

## Developing a model for solving the flight perturbation problem

Amirreza Nickkar<sup>1\*</sup>, Seyedmohammad Seyed-Hosseini<sup>1</sup>, Hassan Javanshir<sup>1</sup>, Hasan Khaksar<sup>2</sup>

<sup>1</sup>Azad University of Tehran (South Branch), Graduate Faculty, Civil Engineering Department, <sup>2</sup>Iran University of Science and Technology, Civil Engineering Department (Islamic Republic Of Iran)

\* Corresponding author: [amirreza.nickkar@gmail.com](mailto:amirreza.nickkar@gmail.com),

[seyedhosseini@iust.ac.ir](mailto:seyedhosseini@iust.ac.ir), [h\\_javanshir@azad.ac.ir](mailto:h_javanshir@azad.ac.ir), [khaksar@iust.ac.ir](mailto:khaksar@iust.ac.ir)

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Received September, 2014

Accepted February, 2015

### **Abstract**

**Purpose:** In the aviation and airline industry, crew costs are the second largest direct operating cost next to the fuel costs. But unlike the fuel costs, a considerable portion of the crew costs can be saved through optimized utilization of the internal resources of an airline company. Therefore, solving the flight perturbation scheduling problem, in order to provide an optimized schedule in a comprehensive manner that covered all problem dimensions simultaneously, is very important. In this paper, we defined an integrated recovery model as that which is able to recover aircraft and crew dimensions simultaneously in order to produce more economical solutions and create fewer incompatibilities between the decisions.

**Design/methodology:** Current research is performed based on the development of one of the flight rescheduling models with disruption management approach wherein two solution strategies for flight perturbation problem are presented: Dantzig-Wolfe decomposition and Lagrangian heuristic.

**Findings:** According to the results of this research, Lagrangian heuristic approach for the DW-MP solved the problem optimally in all known cases. Also, this strategy based on the Dantzig-Wolfe decomposition manage to produce a solution within an acceptable time (Under 1 Sec).

**Originality/value:** This model will support the decisions of the flight controllers in the operation centers for the airlines. When the flight network faces a problem the flight controllers achieve a set of ranked answers using this model thus, applying crew's conditions in the proposed model caused this model to be closer to actual conditions.

**Keywords:** flight perturbation problem, disruption management, rescheduling, optimization

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## 1. Introduction

The use of resource planning optimization techniques in industrial applications is imperative for the present competitive environment of the global economies and can significantly reduce operational costs. Nowadays, Airlines are subject to important challenges with respect to their operational performance that one of them is the scheduling process. Airline scheduling process is characterized by numerous complexities and conventionally done in four steps, including schedule generation, fleet assignment, maintenance routing and crew scheduling. These steps are usually considered continuously and discretely so that the results of each step are considered the input to the subsequent steps (Sherali, Bae & Haouari, 2013). In the final stage of this scheduling process and after maintenance routing, crew scheduling is performed. This step is very important because, after cost of aviation fuel, costs of flight personnel, including pilots, flight crew, etc. has the largest share of operational costs of airline companies (Deng & Lin, 2011).

The purpose of crew scheduling is to assign crew to all scheduled flights of aircrafts; i.e. according to the airline companies' schedule, the number of aircrafts is given as input and, considering other existing limitations of flight team such as their working hours and resting periods and also the objective function which is minimizing wage, a suitable crew is determined for flights. Crew's work schedule is determined by airlines so that all flights of the schedule are covered (Dunbar, Froyland & Wu, 2014; Saddoune, Desaulniers, Elhallaoui & Soumis, 2012).

Airline scheduling process has some defects and problems that usually lead to lack of solution optimality and sometimes it's indefensible. The first problem of this method is that flight time and schedule are first determined and then fleet assignment is solved, which can lead to non-optimum solutions; in contrast, an integrated model specifies both flight arrival and departure times and aircraft type. The second problem is that this approach loses its validity when the above-mentioned disruptions occur (Janic, 2009; Kohla, Larsenb, Larsenc, Rossd & Tiourine, 2007). The objective of solving the problems associated with recovery of flights and crew with delay management approach is to return the whole system to its initial state at the least possible time and minimum cost (Gao, Johnson & Smith, 2009). In airline rescheduling problems, it is well accepted that there is a potential for obtaining superior solutions by integrating the sub-problems of the process. However cost management and schedule planning models are mostly treated independently. The recovery models in this field usually consist of aircraft recovery and crew recovery. This paper focuses on the integrated recovery, which means above-mentioned two recoveries are considered as a whole so we present a more practical model, in which the crew regulations are considered in the current research.

## 2. Research background

In this part, we review some proposed methods for the flight recovery problems in airlines. The issue of airline disruption management as one of the major problems of airlines has been the subject of many investigations since mid 1980s. Firstly, Teodorović and Guberinić (1984) present a method for a new schedule design in situations when not all of planned aircraft are available. This work is later extended by Teodorović and Stojković (1990) by minimizing the total number of cancelled flights and the total passenger delays. Later, they further extended it by including crew considerations. They developed a heuristic model, using the first in, first out (FIFO) principle and a sequential approach based on dynamic programming (Teodorović & Stojković, 1995). Jarrah, Yu, Krishnamurthy and Rakshit (1993) present two network models (for cancellation and re-timing) which can assist in choosing which flights to delay or cancel in the event of unexpected shortages of aircraft due to situations that may arise during the operation of an airline. Their method was successfully implemented in a decision support system named System Operations Advisor (SOA). Talluri (1996) proposed algorithms that find swap opportunities that are guaranteed to satisfy the most important requirements of an assignment, namely flow balance, aircraft count and coverage. Rakshit, Krishnamurthy and Yu (1996) developed a SOA that has saved more than 27,000 minutes of potential delays, which translates to \$540,000 savings in delay costs, and the number of flight delays charged to aircraft controllers in systems operations control has dropped by 50 percent since the date of its implementation (1992). Mathaisel (1996) describes the business process as well as the IT challenges faced with the design and implementation of a decision support system for airline disruption management.

In 1997, Argüello and Bard (1997) presented a time-band model and a greedy randomized adaptive search procedure (GRASP) to reconstruct aircraft routings in response to groundings and delays. The objective was to minimize the total cost for delays and cancellations. The deviation from the original route was not considered. A research by Wei and Yu (1997) was conducted, in which model and solution method of performance management of crew members during disruption were investigated. The model presented in this research was a space-time network model which showed an image of the entire system at any moment. In this state, start time is the current time and end time is the time when the entire system returns to its initial state. In this flight network model, all critical and effective components of flight re-scheduling including airport, crew, flights and also the relationship between origin and destination of flights and the flight crew are considered. The obtained results indicated that, in all scenarios, a response was obtained in less than 20 Sec. In addition, the results of a case study with 6 airports, 51 flights and 18 flight chains showed that the considered model could achieve the response in an acceptable period of time. A binary quadratic programming approach is introduced by Cao and Kanafani (1997a) and (1997b) that integrates delays and

cancellations. Their model maximizes profit while penalizing undesirable outcomes. Thengvall, Bard and Yu (2000) use the models presented in Yan and Yang (1996) and extend them to incorporate the ability to penalize deviations from the original schedule. The model is capable of handling cancellations, delays and flight swaps, but does not consider crew or maintenance issues. Lettovsky, Johnson and Nemhauser (2000) simultaneously updated the crew schedule, which had problem after disruption in the flight chain, by maintaining the existing program in the upcoming flight programs. Their assumption in this model was that crew stayed fixed and unchanged during a flight chain and the reserved flight crew was assumed as an empty flight chain in this model. Crew members were only ready to serve one type of aircraft and served by their maximum allowable flight time (8 h per 24 h). Furthermore, in this study, it was assumed that a flight chain started at the beginning of the day from a particular airport and ended at the same airport at the end of the same night. An integral minimum cost flow model is introduced by Bard, Yu and Argüello (2001). This model aims at reconstructing airline schedules in response to delays by transforming the routing problem into a time-based network in which the overall time horizon is divided into discrete periods. The transformation is polynomial with respect to the number of airports and flights. An optimum of the model corresponds to the optimal solution of the original problem under some slight conditions.

In 2004, Andersson and Värbrand (2004) use a mixed integer multi-commodity flow model with side constraints further reformulated into a set packing model with generalized upper bound (GUB) constraints and using the Dantzig Wolfe decomposition. Disruptions are solved using cancellations, delays and aircraft swaps and the model ensures that the schedule returns to normal within a certain time. The model is tested on real problem data obtained from a Swedish domestic airline and the results show that it is capable of presenting high quality solutions in a few seconds and therefore can be used as a dynamic decision support tool by the airlines. This work is extended by Andersson (2006) by using two meta heuristic methods, a tabu search and a simulated annealing algorithm. These methods use a tree-search algorithm to find new schedules for the aircraft, and utilize a path relinking strategy to explore paths between structurally different solutions. He considered cancellations, delays and aircraft swap in his own work. Results of this research indicate that proposed solution strategies, especially the tabu search, can be successfully used to solve the flight perturbation problem. In this regard, the research of Rafiei, Manzari and Khashei (2012) is strongly related to Andersson (2006). They use the Ant colony metaheuristic approach to solve the Andersson's model. The computational results of this work show that although the proposed method has not better results than tabu search approach, but the statistical tests show the results of the proposed method has not significant difference from it.

Abdelghany, Abdelghany and Ekollu (2008) proposed a model that a rolling horizon modelling framework, which integrates a schedule simulation model and a resource assignment

optimization model, was adopted for a decision support tool for airlines schedule recovery during irregular operations. They express an application scenario where DSTAR saves 8.7% of the total delay. The approach of the paper is very promising when considering larger disruptions, which are foreseeable, a number of hours ahead. In other work, Castro and Oliveira (2009) presented an approach in solving the problem of cost reduction in airlines using Distributed Multi-Agent System (MAS). This approach was based on real data of air control centres and according to case studies. The MAS approach of these researchers aimed to quantitatively calculate operational costs of airline companies. In their study, they demonstrated that the MAS approach was able to significantly reduce delay costs and customer satisfaction without causing direct operational costs. Castro in his PhD thesis proposes a negotiation protocol called Generic Q-Negotiation (GQN) to be used as a decision mechanism in the disruption management process. The results show that the suggested method, not only corroborates existing studies regarding the possible cost reductions that could result from a better disruption management process, but, also, gives the possibility of reaching solutions that balance the utility of the three dimensions of the problem: aircraft, crew and passengers (Castro, 2013).

According to reviewed researches, we do not claim that these are all the works published about the airline integrated recovery problem, However, there has been relatively little work previously done in studying and solving the airline integrated recovery problem. A possible reason for the lack of research methods using an integrated approach is the complexity of the problem when trying to obtain an integrated solution that includes the two or more dimensions. Current research is trying to propose an integrated recovery process as that which is able to recover two important problem dimensions (aircraft and crew) simultaneously.

### **3. A developed mixed integer multicommodity flow model**

This study is performed based on the development of Andersson and Värbrand's model (Andersson and Värbrand, 2004) as one of the flight rescheduling models, with delay management approach. The purpose of their study was to maximize the revenue from ticket selling in airline companies after disruption in flight chain. Their proposed model had two objective functions. The first objective function implied that total revenue from assigning aircrafts to flights was maximized for the company and the second objective function indicated that total cost imposed on the company was the result of the delay. In order to solve the suggested flight perturbation problem, they represented a flight network. This network included three types of nodes: aircraft source nodes, flight nodes, and flight sink nodes. Each node belongs to a station (airport). Each aircraft source node represents a specific airplane and this airplane belongs to a specific station from which the plane is planned to start its tour or to

which the plane will land at the time of perturbation. Each flight node and flight sink node was representative of a specific flight. The position of a node in the network showed the planned departure time and destination. To every flight sink node, an aircraft source node was related. In other words, every flight route started from an aircraft source node and ended in a flight sink node. Considering the type of aircraft source node, to ensure that every route ends in a proper destination, extra constraints should be enforced. Flight sink nodes assigned a specific flight to every aircraft which was specified by the aircraft source node. In other words, these nodes represented the assigned flight to aircrafts in the original scheduled, at the end of perturbation time. Immediately after the perturbation period was being finished, these nodes turn the rescheduled plan back to the original schedule.

Now, Objective function of the developed model in this work minimized airline costs in post-disruption conditions (replanning costs). This developed model has three objective functions. First objective function minimizing the cost of aircraft assignment to flights; second objective function minimizing the total cost for delays (except the costs imposed by crew members) and third objective function suggested that total cost imposed by crew flight on airline companies in the post-disruption conditions were the result of the delay. This cost might include the costs of catering, accommodation, and overtime. In this case, the interval between the start and the end of perturbation is called decision time in which any changes in the original schedule are allowed. For domestic flights, decision time is one day or less. However, in case long haul operations are included, this time is extended to more than a day. Arcs in the network define every possible connection between aircraft source nodes and flight sink nodes. Therefore, if there is an arc between aircraft source node and flight sink node, the available airplane with a certain crew in the source node can be assigned to the flight. The sets, parameters, and variables of the model are as follows:

Sets:

$A$  = set of aircraft source nodes.

$F$  = set of nodes representing flights.

$S$  = set of flight sink nodes.

$K$  = set of aircrafts.

$C$  = set of crews.

Parameters:

$C_{ij}^{kp}$  = cost paid if aircraft k with crew p is assigned to flight j after flight i.

$C_i$  = cost per time unit for delaying flight i.

$s_i^k = 1$  if i is the correct source for aircraft k (i.e.,  $i = k$ ), else 0.

$s_i^p = 1$  if i is the correct source for crew p (i.e.,  $i = k$ ), else 0.

$t_i^k = 1$  if i is the correct flight sink for aircraft k, else 0.

$t_i^p = 1$  if i is the correct flight sink for crew p, else 0.

$D_c$  = cost due to exceeding the existing delay from the maximum allowed delay.

$AD_i$  = departure airport of flight i (represented by a number).

$AA_j$  = arrival airport of flight j (represented by a number).

$TD_j$  = planned departure time for flight j.

$TA_i$  = planned arrival time for flight i (or when aircraft i is available).

$TG_{ij}^{kp}$  = required time between flights if aircraft k is assigned to flight j after flight i.

$D_j$  = maximum allowed delay for flight j.

$C^{kp}$  = capacity of aircraft k.

$h_j$  = average time of flight j.

$T_p$  = the maximum amounts of time flight crews can work.

$P_j$  = number of passengers on flight j.

$M$  = a large positive number.

Variables:

$x_{ij}^{kp}$  = if aircraft k with crew p flies the flight j after the flight i, else 0.

$Z_j = 1$  if the delay of flight j is higher than maximum allowed delay, else 0.

$d_i$  = the amount of delay in flight i.

Considering the mentioned measures, objective function of the proposed mathematical model for crew flight scheduling can be finally provided as in relation (1).

$$Min \sum_{i \in A \cup F} \sum_{k \in A} \sum_{p \in C} c_{ij}^{kp} x_{ij}^{kp} + \sum_{i \in F} c_i d_i + \sum_{j \in F} D_c c_j Z_j \quad (1)$$

S.t.

$$\sum_{k \in A} \sum_{j \in F \cup S} x_{ij}^{kp} = S_i^p \quad \forall i \in A, p \in C \quad (2)$$

$$\sum_{p \in C} \sum_{j \in F \cup S} x_{ij}^{kp} = S_i^k \quad \forall i, k \in A \quad (3)$$

$$\sum_{j \in S} x_{ij}^{kp} - \sum_{j \in A \cup F} x_{ji}^{kp} = 0 \quad \forall i \in F, k \in A, p \in C \quad (4)$$

$$\sum_{j \in F \cup S} x_{ij}^{kp} - \sum_{j \in A \cup F} x_{ji}^{kp} = 0 \quad \forall i \in F, k \in A, p \in C \quad (5)$$

$$\sum_{p \in C} \sum_{j \in A \cup F} x_{ij}^{kp} = t_i^k \quad \forall i \in S, k \in A \quad (6)$$

$$\sum_{k \in A} \sum_{j \in A \cup F} x_{ji}^{kp} = t_i^p \quad \forall i \in S, p \in C \quad (7)$$

$$\sum_{k \in A} \sum_{p \in C} \sum_{j \in F \cup S} x_{ij}^{kp} \leq 1 \quad \forall i \in F \quad (8)$$

$$TA_i + TG_{ij}^{kp} + d_i - (TD_j + d_j) + M(x_{ij}^{kp} - 1) \leq 0 \quad \forall i \in A \cup F, j \in F \cup S, k \in A, p \in C \quad (9)$$

$$x_{ij}^{kp} (AD_j - AA_i) = 0 \quad \forall i \in A \cup F, j \in F \cup S, k \in A, p \in C \quad (10)$$

$$\sum_{k \in A} \sum_{i \in A \cup F} \sum_{j \in F \cup S} h_j x_{ij}^{kp} \leq T_p \quad \forall p \in C \quad (11)$$

$$-M \times Z \leq (D_j - d_j) \quad \forall j \in F \quad (12)$$

$$x_{ij}^{kp} p_j \leq c^{kp} \quad \forall i \in A \cup F, j \in F, k \in A, p \in C \quad (13)$$

$$d_i \geq 0 \quad \forall i \in F \quad (14)$$

$$d_i = 0 \quad \forall i \in A \cup S \quad (15)$$

$$x_{ij}^{kp} = 0, 1 \quad \forall i \neq j \quad (16)$$

$$x_{ij}^{kp} = 0 \quad \forall i = j \quad (17)$$

$$B_j, Z_j \in \{0, 1\}$$

Observing and considering the crew's working hours and rest time are one of the requirements in crew scheduling in post-disruption condition. Every crew member prefers the flights which lead to their living location at the end of their allowed flight time. If their working hours exceed their allowed time, they can sometimes have a no-ticket flight, in which the crew is like other passengers and do not offer any services. Also, this state may occur when the crew must arrive at a station where there is a flight to be served. To this end, a parameter should be defined based on the aviation rules for flight crew to guarantee compliance of allowed flight time of the crew with flight replanning. To do so,  $T_p$  which is the maximum amounts of time flight crews can work should be first defined. Furthermore, parameter  $h_j$  which is the average time of flight  $j$  was defined for the possibility of the allowed flight time of that crew. This parameter should be less than maximum working hours of crew per day for all flights. So, limitations (6), (13), (14) and (15) are added to the objective function of the model.

In this model, constrains (2), (3), (6) and (7) ensure that accurate origin and destination are selected for flight of aircraft  $K$  and crew  $P$ . In other words, an airplane or a crew group is assigned to every source node. Constrains (4) and (5) is survival condition of flow (flow equilibrium) for flight nodes; i.e. the number of input arcs to each flight node is equal to the number output arcs of that node per aircraft. Constrain (8) which hardens solving the model, assigns each flight only to one aircraft and crew. Constrain (9) warrants feasibility of flights in terms of time; i.e. each flight can connect to the next flight to form a flight chain so that landing time plus the needed time between flights (recovery) and their delays are less than the next flight time. Constrain (10) ensures flight feasibility in terms of location; it means that each flight can connect to the next flight which has the same take off location. Constrain (12) ensures the feasibility of the allowed flight time of crew members. Limitation (16) represents that maximum determined amount of delay is greater than or equal to the actual delays. Constrain (13) suggests that  $x_{ij}^{kp}$  can be only one when the capacity of the considered aircraft is greater than or equal to the number of passengers in flight  $j$ . Constrain (14) indicates positive delay. Constrain (15) ensures that each aircraft performs its responsibility in due time one the day after disruption. It is necessary to explain that we assume all crew members could be able to service the all types of aircrafts in current research. In this research, we assumed solution strategies similar to solution strategies that done in the original model.

### 3.1 Dantzig-Wolfe decomposition of the MIP Model

Without constraints (8) in FP-MIMF, the model decomposes into one problem for each aircraft and crew. To separate the delays over the aircraft, let  $d_i^{kp}$  be the delay on flight  $i$  caused by aircraft  $k$  and crew  $p$ . Since at most one aircraft with one crew can be assigned to a flight, we

have:

$$d_i = \sum_{k \in A} \sum_{p \in C} d_i^{kp} \quad i \in F \quad (18)$$

As, for all  $k$  and  $p$ , only one  $d_i^{kp}$  can be nonzero. For one aircraft  $k$  with crew  $p$ , there may however exist two feasible solutions, where the values of  $x_{ij}^{kp}$  are equal but the values of  $d_i^{kp}$  differs. In practice, number of flights and number of crew groups are depended on number of airplanes so, if a given airline have "k" airplanes for covering flight network, number of principle variable ( $x_{ij}^{kp}$ ) of the problem will have a degree equal to  $k^4$ . The solution with the extra delay is clearly dominated by the other solution, since an extra delay will lower the objective function value. Therefore, it is never interesting to consider solutions where the delay is more than required by constraint (9).

Let  $R^{kp}$  be the set of feasible solutions for aircraft  $k$  with crew  $p$  that is not dominated by any other solution and let  $r$  be a point in the set. Furthermore, let  $\bar{x}_{ij}^{kpr}$  and  $\bar{d}_i^{kpr}$  be the values of the variables in the point  $r$ . Now, by using a binary variable  $\lambda^{kpr}$ , the original variables for a certain aircraft  $k$  with crew  $p$  can be expressed as:

$$x_{ij}^{kp} = \sum_{r \in R^{kp}} \lambda^{kpr} \bar{x}_{ij}^{kpr}$$

$$d_i^k = \sum_{r \in R^{kp}} \lambda^{kpr} \bar{d}_i^{kpr}$$

$$\sum_{r \in R^{kp}} \lambda^{kpr} = 1$$

$$\lambda^{kpr} \in \{0, 1\}$$

Using the results above, FP-MIMF can be written as a Dantzig-Wolfe Master Problem:

$$\text{Min} \sum_{k \in A} \sum_{p \in A} \left( \sum_{i \in A \cup F} \sum_{j \in F} c_{ij}^{kp} \bar{x}_{ij}^{kp} + \sum_{j \in F \cup S} c_i \bar{d}_i^{kp} \right) \lambda^{kpr} \quad (19)$$

S.t.

$$\sum_{r \in R^{kp}} \lambda^{kpr} = 1 \quad k \in A, p \in C \quad (20)$$

$$\sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} \left( \sum_{j \in F} \bar{x}_{ij}^{kpr} \right) \lambda^{kpr} \leq 1 \quad (21)$$

$$\lambda^{kpr} \in \{0, 1\} \quad k \in A, p \in C, r \in R^{kp} \quad (22)$$

To make the model easier to understand and work with, let:

$a_i^{kpr} = 1$  if flight  $i$  is included in route  $r$  for aircraft  $k$  with crew  $p$ , else 0.

$$a_i^{kpr} = \sum_{j \in F} \bar{x}_{ij}^{kpr}$$

$$c^{kpr} = \sum_{i \in A \cup F} \sum_{p \in C} \sum_{j \in F} c_{ij}^{kp} \bar{x}_{ij}^{kpr} + \sum_{i \in F} c_i \bar{d}_i^{kpr}$$

Using the substitutions above, the Dantzig-Wolfe Master Problem (DW-MP) is:

$$\text{Min} \sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} c^{kpr} \lambda^{kpr} \tag{23}$$

S.t.

$$\sum_{r \in R^{kp}} \lambda^{kpr} = 1 \quad k \in A, p \in C \tag{24}$$

$$\sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} a_i^{kpr} \lambda^{kpr} \leq 1 \quad i \in F \tag{25}$$

$$\lambda^{kpr} \in \{0, 1\} \tag{26}$$

The problem is now represented by a set packing model with generalized upper bound (GUB) constraints (24), which ensure that each aircraft with certain crew is assigned to exactly one route. The second set of constraints (25) ensures that each flight is not included on more than one route. The feasible routes in each set  $R^{kp}$  define paths from the aircraft source node to the flight sink node. Each route may include flights that the aircraft was not originally assigned to, and may also include delayed flights. The costs for the swaps and the delays are added from the cost that the particular route generates. The objective of the model is to pick one route for each aircraft with certain crew so that the total cost is minimized.

## 4. Solution strategy

### 4.1 LP Relaxation

A straightforward approach for solving DW-MP would be to use B&B and thus iteratively solve the LP relaxation of the problem, i.e., DW-LMP. Then all feasible points  $R^{kp} \forall k \subseteq A, p \subseteq C$  have to be generated, i.e., we need to find all paths in the connection network. Even though it is possible to generate them efficiently, the number of feasible paths increases exponentially with the number of flights. Instead, a restricted version of the DW-LMP can be formulated. Let

$Y^{kp} \subset R^{kp}$  be a subset of columns for each aircraft with certain crew. By replacing  $R^{kp}$  with  $Y^{kp}$  in the model for the DW-MP represented by equations (23) to (26) and perform a linear relaxation, the Dantzig-Wolfe Restricted Linear Master Problem (DW-RLMP) is obtained.

A feasible solution to the DW-RLMP is also a feasible solution to the DW-LMP, and gives a lower bound on the optimal objective function value, i.e.,  $z^*DW-RLMP \leq z^*DW-LMP$ . The solution also provides dual variables: let  $\pi_{kp}$  and  $\mu_i$  be the dual variables associated with constraints (24) and (25) respectively. The increased cost for a new column  $\lambda^{kpr}$  is given by:

$$\bar{c}^{kpr} = c^{kpr} + \pi_{kp} + \sum_{i \in F} \mu_i a_i^{kpr} \tag{27}$$

If a new column with a negative increased cost can be found, it can be added to the DW-RLMP which can be solved once again to provide new dual variables. If no such column can be found, the latest solution to the DW-RLMP is the optimal solution to the DW-LMP. The initial set of columns,  $U_{(k \in A, p \in C)} Y^{kp}$ , needs to be constructed so that it is possible to find a feasible solution to the DW-RLMP.

The problem of finding a new column can be formulated as an IP model. As described above, the set of feasible points separates into one set for each aircraft with certain crew,  $R^{kp}$  which also means one subproblem for each aircraft with certain crew. Using the same definitions as for the FP-MIMF but separating the aircraft with certain crew gives, for example,  $x_{ij}^{kp} \rightarrow x_{ij}$ , where  $x_{ij}$  is equal to one if flight i is followed by flight j.

The objective in the subproblem is to minimize the increased cost for the column, i.e.,  $\min\{c^{kpr} + \pi_{kp} + \sum_{(i \in F)} \mu_i a_i^{kpr}\}$ , where  $a_i^{kpr}$  are used as decision variables. The cost  $c^{kpr}$  is a function of  $a_i^{kpr}$ , but by making the exchange,  $a_i^{kpr} = \sum_{(j \in F \cup S)} x_{ij}$ , a linear objective function is obtained. The subproblem for finding a new column for aircraft k with certain crew p (DW-SUB<sup>kpr</sup>) can be formulated as:

$$\text{Min } \sum_{i \in A \cup F} \sum_{j \in F \cup S} c_{ij}^{kp} x_{ij} + \sum_{i \in F} c_i d_i + \sum_{i \in F} \sum_{j \in F \cup S} \mu_i x_{ij} + \pi_{kp} \tag{29}$$

Subject to

Constraints (2)-(4) and (6)-(11)

Where  $c_{ij}^{kp}$  and  $c_i$  are the same as in FP-MIMF.

If the optimal solution to the problem above has a positive objective function value, a column that may improve the value of the DW-RLMP has been found. The data needed for adding the column to the DW-RLMP is obtained by  $a_i^{kpr} = \sum_{j \in F \cup S} x_{ij}$  and  $c^{kpr} = \sum_{i \in A \cup F} \sum_{j \in F \cup S} c_{ij}^{kp} x_{ij} + \sum_{i \in F} c_i d_i$ .

The column generation strategy used for solving the DW-LMP can be embedded in a B&B procedure, where new columns are generated for each node of the search tree. In this way a solution to the DW-MP can be obtained. The solution technique is called Branch and Price and has proved to be efficient for a large number of problems.

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*Algorithm 1: Restricted Route finder*

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1. Let  $k$  be the current aircraft
  2. Let  $p$  be the current crew
  3. Let  $N = 0$ , be a queue of nodes and  $i=k$
  4. Create a node (the root) for  $kp$  and add it to  $N$
  5. While  $N \neq 0$  do
    6. Let  $M = 0$ ,  $M \leq M_{max}$ , be a set of flights
    7. For all  $j \in F_i^c \cap F_{kp}^p$ 
      8. Calculate the delay needed if  $j$  is to follow  $i$  according to equation (9)
      9. If the delay is within the bound given by (12) and (15)
      10. Calculate the cost of adding the flight to the path
      11. If the cost is better than the best cost for the flights in  $M$
      12. Add the flight to  $M$  and remove the flight with the highest cost from  $M$
    13. For all nodes in  $M$ 
      14. Create a new node and add it to  $N$
    15. Remove  $i$  from  $N$  and let  $i$  equal the next node in  $N$
  16. Traverse all nodes in the tree
    17. If the node can be connected to the flight sink node
      18. Create a column by traversing the path from the new node to the root. Calculate the cost for the path while traversing it and check so that no flight appears twice in the path
      19. If the column has a negative increased cost
      20. Add it to DW-RLMP
- 

If the subproblem is solved to optimality for all aircraft, it will return the paths with the highest reduced costs. If the reduced cost for the optimal path is zero, it is clear that an optimal solution to the DW-LMP has been found. Unfortunately, it is a time consuming process to solve the subproblem to optimality. The subproblem can easily be transformed into a model for the Shortest Path Problem with Time Windows and Linear Node Costs. To solve the subproblem faster, a heuristic can be used. 'Algorithm 1' will build a search tree to find paths with negative increased costs to add to the DW-RLMP.

The cost for a potential connection in Algorithm 1 (Step 10) is calculated as  $c_{ij} + \mu_j + k_{ij}$  where  $c_{ij}$  includes the cost for flying the passengers on flight  $j$  as well as the costs for delaying and swapping the flight.  $\mu_j$  is the dual variable associated with flight  $j$  and  $k_{ij}$  is a component penalizing the idle time the aircraft spends on the ground.  $k_{ij}$  is calculated as the idle time between flight  $i$  and flight  $j$  multiplied by a constant. The new cost component,  $k_{ij}$ , is not used in the total cost for the path (which is calculated in Step 18), but is helpful in the route generation phase to give incentive for constructing longer routes and accepting flights with delays and swap costs into the route. The maximum number of progeny each node can have is

restricted to  $M_{max}$  (Step 6). This limits the tree size as well as the number of columns that are generated.

If no paths with a negative increased cost can be found with *Algorithm 1*, the process terminates and a (not necessarily optimal) solution to the DW-LMP has been found. Unfortunately, it is a time consuming process to solve the subproblem to optimally because this strategy may fail to find the best known solution the model with large data sets probably thus, to avoid these difficulties and find a more efficient way to solving the DW-LMP, we need to use another approach to find optimal solutions.

#### 4.2 A Lagrangian Heuristic for the DW-MP

Another strategy for solving the DW-MP is to use Lagrangian relaxation and subgradient optimization. This method has been successfully by Caprara, Fischetti and Toth (1999) for solving the related set covering problem. Instead of relaxing the binary constraints (26) and obtaining the DW-LMP, the packing constraints (25) are moved to the objective function. For any Lagrangian multipliers  $\mu_i \geq 0$ , the problem DW-MP ( $\mu$ ):

$$\text{Min} \sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} c^{kpr} \lambda^{kpr} + \sum_{i \in F} \sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} (a_i^{kpr} \lambda^{kpr} - 1) \mu_i \quad (30)$$

S.t.

$$\sum_{r \in R^{kp}} \lambda^{kpr} = 1 \quad K \in A, P \in C \quad (31)$$

$$\lambda^{kpr} \in \{0, 1\} \quad (32)$$

is a relaxation to the DW-MP. For a given set of multipliers,  $\hat{\mu}_i$ , the model above describes the Lagrangian Subproblem (DW-LS). The Lagrangian Dual Problem (DW-LDP) is the problem of finding the optimal multipliers:

$$\text{max } h(\mu) = \sum_{i \in F} \mu_i - \text{min} \sum_{k \in A} \sum_{p \in C} \sum_{r \in R^{kp}} (c^{kpr} + \sum_{i \in F} a_i^{kpr} \mu_i) \lambda^{kpr}$$

S.t.

$$\sum_{r \in R^{kp}} \lambda^{kpr} = 1 \quad K \in A, P \in C$$

$$\lambda^{kpr} \in \{0, 1\} \quad K \in A, P \in C, r \in R^{kp}$$

$$\mu_i \geq 0, i \in F$$

It is now easy to see that the sub problem separates into one problem for each aircraft with certain crew. This sub problem can easily be solved by simply picking the column with the lowest cost  $c^{kpr} + \sum_{i \in F} a_i^{kpr} \mu_i$  for each aircraft with certain crew.

In most cases, the solution to the sub problems does not yield a feasible solution to the set packing model. To obtain one, a greedy heuristic can be used which selects the route with the lowest cost, including the multipliers, that does not duplicate any flights. This is repeated until all aircraft with its certain crew is assigned to one route.

The Lagrangian multipliers are updated with the Polyak (1969) subgradient method, and the iterative process continues until the gap between the upper bound given by the Lagrangian dual problem and the lower bound given by the best feasible solution found is sufficiently small or the dual objective function value converges. A local search heuristic is also used to improve the feasible solutions, when such are found, using simple swapping techniques (Andersson and Värbrand, 2004).

Lagrangian multipliers can be used when generating new columns. The principle is the same as for the column generation scheme based on LP relaxation and is described in 'Algorithm 1'. In each iteration, a restricted version of the master problem, DW-RMP, is solved to optimality or near optimality.

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*Algorithm 2: Lagrangian Column Generation*

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1. Let  $\tau = 0$  be an iteration counter
  2. Let  $\varepsilon$  be a sufficiently small number
  3. Initiate the multipliers, i.e.,  $\mu_i^\tau = 0, \forall i$
  4. Generate  $y^{kp} \in R^{kp} \forall k \in A, p \in C$
  5. Repeat
  6.      $\tau = \tau + 1$
  7.      $\mu_i^\tau = \mu_i^{\tau-1}$
  8.     Solve the DW-RMP with  $y^{kp} \forall k \in A, p \in C$ , using the Lagrangian heuristic
  9.     Let  $\mu_i^\tau$  be the optimal multipliers and  $c_{kp}^\tau = c^{kpr} + \sum_{i \in F} a_i^{kpr} \mu_i^\tau$  the cost for the optimal column,  $r$ , for aircraft  $k$  with certain crew  $p$
  10.    For all  $k$  with certain  $p$ , generate new columns,  $r$ , whose cost  $c^{kpr} + \sum_{i \in F} a_i^{kpr} \mu_i^\tau$  lower than  $c_{kp}^\tau$  and add them to  $y^{kp}$
  11. Until  $\sum_{i \in F} |\mu_i^\tau - \mu_i^{\tau-1}|$
- 

The main difference from column generation based on LP relaxation is when to decide whether a column should be added to the problem. In the LP case, the reduced cost for a new column can be calculated, but this is not directly possible when Lagrangian relaxation is used. However, when the DW-LS is solved, the best column as calculated by  $c^{kpr} + \sum_{i \in F} a_i^{kpr} \mu_i^\tau$  is picked for each aircraft. In Step 9, the cost  $c_{kp}^\tau$  is the cost for the best column that can be found with the

optimal multipliers  $\mu^r$ . If a column with lowest cost could be found, that would be picked instead when the subproblem is solved. Therefore, any new column with such cost will improve the solution of the DW-LS and can be added to the DW-RMP.

In Step 10, new columns are generated by *Algorithm 1*, which can be used as described earlier but with minor alterations. Instead of checking if a column has a negative increased cost (Step 19), the check done in Step 10 in *Algorithm 2* is performed. The stop condition in Step 11 is only one possible choice. Another would simply be to stop when no new columns can be found in Step 10.

## 5. Computational results

The aim of providing computational results in this section is to prove the efficiency of the model. Moreover, we show how flight controller can use this method as a decision support tool for assessing different strategies in the time of perturbation. The proposed algorithms have been implemented in MATLAB R2009a environment and a computer with 2.13GHz CPU and 3 GB of RAM is used for the examinations.

### 5.1 Datasets

The sample datasets with which the efficiency of the proposed algorithm and the algorithms available in the literature are compared belongs to Iran's domestic flights data. These datasets are provided under the headings of S1 and S2 and their properties are presented in Table 1. Note that domestic flights are categorized in short range flights with 35 to 120 minutes flight time.

Items	S1	S2
<b>Number of aircraft</b>	11	23
<b>Number of aircraft types</b>	2	4
<b>Number of flights</b>	50	125
<b>Number of airports</b>	16	25
<b>Number of crewgroups</b>	11	24

Table 1. The two data sets of proposed scenarios

According to Andersson and Värbrand (2004), smaller data sets are possible to solve fairly efficiently so, small databases that used in current research are able to give a pragmatic view. These datasets have faced perturbation in two different types: "a" and "b". In perturbation type "a", an airplane is broken down while in perturbation type "b" there is a delay on a group of flights. Combining perturbation types and two datasets, we will have four sets: s1a, s1b,

s2a, and s2b. In s1a, one airplane is not available for 5 hours, and in s1b two flights will face a delay for 25 and 30 minutes respectively. The unavailability of an airplane in s2a is 6 hours and the delay in s2b occurs for 4 flights with 15 to 40 minutes time. The maximum allowed delay for every flight ( $D_i$ ) is 60 minutes and every feasible path has at most 16 flight nodes. The time for every airplane, irrespective of its type, is 10 minutes. This number is changeable and is considered as a fixed number in our tests for comparison purposes only. Every dataset is solved for different weights and their respective costs are calculated.

## 5.2 Prior Generation of Columns

In this section, strategies are tested on the four data sets. Each data set is solved using different weight to calculate the costs. This is to show how a dispatcher would be able to change the weight if the solution obtained dose not meet the needs.

D-Set	Weights					Results	
	can	swp	swa-t	swa	del	Z	UDB
S1a	20	50	100	10	1	76630	76635
S1a	20	50	1000	10	1	77893	77896
S1a	100	50	1000	10	1	383456	383457
S1a	100	50	100	10	1	369211	369217
S1b	20	50	100	10	1	89371	89371
S1b	20	500	100	100	1	87601	87601
S1b	20	1000	100	400	1	85098	85099
S2a	20	50	100	10	1	10348	10357
S2a	20	50	1000	10	1	99627	99630
S2a	100	50	1000	10	1	567239	567242
S2b	20	50	100	10	1	141588	141590
S2b	20	50	100	10	10	111635	111636
S2b	20	500	100	50	1	129414	129419

Table 2. The results of the developed model in sample datasets for the Lagrangian heuristic

The results for the Lagrangian heuristic are presented in Table 2. The first column,  $n$ , contains a run label, so that the individual runs can be easily referred to. The column  $D$ -set notes which data set is used while the Weights are the ones used for calculating the costs in the model.  $can$  is the weight for cancelling a flight,  $swp$  the weight for assigning a flight to a crew of a different flight,  $sw-t$  the weight for assigning a flight to an aircraft of a different type,  $sw$  the weight for assigning a flight to an aircraft in the same fleet as originally planned, and  $del$  is the weight for delaying a flight. In the *Result* section,  $z$  the objective function value for the best

solution found. *UBD* is the upper bound on the optimal solution that is provided by the heuristic

The four different problems provided by the four data sets have been solved with a number of different weights. A variety of weight combinations have been adopted for other data sets whose values may be different, regarding the flight controller opinion. Now, if the flight controller does not like swaps between aircraft types, the corresponding weight may be increased from 100 to 1000, which gives the setting in run 2. In run 2 the solution includes no swaps between aircraft types, but instead the number of passengers affected by a canceled flight has increased to 46, which might be too much for the flight controller who thereby raises the cancellation weight to 100 from 20.

As shown, presented strategy to solve the flight perturbation problem is able to handle any type of data instance and produced good solutions. Reason of this claim is the amount of difference between the amount of the upper bound and the amount of the objective function value. The largest gap between the best solution found and the upper bound for any of the runs occurs in *s2a* that acceptable number. The Lagrangian strategy based on the Dantzig-Wolfe decomposition manage to produce solution within an acceptable time (Under 1 Sec).

## 6. Conclusions

In this research, we could develop and run an efficient and effective model for solving the flight perturbation problem through developing one of the flight scheduling models as we could combine the operational aircraft planning and the crew planning in a unified model. This model will support the decisions of the flight controllers in the operation centers for the airlines. When the flight network faces a problem the flight controllers achieve a set of ranked answers using this model thus, applying crew's conditions in the proposed model caused this model to be closer to actual conditions. In this research, two solution strategies for flight perturbation problem are presented: Dantzig-Wolfe decomposition and Lagrangian heuristic. According to the results of this research, Lagrangian heuristic approach for the DW-MP solved the problem optimally in all known cases. The responses obtained from this developed model and its solving amount may consider as a developing solution technique for large scale problems because these proposed solving methods present a more practical formulation for airline optimal recovery and can be the basis for future studies to investigate on large scale networks in post-disruption conditions. Finally, we recommend passenger handling problem might be incorporated into a larger framework like to propose model in this research as future works.

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