four hours. The orbital body equations are defined as follows [1]

\[
\begin{align*}
A &= (\sqrt{A})^2 \quad \text{Semi-major axis} \\
n_0 &= \sqrt{\frac{\mu}{A^3}} \quad \text{Computed mean motion (rad/sec)} \\
t_k &= t - t_{oe} \quad \text{Time from ephemeris reference epoch} \\
n &= n_0 + \Delta n \quad \text{Corrected mean motion} \\
M_k &= M_0 + nt_k \quad \text{Mean anomaly} \\
M_k &= E_k - e \sin E_k \quad \text{Kepler’s Equation for Eccentric Anomaly (radians)} \\
v_k &= \arctan(\frac{\sin v_k}{\cos v_k}) \quad \text{True Anomaly} \\
E_k &= \arccos(\frac{e + \cos v_k}{1 + e \cos v_k}) \quad \text{Eccentric Anomaly} \\
\dot{\phi}_k &= v_k + w \quad \text{Argument of Latitude} \\
\delta u_k &= c_{us} \sin 2\phi + c_{uc} \cos 2\phi \quad \text{Argument of Latitude Correction} \\
\delta r_k &= c_{rs} \sin 2\phi + c_{rc} \cos 2\phi \quad \text{Radius Correction} \\
\delta i_k &= c_{is} \sin 2\phi + c_{ic} \cos 2\phi \quad \text{Inclination Correction} \\
u_k &= \phi_k + \delta u_k \quad \text{Corrected Argument of Latitude} \\
r_k &= A(1 - e \cos E_k) + \delta r_k \quad \text{Corrected Radius} \\
i_k &= i_0 + \delta i_k + (IDOT)t_k \quad \text{Corrected Inclination}
\end{align*}
\]
\[ \begin{align*}
    x'_k &= r_k \cos u_k \\
    y'_k &= r_k \sin u_k
\end{align*} \] Positions in orbital plane.

\[ \Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e) t_k - \dot{\Omega}_c t_e \] Corrected longitude of ascending node.

\[ \begin{align*}
    x_k &= x'_k \cos \Omega_k - y'_k \cos i_k \sin \Omega_k \\
    y_k &= x'_k \sin \Omega_k + y'_k \cos i_k \cos \Omega_k \\
    z_k &= y'_k \sin i_k
\end{align*} \] Earth-fixed coordinates.

where

\[ \mu = 3.986005 \times 10^{14} \text{ meters}^3/\text{sec}^2 \] WGS 84 value of the earth’s gravitational constant for GPS user.

\[ \dot{\Omega}_c = 7.2921151467 \times 10^{-5} \text{ rad/sec} \] WGS 84 value of the earth’s rotation rate.

Once the equations are correctly computed, we obtain the ECEF coordinates of the satellites that have send us the ephemeris parameters. Of utmost importance is to say that the sensitivity of the satellite antenna phase center to most of the parameters variations is extreme. The sensitivity of the satellite position to angular parameters can be of \( 10^{12} \) meters/semicircle/second [1].

### 6.1.2 Pseudoranges

Once we have the ephemeris data and the satellites positions computed, we have to solve the problem of finding a fix of the receiver position with the knowledge of these satellites positions and the pseudoranges. To compute the receiver position, we need a minimum of 4 equations, i.e. four pseudoranges and four satellite positions. The method used for solving this set of equations is the least squares, which approximates a solution for an overdetermined system, i.e. sets of equations in which there are more equations than unknowns.

But how are the pseudoranges computed in the receiver? As we have explained in the introduction section, every GPS satellite transmits ranging codes and navigation messages by using code-division multiple access (CDMA) on two frequencies, 1575.42 MHz known as L1 that carries both the status and the pseudorandom noise for timing, and 1227.60 MHz known as L2 that is used for a more precise military pseudorandom noise code. Every satellite has a pseudorandom noise (PRN) ranging code associated
6.1 Position Solution

that is transmitted as a part of the navigation message. There is a unique and different C/A PRN for each satellite, and they are almost orthogonal to the others C/A PRN codes of the other satellites. The GPS receivers know what PRN code belongs to every satellite. Therefore, the GPS receiver is capable to discriminate between the different satellite arriving signals by internally creating a nearly identical C/A PRN code like the one from the satellite and correlating them. The time that the internal C/A PRN code has to be sliced to sync the satellite signal approximately the time that the signal has traveled.

Figure 7: PRN code synchronization [6].

So that, multiplying the traveling time for the velocity of the signal, i.e. the speed of light, we obtain the distance between the receiver and the satellite. Since the traveling signal is subject to non desired delays, i.e. satellite clock delay, receiver clock delay, atmospheric delay, multipath delay, etc., a little time error at light speed can become a huge error in meters, so the computed range is called pseudorange.

Figure 8: Pseudoranges graphically explained.

Now that we know how a pseudorange is computed and which non desirable effects has, we present the equation for the pseudorange observation \( P_i^k \) between the receiver \( i \) and the spatial vehicle \( k \)

\[
P_i^k = \rho_i^k + c * (dt_i - dt^k) + T_i^k + I_i^k + e_i^k
\]  

(4)
where the real range or distance between the satellite $k$ and the receiver $i$ is

$$\rho_{ki}^2 = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2} \quad (5)$$

where $(X^k, Y^k, Z^k)$ are the ECEF coordinates of the position of the satellite $k$ and $(X_i, Y_i, Z_i)$ are the ECEF coordinates of the position of the receiver $i$.

The rest of the parameters are defined as follows

- $dt_i$ is the receiver clock offset
- $dt_k$ is the satellite clock offset
- $T_{ki}^k$ is the Tropospheric delay
- $I_{ki}^k$ is the Ionospheric delay
- $e_{ki}^k$ is the observational error of the pseudorange

Once the parameters have been presented, we complete the pseudorange equation introducing (5) in (4), obtaining

$$P_{ki}^k = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2 + c \ast (dt_i - dt_k) + T_{ki}^k + I_{ki}^k + e_{ki}^k} \quad (6)$$

Because the real range $\rho$ between the satellite and the receiver coordinates is a non linear function, to apply the least square method to the set of equations and find the receiver coordinates, we need to linearize the pseudorange equation (6) with some initial guesses or estimates for the receiver’s position, known as the linearization point.

$$f(X_i, Y_i, Z_i) = \sqrt{(X^k - X_i)^2 + (Y^k - Y_i)^2 + (Z^k - Z_i)^2} \quad (7)$$

The linearization starts with a a random initial position $(X_{i,o}, Y_{i,o}, Z_{i,o})$ that will work as the linearization point and will be set up as the origin of coordinates. In every iteration, corrections will be applied to the initials conditions to obtain the receiver’s position and clock offset, so the linearization point will be updated as
6.1 Position Solution

\[ X_{i,1} = X_{i,0} + \Delta X \]
\[ Y_{i,1} = Y_{i,0} + \Delta Y \]
\[ Z_{i,1} = Z_{i,0} + \Delta Z \]

To linearize the real range \( \rho \) equation (7) and hence linearize the pseudorange equation (6), we do the approximation of

\[ f(X_{i,0} + \Delta X, Y_{i,0} + \Delta Y, Z_{i,0} + \Delta Z) \] (8)

Obtaining the first order terms of the Taylor expansion

\[ f(X_{i,1}, Y_{i,1}, Z_{i,1}) = f(X_{i,0}, Y_{i,0}, Z_{i,0}) + \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta X_{i,0}} \Delta X + \]
\[ + \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta Y_{i,0}} \Delta Y + \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta Z_{i,0}} \Delta Z \] (9)

The partial derivatives in (9) are

\[ \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta X_{i,0}} \Delta X = - \frac{X^k - X_{i,0}}{\rho^k_i} \]
\[ \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta Y_{i,0}} \Delta Y = - \frac{Y^k - Y_{i,0}}{\rho^k_i} \]
\[ \frac{\delta f(X_{i,0}, Y_{i,0}, Z_{i,0})}{\delta Z_{i,0}} \Delta Z = - \frac{Z^k - Z_{i,0}}{\rho^k_i} \]

Finally, applying the linearization to the pseudorange equation, we obtain

\[ P^k_i = \rho^k_{i,0} - \frac{X^k - X_{i,0}}{\rho^k_{i,0}} - \frac{Y^k - Y_{i,0}}{\rho^k_{i,0}} - \frac{Z^k - Z_{i,0}}{\rho^k_{i,0}} + \]
\[ + c*(dt_i - dt^k) + T^k_i + I^k_i + e^k_i \] (10)

where the range between the satellite \( k \) and the estimated position of the receiver \( i \) at every iteration is defined as

\[ \rho^k_{i,0} = \sqrt{(X^k - X_{i,0})^2 + (Y^k - Y_{i,0})^2 + (Z^k - Z_{i,0})^2} \]
To solve the set of equations we will apply the least squares method so we need to write the equation (10) in a vectorial form

\[
P_i^k = \rho_{i,0}^k + \begin{bmatrix} -\frac{X^k - X_{i,0}}{\rho_i^k} & -\frac{Y^k - Y_{i,0}}{\rho_i^k} & -\frac{Z^k - Z_{i,0}}{\rho_i^k} & 1 \end{bmatrix} \begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ c\delta t_i \end{bmatrix} - cdt_i^k + T_i^k + I_i^k + \epsilon_i^k
\]

Now we fix the equation system to the least squares formulation

\[
\begin{bmatrix} -\frac{X^k - X_{i,0}}{\rho_i^k} & -\frac{Y^k - Y_{i,0}}{\rho_i^k} & -\frac{Z^k - Z_{i,0}}{\rho_i^k} \end{bmatrix} \begin{bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ c\delta t_i \end{bmatrix} = P_i^k - \rho_{i,0}^k + cdt_i^k - T_i^k - I_i^k - \epsilon_i^k
\]

Let

\[b_i^k = P_i^k - \rho_{i,0}^k + cdt_i^k - T_i^k - I_i^k - \epsilon_i^k\]

Finally we have the structure to solve the position fix and clock offset, where the first three columns of the matrix \(A\) are the components for the three axis of the unit vector pointing from the linearization point to the position of the satellites and the fourth column is all ones. Special mention merits this resolution since it will be used in the later dilution of precision (DOP) computation.

\[
A \Delta x = \begin{bmatrix} -\frac{X^1 - X_{1,0}}{\rho_1^1} & -\frac{Y^1 - Y_{1,0}}{\rho_1^1} & -\frac{Z^1 - Z_{1,0}}{\rho_1^1} & 1 \\ -\frac{X^2 - X_{2,0}}{\rho_1^2} & -\frac{Y^2 - Y_{2,0}}{\rho_1^2} & -\frac{Z^2 - Z_{2,0}}{\rho_1^2} & 1 \\ -\frac{X^3 - X_{3,0}}{\rho_1^3} & -\frac{Y^3 - Y_{3,0}}{\rho_1^3} & -\frac{Z^3 - Z_{3,0}}{\rho_1^3} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{X^n - X_{n,0}}{\rho_1^n} & -\frac{Y^n - Y_{n,0}}{\rho_1^n} & -\frac{Z^n - Z_{n,0}}{\rho_1^n} & 1 \end{bmatrix} \begin{bmatrix} \Delta X_{i,1} \\ \Delta Y_{i,1} \\ \Delta Z_{i,1} \\ c\delta t_{i,1} \end{bmatrix} = b
\]
If $n \geq 4$, means that we have more equations than unknowns and that there is a unique solution for $(\Delta X, \Delta Y, \Delta Z, c\delta t_i)$ which is added to the estimated position of the receiver $i$ of the next iteration 1, and this is used for the next iteration. If we isolate the solution for the position update, we finally have

$$\Delta x = -(A^TWA)^{-1}A^Twb$$

where $W$ works as a weighting matrix that characterizes the reliability of every pseudorange measurement.

6.2 Error Sources in GPS

As it has been explained in the first chapters, the way to measure a position in a satellite-based system, is to determine the distance between the satellite and the location of the object. Two ways have been used so far, the pseudoranges, which are presented and thoroughly explained in the previous section, and the carrier phase method, which involve more difficulty and a brief introduction is done in the first section. Anyway, both systems can be understood as pseudo measurements or biased real distances due to several factors that involve from the satellite to the receiver and the space between them. There are six major causes of ranging errors: satellite ephemeris, satellite clock, ionospheric delay, tropospheric delay, multipath and receiver measurement errors, including software. In this section, the error caused by these factors are presented and modeled.

6.2.1 Satellite Clock Bias $c\Delta t_{sv}$

The inaccuracy of the satellites clock is controlled and monitored by the control segment, which receive the biases and drift of the clocks through the navigation message and then correct it through the master slave station. The main errors from the satellite clock can be divided as clock drift and relativistic errors. The second one, the relativistic error, is due to the clock in orbit will appear to run faster than the clock on Earth. The parameters that describe the behavior of the Satellite clock are the satellite clock bias $a_{f0}$, the drift $a_{f1}$ and drift rate $a_{f2}$. The clock behavior is described by the next polynomial equation:

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^3 \quad (11)$$

where $t$ is the actual time and $t_{oc}$ is the reference epoch for the coefficients in seconds in the GPS week.

To this equation, the relativistic effect $\Delta t_r$ has to be applied to complete the model.

$$\Delta t_r = Fe\sqrt{A}\sin E_k \quad (12)$$
where $F$ is a constant whose value is

$$F = \frac{-2\sqrt{\mu}}{c^2} = -4.442807633 \times 10^{-10} \frac{sec}{\sqrt{meters}}$$

where

$$\mu = 3.986005 \times 10^{14} \frac{meters^3}{second^2}$$

value of Earth’s universal gravitational parameters.

c = 2.99792458 \times 10^8 \frac{meters}{second} speed of light.

So finally, adding (12) to (11), the behavior of the satellite clock can be modeled as

$$\Delta t_{sv} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^3 + \Delta t_r$$

### 6.2.2 Receiver Clock Bias $c\Delta t_r$

GPS receivers, unlike satellites, use cheap crystal oscillators as clocks. These clocks are small, consume less power and are intended to keep a reasonable cost on the receivers. The receiver clock bias is a time error that affects in the same manner to all the pseudoranges calculated. Therefore, it not affects the pseudorange solution and it can be estimated with the least squares solution as it has been explained above.

### 6.2.3 Atmosphere Delays

The speed of the signal from the satellite to the receiver is of utmost importance to fix a position, as it has been explained before. If the signal traveled through the vacuum, the speed would be the speed of light, and with the knowledge of the travel time, we will get the exact range. The main problem of the signal traveling, is that it has to go through the Earth’s atmosphere, so the signal interacts with the particles of these surfaces, slowing its speed and changing its direction. In this section, we divide the atmosphere in two layers: the ionosphere and the troposphere.

The ionosphere is the layer of the atmosphere where the ionization causes sufficient quantities of electrons that can disrupt the propagation of a radio wave. It starts at 50 km above the surface of the Earth until approximated 1000 km. The ionospheric error, which is the bias from the true range to
the range caused by the ionosphere, is frequency dependent and positive in
the pseudoranges and negative in the carrier phase calculation [20]. Dual-
frequency GPS receivers take advantage of the ionosphere dispersive nature,
since a combination between the two signals can be computed to estimate
the ionospheric error.

For single GPS receivers, the Klobuchar Ionospheric Model is often used
to model the ionospheric error [17] and presented below. This model
appeared as a compromise between complexity and correction accuracy, and
is capable of diminish the ionospheric error a 70% [10].
The Klobuchar model uses a set of parameters ($\alpha_0 \ldots 3, \beta_0 \ldots 3$) that are download
together with the latitude and longitude from the navigation message.
Then, the parameters are used in a set of equations with the elevation and
azimuth of the satellite, and the delay is computed.

The ($\alpha_0 \ldots 3$) parameters are known as the coefficients of a cubic equation rep-
resenting the amplitude of the vertical delay and its units are $[\text{semi-circle}]$.

\[ \alpha_0, \alpha_1, \alpha_2, \alpha_3 \]

The ($\beta_0 \ldots 3$) parameters are known as the coefficients of a cubic equation rep-
resenting the period of the model and its units are $[\text{semi-circle}]$.

\[ \beta_0, \beta_1, \beta_2, \beta_3 \]

Latitude: user geodetic latitude [rad]
Longitude: user geodetic longitude [rad]
Elevation: elevation angle between the user and the satellite [rad]
Azimuth: azimuth angle between the user and the satellite, measured clock-
wise positive from the true North [rad]

All these parameters can be download from the satellites to compute the
ionospheric delay. The equations for calculating the delay were published
in [18] and are presented below.

1. Calculate the earth-centered angle (elevation E in semicircles).

\[ \phi = \frac{0.0137}{E + 0.11} - 0.022(\text{semicircles}) \]

2. Compute the subionospheric latitude

\[ \phi_1 = \phi_u + \phi \cos A \]
3. Compute the subionospheric longitude

\[ \lambda_1 = \lambda_u + \frac{\phi \sin A}{\cos \phi_1} \]

4. Find the geomagnetic latitude

\[ \phi_m = \phi_1 + 0.064 \cos (\lambda_1 - 1.617) \]

5. Find the local time

\[ t = 4.32 \times 10^4 \lambda_1 + \text{GPStime} (\text{sec}) \]

6. Compute the slant factor.

\[ F = 1 + 16 \times (0.53 - E)^3 \]

7. Compute the ionospheric time delay.

\[ T_{iono} = F \times \left[ 5 \times 10^9 + \sum_{n=0}^{m} \alpha_n \phi_m^n \times \left( 1 - \frac{x^2}{2} + \frac{x^3}{24} \right) \right] \]

where

\[ x = \frac{2\pi(t - 50.400)}{\sum_{n=0}^{m} \beta_n \phi_m^n} \]

On the other hand, the other layer under study for its disruption on the radio signals is the troposphere. The troposphere is the lower layer of the atmosphere, situated between the surface and 50 km above. It is a non ionized layer and is mostly compound by oxygen and nitrogen. The troposphere error is frequency independent due to it is a non dispersive medium with respect to radio waves up to frequencies of 15 GHz. Empirical models that are used nowadays to compute the troposphere delay can be found in [16], [4] and [26]. Here is presented the model from [8]

\[ T_{tropo} = 2.208 \left( e^{550 \times \text{Height}/6900} - e^{-\text{Height}/6900} \right) / \sin E \]

### 6.2.4 Selective Availability (SA)

Selective Availability was an intentional degradation of the GPS civil performance implemented for United States nation security and it had been of concern to civil GPS users worldwide. In 2005, the United States Government finally disrupted the degradation caused by the Selective Availability and guaranteed that there was no intention to use the Selective Availability
anymore. In 2007, the United Stated Government told that the new generation of GPS satellites, known as GPS III, that were about to be launched, will not have the Selective Availability feature.

6.2.5 Multipath

Multipath is the error caused by the reflection of the signal in objects such as buildings that cause the signal to travel in different paths and arrive to the receiver with different delays which cause interferences. Multipath error affect both pseudoranges and carrier phase detection although the last one in a lower level [20]. It is totally related with the environment of the receiver, its antenna and its tracking loop, and considers any reflective object external to the receiver antenna. Last years receivers are able to reduce significantly the multipath error although is one of the most difficult errors to control.

6.2.6 Dilution of Precision

The term Dilution of Precision (DOP) is used in satellite navigation systems to specify the precision of the measured position. The DOP gives us an idea of the geometrical situation of the satellites that we are receiving. Although it does not give us a whole situation of the satellites, it is a good way to know which satellites we are receiving in a blockage or difficult situation. It is a mathematical function that involves the position of the receiver and the satellites. The more spread are the satellites in the sky, a better position is obtained so a lower DOP. Although the most known DOP values are the position dilution of precision (PDOP) and the geometric dilution of precision, others DOP values are presented in this section [21].

To understand the real nature of the DOP values and its origin, we can use part of the least squares solution used for the pseudoranges.

\[ \Delta x = -(A^TWA)^{-1}A^TWb \]

If we define the covariance matrix of the pseudorange errors \( C_b \) as

\[ C_b = \frac{1}{\sigma_0^2}W \]

where \( \sigma_0^2 \) is the variance of the unit weight and \( W \) is the already presented weighting matrix. Now we can define the covariance matrix of the parameters estimated \( \Delta x \) as

\[ C_{\Delta x} = [(A^TWA)^{-1}A^TW]C_b[(A^TWA)^{-1}A^TW]^T = (A^T C_b^{-1}A)^{-1} \]
If we consider that every observation has the same measurement errors and models, then \( C_b \) is \( \sigma^2 I \) where \( \sigma \) is the standard deviation for all the observations and \( I \) is the identity matrix. Hence

\[
C_{\Delta x} = (A^T C_b^{-1} A)^{-1} = \sigma^2 (A^T A)^{-1} = \sigma^2 \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{xt} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} & \sigma_{yt} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2 & \sigma_{zt} \\
\sigma_{tx} & \sigma_{ty} & \sigma_{tz} & \sigma_t^2
\end{bmatrix}
\]

where the values from the diagonal of the matrix are the variances for the parameters of the solution, and the values outside the diagonal are the correlation between these parameters. From this result we can get the different values for the DOP.

The Position Dilution of Precision

\[
PDOP = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}
\]

The Time Dilution of Precision

\[
TDOP = \sqrt{\sigma_t^2}
\]

The Geometric Dilution of Precision

\[
GDOP = \sqrt{PDOP^2 + TDOP^2}
\]

The Horizontal Dilution of Precision

\[
HDOP = \sqrt{PDOP^2 + \sigma_y^2}
\]

The Vertical Dilution of Precision

\[
VDOP = \sqrt{\sigma_z^2}
\]

6.2.7 UERE

The UERE (User Equivalent Range Error) is the root of the sum of the square of every individual error applied to a pseudorange. This value is then multiplied by the position dilution of precision (PDOP) presented above, and the RMS three dimensional position error is obtained. Although this system considers that the UERE is the same for all the satellites, which is wrong because the atmospheric error cause different errors in the satellites, it is a good indicator to estimate the position accuracy.
6.2.8 Errors Summary

Finally, this table details the typical RMS error for the errors presented above. The UERE, as presented, will be the root of the square of the remaining errors.

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<thead>
<tr>
<th>Error Source</th>
<th>Typical RMS Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selective Availability (SA)</td>
<td>24</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>7</td>
</tr>
<tr>
<td>Troposphere</td>
<td>0.7</td>
</tr>
<tr>
<td>Clock and Ephemeris</td>
<td>3.6</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>1.5</td>
</tr>
<tr>
<td>Multipath</td>
<td>1.2</td>
</tr>
<tr>
<td>Total UERE</td>
<td>25.3</td>
</tr>
</tbody>
</table>

Table 1: GPS Error Sources.

7 Kalman Filter

So far the GPS and the INS theory and mathematical solution and equations have been explained thoroughly to give the reader enough knowledge to understand the further coupling. In this section, the theory of the Kalman filter that later will be applied to the fusion in the algorithm evaluation between the two systems is presented. The explanation include equations and schemes of the Kalman algorithm to facilitate comprehension of the reader. Subsequently, the connection between the Kalman filter and the navigation systems is explained, emphasizing the different implementations or coupling between the systems depending on the used data.

The Kalman filter is a recursive estimator of the state of a discrete-time process $x_k \in \mathbb{R}^n$ of dimension $n$ that can be characterized by the next equation

$$x_k = F_k x_{k-1} + B_k u_k + w_k$$

where the $n \times n$ matrix $F_k$ is the state transition matrix that is applied to the previous state and is related to the nature of the process, $B_k$ is the control matrix that is applied to the control vector $u_k \in \mathbb{R}^n$. Finally, $w_k$ is the noise of the process, that will be characterized as a multivariate gaussian random variable with zero mean and covariance matrix $Q_k$.

$$w_k \sim N(0, Q_k)$$
To complete the Kalman algorithm, we also need a measurement or observation of the true state $x_k$, this will be applied to the estimation depending on its reliability, i.e. its covariance matrix. It is characterized by the next equation

$$z_k = H_k x_k + v_k$$

where $z_k \in \mathbb{R}^m$ is the measurement or observation process with dimension $m$, the $n \times l H_k$ is the observation model matrix that transforms the $m$-space to the $n$-space. Finally, as in the process equation, $v_k$ will be a multivariate gaussian random variable with zero mean and covariance matrix $R_k$.

$$v_k \sim N(0, R_k)$$

Once the process and the measurement are characterized by its equations, the Kalman filter algorithm is presented. The Kalman filter only can determine the next state of the process with the knowledge of the previous state and the observation or measurement, and to do that, it takes a two step algorithm explained below. But before, we need to explain the notation of the “a priori” and “a posteriori” state estimates and covariance matrices. $\hat{x}_{n|m}$ is the “a posteriori” estimate of the state at a certain time $n$ given the observations or measurements up to $m$, included.

The first step, the prediction step, is where the estimate of the state is defined through its initial characterization

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B u_k$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$$

In the second step, the update step, is where the measurement or observation $z_k$ take its part. The measurement residual, i.e. the difference between the observation and the estimate “a priori” state is computed

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1}$$

The updated Covariance is calculated with the “a priori” covariance matrix $\hat{P}_{k|k-1}$ and the covariance matrix of the observation $R_k$.

$$S_k = H_k P_{k|k-1} H_k^T + R_k$$
Then, the Kalman gain is calculated

\[ K_k = P_{k|k-1} H_k^T S_k^{-1} \]

Basically, what this parameter does, is to weight the importance of the measurement observation and its presence in the “a posteriori” state estimate. We can observe that, the bigger the updated covariance \( S_k \) is, the lower the Kalman gain is. Therefore, the bigger the observation covariance matrix \( R_k \) is, the lower the importance of the measurement in the “a posteriori” state estimate. Another way to understand the meaning of Kalman gain parameter is to look at the “a priori” covariance matrix, the lower it is, the lower the Kalman gain is. Therefore, the more similar the “a priori” estimation of the state is to the real value of the state, the less importance of the observation update.

Finally, the Kalman filter gain is applied to the “a posteriori” estimation of the state and to the covariance matrix.

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \\
\hat{P}_{k|k} = (I - K_k H_k) \hat{P}_{k|k-1}
\]

This procedure is repeated iteratively, applying the “a posteriori” estimated state and the covariance matrix of time \( k \) as the “a priori” state estimate and covariance matrix of time \( k + 1 \).

The next figure clarifies the two step Kalman filter algorithm.

**Figure 9:** Kalman Filter algorithm.
7.1 Kalman Filter in Navigation

The Kalman filter has become an important part of the navigation systems solution. This recursive algorithm has been used for the achievement of an optimal solution of fusion navigation systems. The filter uses statistical models to weight correctly the measurements for updating a past state vector. The Kalman filter has been especially popular in GPS/INS fusion or GPS stand alone systems for its versatility, simplicity and for its different approaches.

Many approaches have been developed and tested during the last years, depending on the information used from the sensors into the Kalman filter. The most common approaches are three, the loosely coupling, the tightly coupling and the deeply or ultra tightly coupling. When the output of the GPS is the user position, the most common approach is the loosely or unscented coupling. In this coupling, which is the one used in this thesis, the outputs of the GPS are directly compared with the outputs of the INS (after frame rotations), i.e. both outputs are position. Because of having the need for a GPS position fix, the loosely coupling must have always 4 satellites in sight. This fact gives to this kind of coupling an inclination to be used in clear environments where there is continuously a GPS signal. This simplification carries a suboptimal performance in urban or non clear environments and is outperformed by other coupling approaches like the tightly coupling or the ultra tightly coupling.

In a tightly coupling architecture, also known as unscented, the GPS outputs are the raw data or pseudoranges, that, with the help of the ephemeris are computed to fix the user position. Like in the loosely coupling, the data from the GPS and the data from the INS are fused in a Kalman filter, in this case an extended Kalman filter due to the non linearity nature of the inputs. The tightly coupling is normally preferred because it is less sensitive to satellite dropouts and the Kalman filter models are more exact and simple. Also, the tightly coupling is better in harsh or difficult environments due to the fact that we can play with the signal to noise ratio of every satellite.

![Loosely Coupling Scheme](image)
8 Software and Hardware

For the realization of this thesis both hardware and software interfaces were used. In the software category the applications used were Ublox Center and Matlab. The hardware used were the GPS Evaluation Kit-6T with the Precision GPS Timing LEA-6T module with USB interfaces and a GPS antenna with 5 m of cable, and a wireless foot-mounted inertial navigation module. For the inertial navigation we have used tracking modules developed in the signal processing department at KTH capable of implement inertial navigation and dead reckoning. The tracking modules consist of a 23.2 x 31 x 13.5 mm case that includes a PCB with a μC (microcontroller), four IMU and magnetometers (currently not working) and the Bluetooth module. The PCB communicates through an UART port to the Bluetooth module, which has a 10 m range. The batter life of the modules is approximated to 1.5 h and charging the battery is almost 2 h. For more information of the modules refer to [24].

The Ublox Center is an evaluation software included in the Ublox GPS kits for configuration. The main use of this software was to get hold of and configure the messages received in the GPS and transfer them to the computer through a USB cable. In our case, the NAV-POSECEF message, which give us the GPS position in the X, Y and Z coordinates. Further, the ratio of received messages can be changed too. We selected a 5Hz frequency of GPS messages. Finally we deactivated the internal Extended Kalman Filter of the GPS because it could give us undesirable errors.
8.1 Acquisition Algorithm

The Matlab code developed to implement the system follows the next algorithm.

![Figure 13: Matlab data acquisition algorithm.](image)

The program starts with the acquisition of the IMU step update of the position and of the covariance matrix that are applied following the equations of motion presented in the first part of the thesis. Then, the GPS update were configured to a 5 Hz ratio, therefore, the Kalman filter receives an update from the GPS in a 1:5 ratio respect to the IMU, that works at 1 Hz. The system asks if a GPS update is available every IMU update. If the GPS update is available, the system transforms the ECEF GPS to the navigation module frame and then applies the Kalman filter algorithm, using the position of the IMU as the state estimate and the position of the GPS in the update step as the measurement or observation. If a GPS update is not available, the system continues calculating the position with the IMU as the only system. This process goes on iteratively until the user stops it.

For testing the Fusion, the IMU was situated on the shoe, being capable of clearly identify the movements and phases of the gait. The GPS was situated on the shoulder for a clear vision of the sky, and connected with an USB cable through a computer in a backpack.

![Figure 14: GPS and IMU localization on the body [2].](image)
Part III
Fusion Evaluation

The performance of the fusion is tested in two different environments: a clear sky environment with a continuous reception of the satellite signal and an urban environment situated at the KTH main campus. In both situations the performance is evaluated as follows. First of all, a graphic image of the scenario is shown to give the reader an idea of where the performances are tested. Subsequently, the reference path with the solution of the IMU module and the GPS receiver without the Kalman fusion are plotted. Although they do not give an statistical view of the operation for the two systems, the plots are useful to give the reader a general look of the behaviors of the solutions of them. Next, the plots of the evolution of the errors of the systems compared to the reference path are shown. On one side we have the stand-alone GPS errors with the different fusions evaluated. On the other hand, the evolution of the error of the stand-alone IMU module. These are the plots that give us a solid idea of the performance of the different systems and fusions. The errors are presented in every axis and finally in its RMS solution.

The different fusions are parameterized by its measurement covariance matrix. As it is explain in the Kalman filter section, the Kalman gain parameter determines the reliability of the measurement $z_k$, that is related to its covariance matrix $R_k$. In our case, the measurement $z_k$ is the GPS update, and the covariance matrix $R_k$ is the parameter that we are going to redefine in every fusion. The measurement covariance matrix $R_k$ is defined as

$$
R = \begin{bmatrix}
\sigma^2_x & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma^2_y & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma^2_z
\end{bmatrix}
$$

So in every fusion we are going to apply different covariance matrices to the measurement to see if it is more suitable to use a less or more reliable GPS measurement in every environment. The parameters of the measurement covariance matrix are going to be decide as follows: we are going to assume the same value for the three elements of the diagonal (variances) and we are going to assume as 0 the elements outside the diagonal (covariances). So, when a fusion is named as Fusion Covariance 5 it means that the variances are $\sigma^2_x = \sigma^2_y = \sigma^2_z = 5$, and the measurement covariance matrix of the GPS is defined as

$$
R = \begin{bmatrix}
5 & 0 & 0 \\
0 & 5 & 0 \\
0 & 0 & 5
\end{bmatrix}
$$
Then, the most suitable fusions are presented with the reference path to show graphically how the fusion is improving the solution. Finally, a table summarizes the behaviors of the GPS, the IMU module and the different fusions in terms of maximum error and mean error. To conclude the evaluation, an analysis is done for each environment.

9 Clear Sky Performance

In the clear sky performance, the system was tested in a running track with a clear sky view in all its path. The test covers a 35m walk with 5 rounds to the running track of approximated 420m that makes a total of 2100m.

![Running Track](image)

**Figure 15:** Running Track where the performance was tested [12].

Although the performance of the solution of the GPS was worst than expected because of an error peak of almost 70m, as we can see in the next plots, where the reference position, the GPS solution, the different fusions and the IMU solution are shown, the solution is still enough good to show the benefits of the fusion.

![XY Plot](image)

**Figure 16:** XY plot of the reference path and the GPS solution.
Figure 17: ZY plot of the reference path and the GPS solution.

This two plots demonstrate the theory that have been repeatedly explained during the thesis about the behaviors of the two systems. On one side, the behavior of the GPS shows how the solution remains stable in a long term while it can bounce swiftly in a short term. On the other hand, the behavior of the inertial system show how in a short term it gives a reliable solution, but in a long term the solution is totally unaccurate. Without the error peak in the first 100 steps, the solution of the GPS is much more in accordance of what we will expect in a clear sky view solution, with its maximum error error in 26 m and its mean error in 7 m on the GPS solution.

Here, we can see the evolution of the error in the three axis and the RMS error of the GPS compared to the Kalman fusions parameterized by its covariance.

Figure 18: Evolution of the x-axis GPS and Fusion Error.
Figure 19: Evolution of the y-axis GPS and Fusion Error.

Figure 20: Evolution of the z-axis GPS and Fusion Error.

Figure 21: Evolution of the RMS GPS and Fusion Error.
Here the plots of the evolution of the IMU error related to the reference path. It does not give us too much information but is a good way to see how in each round to the track the error is growing over the time.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{x_axis_error}
\caption{Evolution of the x-axis IMU Error.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{y_axis_error}
\caption{Evolution of the y-axis IMU Error.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{z_axis_error}
\caption{Evolution of the z-axis IMU Error.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{rms_error}
\caption{Evolution of the RMS IMU Error.}
\end{figure}

Below, the plots of the graphical solution of the fusions that previously on the error plots gave us a lower error and the reference path. We can clearly see how the solution is quite more smooth than the GPS solution and that is obviously not drifting like the IMU module solution.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{xy_solution}
\caption{XY plot of the reference path and the Fusion solution.}
\end{figure}
9.1 Results Analysis

The error plots reveal that the errors of the Fusion solution are strongly correlated with the errors of the GPS. So that helps to understand us how this kind of coupling works better in the situations where the GPS solution is quite good. Furthermore, after seeing the results shown in the table, it is fair to say that the most suitable covariance matrix for the observation measurements of the GPS that act as the update or correction of the IMU position solution is a diagonal matrix with the variance of the three axis.
(x,y,z) between 10–15 $m^2$ that will result to an standard deviation of the three axis between 3-4 m that results to be a quite reasonable performance for a GPS solution in a environment where there is a continuous clear signal reception. With a too low covariance, i.e variance of 5 for the three axis, the solution relies to much on the GPS solution. Remember that a variance of 5 $m^2$ for each axis is an standard deviation of 2.23 m approximated which is probably a too precise solution for a traditional GPS. If the variances of the covariance matrix begin to grow (over 15$m^2$), we can see how the mean error starts to grow, that is because we are relying too much on the IMU solution. In the clear sky environment, is easy to find and choose the most suitable covariance matrix because the GPS solution is uniform in all the path apart from the peak so the GPS reliability is almost the same in each point of the track. In this performance the GPS solution is improved a 40% in terms of mean and an 80% in terms of maximum error. It is obvious that the fusion works.

10 Urban Performance

The KTH scenario consists in a 45 minutes walk in a environment which is divided in a building zone an a more clear zone. The walk is divided in a first straight path of approximated 215m (Marker A to B) which is probably the most harsh for the GPS to arrive due to the presence of buildings is added a high volume of car traffic. Then, 3 and a half rounds of approximated 12 minutes and 720 m which makes a total of 3000 m walked. The rounds are divided in 3, from B to C, slightly worse environment due to trees blockage. From C to D point is where the GPS has more difficulties to fix a position due to blockage of high buildings and trees. Finally, from D to C is an environment more or less as B to C. This performance was tested only in a XY frame due to the inability to obtain a vertical reference for the slopes and changes of altitude of the path, another GPS with high accuracy could be a great option to get reliable reference path.

![Figure 28: KTH scenario [12].](image)
In the next plot, the reference path is drawn in red, the IMU solution in green and the GPS solution in blue. It is easy to observe how, like in the clear sky performance, the IMU solution starts drifting dramatically after approximated 100 m in the first straight line. On the other hand, the GPS solution is much better than what we expected, in contrast with the bad performance got in the clear sky performance. We can observe that between points C and D is where that GPS lose track of the position several times, due to the presence of trees and buildings that block the satellite signals.

![XY plot of the reference path and the GPS solution.](image)

**Figure 29:** XY plot of the reference path and the GPS solution.

This XY path plot, like in the other environment, shows the expected behavior for both systems. The GPS solution has a long-term continuous performance but it has its short-terms errors in the places where the signal is bad and there is probably a bad geometry of the satellites (high DOP) too. On the other hand, the IMU stand-alone solution confirms us that is only trustworthy in the first steps since it starts drifting quickly.

In the next three plots we can observe the evolution of the error of the two axis, x and y, of the GPS solution compared to the solution of the different fusions parameterized by its measurement covariance matrix. It is easy to see the epochs when the GPS lose track of the position and that the error of the position is obviously in every axis. Finally the sum of the root of the sum of the squares of the two axis gives us the overall position error.
Figure 30: Evolution of the x-axis GPS Error.

Figure 31: Evolution of the y-axis GPS Error.

Figure 32: Evolution of the RMS GPS Error.
The next three plots describe the evolution of the error of the IMU regarding to the reference path. It is easy to see how it grows quickly and within all the walk arrives to almost 180m error.

**Figure 33:** Evolution of the x-axis IMU Error.

**Figure 34:** Evolution of the y-axis IMU Error.

**Figure 35:** Evolution of the RMS IMU Error.

The last plot shows the graphic solution for the fusions compared to the reference path. As in the clear sky environment, the path is much more smooth than the GPS solution and does not drift as the IMU solution. It is easy to see the main drawback, and is that when the GPS solution is poor for a few seconds, the fusion solution drifts from the reference path.

**Figure 36:** XY plot of the reference path, the IMU and the GPS solution.
Finally, the next tables summarizes the performance of the GPS, the stand-alone IMU and the different fusions.

<table>
<thead>
<tr>
<th>System</th>
<th>Mean Error (m)</th>
<th>Max Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>7.88</td>
<td>79.6</td>
</tr>
<tr>
<td>IMU</td>
<td>97.54</td>
<td>211.76</td>
</tr>
<tr>
<td>Fusion Cov. 5</td>
<td>4.21</td>
<td>20.33</td>
</tr>
<tr>
<td>Fusion Cov. 10</td>
<td>4.10</td>
<td>18.20</td>
</tr>
<tr>
<td>Fusion Cov. 15</td>
<td>4.18</td>
<td>17.31</td>
</tr>
<tr>
<td>Fusion Cov. 20</td>
<td>4.34</td>
<td>16.84</td>
</tr>
<tr>
<td>Fusion Cov. 30</td>
<td>4.72</td>
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</tr>
<tr>
<td>Fusion Cov. 40</td>
<td>5.14</td>
<td>16.25</td>
</tr>
<tr>
<td>Fusion Cov. 50</td>
<td>5.56</td>
<td>16.27</td>
</tr>
<tr>
<td>Fusion Cov. 100</td>
<td>7.41</td>
<td>16.91</td>
</tr>
<tr>
<td>Fusion Cov. 200</td>
<td>10.38</td>
<td>19.33</td>
</tr>
<tr>
<td>Fusion Cov. 500</td>
<td>16.62</td>
<td>28.22</td>
</tr>
</tbody>
</table>

Table 3: Urban Environment Performance Resume.

10.1 Results Analysis

First of all, comparing the two performances in terms of error is not the best way to evaluate the system because the errors of the clear sky environment are in a three axis frame and in the urban environment are in a two axis frame. In this urban performance we can observe in the errors plots how the errors of the fusion are strongly correlated with the errors from the GPS. When the GPS has a peak of error, the fusion has a reduced peak of error compared to the GPS. The less we trust in the GPS, i.e the bigger the variances in the measurement covariance matrices, the smaller the peaks of error are going to be. But, unlike the clear sky performance, here we have two find a covariance matrix that balances this peaks of error and lose of signal, with the parts where the GPS signal is enough good to trust in it. That is because the reception of the GPS is different on every situation of the path where a different covariance matrix should be used (the variance of the GPS Error is bigger). Thats the main reason why in the table we can observe how if we trust less in the GPS the error peaks are going to be lower but the mean error is going to be bigger. In this environment, the mean error has been reduced a 43% in the best case and the peak error 80% in the best case compared to GPS. Although the overall performance of the system is quite good, is obvious that in this environment, another coupling like the tight or the ultra tight would be much better, because in these couplings, the reliability of the GPS is intrinsic to the pseudoranges or carrier phase used and its signal to noise ratio.
11 Conclusions

The aim of the project was to develop a real time loosely coupling between a GPS receiver and a foot-mounted IMU using a Kalman Filter. The thesis covers and explanation of both systems, GPS and Inertial Navigation Systems, through a theory explanation, its mathematical equations and its sources of errors, to make it easier for the reader to understand the later fusion. The combination of both systems requires to fuse and synchronize the data in the same reference frame. The Kalman filter that fuse the data is presented and different systems of coupling are presented, choosing finally the loosely coupling. Finally, the system is tested in two environments, in a clear sky performance were although the GPS solution was expected to be better, the fusion works perfectly, giving a smooth path to the user and diminishing the IMU and GPS error. The second performance is tested in a Urban environment at KTH, where the system works as expected. In terms of percentage the system performance is enhanced as in the clear sky environment but showing weaknesses where the GPS signal gives a bad position fix. The evolution of the error for both systems is presented in different the three axis and RMS for the clear sky environment, and for two axis and RMS for the urban environment. The performance of the system is studied trough the study of its errors after parameterizing the covariance matrix of the measurement. A table is presented for each performance, listing the differents covariance used and its mean and maximum error. It is demonstrated that in a continuous reception of the GPS signal a loosely-coupling is enough but in a harsh or urban environment a tight-coupling should be used for taking the system to the next level.

12 Future Work

A lot of work had already been done in this field, and loosely coupling is almost obsolete in urban environments due to other couplings like the tightly or ultra tightly coupling. The first objective of the thesis was to apply a tightly coupling to the systems using pseudoranges, but for technical reasons it was not possible. So an obvious next step should be to work on a real time system that instead of a loosely coupling uses a tightly coupling, applying this in environments that mix urban situations and totally lose of the GPS signal like indoor environment, studying that way the dependability of the foot-mounted IMU in short-time GPS signal lose. Following the work in this thesis, an Android application could be implemented to use the system real time in a more suitable platform like a mobile phone or a tablet. The same should be applied to a tightly coupling study. Other things possible to add to the thesis would be a HSGPS High sensitivity GPS to track a more trusty reference path and therefore have an exact systems errors.
References


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