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A ROBUST AND EFFICIENT 3D CONSTITUTIVE LAW TO DESCRIBE THE RESPONSE OF QUASI-BRITTLE MATERIALS SUBJECTED TO REVERSE CYCLIC LOADING: FORMULATION, IDENTIFICATION AND APPLICATION TO A RC SHEAR WALL

B. RICHARD[*], M. VASSAUX[†], F. RAGUENEAU[*,†] AND A. MILLARD[‡]

[*] CEA, DEN, DANS, DM2S, SEMT, Laboratoire d'Etudes de Mécaniques Sismiques
F-91191 Gif-sur-Yvette, France
e-mail: Benjamin.Richard@cea.fr

[†]LMT-Cachan (ENS Cachan/CNRS/Université Paris-Saclay)
61, avenue du Président Wilson, F-94230 Cachan, France e-mail:
Maxime.Vassaux@lmt.ens-cachan.fr

[*,†]LMT-Cachan (ENS Cachan/CNRS/Université Paris-Saclay)
61, avenue du Président Wilson, F-94230 Cachan, France e-mail:
Frédéric.Ragueneau@lmt.ens-cachan.fr

[‡]CEA, DEN, DANS, DM2S, SEMT, Laboratoire de Modélisation des Systèmes et des Structures
F-91191 Gif-sur-Yvette, France
e-mail: Alain.Millard@cea.fr

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Abstract. In this paper, a new constitutive law aiming at describing the quasi-brittle behavior when subjected to cyclic loading is presented. The proposed model is formulated within the framework of isotropic continuum damage mechanics. The Cauchy stress tensor is split into two contributions: one related to the matrix (without cracks) behavior the other related to the crack behavior. This strategy allows accounting for both the crack closure effect and the hysteretic effect in an efficient way making possible large-scale computations. In addition, a specific attention is paid to the way of identifying the material parameters, often requiring complex experimental tests not always easy to carry out. A virtual testing approach based on the use of a discrete element model is used for this purpose.

1 INTRODUCTION

Recent events that occurred in the world have once again shown the importance to predict accurately the structural behavior of reinforced concrete structures when subjected to cyclic loadings. The numerical description of the complex cyclic behavior of quasi-brittle materials remains an open question and for this reason, this issue keeps on feeding an intense research field. Toward the completion of the simulation of three-dimensional structures made of concrete subjected to cyclic loading, phenomenological models have inherent advantages. However, the constitutive equations complexity increases rapidly when trying to capture consequences of mechanisms induced under uniaxial cyclic loading. Indeed even recently developed models are either not robust enough to simulate the complete response of structures subjected to cyclic loading, or do not reproduce accurately phenomena related to cracks closure and friction, as outlined by results of the ConCrack benchmark. Indeed, a low regularity of the constitutive laws (discontinuous stiffness variations at stress sign changes) due to the consideration of stiffness recovery under reverse loading leads to numerical robustness issues [1]. In addition, the inaccurate description of hysteresis loops induces poorly estimated hysteretic dissipation and therefore, specific issues when dealing with dynamic loadings. The first purpose of this study is to enhance the efficiency of the model proposed in [2] to achieve structural simulations. A specific attention is paid to phenomena observed under cyclic loading. Issues encountered with the actual model are partly induced by a lack of experimental data, preventing from establishing a finer mechanical description of the material's behavior. Experimental data on quasi-brittle materials Representative Volume Element (RVE) subjected to reverse cyclic loading are sparse due to control and repeatability issues. Therefore the replacement of part of laboratory experimentation by virtual testing is investigated. A RVE is considered here to be approximately of a 0.1 m characteristic length, such as the material can be considered homogeneous with respect to different phases. The second purpose of this paper is thus to illustrate the use of virtual testing as a complement to laboratory experimentation to develop a regularized and accurate macroscopic constitutive model for quasi-brittle materials fitted for cyclic loading. A microscopic model based on the Discrete Element Method (DEM) is used as a virtual testing machine.

2 CONSTITUTIVE LAW FORMULATION

The phenomenological macroscopic model is formulated using a rather classic decomposition of the total stress in the RVE. It is considered that the total stress $\underline{\underline{\sigma}}$ can be split in two independent parts:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^m + \underline{\underline{\sigma}}^f \tag{1}$$

with the stress in the cracked continuous media $\underline{\underline{\sigma}}^m$, neglecting any interaction between the cracks, classically modeled with a damage model; and the stress in the cracks when closed $\underline{\underline{\sigma}}^f$.

Free energies Ψ^m and Ψ^f respectively associated to the two stress tensors are defined, and compose the total free energy of the specimen:

$$\Psi = \Psi^m + \Psi^f \tag{2}$$

At this point, both thermodynamic potentials remain completely unspecified. The free energy Ψ^m is completely detailed in the next paragraph based on existing models. In contrast, the free energy Ψ^f is only roughly introduced since it is not totally determined already, a virtual testing step will be required to complete its formulation.

2.1 CRACKING PROCESS

Fracture processes are modeled by means of the continuum damage theory. The simpler the damage variable is kept, the more robust the proposed macroscopic model is. Therefore, in view of the structural applications of the proposed model, an isotropic damage model is formulated, implying a unique scalar damage variable. The free energy associated to the cracked continuous media simply writes:

$$\Psi^m = \frac{1}{2}(1-D)\,\underline{\underline{\epsilon}} : \boldsymbol{C} : \underline{\underline{\epsilon}} + \Psi^{m,D}(z) \tag{3}$$

with D the isotropic damage variable, $\underline{\underline{\epsilon}}$ the second-order total strain tensor, $\boldsymbol{C}$ the fourth-order Hooke's tensor, z the isotropic hardening variable, and $\Psi^{m,D}$ the free energy related to damage. The formulation of the non-associated pseudo-potential of dissipation φ^m is based on the Mazars failure criterion and expressed in terms of thermodynamical variables.

$$\varphi^m = \bar{Y} - (Y_0 + Z) \tag{4}$$

where Z stands for the thermodynamic force associated to z, $\bar{Y}$ the energy rate $\bar{Y} = \frac{1}{2}E\epsilon_0\epsilon^{eq}$ which is written as a function the equivalent strain $\epsilon^{eq} = \sqrt{\langle\underline{\underline{\epsilon}}\rangle_+ : \langle\underline{\underline{\epsilon}}\rangle_+}$, ϵ_0 the elastic limit strain, and Y_0 the elastic limit energy rate written in a similar fashion $Y_0 = \frac{1}{2}E\epsilon_0^2$. The asymmetry between traction and compression loading is only considered through its consequence on the peak load and the softening behavior of the material, and is introduced in the damage variable evolution law derived from $\Psi^{m,D}$. An additional variable κ is introduced to consider the effect of a confining pressure on cracks propagation, namely a higher peak load value and a more ductile behavior:

$$\frac{\mathrm{d}\Psi^{m,D}}{\mathrm{d}z}(z) = -\frac{\kappa}{B_0} ln\left[\frac{Y_0}{\bar{Y}}(1+z)\right] \tag{5}$$

where B_0 stands for a parameter controlling the softening behavior and κ computed as follows:

$$\kappa = 1 + k_0\left(\frac{\langle\underline{\underline{\sigma}}^m\rangle_- : \langle\underline{\underline{\sigma}}^m\rangle_-}{(\underline{\underline{\sigma}}^m) : (\underline{\underline{\sigma}}^m)}\right)^{1/2} = 1 + k_0\left(\frac{\langle\boldsymbol{C} : \underline{\underline{\epsilon}}\rangle_- : \langle\boldsymbol{C} : \underline{\underline{\epsilon}}\rangle_-}{(\boldsymbol{C} : \underline{\underline{\epsilon}}) : (\boldsymbol{C} : \underline{\underline{\epsilon}})}\right)^{1/2} \tag{6}$$

where k_0 stands for a parameter measuring the influence of the confining pressure, and therefore only influences the failure behavior when cracks are induced indirectly (e.g. in compression). The direct tension behavior remains uninfluenced, indeed κ is then equals to 1. Based on consistency conditions, the damage variable evolution law finally writes:

$$D = 1 - \frac{Y_0}{\bar{Y}}\exp\left[-\frac{B_0}{\kappa}\left(\bar{Y} - Y_0\right)\right] \tag{7}$$

2.2 CRACK CLOSURE EFFECT

2.2.1 STIFFNESS RECOVERY

Cracks mechanical behavior described by the stress tensor $\underline{\underline{\sigma}}^f$ is first considered elastic. $\underline{\underline{\sigma}}^f$ is defined as non-linear function of the strain tensor $\underline{\underline{\epsilon}}^f = D \times \underline{\underline{\epsilon}}$, which could be called the homogenized contribution of cracks opening to the total strain of the RVE. Such strain tensor definition is found as well in [3]. The following assumption is made on the evolution of $\underline{\underline{\sigma}}^f$ with respect to $\underline{\underline{\epsilon}}^f$:

$$\dot{\underline{\underline{\sigma}}}^f = \vartheta \left(\dot{\underline{\underline{\epsilon}}}^f \right) \boldsymbol{C} : \dot{\underline{\underline{\epsilon}}}^f \tag{8}$$

The function ϑ is scalar, in other words, the tangent modulus of the cracks stress-strain relationship is proportional to the undamaged Hooke's elastic tensor. Therefore ϑ represents the part of the lost stiffness due to cracking which is recovered thanks to cracks closure, and can only take values ranging from 0, when cracks are completely opened, to 1, when cracks are completely closed. Since ϑ evolves according to the materials solicitation, it is considered to be dependent on $\dot{\underline{\underline{\epsilon}}}^f$. Set aside its physical meaning, ϑ can be considered as a numerical regularization of the multiple Signorini's contact problem induced by cracks closure. ϑ should then be formulated to evolve from 0 to 1 in sufficiently regular manner to avoid discontinuities of the constitutive laws or of their derivatives. The final formulation of ϑ is proposed and physically justified in the next section by means of a virtual analysis of the evolution of the proportion of closed cracks during a uniaxial cyclic test. The elastic part of the free energy associated to cracks behavior is then written as:

$$\Psi^{f,e} = \int \left(\int \vartheta \left(\underline{\underline{\epsilon}}^f \right) \boldsymbol{C} : \mathrm{d}\underline{\underline{\epsilon}}^f \right) \mathrm{d}\underline{\underline{\epsilon}}^f \tag{9}$$

A first condition on ϑ can be set, in order to ensure continuity of the free energy $\Psi^{f,e}$, such condition would be that ϑ be at least of class $\mathcal{C}^0$.

2.2.2 HYSTERETIC LOOPS

The explanation of hysteresis effects relying on frictional sliding occurring at the cracks surfaces justifies a modeling method based on plasticity theory. In consequence, a perfect plasticity model along with a Drucker-Prager criterion is utilized. Because of perfect plasticity, the free energy Ψ^f is reduced to an elastic part $\Psi^{f,e}$ and introduces a single internal variable, the plastic strain accumulated through sliding between the cracks $\underline{\underline{\epsilon}}^{f,p}$, defined as $\underline{\underline{\epsilon}}^f = \underline{\underline{\epsilon}}^{f,e} + \underline{\underline{\epsilon}}^{f,p}$:

$$\begin{aligned}\Psi^f &= \Psi^{f,e} \left(\underline{\underline{\epsilon}}^f - \underline{\underline{\epsilon}}^{f,p} \right) \\ &= \int \left(\int \vartheta \left(\underline{\underline{\epsilon}}^f - \underline{\underline{\epsilon}}^{f,p} \right) \boldsymbol{C} : \mathrm{d} \left(\underline{\underline{\epsilon}}^f - \underline{\underline{\epsilon}}^{f,p} \right) \right) \mathrm{d} \left(\underline{\underline{\epsilon}}^f - \underline{\underline{\epsilon}}^{f,p} \right)\end{aligned} \tag{10}$$

Yet $\underline{\underline{\epsilon}}^{f,p}$ is independent on $\underline{\underline{\epsilon}}^f$. In addition, $\underline{\underline{\epsilon}}^{f,p}$ as a plastic strain refers to an isochoric transformation, therefore its first invariant is null. Then $\vartheta \left(\underline{\underline{\epsilon}}^f - \underline{\underline{\epsilon}}^{f,p} \right) = \vartheta \left(\underline{\underline{\epsilon}}^f \right)$, hence:

$$\Psi^f = \int \left(\int \vartheta \left(\underline{\underline{\epsilon}}^f \right) \boldsymbol{C} : d\underline{\underline{\epsilon}}^f \right) \mathrm{d}\underline{\underline{\epsilon}}^f - \frac{1}{2} \vartheta \left(\underline{\underline{\epsilon}}^f \right) \underline{\underline{\epsilon}}^{f,p} : \boldsymbol{C} : \underline{\underline{\epsilon}}^{f,p} \tag{11}$$

Thus:

$$\Psi^f = \Psi^{f,e}\left(\underline{\underline{\epsilon}}^f\right) - \frac{1}{2}\vartheta\left(\underline{\underline{\epsilon}}^f\right)\underline{\underline{\epsilon}}^{f,p} : \boldsymbol{C} : \underline{\underline{\epsilon}}^{f,p} \tag{12}$$

Conditions of continuity of the free energy Ψ^f are not changed when introducing frictional sliding, it is still required that ϑ be $\mathcal{C}^0$. The pseudo-potential of dissipation is a Drucker-Prager criterion:

$$\varphi^f = \sqrt{\mathrm{J}_2\left(\underline{\underline{\sigma}}^f\right)} + \mu_0 \mathrm{I}_1\left(\underline{\underline{\sigma}}^f\right) \tag{13}$$

where μ_0 stands for a parameter which could be assimilated to a friction coefficient. Regarding the physical significance of the chosen criterion, the J_2 part refers directly to shear occurring in the cracks, while I_1 rather refers to cracks surface normal pressure. In consequence, when J_2 exceeds $\mu_0\mathrm{I}_1$, frictional sliding is observed. Furthermore, the J_2 part depends on ϑ through $\underline{\underline{\sigma}}^f$, therefore depends on the proportion of closed cracks. Thus when all the cracks are open the J_2 is negligible and as expected no frictional sliding is observed.

3 IDENTIFICATION BY VIRTUAL TESTING

3.1 UNILATERAL EFFECT

The formulation of the continuum model accounting for cyclic effects has almost fully been presented, only remains the function ϑ to be defined. Its physical sense has already been explained, namely the evolution of the proportion of closed cracks with respect to the cracks strain tensor $\underline{\underline{\epsilon}}^f$. However, ϑ remains to be characterized. This process is undertaken using the virtual testing machine aforementioned and the discrete simulations of the uniaxial cyclic test [4]. The function ϑ, as the proportion of closed cracks, represents the proportion of cracks in which forces transit and contribute to the stiffness recovery of the specimen. The evolution of ϑ is characterized analyzing the evolution of the ratio of number of contacts detected and the number of cracks initiated in the virtual material sample during the simulation of the uniaxial cyclic test. The analysis is led for different damage levels, that is different maximal cracks strains $\epsilon^f_{\max}$. The maximal cracks strains tensor is defined as $\underline{\underline{\epsilon}}^f_{\max} = \underline{\underline{\epsilon}}^{f,t_m}$, where t_m stands for the pseudo-time such as $\mathrm{I}_1\left(\underline{\underline{\epsilon}}^{f,t_m}\right) = \max\limits_{\forall t}\left[\mathrm{I}_1\left(\underline{\underline{\epsilon}}^{f,t}\right)\right]$. The figure 1 shows the evolution of the proportion of closed cracks during unloading phases.

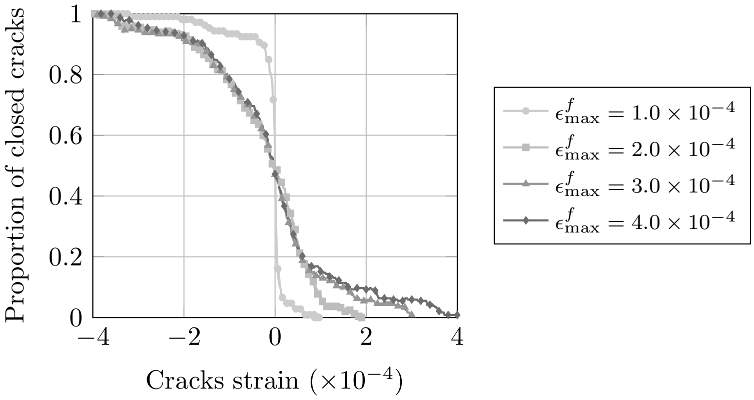


Figure 1: Dependency of the proportion of closed cracks to $\epsilon^f_{\max}$ evaluated with the microscopic model.

Independently of $\epsilon^f_{\max}$, the proportion of closed cracks follows a sigmoidal evolution with respect to ϵ^f. To determine an analytic expression for ϑ let us consider the probabilistic event "a crack closes", ϑ is the distribution function of this event. From the results obtained with the microscopic model, it appears that this event follows a symmetrical distribution centered in $\epsilon^f = 0$, therefore it could be assumed that the event "a crack closes" follows a Gaussian distribution of zero mean. ϑ is then expressed as the distribution function of a Gaussian distribution:

$$\vartheta = 1 - \frac{1}{1 + \exp\left[-f \times \mathrm{I}_1\left(\underline{\underline{\epsilon}}^f\right)\right]} \tag{14}$$

where the function f is associated to variance of the Gaussian distribution. The maximal cracks strain $\epsilon^f_{\max}$ affects the evolution of the proportion of closed cracks (see figure 1). The more damaged the specimen, the bigger the variance of the event "a crack closes". The function f is finally chosen to account for this dependency:

$$\vartheta = 1 - \frac{1}{1 + \exp\left[-\frac{\alpha_0}{\mathrm{I}_1\left(\underline{\underline{\epsilon}}^f_{\max}\right)}\mathrm{I}_1\left(\underline{\underline{\epsilon}}^f\right)\right]} \tag{15}$$

where α_0 stands for a parameter controlling a reference variance of the event "a cracks closes".

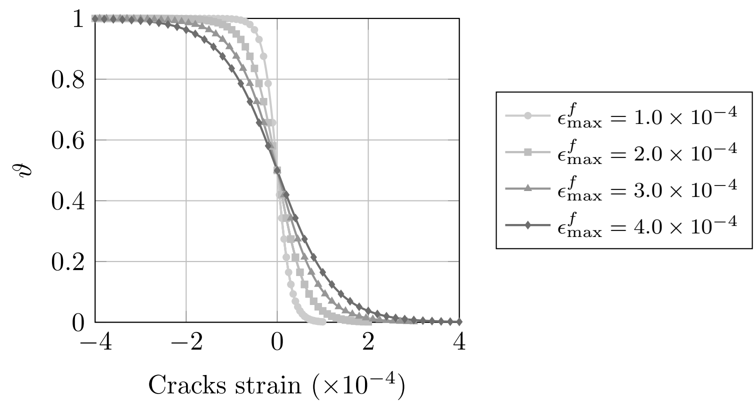


Figure 2: Dependency of the proportion of closed cracks to $\epsilon^f_{\max}$ evaluated with the function ϑ for $\alpha_0 = 6.5$.

The formulated expression of ϑ is a sigmoidal function, therefore it is at least of class $\mathcal{C}^0$. Continuity of the total free energy is therefore ensured.

3.2 HYSTERETIC EFFECT

Even though, unlike cracks closure, efficient and robust modeling techniques already exist to reproduce frictional sliding at the macroscopic scale, based on plasticity theory, the parameters introduced remain to calibrated. As the Drucker-Prager criterion for the initiation of sliding between the cracks has been presented beforehand (see equation 13), the parameter μ_0 has to be evaluated. Instead of calibrating the value of μ_0 arbitrarily, a methodology based on the virtual testing machine is once again proposed. By means of the microscopic model it is possible to estimate the dissipated energy specific to frictional sliding, among other dissipation mechanisms. Therefore, the parameter μ_0 is calibrated in order to observe an identical friction specific dissipation in concrete RVEs modeled using the microscopic model and the macroscopic continuum model (i.e a Gauss point). The comparison of both models is realized during the simulation of a complete cycle of uniaxial cyclic test, namely loading, unloading, reloading. The amplitude of the cycle is arbitrarily chosen to vary from $\epsilon^f_{\max} = 2.0 \times 10^{-4}$ to $\epsilon^f_{\max} = -1.0 \times 10^{-4}$ back to $\epsilon^f_{\max} = 2.0 \times 10^{-4}$. Friction related dissipated energies are respectively computed from the following formulations:

- for the microscopic discrete model, the dissipated energy is incrementally computed as the sum of the variations of dissipated energies for every contact between particles i and j detected, thus:

$$E_\mu^{d,t+1} = E_\mu^{d,t} + \sum_{i=1,\ldots,n_{particules}} \left[\sum_{j=1,\ldots,n^i_{contact}} \frac{1}{2} \left({}^T \left(\underline{F}^{t+1}_{fric,ij} + \underline{F}^{t}_{fric,ij} \right) \cdot \left(\Delta\delta u_{s,ij} \underline{t}_{c,ij} \right)^{t+1} \right) \right] \quad (16)$$

where $\underline{F}_{fric,ij}$ and $\Delta\delta u_{s,ij}$ respectively stand for the friction force and the increment of relative sliding displacement between particles i and j, t the current time-step, and $n^i_{contact}$ the amount of contacts detected on the particle i;

- for the macroscopic continuum model, the dissipated energy is incrementally computed as the integral over the representative volume Ω of the tensor product of the cracks stress $\underline{\underline{\sigma}}^f$ and the increment of plastic cracks strain $\Delta\underline{\underline{\epsilon}}^f$, thus:

$$E_\mu^{c,t+1} = E_\mu^{c,t} + \int_\Omega \left[\frac{1}{2} \left(\underline{\underline{\sigma}}^{f,t+1} + \underline{\underline{\sigma}}^{f,t} \right) : \Delta\underline{\underline{\epsilon}}^{f,p,t+1} \right] dV \tag{17}$$

Evolutions of the computed energies during the aforementioned loading cycle are presented in figure 3. Both dissipated energies remain null during the loading step, cracks are completely open. Then, an important growth is observed during unloading, while cracks progressively close. During reloading, both energies evolutions stagnate first since cracks are completely closed, before opening progressively leading to another growth of the dissipated energy related to friction.

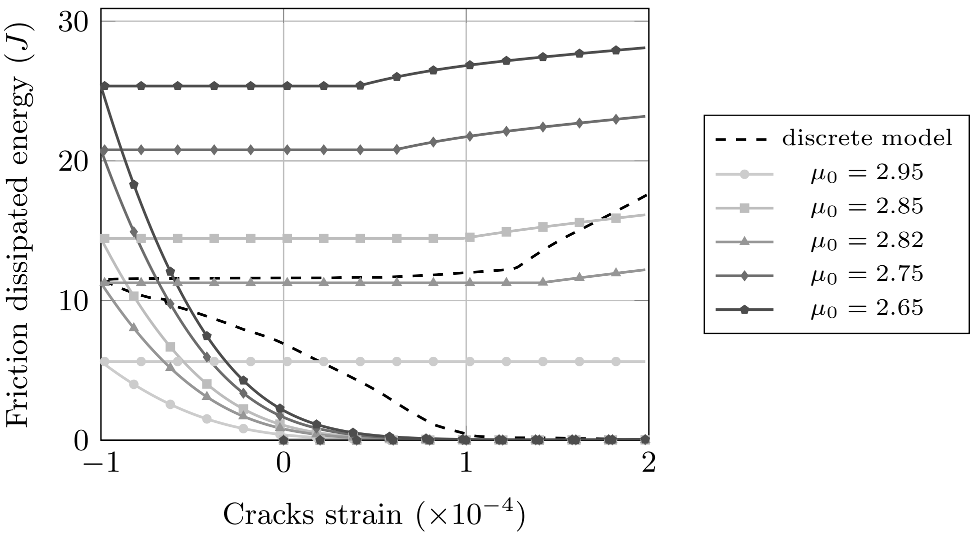


Figure 3: Sensitivity of the friction related dissipated energy of the continuum model to the parameter μ_0.

The parameter μ_0 is then evaluated at $\mu_0 = 2.82$. It might be added that the friction related dissipated energies computed with both models present similar trends, which comforts the choice of the perfect plastic modeling, along with a Drucker-Prager criterion, of the cracks frictional sliding mechanism.

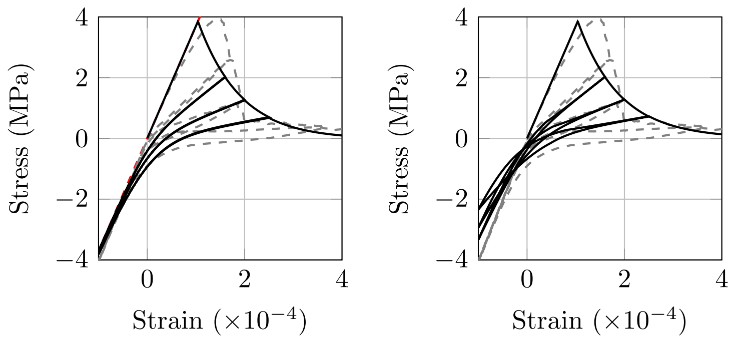
4 LOCAL RESPONSE

The continuum model has been implemented in the finite element software Cast3M [5], developed by the French Sustainable Energies and Atomic Energy Commission (CEA). Parameters of the continuum media part of the model are calibrated by equivalence with macroscopic reference results provided by the virtual testing machine on a square sample of 0.1 m side length. The tension resistance, the tension fracture energy, and the compression resistance are respectively utilized to calibrate ϵ_0, B_0 and k_0 (see table 1). In the whole section results plotted with dashed lines refer to the results provided by the virtual testing machine, and plain lines refer to results obtained with the continuum model.

E (GPa)	ϵ_0	B_0 (kJ^{-1}.m^3)	k_0	α_0	μ_0
37	1.0×10^{-4}	4 ($G_f = 56$ J.m^{-2})	4.5	6.5	2.82

Table 1: Calibrated values of the continuum model's parameters for the local validation.

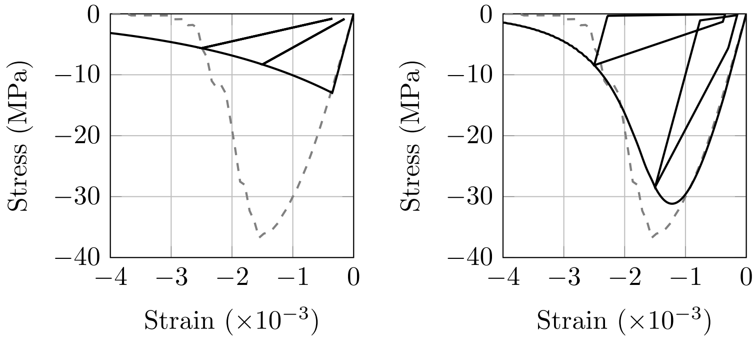
The uniaxial reverse cyclic test response obtained with the macroscopic model is fairly close to the reference response obtained with the virtual testing machine. First, the addition of the non-linear model of cracks behavior to a classic model, utilized to describe continuum media's behavior, allows to reproduce the progressive stiffness recovery as well as residual strains which disappear in compression (see figure 4a). Second, the addition of the plastic model of cracks behavior, enables the emergence of the hysteretic behavior, crescent-shaped hysteresis loops are observed at accurate stress levels (see figure 4b). Thus, dissipative mechanisms are activated for appropriate loading amplitudes.



(a) with elastic cracks behavior (b) with elasto-plastic cracks behavior

Figure 4: Uniaxial reverse cyclic response of the continuum model.

A uniaxial cyclic compression test is simulated. Friction between loading supports and the sample influences highly the inelastic compression response. Therefore, so as to ease the comparison, for both models, no friction in between the sample and the loading supports has been considered. The overall compression response, peak-value and softening behavior, obtained with the continuum model is similar to the response obtained with the virtual testing machine. Such results are interesting, knowing that a single scalar damage variable is introduced in the continuum model. It is also quite interesting to note the contribution of the cracks behavior to the compression response. Hysteresis loops can be observed as well as in tension. But above all, cracks behavior increases the value of the compression resistance, due to the formulation of the function ϑ, which depends on the first invariant of the cracks strain tensor.



(a) without cracks behavior (b) with cracks behavior

Figure 5: Uniaxial compression cyclic response of the continuum model.

5 STRUCTURAL CASE STUDY

The structural validation is undertaken simulating the response of a reinforced concrete wall submitted to horizontal shear forces. This structure has been tested in the ConCrack benchmark as part of the CEOS.fr project. The simulation of this case study allows the testing of two aspects of the continuum model: (i) its numerical robustness, because the setup implies local shear loading, multiple cracking locations, structural size dimensions, besides few accurate numerical results are available according to conclusions drawn from the ConCrack benchmark and (ii) its representativeness with respect to phenomena observed under cyclic loading, because alternate cyclic loading is simulated. The concrete structure is modeled using eight-nodes cubic finite elements. Top and bottom massive parts of the wall are considered to remain elastic (in green, see figure 6). The central thinner part of the wall is modeled with the proposed model (in gray, see figure 6). Inelastic parameters are entirely calibrated from results provided by the virtual testing machine, with the exception of the elastic strain limit, along with elastic parameters, which were given in the benchmark's report. Calibrated parameters value are given in the table 2. A regularization of the fracture energy depending on elements size is implemented.

E (GPa)	ϵ_0	B_0 (kJ^{-1}.m^3)	k_0	α_0	μ_0
22	1.5×10^{-4}	4	4.5	6.5	2.82

Table 2: Calibrated values of the continuum model's parameters for the member-scale simulations.

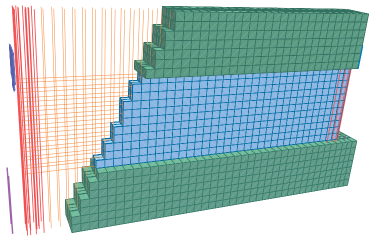
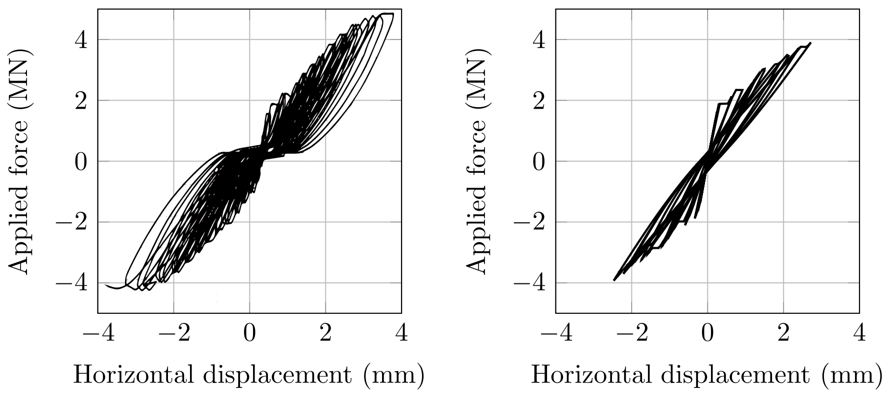


Figure 6: Mesh of the shear-wall (with $h^c = 0.1$ m).

The global response obtained with the proposed model is satisfactory on the two aforementioned aspects. The complete response to the prescribed load has been successfully simulated (see figure 7b), which confirms the inherent numerical robustness of the continuum model. The achieved robustness is mostly explained by the continuity of the established constitutive laws, to which can be added to a fully explicitly local integrated continuum model with the exception of plasticity. Enhanced representativeness is also noticed. Regarding the fracture mechanisms, simulated results show that the global response is well estimated, either the elastic limit or the stiffness of the cracked structure, are accurately evaluated. Regarding the cyclic effects, the global response is symmetrical, indeed stiffness recovery is accurately observed when the horizontal loading direction is inverted and peak loads are identical on both loading directions. This observation implies that the crack closure mechanism is accurately accounted for and reproduced even for member-scale simulations. Because of the isotropic damage description, if stiffness recovery had not been accurately reproduced, a symmetrical global response could not have been observed. This consists in a first validation of the chosen ϑ formulation. The global response presents hysteresis loops, yet the steel-concrete interface is non-degradable and steel rebars have remained elastic during the whole simulation. Consequently, the observed hysteresis loops are induced by the continuum model and confirms that frictional sliding is reproduced in member-scale simulations.



(a) experiment

(b) simulation

Figure 7: Global response of the shear-wall to alternate loading.

6 CONCLUDING REMARKS

The purpose of the present paper was to propose a continuum model for quasi-brittle materials able to reproduce phenomena observed under cyclic loading, while remaining sufficiently robust to simulate the behavior of massive structures. The formulation of macroscopic constitutive laws has been established using a virtual testing approach. A model framework, quasi-brittle materials submitted to cyclic loading has been proposed on the basis of the model proposed in [2]. The continuity of the original model has been improved introducing the distribution function of a statistical Gaussian process to regularize the homogenized contact problem of closing cracks. Such smoothing function introduction has been physically justified by the analysis of micro-cracks opening and closure with the microscopic model. An homogenized cracks stress tensor, on which a perfect plastic model has been applied, to reproduce phenomena related to frictional sliding of cracks surfaces. Such modeling choices led to satisfying results, namely crescent-shaped and accurately positioned hysteresis loops. Quality of the resulting and identified model has been verified at the RVE scale and validated at the member scale by the simulation shear wall under alternate loading. The simulation of the complete loading path has been achieved. This simulation served as a validation of the numerical robustness of the proposed continuum model. The accurate consideration of cracks closure and friction has also been observed at the member scale which served as a validation of the cyclic effects modeling.

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