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OPTIMIZATION OF THE LOGISTICS OF EXTRA-CURRICULAR ACTIVITIES

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ABSTRACT: *Most of the children develop extra-curricular activities in the afternoon after the school timetable. These activities are generally organized in each school, but sometimes the amount of scholars interested in one activity is very limited; the school has a few available resources or the preferences of their students do not match with the facilities. To expand the number of activities offered to the students in a school the creation of clusters that group different schools is proposed. But also a transport problem appears as the children should be moved from one school to another, where they can enjoy their preferred activities. Therefore, this can be considered a School Bus Routing Problem (SBRP). We define a procedure of several steps. First, a heuristic is implemented to obtain the clusters. Subsequently, the assignment of activities and the optimal path between cluster's members are determined. The proposed children transport is based on the availability of one or two buses per cluster. This model has been applied to the schools in Barcelona.*

KEYWORDS: *School Bus Routing, Set Covering Problem, Clusters, Transport.*

1 INTRODUCTION

A city has to cover the basic service of education, either via public or private schools, to improve and to promote social cohesion through its educational network. These last years the extracurricular activities have become a complement of the school day for many children, as they are necessary, for instance, if both parents work at the time children come out from school.

Most of the children want to develop extracurricular activities (Rosenfeld and Wise, 2002) after the usual school timetable, in the afternoons. However, the demand of extracurricular activities is often insufficient in a single school to cover the expenses of the trainer... This can be analysed from several points of view: activities in fashion require facilities not available in schools, activities that have an affordable price, lack of facilities for activities of potential interest, excess of offer in other cases, other leisure organizations or clubs offering similar activities than those organized in schools, activities that do not meet the minimum target for profit, activities with groups of different levels...

Extracurricular school activities which could potentially be done in a school often are finally not carried out for lack of a minimum number of students. Moreover, there is no coordination of activities between schools and therefore only the demand of a specific number of activities is offered in each centre. Thus, the model proposed enlarges the range of activities considering not only the ones in the school where students study, but also other

schools or neighbourhood organizations, thus forming a network of associations. With the clustering of the neighbour schools, the offer and the demand can be extended.

Once this agreement between different schools is established, the transport for the displacement of children must be provided.

A proposed clustering between different schools, that could include other organizations (sport clubs,...), is presented in this study, which continues the work of Martínez and Figueras (2011) for the public schools in Barcelona. In order to transport students, an efficient transport network must be implemented. This second side of the problem is complemented by the determination of the routes for the movement of these children between the facilities of each activity. They only consider the clustering of schools and propose a standard route for each of the clusters. Finally, the economic aspect is considered establishing a minimum amount of children per activity and in a bus route.

The definition of the problem is presented in Section 2. Section 3 shows the key elements of the model. Section 4 describes briefly the procedure. In the following sections, each of the steps is developed: Section 5 is for the clustering; Section 6 defines the assignment of the activities for each of the clusters, and in Section 7 the ability of transport flows between schools is designed. The paper concludes with the discussion of the study case, considering the public schools in Barcelona in Section 8,

some conclusions and further work are shown in Section 9.

2 DEFINITION OF THE PROBLEM

This work can be classified as a work on the school bus routing problem (SBRP). Park and Kim (2010) wrote recently a review on this problem. We are given the demand for a number of activities and the places where each of these activities can be realized. So, compared to other variants of SBRP, the destination of each child is not known in advance.

For the routing, we must take in mind that this problem is different to the conventional Vehicle Routing Problem (VRP), because “school students are not simple packages, as in the case of pick-up and delivery of goods” (Bowerman et al, 1995).

The objective is to create clusters of nodes, i.e. form subgroups of nodes. When the clusters are created, one important constraint is the maximum waiting time, which is the maximum time a boy or girl can wait in a school to be picked up by a bus and moved to another school.

The Set Covering Problem (SCP) is defined as follows: given a universe U (here it consists of the set T of all schools), we look for a family of subsets (here, school clusters) such that each student in the cluster gets access to the selected activity. This problem has been widely studied, since the work of (Schilling et al, 1993) to the more recent ones (Farahani et al, 2012).

The coverage or solution to the problem is, therefore, a subfamily $S \subseteq T$ which in binding results in T . As a first step, we plan to solve the case in which each one of the m clusters created are disjoint sets. Therefore, we will not allow that a node belongs to more than one cluster $S = \{S_1, \dots, S_m\}$ such that $S_1 \cup \dots \cup S_m = T$.

In our case, we determine the subsets of schools, such that the distance between a school and the closest school is no greater than t_{max} . This comes conditioned by the travel time of the bus between both locations. Any school is initially assimilated as a depot or starting point of a bus route. From each one, we seek coverage for other schools in the area, and so a potential subset is defined. This should be done considering each school as a depot.

Let us consider a route which starts from a school and goes to the rest of schools in the cluster. There are two possibilities: the same bus is going to the starting point following the opposite route, or another bus is necessary. As we consider all the activities will begin after t_{max} minutes the classes have finished, the maximum travel time for a student, i.e., between two of the nodes in the cluster, must respect this constraint.

If a single bus is assigned to the route formed between all nodes in the cluster, the time should be done less than $t_{max}/2$ minutes, but the time may be doubled, i.e. to t_{max} minutes, if two buses are assigned to the transport between the nodes of the cluster (one for the route in one way and the other for the reverse route).

Usually, SBRP consists of smaller sub-problems. Desrosiers et al (1981) proposed five steps for their resolution: data preparation, bus stop selection, bus route generation, school bell time adjustment and route scheduling.

The data preparation is necessary to consider the schools involved in the problem, the number of students asking for the activities, the potential activities to be carried in each school, the distances between origins and destinations and the vehicles for the transport.

For the sources of the student's demand, we consider each of the schools in an urban area. On the other hand, for the destinations of the students a selection of which activity will be done at each school is necessary. Therefore, this is a location problem, the representation of which may be done through a graph (Francis and White, 1974), where each node represents a school in which the activities can be realized.

In our case, the bus stop selection is skipped, as in most of the SBRP papers, a bus stop is given by each school, to take and/or leave the students.

The bus route generation leads to the construction of the routes. The algorithms can be classified, according to Bodin and Berman (1979) in route-first, cluster-second or cluster-first, route-second. The first option builds an initial large route, which is divided in parts or clusters. The second option groups the students into clusters and then determines the route.

Another step, the school bell time adjustment, has no sense in our case as the classes at all the schools finish at the same time, and also the extra-curricular activities start simultaneously.

Finally, the route scheduling will determine the exact starting and ending time of each route. In our problem, as the initial time for any route is the same, a single bus will be used for a single route and time 0 will be the ending time for classes.

According to the classification on the problem characteristics presented by Park and Kim (2010), our problem is classified as an urban service for multiple schools, not frequent in literature; the problem scope in our case is for the afternoons; mixed loads, as traditionally this has been considered when students form different schools share the same bus (this is one of the objectives to reduce globally costs); a heterogeneous fleet of buses as we assume that each bus (in different clusters) has dif-

ferent own characteristics, as capacity, fixed cost and per unit distance variable cost.

For the different objectives and constraints considered in the SBRP, most of the studies aim to minimize the number of buses needed and the total bus travel distance. Only few papers consider alternative objectives as how well the demand is satisfied, balancing the loads and travel times for buses, the riding time for students... Braca et (1997), Li and Fu (2002), and Park and Kim (2010) detail them.

Generally the economic objective does not appear (Park and Kim, 2010), as it is considered an obliged transport because children must go to the school. But as the extra-curricular activities are not obliged and a company should provide the service, it has to take into account the balance between the cost (fixed cost per bus and variable cost per distance plus the instructor's cost) and the price paid by the families for the activity.

Various constraints have been considered for SBRP since Braca et al (1997) and summarized in Park and Kim (2010). We consider the vehicle capacity constraint (i.e. the maximum number of students in a bus); the maximum riding time (the maximum time in a bus for children), the minimum number of children to create a route (which comes from the balance between costs and activity prices).

We adopt the usual hierarchy in the planning process: the strategic planning of clustering is first solved and we leave the scheduling issue of the activities to a later phase. This supposes that the cluster-first, route-second approach is followed.

The resolution of the general problem, which allows a clustering, does not guarantee a preferable magnitude in the number of schools per subset.

3 MODEL OF THE PROBLEM

The first element in the model is the demand and the second one, the potential offer to cover it.

At the beginning of the course, each student of all the schools is asked about his/her preferences on extra-curricular activities. We define up to p activities ($k=1, \dots, p$).

There are m schools, which together are included in the called set T of schools; we use i to denote one of the schools ($i=1, \dots, m$). Each school has a location in the city map.

We define $d_{i,k,t}$ as the demand in the school i of activity k for children in the age t ($t=1, \dots, q$), given q levels in the primary school.

On the other hand, there are n potential facilities in schools in which an activity k can be developed. They are included in a set F of facilities, and we use j to denote one of the facilities ($j=1, \dots, n$). Each one has a location in the city map.

Initially we define $a_{j,k,t}$ as the availability of the facility j to carry out the activity k for children in the age t ($t=1, \dots, q$), given q levels in the primary school. But we can suppose that the facilities usually do not depend on the age, so it can be simplified to $a_{j,k}$.

Associated to the facility, there is a location in the map and a maximum number of participants or capacity of the activity k in school j , called as $P_{j,k}$. As a first approximation to the problem, we will not consider it as a constraint.

Note that a school i , for example, may have two basketball courts and so, there will be two "schools" j .

Finally, $t_{i,j}$ is the time (in minutes) of the travel from the school i to a facility j . Nevertheless, in a first model we only consider a single facility per school; in this case the travel is between a school i and another school j can be also represented as $t_{i,j}$, being $i, j = 1, \dots, m$.

The estimated maximum time a child can wait in a school to be picked up by a bus, or a generic transport mean, to take him or her to another facility or be in his/her way to where the activity takes places, what is called "waiting time", is 15 minutes. Therefore, below $t_{\max}=15$.

This implies that the round trip between two nodes in the graph is equal to or less than 15 minutes, as this is estimated maximum time to start an activity after classes are finished. Therefore, the initial condition is set as no interrelation between nodes exceeds 7.5 minutes. We should not forget that the longest the travel time between nodes is, the least the reliability of meeting schedules is, for example due to the traffic.

In addition, the ability to have up to two buses per cluster is also taken into account, which means that the clusters with an optimal route to go from a first node to the last node and back is not greater than 30 minutes. That is, the value associated with an arc of the graph should have now a value of 15, at most.

This time constraint permits to greatly diminish the inter-relationships between nodes, for simplicity of clustering. Consequently, all those arcs connecting two nodes exceeding in the one way or the other one the t_{\max} minutes, in this case, are deleted. We recall that the traffic ways of the streets imply that not always the time to travel between node i and node j is the same as performing the reverse route (between node j and node i).

Another time constraint is related to the capacity of the buses.

The objectives will be first to minimize the total bus time of travel for all the created routes. And as the least important criteria, the number of pupils transported between schools is going to be minimized.

4 PROCEDURE

The procedure will solve a set of problems.

The first one is the clustering problem, whose objective is to divide the set of schools into clusters in order to group the closest points. At the end of this phase, and considering a single or double bus per cluster, we have an upper bound for the number of buses.

The second one, given a small number of schools, is the assignment problem. The objective is to select one of the school candidates to develop an activity, if there is any possibility. This is done for each of the activities, in each of the clusters.

Finally, once given the assignment of activities, the children's destination is known and the movement of boys and girls can be determined, as well as the number of buses necessary per cluster. If we have the fixed cost of the buses, the variable cost per distance and the established prices paid by the children for each activity, a selection of profitable activities may be carried out.

5 CLUSTERING PROCEDURE

Let G the graph consisting of m nodes, which are to be grouped into clusters. Considering the location of nodes, the constraints for the "waiting time" of students in bus stops (schools) and a study of the demand, we will only accept subsets or clusters between a minimum of n_{min} nodes and a maximum of n_{max} nodes.

The procedure should be a partition and, therefore, small subsets satisfying the conditions in the previous paragraph will be formed. First, we present the linear program and then the heuristic procedure, which guarantees good quality in the solution obtained.

5.1 Linear Program

A mathematical program finds the minimum set of arcs between nodes with no isolated node.

We define the binary variables $x_{i,j,c}$ indicating if the arc between node i and node j is chosen for cluster c ($x_{i,j,c}=1$) or not ($x_{i,j,c}=0$). These variables express the suitability of having or not any possible path between two nodes. The value 1 will be taken when the pair of nodes is directly related, i.e. a bus will go from school i to school j and vice versa.

We should add that the arcs that do not meet the constraint of maximum travel time or also "waiting time" between nodes are excluded and no corresponding binary variable is defined. Therefore:

$$\exists x_{i,j,c} \in \{0,1\} \text{ if } i \neq j \text{ and } t_{i,j} \leq t_{max} \text{ for } i,j=1,\dots,m$$

The linear program is the following:

$$[\text{MIN}] z = \sum_{i=1}^m \sum_{j=1}^m \sum_{c=1}^{\max c} t_{i,j} \cdot x_{i,j,c} \quad (1)$$

subject to

$$\sum_{j=1/i \neq j}^m \sum_{c=1}^{\max c} x_{i,j,c} \leq 2 \quad i = 1, \dots, m; \forall c \quad (2)$$

$$\sum_{j=1/i \neq j}^m \sum_{c=1}^{\max c} x_{i,j,c} \geq 1 \quad i = 1, \dots, m; \forall c \quad (3)$$

$$\sum_{i=1}^m \sum_{j=1/i \neq j}^m x_{i,j,c} \geq 2 \cdot (n_{min} - 1) \quad \forall c \quad (4)$$

$$\sum_{i=1}^m \sum_{j=1/i \neq j}^m x_{i,j,c} \leq 2 \cdot (n_{max} - 1) \quad \forall c \quad (5)$$

$$x_{i,j,c} = x_{j,i,c} \quad i, j = 1, \dots, m; \forall c \quad (6)$$

$$x_{i,j,c} \in \{0,1\}$$

The objective that will be minimized is the total time consumed in the transport in all the clusters (1).

There are three types of constraints. The first ones correspond to equation 2, which impose that the number of arcs emerging from one node to the rest is less than or equal to 2 (for the case when a node is an intermediate point of a tour), and equation (3), which impose a minimum number of one arc emerging from one node to the rest of nodes (thus, no node will be isolated and can be the edge of a tour). The second ones are expressed in equation (4) and equation (5), for the minimum and maximum number of nodes in a cluster ($n_{min}=4$ and $n_{max}=5$, respectively), which turns in an arc less. This is evaluated for the viability and impact on the creation of the clusters. We should keep in mind that the arcs that do not meet the constraint of the maximum travel time (t_{max} minutes) are excluded. The stop time at each school is neglected. Finally, the constraints of equation (6) require that if an arc from a node i to another node j is selected, we must also choose the reverse arc, emerging from j to i .

5.2 Heuristic procedure

The method used searches the clustering in the graph, starting from the location edges. The heuristic follows these rules:

- Graph resolution with the beginning in the edges of the school map, and take one of the closest schools for a cluster going towards the city center.

- The clusters can be formed by 4 or 5 nodes in the graph. Therefore, the existing chains may be broken to avoid excluding clusters with less than 4 nodes.

Once tested the inclusion of new nodes to a current cluster the result of the route is evaluated according to the objective function, and compared with other possible association between pairs of arcs which do not isolate any node.

Given the condition of size in the graph, a cluster is considered as provisional and the process continues to evaluate the suitability of alternative clusters.

6 ACTIVITY ASSIGNMENT

Let Y the set of m' nodes in a cluster c ($n_{\min} \leq m' \leq n_{\max}$) and let U' the set of arcs in the cluster that meet the constraint of maximum time for feasible routes. The graph $G(Y,U')$ associated with the problem is oriented and connected.

A mathematical program establishes in which node in the graph an activity will be done. Previously to solve the problem, if an activity can be realized in none of the schools, this is deleted from the list.

We define the binary variables $z_{i,k}$, which will take a value 1 if the activity k will be assigned in school i .

The number of children $s_{i,k}$ moved to school i for activity k is defined as:

$$s_{i,k} = \sum_{t=1}^{m'} \sum_{t=1}^q d_{i',k,t} - \sum_{t=1}^q d_{i,k,t} \quad (7)$$

Finally, there can be a maximum number act_{max} of activities per school.

The linear program is the following:

$$[MIN] z = \sum_{i=1}^{m'} \sum_{k=1}^p s_{i,k} \cdot z_{i,k} \quad (8)$$

subject to

$$\sum_{i=1}^{m'} z_{i,k} = 1 \quad k = 1, \dots, p \quad (9)$$

$$\sum_{k=1}^p z_{i,k} \leq act_{max} \quad i = 1, \dots, m' \quad (10)$$

$$z_{i,k} \in \{0,1\}$$

The objective function (8) seeks to minimize the number of students who have to take the bus and go to another school. The constraint 9 looks for the best places for each activity. The constraint (10) is not obliged, but it may be imposed if the number of activities per school is limited.

7 TRANSPORT CAPACITY

The last stage to define the feasibility of the proposed solution is to evaluate the capacity in the buses, or alternative means of transport.

A depot is defined in any of the 2 ends if the route needs just a bus or two depots, one at each end side, for 2 buses. The maximum bus travel is estimated at t_{max} minutes. However, the ratios between time and route travel time are compared to round the number of buses. Given the cost of a bus, the maximum deviation percentage before choosing another bus is 15%. Thus, for ratios greater than 1.15, two buses are chosen; otherwise, only a bus is assigned to a cluster.

7.1 Required capacity in the case of one depot

The demand matrix determines the flow in each arc, considering the loading and unloading of children at each node (i.e. school). The required capacity for transport (the minimum number of required seats in the bus) will correspond to that arc of the graph with the maximum flow. Consequently, we can say that the most efficient situation would be if all the values in the flow matrix were equal. This way, you can move the highest number of children given a certain capacity in number of seats (Hall, 2003).

Let $y_{i,j}$ the number of children who must be transported from node i to node j . The flow leaving a node can be calculated using the equation for one way (equation 11) and the equation for the back way (equation 12) as follows:

$$f_l = \sum_{i=1}^l \sum_{j=l+1}^{m'} y_{i,j} \quad l = 1, \dots, m'-1 \quad (11)$$

$$g_{l+1} = \sum_{j=1}^l \sum_{i=j+1}^{m'} y_{i,j} \quad l = 1, \dots, m'-1 \quad (12)$$

Given the maximum flow in one way (maximum of the $m'-1$ values f_l) and in the back way (maximum of the $m'-1$ values g_l), the maximum flow in any way is determined with equation (13):

$$Maxflow = \text{Max}_{l=1, \dots, m'-1} \{f_l; g_{l+1}\} \quad (13)$$

7.2 Required capacity in the case of two depots

If the route in the graph requires 2 depots, each bus computes f_l and g_{l+1} using equations 11 and 12 from the matrix of demand and the minimum capacity of each vehicle is determined with equation 14 and 15. I.e. we obtain the maximum of both kinds of flows without finding the maximum between them).

$$Maxflow^1 = \text{Max}_{l=1, \dots, m'-1} \{f_l\} \quad (14)$$

$$Maxflow^2 = \text{Max}_{l=1, \dots, m'-1} \{g_{l+1}\} \quad (15)$$

Finally, the remaining capacity or the slack is computed. This variable depends on the capacity of the bus and the demand on each node. It is useful to accept or refuse a future increase of passenger flows according to the economic or strategic viability.

8 CASE STUDY

In Barcelona there are 164 public schools. However, considering that in a few cases the distance between them is minimal in order to share facilities, the number of locations can be simplified up to 160. Some initial considerations are made to see if any node can be discarded.

We impose that in the clusters a travel between any couple of nodes is less than $t_{max}/2=7.5$ minutes (if a single bus is assigned to the cluster) or $t_{max}=15$ minutes (if two buses are assigned).

First, we plot the graph with the 160 nodes and the set of arcs which accomplish the constraint on t_{max} . We can see that the graph is not connected, since there are four different subgraphs that should be evaluated separately. It is also noted that four nodes are sufficiently isolated from the rest of nodes; thus, we decide to exclude them and the graph has finally 156 nodes.

For our case, we take $n_{min}=4$ and $n_{max}=5$. If we apply the heuristic procedure presented in Section 5, we obtain a total number of 37 clusters. Later, the best route between the nodes of each one of the clusters is determined, using the linear program shown in Section 5 for the nodes in a cluster.

In Table 1 we can see the 37 clusters, the routes between the nodes and the total travel time to cover the transport in both ways. Two types of routes can be distinguished. The first group includes those whose travel time is up to 15 minutes, in which case a single bus is needed to cover the transport in the cluster. The second group includes those others, whose travel time is between 15 and 30 minutes, in which two buses cover the transport in the cluster in both ways.

For example, we will consider the cluster 5 composed by the schools 102, 103, 104 and 105. The objective is to evaluate the required capacity to perform the transport in a cluster. The data for demand at each school i for each activity k , considering the children of any age t , are shown in Table 2.

Cluster	Nodes	Total time (min)
1	157-156-159-158	28
2	48-47-46-44	14
3	43-39-34-35-27	28
4	140-135-142-139	18
5	105-104-102-103	11
6	109-111-110-89-86	18
7	106-116-122-121	15
8	137-143-149-150-136	18
9	90-108-85-84	15
10	38-51-52-42-41	23
11	60-57-58-54	15
12	99-88-98-83-59	22
13	126-125-123-112	16
14	65-64-82-87	10
15	28-29-18-25	18
16	117-115-114-113	19
17	100-77-76-61-75	30
18	133-134-132-131	19
19	7-8-11-16	13
20	73-81-63-72-66	17
21	160-1-5-9	17
22	96-93-95-92-94	10
23	153-152-154-155	13
24	36-30-37-53	19
25	19-17-24-26	19
26	141-147-146-151	14
27	31-32-33-40	10
28	10-12-13-14	22
29	138-145-144-148	8
30	62-45-55-56	24
31	101-78-74-70	16
32	119-120-118-124	22
33	127-130-129-128	9
34	22-21-20-23	15
35	3-2-4-6	10
36	69-68-67-79	18
37	80-91-97-107	11

Table 1: Optimal routes for the 37 clusters.

If we have many activities and in order to obtain an initial solution, we propose to group them into the number of schools that can act as facilities. In this case, if we have 10 different activities, those are grouped in 4 generic activities (Table 2): team sports (Ts), individual sports (Is), cultural activities (Ch) and foreign languages (Lg).

	102	103	104	105
Ts	6	4	8	5
Is	7	8	4	4
Cu	5	5	5	3
Lg	6	9	7	1

Table 2: Demand of each activity in the schools of cluster 5.

The next step is to choose the school where each one of the activities will take place, acting as a facility. We solve an assignment problem for each cluster.

Using the data shown in the Table 2, if $act_{max}=1$ the optimum assignment is: activity Ts in school 104; activity Is in school 102; activity Cu in school 105 and activity Lg in school 103.

Once the assignment is done, we have a matrix of times in minutes between schools (nodes in the graph) as the route for the cluster was done earlier (see the transport times in Table 3).

	103	102	104	105
103	-	2	-	-
102	1	-	2	-
104	-	2	-	3
105	-	-	1	-

Table 3: Matrix of transport times (in minutes) between schools in cluster 5.

The matrix with the flows is shown in Table 4. Note that in this matrix we sequence the schools in the rows and columns according to the route.

	103	102	104	105
103	0	8	4	5
102	6	0	6	5
104	7	4	0	5
105	1	4	3	0

Table 4: Matrix of flows between schools in cluster 5.

The next step is to choose if one or two buses are necessary for the transport in a cluster. The determination of the number of buses, last column in Table 5, is done as follows:

- From the optimal routes (Table 1), we have the total time for transport, called tt and measured in minutes.
- We obtain a ratio between tt and t_{max} , the maximum waiting time.
- As described in Section 7, for a ratio greater than 1.15, two buses are chosen; otherwise, one bus is chosen.

Example for a single bus. Given the required flows in Table 4, we apply equations 11 and 12 to obtain the required capacity of the bus at each part of the circuit. These results are detailed in Table 6 in the new column f_l and the new row g_l . We add another column with the remaining capacity in the bus, calculated as the difference between the bus capacity BC and the required capacity. Similarly, we add another row with the remaining capacity in the other way. We associate each one of the capacity requirements to the origin of the arcs in the graph.

Cluster	tt (min)	tt/ t_{max}	Buses
1	28	1.87	2
2	14	1.67	2
3	28	1.87	2
4	18	1.20	2
5	11	0.73	1
6	18	1.20	2
7	15	1.00	1
8	18	1.20	2
9	15	1.00	1
10	23	1.53	2
11	15	1.00	1
12	22	1.47	2
13	16	1.07	1
14	10	0.67	1
15	18	1.20	2
16	19	1.27	2
17	30	2.00	2
18	19	1.27	2
19	13	0.87	1
20	17	1.13	1
21	17	1.13	1
22	10	0.67	1
23	13	0.87	1
24	19	1.27	2
25	19	1.27	2
26	14	0.93	1
27	10	0.67	1
28	22	1.47	2
29	8	0.53	1
30	24	1.60	2
31	16	1.07	1
32	22	1.47	2
33	9	0.60	1
34	15	1.00	1
35	10	0.67	1
36	18	1.20	2
37	11	0.73	1

Table 5: Determination of the number of buses per cluster.

l	Node	103	102	104	105	f_l	$BC-f_l$
1	103	0	8	4	5	17	3
2	102	6	0	6	5	20	0
3	104	7	4	0	5	15	5
4	105	1	4	3	0	-	-
	g_l	-	14	16	8		
	$BC-g_l$	-	6	4	12		

Table 6: Required capacity and remaining capacity between schools in cluster 5.

Remember that the requirements in one way are shown in column f_l , between node l and node $l+1$ (i.e. the students between school 103 and school 102 are 17). And in the other way they are shown in row g_l , between node l' and node $l'-1$ (i.e. the students between school 104 and

school 102 are 16). Here we considered a mini-bus with 20 seats, so the capacity is $BC=20$.

In this case, the transportation between schools in cluster 5 is reflected in this graph with 4 nodes (Figure 1). The transport is done with a single bus, whose depot may be school 103 (i.e. the route starts in 103, goes to 105 and comes back to 103) or school 105 (i.e. the first part of the route goes from 105 to 103 and comes back to 105). It has a total run time of 11 minutes and carries a total of 58 children (the addition of all the flows in the matrix shown in Table 4).

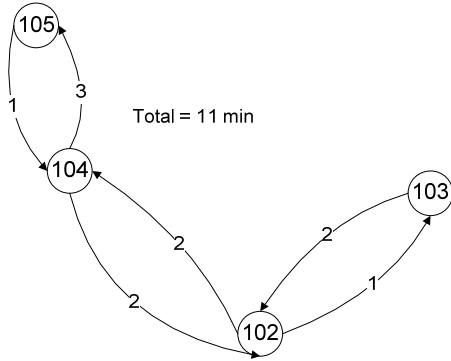


Figure 1: Example of transport in cluster 5 with a single bus (one depot, in school 103).

Example for two buses. In this case we take as example the cluster 3, composed by the schools 27, 34, 35, 39 and 43. Once the assignment of activities to facilities is done, we have a matrix of times between schools (nodes in the graph) as the route for the cluster was 43-39-34-35-27 (see Table 1). The total transport time in both ways is 14 minutes, although the time between two of the nodes may be different in both ways (see Table 7).

	43	39	34	35	27
43	-	3	-	-	-
39	3	-	2	-	-
34	-	3	-	6	-
35	-	-	6	-	3
27	-	-	-	2	-

Table 7: Matrix of transport times between schools in cluster 3.

The matrix with the flows is shown in Table 8 (we sequence again the rows and columns according to the determined route, Table 1).

Given the required flows in Table 8, we also apply equations 11 and 12 to obtain the required capacity of the bus at each part of the circuit (in Table 9): the new column f_i and the new row g_r . The last column and the last row have the remaining capacity in the bus. Here we consider a bus with 50 seats of available capacity, what is $BC=50$.

	43	39	34	35	27
43	0	8	3	9	5
39	10	0	4	6	2
34	3	6	0	5	8
35	8	5	6	0	9
27	9	7	6	4	0

Table 8: Matrix of flows between schools in cluster 3.

	l'	1	2	3	4	5		
L	Node	43	39	34	35	27	f_i	$BC-f_i$
1	43	0	8	3	9	5	25	25
2	39	10	0	4	6	2	29	21
3	34	3	6	0	5	8	35	15
4	35	8	5	6	0	9	24	26
5	27	9	7	6	4	0	-	-
	g_r	-	30	38	41	26		
	$BC-g_r$	-	20	12	9	14		

Table 9: Required capacity and remaining capacity between schools in cluster 3.

In this case, the transportation between schools in cluster 3 is reflected in Figure 2 (a graph with 5 nodes). The transport is done with two buses. The first one has the starting point in school 43 (i.e. the route starts in 43 and finishes in 27, 14 minutes later). The second one has the starting point in school 27 (i.e. the route goes from 27 to 43, and lasts 14 minutes). In the cluster the total run time is 28 minutes, and carries a total of 123 children (Table 8). The maximum required capacity is 41 in the first route, so the remaining required capacity goes from 9 (between schools 35 and 34) to 26 (between schools 35 and 27).

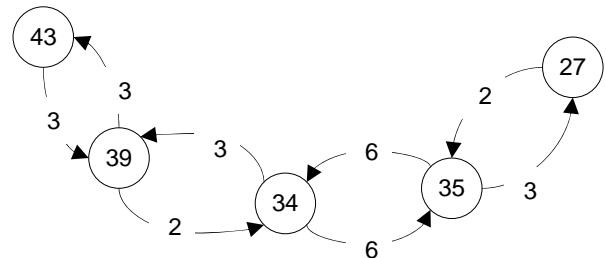


Figure 2: Example of transport in cluster 3 with two buses (the first depot in school 43 and the second one in school 27).

9 CONCLUSIONS

Nowadays there are many extra-curricular activities. Not always an activity can be realized in the same school of the children interested in it. Another problem that appears is that the number of children interested in one activity does not reach the minimum to cover its cost. For this reason, a solution to these problems is to provide transport to the students of a school to go to another school and there practice their favorite sports, cultural activity... This has led to model the schools in a city as the nodes of a graph and cluster them in subsets grouping the nearest public schools. Another interesting deci-

sion is the assignment of activities to the facilities in each school.

The clustering will allow both reducing the costs of activities and increasing the number of activities, but a transport is required. The objective is to provide the most efficient transport in terms of passengers carried according to the bus capacity. We consider one or two buses per cluster, which means in the first case a bus moves in both ways, and in the second case, which a bus moves in one way and the second in the other. The following decision is to determine the best route between schools in the cluster. Once determined, and considering the demands of activities at each node and the movement of children, we can determine the flows between schools, nodes in the graph, and the remaining capacity of the buses.

For a future research, we should analyze the possibility of that a node in the graph can form part of more than one cluster, according to the demand on the different activities; the demand according to the ages of the children; and the assignment of individual activities to facilities in schools instead the assignment of groups of activities.

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