Design and construction of optical superlattices for a quantum gas experiment

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Abstract

In recent years ultracold atomic gases have emerged as a new and promising platform for exploring the physics of quantum many-body systems, allowing researchers to revisit fundamental problems of condensed-matter physics such as superconductivity or magnetism. By trapping ultracold atoms in the periodic potential created by interfering laser beams (a so-called optical lattice), it has indeed become possible to engineer very clean “materials” whose parameters can be tuned in a wide range. This bottom-up approach comes very close to Feynman’s idea of a “quantum simulator”, and should give a new access to long standing open problems such as frustrated quantum magnetism or high-temperature superconductivity. Furthermore, these systems also allow for the realization of completely novel situations without counterpart in the solid-state context.

The goal of this project is to design and construct an optical lattice setup for a quantum gas experiment using a superlattice approach. In this configuration several periodic potentials are overlaid, leading to a crystalline structure of (dynamically) adjustable geometry. The project has a theoretical and an experimental part. In a first part, several possible beam configurations (depending on the geometrical arrangement of the beams, their exact frequencies and their trapping or anti-trapping character) have been analyzed and the corresponding band structures calculated. In a second part, a test laser setup has been designed and constructed, and the required stabilization systems (intensity and phase) have been developed.
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Introduction

During the last decade ultracold quantum gases have become important tools in the simulation and study of fundamental phenomena in quantum many-body physics. At ultracold temperatures, atoms reach a state known as quantum degeneracy where quantum effects become essential. These systems are easily described and highly controllable. In particular, the interactions among atoms can be tuned and the potential landscape they see can be adjusted.

Optical lattices offer a way of creating periodic potential landscapes in an ultracold quantum gases experiment. An optical lattice is formed by the interference of laser beams. This interference generates a spatial periodic potential capable of trapping the atoms. In this way, the optical lattice forms a potential similar to a crystalline structure with the atoms trapped in it acting as the electrons in a solid. Somehow, it is like creating “artificial solids” that can be easily controlled and studied.

This technique has become an useful way for simulating and understanding the complex interactions of electrons and the underlying crystalline structure. The Hamiltonian governing the “artificial solid” can be easily derived and its parameters (e.g. interaction energy, kinetic energy, lattice geometry, etc.) can be tuned almost freely. This goes back to the idea of R. P. Feynman of studying a quantum system by choosing a physically different, but fully controllable system with the same properties [1], an approach known as quantum simulation.

In order to build this kind of “quantum simulator”, it is necessary to have atoms in a quantum degenerate state and this can only be achieved when their temperature is in the range of nanokelvins, close to the absolute zero.

Reaching this low temperatures is the largest obstacle in the way towards obtaining degenerate quantum gases. Only with the invention of laser cooling and evaporative cooling techniques this could be achieved. In fact, it was not until 1995 when the first quantum degenerate gas of bosons (Bose-Einstein condensate) was produced [2, 3]. The Nobel Prize in physics was awarded to E. A. Cornell, W. Ketterle and C. E. Wieman for “the achievement of Bose-Einstein condensation in dilute gases of alkali
atoms, and for early fundamental studies of the properties of the condensates\textsuperscript{[1]}. Later in 1999 the same was achieved for a gas of fermions \textsuperscript{[2]}. This experimental progress made possible the use of optical lattices to begin developing quantum simulators.

The objective of this project is the design and construction of an optical lattice setup for a quantum gas experiment which is being developed at ICFO by the Ultracold Quantum Gases group.

The purpose of the Ultracold Quantum Gases group at ICFO is to obtain quantum degenerate gases (Bose Einstein condensates and degenerate Fermi gases) and use them to study a broad range of physics problems. In particular, one of the objectives of the group is to study quantum magnetism and quantum phase transitions. To this end, the experiment requires tunable optical lattices.

The laser beams can be tuned to obtain a wide range of different lattice structures. Depending on the geometrical arrangement of the beams, their wavelengths and their trapping or anti-trapping character, the potential landscape will vary. Some of these arrangements have an special interest in terms of physics research. In this project, methods for generating some of these configurations have been analyzed.

In order to generate the optical lattice, some of the parameters of the laser beams (intensity and phase) need to be controlled. An optical lattice test setup has been constructed during the project, to develop the required stabilization systems.

In this project report, we will first give an overview of the experimental setup developed for producing quantum degenerate gases at the Ultracold Quantum Gases group at ICFO (Chapter\textsuperscript{1}). Next we will go through the theory that lays behind the optical lattice potentials and we will design several interesting potential landscapes, analyze them and obtain their band structure (Chapter\textsuperscript{2}). Finally, we will talk about the experimental development of the optical lattice test setup and the required stabilization systems (Chapter\textsuperscript{3}).

\textsuperscript{1}http://www.nobelprize.org/nobel_prizes/physics/laureates/2001/
Chapter 1

Overview of the quantum gas experimental setup

In this section we will overview the fundamentals of quantum degenerate gases and we will give some details of the quantum gases experiment at ICFO, in which this project is comprised.

1.1 Quantum degenerate gas

Particles can be classified either as bosons, if they have an integer spin, or fermions, if they have a half-integer spin. Depending on this, they follow one or another energy distribution statistics: Bose-Einstein or Fermi-Dirac statistics. Bosons differentiate from fermions mainly in the fact that multiple indistinguishable bosons can occupy the same quantum energy state whereas identical fermions cannot.

In 1924, S. N. Bose and A. Einstein theoretically described that bosons should undergo a phase transition at low temperatures even if there is no or negligible interaction between them [5, 6]. This phase transition would not rely on an interaction between the particles but occur only due to quantum statistical effects relying on the indistinguishable nature of bosons.

Figure 1.1: Bose-Einstein condensation appears when atoms are cooled below a critical temperature and a large fraction of atoms occupies the lowest quantum state.
Bose-Einstein condensation (BEC) appears when a dilute gas of bosons is cooled down below a critical temperature, close to the absolute zero. In this regime, as depicted in Figure 1.1, a large fraction of particles occupy the lowest quantum state of the system.

Fermions cannot undergo this phase transition, due to the Pauli exclusion principle. Nevertheless, they can still be cooled until quantum statistical effects dominate. In this situation, they occupy successively each low-energy state of the system (one identical fermion per state), forming a "Fermi sea" or degenerate Fermi gas (DFG), which has some unique properties too.

Bose-Einstein condensation has been described as a new state of matter. This state allows observing quantum mechanics effects and wave-particle duality on a macroscopic scale.

From a theoretical point of view, according to the de Broglie hypothesis of matter waves, atoms at temperature $T$ and with mass $m$ can be described as quantum-mechanical wave packets with a wavelength $\lambda_{DB}$ of:

$$\lambda_{DB} = \frac{h}{\sqrt{2\pi mk_B T}}$$

As stated in the previous formula, the value of $\lambda_{DB}$ increases with decreasing temperature. Bose-Einstein condensation is reached when the atoms are cooled down to the point where $\lambda_{DB}$ becomes comparable to the interatomic distance $d$ (see Figure 1.2). In this state, the neighbour matter waves overlap and form a macroscopic quantum system of indistinguishable particles.

The critical temperature $T_{\text{crit}}$ depends on the density of the atomic cloud, which is different from one experiment to another. Then, it is more convenient to discuss in terms of the phase space density (PSD) of a sample. If $n$ is the spatial density of atoms, then the phase space density is defined as the dimensionless quantity $\text{PSD}$:

$$\text{PSD} = n\lambda_{DB}^3$$
For an ideal gas, Bose-Einstein condensation occurs when the phase space density is greater than 2.612. This condition is achieved by lowering the temperature of the atomic gas while simultaneously increasing its spatial density.

1.2 Experimental procedure

Quantum degeneracy can be experimentally achieved by confining an ensemble of atoms in magnetic or optical traps and performing a combination of laser cooling followed by evaporative cooling until quantum degeneracy is reached at temperature in the scale of nanokelvins. Reaching this temperature requires a series of sophisticated cooling procedures.

The first step consists in laser cooling the gas using light at frequencies close to atomic resonances. There are several procedures engaged in this technique but the main one is Doppler cooling, which requires light with a frequency slightly lower than that of an electronic transition. As the name states, the cooling is done thanks to Doppler effect: the atoms moving towards the light source will absorb more photons than the rest as they see the light with a higher frequency. When the photon is absorbed the atom will lose momentum to ensure momentum conservation. The atom will later reemit the photon in a random direction by spontaneous emission, therefore on average the atom will have lost kinetic energy.

With the laser cooling techniques, a minimum temperature of microkelvins can be reached. However, this is still far from the desired nanokelvin temperature. To reach this lower temperature another technique comes in: evaporative cooling. Using this technique requires to cool the atoms in a conservative trap, where they do not scatter photons anymore. Due to the interaction of the atom with an external magnetic field, its energy will vary depending on the magnetic field. Using an inhomogeneous
magnetic field with a local minimum, one can create a potential well where the atoms will be trapped. Nevertheless, only the atoms with a negative magnetic moment (low field seekers) will be trapped. The rest (high field seekers) will tend to go to high fields and so to leave the trap. Therefore, before trapping the atoms, they must be driven to states where their magnetic moment is negative.

Once the atoms are trapped in the right state, the cooling will be done by getting rid of the atoms with higher kinetic energies. In order to do this, a radiofrequency signal must be applied. This signal will flip the spin of certain atoms making them high field seekers and thus making them leave the trap. In order to only make the high energy atoms leave, the external electromagnetic signal must be set to a frequency so that the flip is done in points of the trap with high field, which only atoms with higher energy can access. By rethermalization the remaining atoms will become cooler.

Finally, the atoms are trapped in an optical trap where by further evaporation, the quantum degeneracy regime is attained. Then, next step towards obtaining a quantum simulator comes with optical lattices. That is, the quantum gas has to be subjected to an optical lattice potential. This is done by shining an arrangement of laser beams through the atomic cloud.

The read out of the system is performed via absorption imaging. A laser beam resonant with the atomic transition is sent through the cloud. The atoms absorb light and the shadow they cast on the beam is imaged on a camera. Processing the captured image allows to obtain information about the atomic cloud such as number of atoms, temperature, particle velocity distribution and cloud size.

1.3 Experimental apparatus

In the Ultracold Quantum Gases group at ICFO, an apparatus for obtaining quantum degenerate gases (either BEC and DFG) has been built. The objective is to use quantum gases to study a broad range of physics problems, mainly related to solid state physics.

The atomic species used in this experiment is potassium. The two main reasons for this choice are: that this element has two available isotopes, $^{39}$K and $^{41}$K, which are bosons and another one, $^{40}$K, which is a fermion; and that interatomic interactions can be controlled at accessible magnetic fields for Bose gases, Fermi gases and Fermi-Bose mixtures using so-called Feshbach resonances\footnote{Feshbach resonances allow a tuning of the scattering properties of a degenerate atomic gas to arbitrary repulsive or attractive values. This enables the creation of strongly interacting atomic quantum gases.}.

\textsuperscript{\textdagger}
The heart of the experiment apparatus is the vacuum setup. It consists of two stainless steel chambers as depicted in Figure 1.4. The two chambers are connected via a differential pumping tube (4) which allows having ultra high vacuum in the “science chamber” (6) while maintaining a high atomic pressure on the 2D MOT chamber (3).

The “science chamber” provides multiple optical accesses, through which the laser beams can reach the atomic cloud inside from different angles (for example, for creating an optical lattice potential or for obtaining images of the atomic cloud).

Another fundamental part of the experiment is the laser system and the magnetic coils. The laser system generates all the laser beams with the appropriate wavelengths and power levels needed to manipulate the atomic gas. The coil system is in charge of generating the magnetic fields capable of trapping the atoms as well as the Feschbah resonances capable of tuning the atomic interactions.

The quantum degenerate gas is obtained by following a very precise sequence of operations involving a large amount of devices that need to be precisely and synchronously controlled. All this process is managed using a complex computer control software and multiple controllers.

After the quantum degenerate gas is obtained, several cameras are used to image the atomic cloud. The captured images are processed using a specific software capable of extracting the characteristic parameters of the atomic cloud.
My main participation in the experiment has consisted in the development of a test setup for generating optical lattices, which was the objective of this project. Nevertheless, besides this contribution, I have also participated in the development of some other parts of the experiment.

I was in charge of setting and testing the signal generator used in the evaporative cooling process. We use a high-performance arbitrary waveform generator manufactured by Signadyne, an ICFO spin-off. This kind of signal generator fulfills the specific requirements of the experiment. In order to control it and synchronize it with the rest of the experiment modules, I integrated it in the experiment control software.

Furthermore, I also tested and integrated a high-speed low-noise camera in the experiment. The development of the imaging system is described in detail in [9].

### 1.4 Achieving a Bose-Einstein condensate at ICFO

Following the experimental procedure and using the apparatus described previously, in June 2015 the Ultracold Quantum Gases group obtained for the first time a Bose-Einstein condensate of $^{41}$K.

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2Signadyne SD AWG-H3344
3Andor iXon Ultra 897
Figure 1.6 shows the absorption images of four expanding atomic clouds obtained sequentially. The images are taken after releasing the cloud from the optical trap and letting it expand for a few milliseconds. This gives access to the momentum distribution of the cloud, which is very different for a thermal gas when compared to a Bose-Einstein condensate (Gaussian distribution vs. inverted parabola).

The images show the emergence of a Bose-Einstein condensate from the thermal cloud. The first image shows only thermal atoms. In the second and third image, with further cooling of the atomic cloud, one can see the coexistence of a thermal cloud and BEC. Finally, the fourth image shows a pure BEC.
Chapter 2

Optical lattices: basic theoretical concepts and design of lattice structures

2.1 Basic theoretical concepts of optical lattices

In this section we will go over the theoretical concepts behind optical lattices and their application to ultracold quantum gases.

2.1.1 Crystalline structures and optical lattices

Crystalline structures can be described as periodic arrangements of potential wells. Studying these structures has become fundamental in solid state physics. However, accessing and tuning the Hamiltonian of a real solid is something almost impossible. Optical lattices offer an opportunity to obtain arrays of trapped atoms that can simulate solid state systems. These “simulated systems” can be easily described and their parameters can be tuned almost freely.

An optical lattice is formed by the interference of laser beams propagating with different phases. This interference generates a spatially distributed periodic potential capable of trapping atoms that are at extremely low temperatures. Tuning the interferences between lasers can yield to lattices of trapped atoms similar to crystal lattices.

2.1.2 Optical dipole force

From a physical point of view, optical lattices are possible thanks to the fact that when an atom is placed in a spatially inhomogeneous light field it experiences a force.
For a basic understanding of the interaction between atoms and light fields, we consider a simplified system, the so-called two-level atom. It describes accurately enough the complex multi-level systems in real atoms if one considers a specific atomic transition between a ground state $|g\rangle$ and an excited state $|e\rangle$ with the differential energy:

$$\Delta E_{eg} = \hbar(\omega_e - \omega_g) = \hbar \omega_{eg}$$

and the incident light frequency $\omega_L$ is close to the atomic transition resonance $\omega_{eg}$.

![Figure 2.1: Two-level atom model.](image)

An important parameter is the laser detuning $\Delta = \omega_L - \omega_{eg}$, where $\omega_L$ is the frequency of the light field. A light field of frequency $\omega_L$ induces an oscillating dipole moment $\vec{p} = \alpha(\omega_L)$, being $\alpha(\omega)$ the complex polarizability and $\vec{E}$ the electric field vector of the light field. This oscillating dipole moment interacts with the light field and averaged over time, gives rise to a conservative dipole potential:

$$V_{dip} = -\frac{1}{2} \langle \vec{p} \cdot \vec{E} \rangle = -\frac{1}{2}\epsilon_0 c \text{Re}(\alpha) I$$

with $I = \frac{1}{2}\epsilon_0 c |\vec{E}|^2$ being the laser intensity.

The complex polarizability is connected to the spontaneous scattering rate by:

$$\Gamma_{sc} = -\frac{1}{\hbar \epsilon_0 c} \text{Im}(\alpha) I$$

In a two-level atom, the on-resonance damping $\Gamma$ of the oscillating dipole is equal to the spontaneous decay rate. It is given by the electric dipole matrix element between the excited state $|e\rangle$ and the ground state $|g\rangle$:

$$\Gamma = \frac{\omega_0^3}{3\pi\epsilon_0 \hbar c^2} \langle e|D|g \rangle$$

with $D = -e\vec{r}$ being the dipole operator.
According to [10], in the limit where the laser light is not too far-detuned from resonance and its intensity is low enough for saturation to be negligible, the dipole potential can be expressed by:

\[
V_{\text{dip}}(r) \approx \frac{3\pi c^2}{2\omega_0^2} \frac{\Gamma}{\Delta} I(r)
\]

Therefore, from this formula we can infer that attractive dipole potentials can be created with a red-detuned light field (\(\Delta < 0\)) and repulsive ones with blue detuning (\(\Delta > 0\)).

Similarly, the spontaneous scattering rate can be expressed as:

\[
\Gamma_{\text{sc}}(r) \approx \frac{3\pi c^2}{2\hbar \omega_0^2} \left( \frac{\Gamma}{\Delta} \right)^2 I(r)
\]

This implies that in order to have as little inelastic scattering processes as possible, larger detunings should be used. Nevertheless, for a fixed intensity there is a trade-off because the depth of the potential decreases linearly with increasing detuning [11].

### 2.1.3 Optical lattice potentials

The intensity dependence of optical dipole potentials makes possible the creation of arrays of spatially periodic potential wells. The simplest example of optical lattice consists of two counter-propagating laser beams of Gaussian spatial profile with equal wavelengths \(\lambda\) and linear polarizations. This can be implemented with a laser beam passing through the atomic cloud and then being retro-reflected with a mirror. The electric field distribution seen by the atoms then reads:

\[
\vec{E}(r, z) = \vec{e}_z \left( E_0 e^{-\frac{r^2}{w_0^2}} \left( e^{i(kz+\phi_1)} + e^{i(-kz+\phi_1+\phi_2)} \right) e^{i\omega_L t} \right)
\]

The radial envelope is given by the cylindrical symmetric Gaussian beam profile with a waist \((1/e^2\) radius\) of \(w_0\), \(k = 2\pi/\lambda\) is the wave vector of the laser light, \(\phi_1 - \phi_2\) is the phase difference between the back and forth beams and \(\vec{e}_z\) is the directions of the beam’s polarization.

The dynamical time scale of the atoms in an optical lattice is of the order of kHz. In exchange, the time dependent component of the electric field is in the magnitude of THz. Therefore, the atoms are not able to follow the oscillating potential created by the time dependent component of the electric field. So, when deriving the effective potential experienced by the atoms, the time average of it needs to be considered.
Since the atomic potential is proportional to the squared modulus of the electrical field, the following standing wave interference pattern is obtained:

\[ V(r, z) \propto V_0 e^{-\frac{2r^2}{w_0}} \cos^2(kz) \]

with \( V_0 \) being four times the dipole potential in the intensity maximum \( (V_0 = 4V_{\text{dip}}) \), as the two beams interfere constructively and the electric field vectors are added coherently.

Note that the standing wave also produces a potential due to the radial intensity profile of the beam. This breaks the translational lattice symmetry and therefore alters the situation with respect to a real solid, where the energy on every lattice site is equal everywhere and only falls off abruptly at the edge of the sample. However, for the sake of simplification the system can be considered to be locally homogeneous and the radial profile can be neglected [12]:

\[ V(r, z) \propto V_0 \cos^2(kz) \]

For a red-detuned lattice, the intensity maxima correspond to potential minima, which form a linear array of traps. When an additional orthogonal standing wave is superimposed, the resulting intensity pattern becomes a planar array of tubes with simple square periodicity. Adding another orthogonal standing wave, the result resembles a simple cubic crystal structure [11].

\[ V(x, y, z) = V_{0,x} \cos^2(kx) + V_{0,y} \cos^2(ky) + V_{0,z} \cos^2(kz) \]

Figure 2.2: Lattice structures in 1D, 2D and 3D produced by orthogonal optical standing waves. Adapted from [11].

The geometry of the lattice can be changed in different manners by modifying the respective intensities of the standing waves, the polarization and relative phase.
of the beams and/or the intersection angles. Later in this chapter we will discuss how to generate different lattice geometries (e.g. triangular lattice) that may result interesting for a quantum simulation experiment.

2.1.4 Bravais lattice and reciprocal lattice

A fundamental concept in the description of any crystalline solid is the Bravais lattice, which specifies the periodic array in which the repeated units of the crystal are arranged. It consists of all points with position vectors $\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$, being $n_i$ all integers, and $\mathbf{a}_i$, the primitive vectors that generate the lattice. The Bravais lattice defines the real space crystalline structure of a solid.

When analyzing crystalline lattices, it is also convenient to work in terms of the reciprocal lattice. Considering the points of the Bravais lattice $\mathbf{R}$ and a plane wave $e^{i\mathbf{k} \cdot \mathbf{r}}$, the set of all wave vectors $\mathbf{K}$ that yield plane waves with the periodicity of the Bravais lattice is known as its reciprocal lattice. Then, any wave vector $\mathbf{k}$ can be written as:

$$\mathbf{k} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2 + k_3 \mathbf{b}_3,$$

where $\mathbf{b}_i$ are vectors that satisfy $\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi \delta_{ij}$ [13].

An optical lattice, formed by a set of beams with their respective wave vectors $\mathbf{k}_i$, will generate a reciprocal lattice by the possible momentum transfers between the lattice beams, as a trapped atom can absorb a photon from the light mode of one beam and re-emit it into another. Therefore, the reciprocal lattice vectors $\mathbf{b}_i$ can be defined as:

$$\mathbf{b}_i = \epsilon_{ijk} (\mathbf{k}_j - \mathbf{k}_i)$$

where $\epsilon_{ijk}$ is the Levi-Civita symbol.

2.1.5 Band structure

As the ions in a crystalline solid are arranged in a regular periodic array, we need to consider the problem of an electron in a potential $V(\mathbf{r})$ with the periodicity of the underlying lattice. That is:

$$V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R})$$

for all lattice vectors $\mathbf{R}$.

Let’s study then the Hamiltonian governing an electron in this periodic potential and the properties that arise from Schrödinger’s equation:

$$H \Psi = \left( -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \Psi = \varepsilon \Psi$$
According to Bloch’s theorem\textsuperscript{1}, the eigenstates $\Psi$ of this Hamiltonian with a periodic potential can be chosen to have the form of a plane wave times a function with the periodicity of the lattice ($u_{n,k}(\vec{r}) = u_{n,k}(\vec{r} + \vec{R})$):

$$\Psi_{n,k}(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_{n,k}(\vec{r})$$

Therefore, this implies that:

$$\Psi_{n,k}(\vec{r} + \vec{R}) = e^{i\vec{k} \cdot \vec{R}} \Psi_{n,k}(\vec{r})$$

This concept can be equally applied to the ultracold atoms trapped in an optical lattice. Therefore, the results of this equation will also determine the energy levels of the system formed by the ultracold atoms and the optical lattice.

In order to solve this equation, let’s expand the wave functions as a Fourier series\textsuperscript{2} in terms of the wave vectors $\vec{q} = \vec{k} - \vec{K}$:

$$\Psi_n(\vec{r}) = \sum_{\vec{q}} c_{n,\vec{q}} e^{i\vec{q} \cdot \vec{r}}$$

As the potential $V(\vec{r})$ is periodic, it can be also expanded in a Fourier series only with the plane waves with the periodicity of the lattice and therefore with wave vectors that are vectors $\vec{K}$ of the reciprocal lattice:

$$V(\vec{r}) = \sum_{\vec{K}} V_{\vec{K}} e^{i\vec{K} \cdot \vec{r}}$$

Placing both of these expansions in Schrödinger’s equation, we obtain the following equation:

$$\sum_{\vec{q}} \frac{\hbar^2}{2m} \vec{q}^2 c_{n,\vec{q}} e^{i\vec{q} \cdot \vec{r}} + \left(\sum_{\vec{K}} V_{\vec{K}} e^{i\vec{K} \cdot \vec{r}}\right) \left(\sum_{\vec{q}} c_{n,\vec{q}} e^{i\vec{q} \cdot \vec{r}}\right) = \varepsilon_{n,\vec{q}} \left(\sum_{\vec{q}} c_{n,\vec{q}} e^{i\vec{q} \cdot \vec{r}}\right)$$

And simplifying:

$$\sum_{\vec{q}} e^{i\vec{q} \cdot \vec{r}} \left\{ \left(\frac{\hbar^2}{2m} \vec{q}^2 - \varepsilon_{n,\vec{q}}\right) c_{n,\vec{q}} + \sum_{\vec{K}'} V_{\vec{K}'} c_{n,\vec{q} - \vec{K}} \right\} = 0$$

All plane waves $e^{i\vec{q} \cdot \vec{r}}$ are orthogonal, so that means that the term $\left(\frac{\hbar^2}{2m} \vec{q}^2 - \varepsilon_{n,\vec{q}}\right) c_{n,\vec{q}} + \sum_{\vec{K}'} V_{\vec{K}'} c_{n,\vec{q} - \vec{K}}$ must be 0 for all possible wave vectors $\vec{q}$. Solving this equation, we

\textsuperscript{1}See Chapter 8 in [13] for more details and a demonstration of Bloch’s theorem.

\textsuperscript{2}This is always possible if the Born-von Karman boundary conditions are obeyed, which happens to be true in a lattice like the ones we are studying.
obtain the energies $\varepsilon_{n,\vec{q}}$, which define the system’s band structure. Intuitively, the 
band structure describes the ranges of energy that a particle within the lattice can 
have (called energy bands) and ranges of energy that it cannot have (called forbidden 
bands). The plane wave coefficients $c_{n,\vec{q}}$ that define the wave function $\Psi_{n,\vec{q}}$ are also 
obtained by solving this equation.

As an example, we can compute the band structure of the 1D optical lattice:

$$V(r, z) \propto V_0 \cos^2(kz)$$

For this case, analytical expressions can be obtained from Schrödinger’s equation, 
but it is more convenient and easy to obtain the band structure by numerical com-
putation. Using a Matlab algorithm, we obtain the band structures in Figure 2.3 for 
different potential depths $V_0$.

![Figure 2.3: Band structure of 1D optical lattice for different lattice depths $V_0$.](image)

In order to avoid units, note that the potential depth is expressed in units of the 
atomic recoil energy $E_r = (\hbar k)^2/(2m)$. The energy values are expressed as a function
of the quasimomentum ($\hbar q$), that is, the momentum of the particles in the lattice whose admissible values reflect the periodicity of the lattice.

2.2 Designing optical lattice potentials

In this section we will design several interesting optical lattice potentials considering the electric field created by laser beams and the dipole force that they generate. We will derive the analytical expression of the potentials, study the lattice they generate and compute its band structure. In the design, we will take into account the constraints of the quantum gas experiment at ICFO.

2.2.1 Lattice constraints

Angles

The “science chamber”, in the center of which the degenerate gas is trapped, has 6 viewports around the frontal structure. This allows to send beams to the atomic cloud in three different directions ($x$, $y$ and $w$). All beams are coplanar. The $x$ and $y$ beams form an angle of 90° and $w$ runs through the bisecting line. Additionally, there are two viewports that allow to access the atomic cloud in the transversal plane ($z$ axis).

![Accessible beam directions in the science chamber. Plan view.](image)

Wavelengths

The experiment’s laser system can produce single frequency laser beams with wavelengths of 1064 nm and 532 nm. Recalling that the potentials can be either attractive or repulsive depending on the sign of the detuning $\Delta$ and considering
that the main atomic transition of potassium $^{41}$K is at 767 nm, then we can infer that 1064 nm beams will produce attractive potential wells and 532 nm will produce repulsive potentials.

Furthermore, one must note that for equal angles the spacing between lattice sites created with 1064 nm light will be twice as large as for 532 nm beams.

**Potential depths**

Potential depths of the optical lattice can be tuned by adjusting the intensity of the beams. There is a proportionality relation between the potential depth $V_0$ and intensity of the beam $I_0$ given by $V_0 = 4V_{\text{dip}} = \frac{6\pi c}{\omega_0^2} \Delta I_0$. The potential depths of the different overlapping potentials created by several interfering or non-interfering beams will affect to a great extent the lattice geometry.

**Phase**

The interference phase of each beam will determine the relative position between the potential landscapes created by each beam. In configurations where laser beams along different directions interfere with each other, it will also have an influence in the overall lattice structure.

### 2.2.2 Lattice geometries

#### 2.2.2.1 Running-wave lattices

**Triangular lattice**

This first lattice geometry will be obtained by overlapping three running wave laser beams in the $x$, $y$ and $w$ directions, all with the same red-detuned frequency and with linear polarization in the $z$ direction (see Figure 2.5). Note that, due to the fact that the lasers are red-detuned, the atoms are attracted to the intensity maxima of the laser field. The electric fields of the interfering beams (neglecting the time dependence, since it is averaged in the effective potential) read:

\[
\vec{E}_x(x, y) = E_x e^{i(kx + \phi_1x)}\hat{e}_z \\
\vec{E}_y(x, y) = E_y e^{i(ky + \phi_1y)}\hat{e}_z \\
\vec{E}_w(x, y) = E_w e^{i(k(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}) + \phi_1w)}\hat{e}_z
\]

Figure 2.5: Beam arrangement for the triangular lattice.
Therefore, the effective atomic potential is:

\[
V_{at}(x,y) = -|\vec{E}_x(x,y) + \vec{E}_y(x,y) + \vec{E}_w(x,y)|^2
= -2E_x E_y \cos(k(x-y) + \phi_{1x} - \phi_{1y})
- 2E_x E_w \cos \left( k \left( \frac{2 + \sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y \right) + \phi_{1x} - \phi_{1w} \right)
- 2E_y E_w \cos \left( k \left( \frac{\sqrt{2}}{2} x + \frac{2 + \sqrt{2}}{2} y \right) + \phi_{1y} - \phi_{1w} \right)
\]

As we will see in next chapter, all the beams will be phase locked in order to prevent displacements of the lattice due to phase noise. Therefore, we can consider that \(\phi_{1x} = \phi_{1y} = \phi_{1w}\). Taking into account this and rewriting the potential in terms of the potential depths, we obtain the following expression:

\[
V_{at}(x,y) = -\sqrt{V_x V_y} \cos(k(x-y)) - \sqrt{V_x V_w} \cos \left( k \left( \frac{2 + \sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y \right) \right)
- \sqrt{V_y V_w} \cos \left( k \left( \frac{\sqrt{2}}{2} x + \frac{2 + \sqrt{2}}{2} y \right) \right)
\]

Using a Mathematica script, we can obtain a plot of the potential. Figure 2.6 shows the triangular lattice potential obtained with this beam configuration.

![Triangular lattice potential](image)

Figure 2.6: Triangular lattice potential. Note that the atoms occupy the blue regions in the contour plot. Parameters: \(V_x = V_y = V_w = 2E_r\).

It is interesting to calculate and analyze the band structure of these lattice geometries. To do so, we take advantage of a program for Matlab\(^3\) that calculates band

\(^3\)Wannier states for optical lattices. Developed in the group of Prof. Dieter Jaksch at the University of Oxford. [https://ccpforge.cse.rl.ac.uk/gf/project/mlgws/](https://ccpforge.cse.rl.ac.uk/gf/project/mlgws/)

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structures of optical lattices, among others things. It has been adapted for our specific parameters as well as for obtaining plots of the band structures. The band structure for this lattice is plotted in Figure 2.7.

![Triangular lattice band structure](image)

Figure 2.7: Triangular lattice band structure. Parameters: $V_x = V_y = V_w = 2E_r$.

The band structures show a large energy gap between the first and second band. This is characteristic of materials with this kind of geometry, such as quartz.

**Hexagonal lattice**

With the same beam arrangement but using 532 nm laser light instead of 1064 nm (see Figure 2.8), we can obtain a hexagonal lattice. In this case, the laser light is blue-detuned and therefore, the atoms occupy the regions of zero intensity.

Hence, the atomic potential reads:

$$V_{\text{at}}(x, y) = \sqrt{V_x V_y} \cos(k(x - y))$$

$$+ \sqrt{V_x V_w} \cos \left( k \left( \frac{2 + \sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y \right) \right)$$

$$+ \sqrt{V_y V_w} \cos \left( k \left( \frac{\sqrt{2}}{2} x + \frac{2 + \sqrt{2}}{2} y \right) \right)$$

![Beam arrangement for the hexagonal lattice](image)
Figure 2.9 shows the hexagonal lattice potential obtained with this beam configuration.

![Hexagonal lattice potential](image)

Figure 2.9: Hexagonal lattice potential. Parameters: $V_x' = V_y' = V_w' = 2E_r$.

The band structure for this lattice is plotted in Figure 2.10.

![Hexagonal lattice band structure](image)

Figure 2.10: Hexagonal lattice band structure. Parameters: $V_x' = V_y' = V_w' = 2E_r$.

The symmetry of the hexagonal lattice makes both energy bands touch in a pair of points, the so-called Dirac points. In nature we can find this same band structure in materials like graphene. With this technique we can obtain “artificial solids” with this geometry and move their Dirac points at will by tuning the potential depths.
2.2.2.2 Retro-reflected superlattice

Next lattice geometry is obtained by using two retro-reflected perpendicular beams of same frequency in the $x$ and $y$ directions, which interfere with each other, and overlapping a slightly detuned beam $x'$ in the $x$ direction, which does not interfere with the other beams. All the beams used have the same polarization, linear and in the $z$ direction.

As one can see in Figure 2.11, the electric fields of the interfering beams (neglecting the time dependence) are:

$$\vec{E}_x(x, y) = E_x e^{i(kx + \phi_1x)}$$
$$\vec{E}_y(x, y) = E_y e^{i(ky + \phi_1y)}$$
$$\vec{E}_{x'}(x, y) = E_{x'} e^{i(k'x + \phi_1x')}$$

with $\phi_{1i}$ being the phase of the beam when it reaches the atomic cloud and $\phi_{2i}$ the phase picked up when it is reflected. Then, for red-detuned light the potential seen by the atoms is:

$$V_{at}(x, y) \propto -|\vec{E}_x(x, y) + \vec{E}_y(x, y) + \vec{E}_{x'}(x, y)|^2$$

The detuning of $x'$ makes the interference with the other beams negligible, as it moves rapidly and the atoms cannot follow it. We can further simplify by considering $k' = k$, because if a small detuning is chosen, the wavelength change is minimal. Hence, the effective atomic potential reads:

$$V_{at}(x, y) = -|\vec{E}_x(x, y) + \vec{E}_y(x, y)|^2 - |\vec{E}_{x'}(x, y)|^2$$

$$= -2E_x^2(1 + \cos(2kx - \phi_{2x})) - 2E_y^2(1 + \cos(2ky - \phi_{2y}))$$
$$- 8E_xE_y \cos(kx - \phi_{2x}/2) \cos(ky - \phi_{2y}/2) \cos(\phi_{1y} - \phi_{1x} + \phi_{2y} - \phi_{2x}/2)$$
$$- 2E_{x'}^2(1 + \cos(2kx - \phi_{2x'}))$$

A coordinate change can be done to simplify this expression [14], being the new coordinate system:

$$\tilde{x} = x - \frac{\phi_{2x}}{2k}$$
$$\tilde{y} = y - \frac{\phi_{2y}}{2k}$$
$$\phi = \phi_{1y} - \phi_{1x} + \frac{\phi_{2y} - \phi_{2x}}{2}$$
$$\theta = \phi_{2x} - \phi_{2x'}$$
The resulting expression is:

\[
V_{\text{nl}}(x,y) = -V_x \cos^2(k\tilde{x}) - V_y \cos^2(k\tilde{y}) - V_{x'} \cos^2(k\tilde{x} + \theta/2) - 2\sqrt{V_x V_y} \cos(k\tilde{x}) \cos(k\tilde{y}) \cos(\phi)
\]

with \(V_i\) being the potential depths for each beam.

This potential can be seen as a superposition of two lattices: a square lattice (\(V_x = 0\)) and a chequerboard lattice (\(V_{x'} = 0\) and \(\phi = 0\)). Fluctuations in the phase \(\phi\) between the \(x\) and \(y\) beams modify the checkerboard structure, which for \(\phi = \pi/2\) becomes a square structure. Thus, it is crucial to stabilize this relative phase (in the next chapter we will discuss about the stabilization system).

For \(\phi = 0\) and \(\theta = \pi\), the most interesting lattice geometries are obtained. Different geometries are possible depending on the relative power of the laser beams. As we can see in Figure 2.12, multiple structures can be created by tuning the potential depth of \(x'\).

Figure 2.12: Retro-reflected optical lattice potentials for different values of \(V_{x'}\). The rest of the parameters are set constant to: \(V_x = 0.28E_r\), \(V_y = 1.8E_r\), \(\phi = 0\) and \(\theta = \pi\).
The band structures for these lattice geometries are plotted in Figure 2.13.

(a) Brick wall lattice. \( V_x' = 3.6E_r \)

(b) Triangular lattice. \( V_x' = 0.8E_r \)

(c) Dimers. \( V_x' = 1.8E_r \)

(d) 1D chains. \( V_x' = 8E_r \)

Figure 2.13: Retro-reflected superlattice band structures. Parameters: \( V_z = 0.28E_r \), \( V_y = 1.8E_r \), \( \phi = 0 \) and \( \theta = \pi \).

As the band structures show, the triangular lattice obtained with this beam arrangement has close similarities to the one obtained with the running-wave configuration, and the brick wall lattice has also great similarities with the hexagonal lattice obtained before.
Chapter 3

Experimental implementation of an optical lattice

This section is devoted to the implementation of an optical lattice setup to produce potential landscapes like the ones described in Section 2.2. During this project the main effort has been put in experimentally building a test setup that is capable of stabilizing and adjusting the parameters of the optical lattice beams.

3.1 Optical setup

The laser light used to create the optical lattice is derived from a high power 1064 nm Nd:YAG laser.

The test setup is built in a different lab than the one where the experiment setup is built and where the laser light is produced. Therefore, a 20 meters long optical fiber is used to transport the laser light to the optical table in the other lab. The laser light is coupled into the fiber using an aspheric lens \( f = 6.24 \) mm and a pair of mirrors to properly align the laser beam into the fiber. The coupling efficiency obtained is about 80% for an input power of 35 mW. At the end of the fiber, another collimating lens with the same characteristics has been used to collimate the laser beam exiting the fiber.

The optical fiber used is polarization maintaining, it maintains the polarization of the laser beam going through the fiber. However, due to the long distances of the fiber, it is only maintained if the polarization is aligned with the fiber in the right angle. Two half-wave plates (\( \lambda/2 \)), one at the input and another at the output, have been used to adjust the right polarization of the beam passing through the fiber. This avoids polarization fluctuations in the setup that due to the use of polarization dependent optics would yield to intensity fluctuations.
3.2 Phase stabilization system

Lattice structures are influenced by the phase of the laser beams used to generate them. As seen in the previous chapter, fluctuations in phase can modify the lattice structure or displace it, producing detrimental effects to the experiment. Vibrations of the fibers and the optical components as well as fluctuations in air pressure, room temperature and humidity lead to changes in phase of up to $2\pi$ in timescales as short as milliseconds. For this reason, it is essential that the beams used in the optical lattice are actively stabilized in phase.

### 3.2.1 Basic scheme

#### 3.2.1.1 Mach-Zehnder interferometer

The basic setup used to detect phase fluctuations is based on a Mach-Zehnder interferometer scheme. As pictured in Figure 3.2, the laser beam is split into two beams using a beam sampler (BS). Part of the light is used as a reference and the rest is sent to the atomic cloud. Both beams are retro-reflected and recombined in the beam sampler. The recombination produces a interference pattern that can be detected with a photodiode (PD). The obtained signal will be directly related to the phase difference between both beams.
3.2.1.2 Heterodyne interferometer

Starting from this basic scheme, we use an heterodyne interferometer and a phase-locked loop in order to be able to actively phase stabilize the beams. The idea consists in mapping the phase noise into an RF signal, comparing this to a stable oscillator at the same frequency and feedbacking the error signal onto the laser beam. This scheme is inspired by the work in [15].

In order to feedback the error signal onto the laser beam, an acousto-optic modulator (AOM) is used. This is an optical device that uses the acousto-optic effect to diffract and shift the frequency of light using sound waves. It consists of a piezoelectric transducer attached to a piece of glass. An oscillating signal drives the transducer to vibrate, which creates sound waves in the material. This produces an oscillation of the refraction index capable of diffracting an incident laser beam passing through the glass. The diffracted beam emerges at a certain angle $\theta$ and with a frequency shift equal to the frequency of the sound wave. These frequency and angle shifts are required by the fact that energy and momentum are conserved in the process. The intensity of the diffracted beam can be tuned adjusting the intensity of the radio-frequency signal exciting the glass. See [16] for further information.

**Optical scheme**

![Optical scheme of the heterodyne interferometer.](image)

As one can see in Figure 3.3 in order to obtain the heterodyne interferometer we need to add to the basic interferometer setup an AOM in charge of producing the frequency shift and feedbacking the error signal. The reference beam $I_1$ is sent through the first arm and is retro-reflected using a mirror. It passes through the beam sampler again and hits the photodiode with a phase difference $\Delta \phi_1$. The second beam $I_2$ passes through the AOM, which produces a first order beam with a frequency shift
of $f_{AOM}$ and a not refracted beam (0th order) that is blocked. The frequency-shifted beam is sent to the atomic cloud through an optical fiber and after passing through the atomic cloud, it is retro-reflected and sent back through the optical fiber. It passes again through the AOM, having its frequency shifted $f_{AOM}$ again, and arrives to the photodiode with a frequency shift of $2f_{AOM}$ and a phase difference $\Delta \phi_2$.

**Electronics scheme**

The beat of both beams produces the following electric signal in the photodiode:

$$V_{PD} \propto |\sqrt{I_1} \exp(i(-2\pi f_0 t + \Delta \phi_1)) + \sqrt{I_2} \exp(i(-2\pi (f_0 + 2f_{AOM}) t + \Delta \phi_2))|^2$$

$$\propto I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(2\pi 2f_{AOM} t + \Delta \phi_1 - \Delta \phi_2)$$

This signal needs to be electronically treated and fed back into the AOM. As it is shown in Figure 3.4, the photodiode signal is mixed down with an RF source operating at $2f_{AOM}$ and with an amplitude $A_{osc}$. And the resulting signal is filtered with a low pass filter obtaining a DC signal that depends on the phase difference:

$$V_{DC} \propto \sqrt{I_1 I_2} A_{osc} \cos(\Delta \phi_1 - \Delta \phi_2)$$

![Figure 3.4: Electronics scheme of the heterodyne interferometer.](image)

The easiest way to maintain the phase stable is to lock it to 0, as this point only depends on the phase difference and not on the laser intensities and the RF
source amplitude. This DC signal is fed into the generator that produces the RF signal exciting the AOM. This feedback signal is used as an FM modulation of the output RF signal (centered at $f_{AOM}$). The frequency variations $\Delta f$ produced by this FM modulating signal compensate the phase changes $\Delta \phi$ due to the vibrations or fluctuations of the environment parameters. The frequency shift $\Delta f$, when integrated over time, will be equivalent to a phase shift and this will compensate the phase fluctuations, as there is a proportionality between the frequency shift and the phase difference. This feedback scheme is analogous to a phase-locked loop (PLL).

3.2.2 Experimental implementation

The simplest way to implement the phase stabilization system consists in using as a reference the beam that is reflected in the beam sampler and retro-reflecting it using a mirror located at a fixed distance from the beam sampler. This setup gives us a fixed and stable reference beam with which the interferometry is done. This was the first setup that was implemented during the project.

3.2.2.1 Optics setup

Light coming from the 1064 nm Nd:YAG laser through the optical fiber is split into two beams using a plate 90:10 beam splitter. The light beam that is reflected from the beam splitter (10% of the original laser beam) travels for a distance of approximately 10 cm and is retro-reflected back using a pair of mirrors.\(^1\) This beam is used as the reference.

The beam that is transmitted through the beam splitter (90% of the original laser beam) is directed towards an acousto-optic modulator (AOM). In the setup we use

\(^1\)Each optical element used in the setup is preceded by two mirrors that allow to control all variables of the alignment of the beam and to adjust the beam path. With two successive mirrors, one can adjust position as well as angle of one part of the setup with respect to the next part. In a beam walk, the first mirror will have a larger influence on the position of any point downstream in the beam compared to the second mirror. The second mirror can be used to adjust predominantly the angle $\theta$.\(^1\)
an AOM optimized for 1064 nm laser light and for a diffraction frequency of 80 MHz. This AOM produces a first-order diffraction with a maximum efficiency of 85% for an incident RF drive power of 1.4 W. The AOM is mounted in a custom made mount.

![AOM Scheme](image)

Figure 3.6: Acousto-optic modulator (AOM) scheme. Adapted from IntraAction ATM-801A2 AOM datasheet.

After passing the AOM, the beam is split into two beams: the 0th order, which is not shifted in frequency, and the 1st order, which has a frequency shift of $f_{\text{AOM}} = 80$ MHz. The 0th order is blocked and the 1st order is coupled using an aspheric lens ($f = 6.24$ mm) into a 2 meter long optical fiber patch cable. At the other end of the fiber, the beam is collimated back using an aspheric lens again, and with a pair of mirrors it is retro-reflected back into the fiber. In the final setup, the beam coming out from the fiber will be transmitted through one of the windows of the science chamber, it will pass through the atomic cloud and it will come out through another window, from where it will be retro-reflected.

The retro-reflected beam is transmitted again through the optical fiber and enters again the AOM, which produces in the light an additional frequency shift of $f_{\text{AOM}} = 80$ MHz (in total, $2f_{\text{AOM}} = 160$ MHz). The beam arrives to the beam splitter again and part of it is split towards a photodector, along this path it mixes with the reference beam which is split and retro-reflected initially.

Table 3.2 lists all the optical devices used in the construction of this part of the setup.

<table>
<thead>
<tr>
<th>Part</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Splitter</td>
<td>Thorlabs BSN11</td>
</tr>
<tr>
<td>AOM</td>
<td>IntraAction ATM-801A2</td>
</tr>
<tr>
<td>Aspheric lens</td>
<td>Thorlabs C110TME-1064</td>
</tr>
<tr>
<td>Fiber optics patch cable</td>
<td>Thorlabs P3-780PM-FC-2</td>
</tr>
</tbody>
</table>

Table 3.2: List of devices and their reference.
3.2.2.2 Electronics setup

A high sensitivity InGaAs PIN photodetector is used to convert the light beams into an RF signal. This component comes with a SC connectorized fiber pigtail in where the beams must be coupled. In order to couple the beams into the photodiode’s fiber, first both beams are coupled into a short fiber optic patch cable with regular FC connectors and then, using a FC/SC adapter, it is connected to the photodiode input connection.

The photodetector includes a low noise pre-amplifier too and comes in a hermetic metal package with a coplanar output. In order to power it and to obtain the output signal easily, a PCB board and a rack box were designed and manufactured. The board includes two photodetector modules and the front panel has the corresponding connectors for the input fibers and SMA connectors for the RF output signal.

Figure 3.7: SC (left) and FC (right) connectors.

Figure 3.8: Photodetector PCB schematic.

Figure 3.9: Rack box and PCB for the photodetector.
The photodetector converts the beat of the reference beam and the reflected light from the beam we want to stabilize into an RF signal. That mixing yields to two sinusoidal signals, one at \( f_2 - f_1 = f_0 + 2f_{AOM} - f_0 = 2f_{AOM} \) and another at \( f_2 + f_1 = f_0 + 2f_{AOM} + f_0 = 2f_0 + 2f_{AOM} \), being \( f_0 \) the original frequency of the beam coming from the laser. The second one is too high in frequency (THz) so it cannot be detected by the photodetector. So, at the end, the signal coming out from the photodetector has a frequency of \( 2f_{AOM} = 160 \) MHz. As stated in the previous section, the amplitude of that signal is dependent on the phase difference between both beams.

![Figure 3.10: Active phase stabilization: electronics setup.](image)

Even though the photodetector module includes a pre-amplifier, the output power is not enough. So another amplifying stage needs to be used. This amplifier increases the signal power by about 20 dB. To avoid high order harmonics that may appear in the amplifier, the resulting signal is filtered with a 190 MHz cut-off frequency low pass filter. It is interesting to monitor the spectrum of the signal to check the output frequencies. For this reason part of the signal is fed to a spectrum analyzer via a directional coupler.

In order to obtain an error signal that depends on the phase, we need to downconvert this RF signal to a DC signal that maps the phase noise. To achieve this (see Figure [3.10]), the RF signal is mixed with an RF source oscillating at \( 2f_{AOM} = 160 \) MHz. As RF source we use an USB controllable RF signal generator. The RF signal generator is referenced with a common 10 MHz source produced by a highly stable oven-controlled crystal oscillator (OCXO).²

The photodetector signal is injected through the LO port of the mixer, the RF source signal is injected through the RF port and the output signal is obtained from the IF port. The output signal has a DC component and a \( 4f_{AOM} = 320 \) MHz.

²All signal generators of the system and the spectrum analyzer are synchronized with the same 10 MHz OCXO signal, which is included in the Rohde & Schwarz SMC 100A signal generator.
component, as a result of the mixing. The signal is low-passed in order to retain only the DC signal.

An oscilloscope is used to monitor this DC output. As we can see in Figure 3.11, the signal coming out from the phase detector varies substantially over time. Any vibration in the optical table or the optical fibers produces big changes in the phase difference.

![Figure 3.11: Sample of the error signal without any feedback control. Note that the timescale is 2 seconds per division and that there are variations of over 5 V_{pp}, meaning that there are great phase changes.](image)

This resulting DC signal has a sinusoidal dependence with the phase difference of the beams. As stated in Section 3.2.1.2, our goal is to lock the error signal to 0 meaning that the phase difference is 0. To do this, the signal is fed into the signal generator that produces the RF signal injected in the AOM. The signal generator is configured to produce an RF signal with a carrier frequency of $f_{AOM} = 80$ MHz and a frequency modulation around it proportional to the feedback error signal (the gain can be adjusted to optimize the response). Therefore, this acts as phase-locked loop system.

The signal generator changes its frequency and this produces a change in the phase of the beam sent to the atomic cloud. The frequency is adjusted so the phase difference with respect to the reference beam tends to zero always. In Figure 3.12 one can see how the error signal becomes 0 when the feedback system is turned on.
Figure 3.12: Error signal before turning on the feedback control system (first half) and after (second half). Note that when the feedback control is turned on, the error signal becomes 0.

The output power of the signal generator has a magnitude of milliwatts whereas the AOM requires an input power of approximately 1 W. Hence, the RF driving signal is amplified before being injected in the AOM.

Table 3.3 lists all the devices used in the construction of this part of the setup.

<table>
<thead>
<tr>
<th>Part</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>InGaAs PIN photodetector</td>
<td>Oclaro PT10GC-J</td>
</tr>
<tr>
<td>Fiber optic patch cable</td>
<td>Thorlabs P5-780PM-FC-2</td>
</tr>
<tr>
<td>FC/SC adapter</td>
<td>Thorlabs ADAFCSC1</td>
</tr>
<tr>
<td>Amplifier (photodiode output)</td>
<td>Minicircuits ZFL-500LN+</td>
</tr>
<tr>
<td>190 MHz low pass filter</td>
<td>Minicircuits SLP-200+</td>
</tr>
<tr>
<td>Spectrum analyzer</td>
<td>Rigol DSA 815</td>
</tr>
<tr>
<td>Directional coupler</td>
<td>Minicircuits ZEDC-15-2B</td>
</tr>
<tr>
<td>Mixer</td>
<td>Minicircuits ZFM-3-S+</td>
</tr>
<tr>
<td>RF source</td>
<td>Windfreaktech MixNV</td>
</tr>
<tr>
<td>30 MHz low pass filter</td>
<td>Minicircuits SLP-30+</td>
</tr>
<tr>
<td>Oscilloscope</td>
<td>Agilent DS01702B</td>
</tr>
<tr>
<td>Signal generator</td>
<td>Rohde &amp; Schwarz SMC 100A</td>
</tr>
<tr>
<td>Amplifier (AOM input)</td>
<td>Minicircuits ZHL-1-2W-S+</td>
</tr>
</tbody>
</table>

Table 3.3: List of devices and their reference.
In Figure 3.13 one can see a scheme of the complete setup (optics and electronics).

Figure 3.13: Active phase stabilization scheme using a fixed path reference. See Appendix A.2 for a picture of the setup.

3.2.3 Characterization of the phase stability

Figure 3.14: Comparison of the frequency spectrum of the beat signal seen by the photodetector in the unlocked and locked state.

(a) Unlocked state  (b) Locked state

Figure 3.14: Comparison of the frequency spectrum of the beat signal seen by the photodetector in the unlocked and locked state.

A simple way of checking the proper functioning of the phase lock consists in
observing the frequency spectrum of the heterodyne beat signal obtained from the
photodetector using a spectrum analyzer (see Figure 3.14). The phase noise compo-
nents produce a spreading of the carrier frequency to adjacent frequencies, resulting
in noise sidebands. When the phase lock is turned on, one can see that the phase
noise around the carrier is reduced substantially.

Phase noise has a random behaviour, it depends on multiple environmental fluctua-
tions and vibrations. Therefore it becomes difficult to characterize the phase lock
taking into consideration only the phase noise produced by the environment. For
instance, someone opening the lab door produces big changes in phase noise for a
short time that are hard to characterize.

As a solution to this characterization problem, we came up with an idea: produc-
ing large phase oscillations in the beam that can be controlled easily and that are
large enough to mask any other uncontrolled phase noise. The solution consisted in
attaching a piezoelectric actuator to one of the mirrors in charge of retro-reflecting
the laser beam that goes to the atomic cloud.

Using an arbitrary waveform generator, the piezoelectric is excited periodically
and that produces a periodic oscillation of the beam phase. In Figure 3.16a, one can
see how the actuator introduces large phase noise sidebands in the frequency spec-
trum. As expected, when the phase lock is turned on, the phase noise is substantially
reduced (Figure 3.16b).

Figure 3.15: Mirror attached to a piezoelectric actuator.

<table>
<thead>
<tr>
<th>Part</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric actuator</td>
<td>Piezomechanik HPSt 150/14-10/12</td>
</tr>
<tr>
<td>Arbitrary waveform generator</td>
<td>Rigol DG 1022</td>
</tr>
</tbody>
</table>

Table 3.4: List of devices and their reference.
This controlled phase noise allows us to characterize the phase lock by measuring the power difference between the carrier and the main phase noise sideband. As an example, in the measure of Figure 3.16, the power ratio is only 0.23 dB in the unlocked state and when the phase lock is on, the ratio goes up to 53.93 dB. As we want as much spectral purity as possible, we need to optimize this phase noise to carrier ratio.

![Figure 3.16: Comparison of the frequency spectrum of the beat signal seen by the photodetector in the unlocked and locked state with the phase noise produced by the piezoelectric actuator (200 Hz).](image)

### 3.2.3.1 Frequency response

First thing that was considered during the characterization process was which phase noise bandwidth is able to compensate the phase lock. It is not the same compensating the phase noise produced by a shake of one of the fibers (high frequency) than compensating the one produced by the room temperature fluctuation over one day (low frequency).

The results of the measurement of the ratio between the phase noise sideband and the main carrier for different frequencies of the piezoelectric actuator can be seen in Figure 3.17. As expected, the phase lock is able to reduce more effectively the phase noise produced by low frequency oscillations. For frequencies up to 1 kHz, the phase noise to carrier ratio is higher than 30 dB. According to previous experience, this value seems to be large enough to not affect appreciably the atoms when they are in the optical lattice. In fact, if the optical fibers and the optical tables are well isolated from vibrations, most of the phase noise produced due to the environment fluctuations will be in that range of frequencies.
Another parameter that can be adjusted in order to optimize the phase lock is the FM gain ($S_{FM}$), that is, the frequency variation with respect to the carrier frequency that the signal generator produces for a given input error signal.

$$\Delta f = S_{FM}[\text{Hz/V}] \cdot V_{\text{error}}[\text{V}]$$

This parameter can be adjusted in the signal generator. A higher FM gain means a faster response to small variations in phase and therefore a more efficient reduction of phase noise. For this reason as the FM gain is increased, the phase noise should steadily decrease, giving a higher phase noise to carrier power ratio.

In Figure 3.18 one can see the phase noise to carrier ratio as a function of the FM gain parameter. The measurements show this reduction on phase noise as the FM gain is increased, but the plot shows also that the increasing unexpectedly ceases at a certain FM gain (around 6.5 kHz/V in this case).

This break-down occurs due to the appearance of an instability in the phase lock system. Two resonance frequencies appear at approximately $\pm 58.8$ kHz off the central carrier. The origin of this resonance is not clear but it may be related to the natural resonance of the integral control system.
3.2.3 Intensity dependance

Finally, another thing we need to take into consideration is the efficiency of the phase lock for different intensities of the stabilized beam. The intensity of the beam is regulated via the AOM RF signal power. The highest diffraction efficiency in the AOM is reached when a specific input power is injected. The efficiency as a function of the input power follows a Bessel function. Hence, for lower and higher powers the efficiency will be worse than in the optimal point. In this case, the optimal efficiency is obtained when the signal generator output power is -0.5 dBm and therefore, the
AOM input power is about 1.4 W (note that the RF signal coming out from the signal generator is amplified by a 30 dB amplifier before being injected in the AOM).

The power of the beam that is being stabilized is directly related to the diffraction efficiency and therefore to the AOM input power. In this sense, the amplitude of the signal received in the photodiode and the error signal have also a direct relation with the AOM input power. As we saw previously:

$$V_{\text{beat}} \propto \sqrt{I_1 I_2} \cos(2\pi 2f_{\text{AOM}} t + \Delta \phi_1 - \Delta \phi_2)$$

As $I_1$ (the reference beam intensity) is kept constant, any variation of $I_2$ will make the beat signal amplitude to change. In Figure 3.20 and Figure 3.21 one can see the evolution of the frequency spectrum of the beat signal for different AOM input power values.

Figure 3.20: Spectrum evolution for different AOM input power values.
This variation in the beat signal affects the phase lock performance. The measurements\(^3\) in Figure 3.22 show that for low intensities (low AOM input power), the phase noise cannot be reduced properly and hence the noise to carrier power ratio is reduced.

\(^3\)For the sake of simplicity, all power measurements are taken from the signal generator, before the 30 dB amplification stage.
As a matter of principle, if the intensity of the beam were to be kept constant there should be no problem because even for small intensity values, a good amplifying stage would compensate the low power and the stabilization could be well performed. But the experiment requirements make this impossible. The atomic cloud must be loaded into the optical lattice adiabatically. That is, the power of the lattice beams needs to be increased progressively from zero to the final value (see Figure 3.23).

![Figure 3.23: Optical lattice power evolution over time.](image)

One solution to this problem could be the implementation of a variable gain amplifier, capable of adjusting its gain to the power of the beat signal. However, this requires more complex electronics that is not always as reliable as desired. A simpler and more robust solution was developed during the project but it required a redefinition of the setup scheme, as it will be explained in the next section.

### 3.2.4 AOM 0th order reference beam

#### 3.2.4.1 Concept

As we saw in the previous section, decreasing the beam intensity reduces the heterodyne beat signal’s amplitude and at the same time that reduces the performance of the phase-locked loop system. To find a solution for this, let’s recall the beat signal function:

$$V_{\text{beat}} \propto \sqrt{I_1 I_2} \cos(2\pi f_{\text{AOM}} t + \Delta \phi_1 - \Delta \phi_2)$$

In the previous scheme we kept a fixed reference beam ($I_1$) and therefore, decreasing the stabilized beam intensity ($I_2$) would drop down the beat signal amplitude. But, what if we could compensate this variation in $I_2$ by changing the intensity of $I_2$?

As we explained previously, the AOM used in the setup produces two beams: a diffracted beam (1st order), which has the desired frequency shift and is sent to the atomic cloud, and the not-diffracted beam (0th order), which has the same frequency
as the incoming beam. In the previous setup, the 0th order beam was blocked because it was not useful, as we already had a reference beam, obtained from the beam splitter, and the 1st order beam that was sent to the atomic cloud.

But, what if instead of using the beam from the splitter, we use the 0th order beam as the reference beam? As it has been explained, the intensity of the 1st order beam depends on the power of the RF signal injected in the AOM. Lowing down this power value, makes the intensity of the 1st order beam decrease due to the fewer phonons in the AOM crystal. In exchange, the 0th order beam increases, as more light passes without being diffracted. One can see in Figure 3.24 that both beams intensity values add up to a constant value for any AOM input power.

Therefore, using the 0th order as reference will make $I_1$ to increase as $I_2$ is decreased, making the beat signal amplitude to remain at higher values for low $I_2$ intensities than in the previous setup.

### 3.2.4.2 AOM 0th order reference beam implementation

To implement this new setup, the beam splitted initially needs to be blocked and the 0th order has to be retro-reflected back to the splitter and towards the photodetector, where it will be mixed with the 1st order. In Figure 3.25 the changes in the scheme are represented.
3.2.4.3 Schemes comparison: fixed reference beam and AOM 0th order reference beam

Even though the variation of the beat signal amplitude is reduced significantly with this setup, the mean amplitude is lower than the fixed reference beam setup due to the loss of power in the beam splitter. For this reason, to compare both setups, the fixed reference beam setup has been adjusted to have a similar reference power than that of the 0th order. It should be noted that what is important is to reduce the variation of the detected beat signal in all the intensity range of the stabilized beam, and not the mean power received over all the different intensity values. As it was explained, a good amplifying stage can solve that but not the fact that the variation range is too large.

In Figure 3.26 one can see the comparison of the phase noise to carrier ratio for both setups. The ratio of the fixed reference beam setup decreases steadily as the RF power injected in the AOM is lowered down. In exchange, the 0th order setup keeps the ratio in good figures (between 30 and 50 dB) even for very low input power values.
Figure 3.26: Comparison of the phase noise to carrier ratio as a function of the AOM RF signal power (before amplification) for the fixed reference beam setup and the 0th order reference beam setup. Configuration: 6 kHz FM gain and 100 Hz piezo modulation.

This setup seems to give a good solution to the intensity variations problem. But it still has some drawbacks. It implies high optical power losses of the reference beam due to the path it follows. And besides this, that can be compensated increasing the laser power and the amplification stage, there are also some problems in the coupling of both beams into the photodetector fiber. The intensity changes of the AOM driving power create a slight change in the angles of the outcoming beams, producing at the same time a misalignment and a loss of coupling efficiency.

### 3.2.5 EOM sideband reference

#### 3.2.5.1 Concept

An electro-optic modulator (EOM) is an optical device consisting of a crystal whose refractive index varies as a function of an external electrical field. In this way, the phase of the light that passes through can be controlled with an applied electric field (electro-optic effect). The electric field is created by placing a parallel plate capacitor across the crystal. An inductor connected in parallel can make it resonate at a certain frequency when excited externally with an RF source. See [16] for further information.

The EOM can be used to create sidebands in the laser beam spectrum, that is, to add to the carrier frequency $f_0$ two sidebands at $f_0 \pm f_{EOM}$. It is important to remark that the power of this sidebands can be easily controlled by adjusting the power of
the EOM driving RF signal. This will be an asset from which we took advantage in developing a better phase stabilization scheme.

Figure 3.27: Electro-optical modulator (EOM) scheme. Adapted from [16].

As explained in previous sections, the experiment requires an optical lattice that is stable in phase for a wide range of power values. One of the main issues that have been faced in the other setups has to do with maintaining a high enough amplitude of the heterodyne beat signal used to phase lock the system for a range of different beam intensities.

An improved solution for this problem consists in using an EOM to introduce sidebands in the main beam and use one of them for the phase-locking. The amplitude of this sideband can be controlled electronically in order to keep it always with a constant amplitude and hence, this would keep the phase-lock beat signal always stable.

3.2.5.2 EOM sideband reference implementation

Optics setup

In terms of optical devices, the new scheme has the same configuration as the fixed reference beam setup but with an EOM located between the beam splitter and the AOM.

Figure 3.28: Active phase stabilization scheme using an EOM sideband as reference: optical setup.
Electronics setup

The major changes appear in the system’s electronics. To understand the new situation and the changes that need to be made in the electronics, let’s take a look at the new spectrum of the beat signal captured by the photodetector (see Figure 3.29).

![Figure 3.29: Spectrum of the beat signal with the EOM setup.](image)

We see the same frequency component at $2f_{\text{AOM}} = 160$ MHz (marker 1) as the original scheme but now we have two additional components due to the EOM. The EOM operates at a frequency of 469 MHz, so the sidebands are located at $|2f_{\text{AOM}} + f_{\text{EOM}}| = 629$ MHz (marker 4) and $|2f_{\text{AOM}} - f_{\text{EOM}}| = 309$ MHz (marker 2). The $f_{\text{EOM}} = 469$ MHz component (marker 3) is due to a leakage of the RF signal used to drive the EOM.

We want to use as reference beat signal the higher sideband ($|2f_{\text{AOM}} + f_{\text{EOM}}| = 629$ MHz), therefore, we need now an RF source oscillating at that frequency to downconvert it (see Figure 3.30). That downconverted signal is low passed in order to get rid of the other components and following the same approach as before, it is fed back into the signal generator that excites the AOM and compensates the phase fluctuations. Additionally, a RF source needs to generate the driving RF signal of the EOM, at $f_{\text{EOM}} = 469$ MHz. The RF source signal passes through a 30 dB amplification stage, as the EOM requires an input power of around 1 W.
In order to keep the amplitude of the sideband constant, we use a feedback controller system that actuates on the power of the RF signal that drives the EOM with the finality of keeping it constant. This controller takes advantage of the second sideband introduced by the EOM ($|2f_{\text{AOM}} - f_{\text{EOM}}| = 309$ MHz), it mixes it down and low passes it in order to obtain a DC signal proportional to the sideband amplitude. This signal is fed into a proportional-integral-derivative (PID) controller device that compares it to a setpoint (calculating an “error value”) and acts on the RF driving power to minimize the error.

Similarly to the phase stabilization control system, this control system requires to split part of the photodetector signal, mix it down with an RF signal oscillating at $|2f_{\text{AOM}} - f_{\text{EOM}}| = 309$ MHz and low pass it. This signal is fed into the PID (ICFO custom made) and the PID acts on a voltage variable attenuator connected between the EOM RF source output and the amplifier that amplifies the RF signal before being injected in the EOM (see Figure 3.31).
Figure 3.31: Active phase stabilization scheme using an EOM sideband as reference: EOM sidebands power control electronics setup.

This last feedback control system was implemented but by the time this report was written, it was not fully operational yet.

The additional devices used in this setup are listed in Table 3.5.

<table>
<thead>
<tr>
<th>Part</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOM</td>
<td>QuBig EO-K39M3</td>
</tr>
<tr>
<td>EOM RF source</td>
<td>Windfreaktech MixNV</td>
</tr>
<tr>
<td>Voltage variable attenuator</td>
<td>Minicircuits ZX73-2500</td>
</tr>
<tr>
<td>PID</td>
<td>ICFO TE2014.373</td>
</tr>
<tr>
<td>Amplifier (EOM input)</td>
<td>Minicircuits ZHL-1-2W-S+</td>
</tr>
</tbody>
</table>

Table 3.5: List of devices and their reference.
The scheme in Figure 3.32 shows the complete setup.

Figure 3.32: Active phase stabilization scheme using an EOM sideband as reference. See Appendix A.4 for a picture of the setup.
3.2.6 Retro-reflected and running-wave configurations

In Chapter 2.2 we discussed that depending on the optical lattice structure that we wanted to obtain, we would need either retro-reflected beams or running-wave beams (see Figure 3.33). As we explained in the previous section, in order to obtain a phase locked beam, we need to have the retro-reflected light to do the beat with a reference beam. But in the case of a running-wave beam, we cannot have this light back.

In this project we developed a solution for this lack of retro-reflected light in the case of running-wave beams taking advantage of fiber connectors characteristics.

Optical fiber cables have different types of mechanical connectors and this determines the quality of lightwave transmission. Most common technology in this kind of experiments is the APC connector. The end face is curved and it is angled at an industry-standard 8°. This reduces back reflections to about −70 dB and avoids optical etaloning in the fiber (interferences between the two fiber facets).
However, another common connector in industry is the PC connector. In the PC connector, the end faces are polished flat and slightly curved. The fact that the connector is not angled makes the back reflection to be about $-40$ dB, which is still very low but reflects enough light to make the interferometer work. To avoid etaloning along the fiber, we use an APC polishing at the entrance tip of the fiber and a PC connection at the exit tip.

The problem in this case is that the phase is only stabilized up to the fiber end and not in the atoms. This requires passive stability of the optical path between the fiber exit tip and the atoms.

3.3 Intensity control and stabilization system

Besides the phase stabilization, the experiment also requires control and stabilization of the optical lattice beam intensities. Lattice intensities need to be adjusted progressively so the atomic cloud is loaded adiabatically in the optical lattice. And once the intensities reach their maximum value, they need to remain very stable during the duration of the measurements.

3.3.1 Experimental implementation

Just after exiting the fiber that leads to the atomic cloud and before the photodiode, the beam polarization needs to be cleaned, as optical fibers produce polarization fluctuations that can become intensity variations. A half-wave plate ($\lambda/2$) and a polarizing beam splitter (PBS) are adjusted to obtain only linear polarization in one direction (see Figure 3.35).

Additionally, active stabilization needs to be carried out. The intensity stabilization system was designed similarly to the EOM sideband stabilization system. The setup is implemented at the exit of the fiber that leads to the atomic cloud, as this is where the intensity needs to be stable. Using a beam sampler, some of the light is split towards an InGaAs PIN photodiode, which is in charge of monitoring the beam power. The captured signal is sent to a PID controller and compared to a computer controllable setpoint. The PID obtains an error signal from this and actuates over the power of the RF signal sent to the AOM, that as we explained in previous sections, changes the intensity of the beam being transmitted. A voltage variable attenuator connected between the signal generator and the AOM is used as the actuator.
In Figure 3.36 one can see an scheme of this intensity control and stabilization setup.

![Intensity control and stabilization scheme](image)

Figure 3.36: Intensity control and stabilization scheme. See Appendix A.5 for a picture of the setup.

With this setup, the beam power can be easily controlled by injecting a setpoint voltage in the PID from an external source (e.g. a computer controlled analog output).

### 3.3.2 Characterization

The control system was tested in order to obtain an idea of the speed of response of the system. As one can see in Figure 3.37 the system response is quite acceptable for values as high as 10 kHz.

![Response of the control system for different frequencies](image)

(a) 1 kHz  
(b) 10 kHz  
(c) 100 kHz

Figure 3.37: Response of the control system for different frequencies. In yellow, setpoint signal, and in green, response signal.
Conclusions

During my bachelor’s final project, I have worked in the Ultracold Quantum Gases group at ICFO, that is developing an experiment for generating quantum degenerate gases and using them to study condensed matter physics problems. In particular, I have been in charge of the design and development of the optical lattice system that will be used to subject the quantum degenerate gases to periodic potential landscapes, analogous to the crystalline structure of a solid.

In the first part of the project, I analyzed the lattice structures that can be created with the constraints of our experiment. I designed some interesting potential landscapes considering the electric fields created by laser beams, derived the analytical expressions of the potentials and computed the lattice band structures using a Matlab script, that was adapted to our requirements.

In the second part of the project, I designed and constructed a test setup for generating optical lattice potentials with laser beams. The two main requirements that needed to be fulfilled are phase stability as well as intensity controllability and stability.

On one hand, I built and tested three active phase stabilization setups, beginning with a simple heterodyne interferometer with an AOM and a phase-locked loop controller, and ending with a more complex stabilization system based in an EOM. In this report one can find the characterization results of each setup and the improvements introduced by each new scheme. On the other hand, I constructed and tested a intensity control and stabilization system based in a PID controller.

In June 2015, the first Bose-Einstein condensate was produced at ICFO while this project was being developed. In the near future, the results of my project will be integrated with the main experiment in order to subject the quantum degenerate gas to optical lattice potentials.

Besides my work in the optical lattices system, I also participated in the development of other parts of the experiment. Specifically, I worked with the control system and the software in charge of managing all the experiment devices. I integrated in this system a signal generator that is used to produce the RF signals for evaporative cooling as well as a camera that is used for imaging the atomic cloud.
Appendix A

Pictures of the experimental setup

In this appendix, pictures of the experimental setups are shown. Note that 1064 nm laser light is not visible, therefore, for a better understanding beams have been drawn over the pictures.

A.1 Laser setup

(a) Laser beam coupled into the 20 meters long fiber.  
(b) Nd:YAG laser.

Figure A.1: Laser setup.
A.2 Phase stabilization setup with fixed reference beam

Figure A.2: Phase stabilization setup with fixed reference beam.

Figure A.3: Optical devices in the setup.
A.3 Phase stabilization setup with AOM 0th order reference beam

Figure A.4: Phase stabilization setup with AOM 0th order reference beam.
A.4 Phase stabilization setup with EOM sideband reference setup

Figure A.5: Phase stabilization setup with EOM sideband reference.

Figure A.6: The EOM and the AOM in the setup.
A.5 Intensity control and stabilization setup

Figure A.7: Intensity control and stabilization setup.
A.6 Complete setup

Figure A.8: Complete setup in the optical table.
Bibliography


