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IMPLEMENTATION OF ZERO FORCING AND MMSE EQUALIZATION TECHNIQUES IN OFDM

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1. INTRODUCTION TO OFDM

The technology we call OFDM (Orthogonal Frequency Division Multiplexing) in this report is usually viewed as a collection of transmission techniques. When applied in a wireless environment, such as radio broadcasting, it is usually referred to as OFDM. However, in a wired environment, such as in asymmetric digital subscriber-lines (ADSL), the term discrete multitone (DMT) is more appropriate.

The history of OFDM dates back to the mid 60's, when Chang published his paper on the synthesis of band limited signals for multichannel transmission [1]. He presents a principle for transmitting messages simultaneously through a linear band limited channel without interchannel (ICI) and intersymbol interference (ISI). Shortly after Chang presented his paper, Saaltzberg performed an analysis of the performance [2], where he concluded that "the strategy of designing an efficient parallel system should concentrate more on reducing crosstalk between adjacent channels than on perfecting the individual channels themselves, since the distortions due to crosstalk tend to dominate". This is an important conclusion, which has proven correct in the digital baseband processing that emerged a few years later.

A major contribution to OFDM was presented in 1971 by Weinstein and Ebert [3], who used the discrete Fourier transform (DFT) to perform baseband modulation and demodulation. This work did not focus on "perfecting the individual channels", but rather on introducing efficient processing, eliminating the banks of subcarriers oscillators. To combat ISI and ICI they used both a guard space between the symbols and raised-cosine windowing in the time domain. Their system did not obtain perfect orthogonality between subcarriers over a dispersive channel, but it was still a major contribution to OFDM.

Another important contribution was due to Peled and Ruiz in 1980 [4], who introduced the cyclic prefix (CP) or cyclic extension, solving the orthogonality problem. Instead of using an empty guard space, they filled the guard space with a cyclic extension of the OFDM symbol. This effectively simulates a channel performing cyclic convolution, which implies orthogonality over dispersive channels when the CP is longer than the impulse response of the channel. This introduces an energy loss proportional to the length of the CP, but zero ICI generally motivates the loss.

OFDM systems are usually designed with rectangular pulses, but recently there has been an increased interest in pulse shaping [5, 6, 7]. By using pulses other than rectangular.

The spectrum can be shaped to be well-localized in frequency, which is beneficial from an interference point of view.

OFDM is currently used in the European digital audio broadcasting (DAB) [19] standard [8]. Several DAB systems proposed for North America are also based on OFDM, and its applicability to digital TV broadcasting is currently being investigated [9, 10, 11, 12, 13]. OFDM in combination with multiple-access techniques are subject to significant investigation, see e.g., [14, 15, 16]. OFDM, under the name DMT, has also attracted a great deal of attention as an efficient technology for high-speed transmission on the existing telephone network, see e.g., [17, 18].

2. OBJECTIVES

The objective of this project is to study a communication system using orthogonal carriers, OFDM. The focus of the study is the implementation of Zero Forcing and MMSE equalization techniques in order to reduce the interference mitigation. Then, proceed to the comparison between both equalizers.

A discrete-time OFDM system will be tested and analysed with the both equalizers. What is expected is to compare de Bit Error Rate (BER) of the system using both equalizers. Before to do that, the symbol constellation obtained after the equalization will be also analysed.

The main objective is to know which equalizer is better in determinates conditions.

3. METHODOLOGY

To design the system Matlab will be used. A discrete-time 4-QAM OFDM system using Cyclic Prefix, Equalization and AWGN will be implemented.

The project starts with a test of a theoretical system, with an ideal channel in order to verify the good implementation in Matlab.

Once the theoretical system is verified that works correctly, the system will be analysed using three different channels and the two equalizers. The BER and symbol constellation after using each equalizer will be compared and commented in each one of the three channels.

This project is organised as follows: In Section 4 we present common OFDM models, included continuous-time and discrete-time. The OFDM system which we will work with in our study is explained and showed in Section 5. The results of our OFDM system's test are in Section 6. In Section 7 we discuss and summarize the contents of this report. Finally, Sections 8 and 9 are composed by the annexes and the references.

4. OFDM SYSTEM

The basic idea of OFDM is to divide the available spectrum into several sub channels (subcarriers). By making all sub channels narrowband, they experience almost flat fading, which makes equalization very simple. To obtain a high spectral efficiency the frequency response of the sub channels are overlapping and orthogonal, hence the name of OFDM, This orthogonality can be completely maintained, even though the signal passes through time-dispersive channel, by introducing a cyclic prefix. There are several versions of OFDM, see e.g. [3, 6, 17], but we focus on systems using such a cyclic prefix [4]. A cyclic prefix is a copy of the last part of the OFDM symbol which is pretended to the transmitted symbol, see figure 4.1.

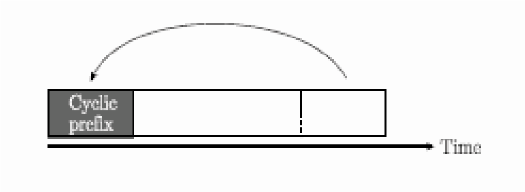


Figure 4.1. The cyclic prefix is a copy of the last part of the OFDM symbol. [4]

This makes the transmitted signal periodic, which plays a decisive roll in avoiding intersymbol and intercarrier interference [17]. This is explained later in this section. Although the cyclic prefix introduces a loss in signal-to-noise ratio (SNR), it is usually a small price to pay to mitigate interference.

A schematic diagram of baseband OFDM system is shown in Figure 4.2.

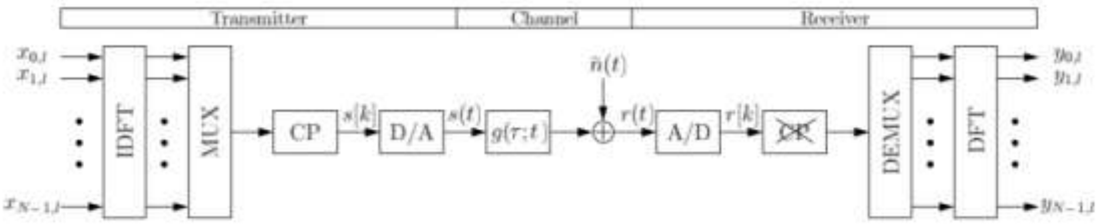


Figure 4.2 A digital implementation of a baseband OFDM system. [23]

For this system we employ the following assumptions:

- A cyclic prefix is used.
- The impulse response of the channel is shorter than the cyclic prefix.
- Transmitter and receiver are perfectly synchronized.
- Channel noise is additive, white, and complex Gaussian.

The difficulties in a complete analysis of this system make it rather awkward for theoretical studies. Therefore, its common practice to use simplified models resulting in a tractable analysis. We classify these OFDM system models into to different classes: continuous-time and discrete-time.

4.1. CONTINUOUS-TIME MODEL

The first OFDM systems did not employ digital modulation and demodulation. Hence, the continuous-time OFDM model presented below can be considered as the ideal OFDM system, which in practice is digitally synthesized. Since this is the first model described, we move through it in step-by-step fashion. We start with the waveforms used in the transmitter and proceed all the way to the receiver. The baseband model is shown in Figure 4.3.

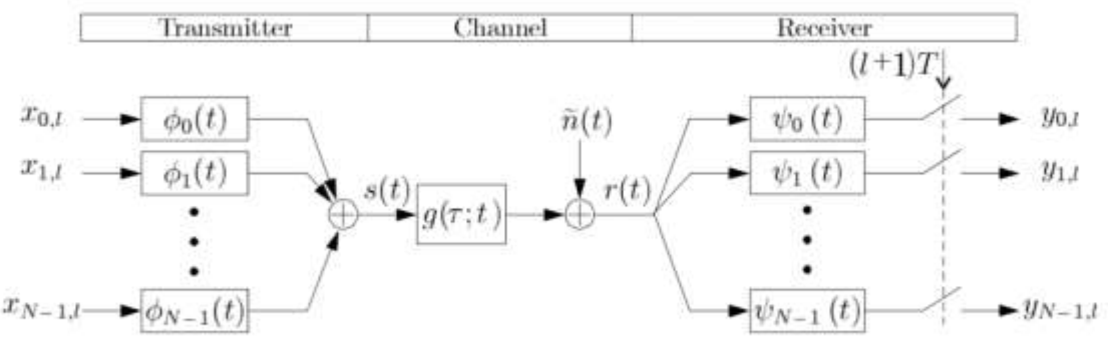


Figure 4.3. Base-band OFDM system model [23].

4.1.1 Transmitter

Assuming an OFDM system with N subcarriers, a bandwidth of W Hz and symbol length of T seconds, of which T_{cp} seconds is the length of the cyclic prefix, the transmitter uses the following waveforms [23]

$$\emptyset_k(t) = \begin{cases} \frac{1}{\sqrt{T - T_{cp}}} e^{j2\pi \frac{W}{N} k(t - T_{cp})} & if\ t \in [0, T] \\ 0 & otherwise \end{cases} , \quad (2.1)$$

where $T = N/W + T_{cp}$. Note that $\emptyset_k(t) = \emptyset_k(t + N/W)$ when t is within the cyclic prefix $[0, T_{cp}]$. Since $\emptyset_k(t)$ is a rectangular pulse modulated on the carrier frequency KW/N, the common interpretation of OFDM is that it uses N subcarriers, each carrying a low bit-rate. The waveforms $\emptyset_k(t)$ are used in the modulation and the transmitted base band signal for OFDM symbol number l is [23]

$$s_l(t) = \sum_{K=0}^{N-1} x_{k,l}\, \emptyset_l(t - lT) ,$$

where $x_{0,l}, x_{1,l}, ..., x_{N-1,l}$ are complex numbers from a set of original constellation points, When an infinite sequence of OFDM symbols is transmitted, the output from the transmitter is a juxtaposition of individual OFDM symbols [23]:

$$s(t) = \sum_{l=-\infty}^{\infty} s_l(t) = \sum_{l=-\infty}^{\infty} \sum_{k=0}^{N-1} x_{k,l}\, \emptyset_k(t - lT) , \quad (2.2)$$

4.1.2 Physical channel

We assume that the support of the (possibly time variant) impulse response g(τ;t) of the physical channel is restricted to the interval $\tau \in [0, T_{cp}]$, i.e., to the length of the cyclic prefix. The received signal becomes [23]

$$r(t) = (g * s)(t) = \int_0^{T_{cp}} g(\tau; t)\, s(t - \tau) dT + \tilde{n}(t) , \quad (2.3)$$

where ñ(t) is additive, white, and complex Gaussian channel noise.

4.1.3 Receiver

The OFDM receiver consists of a filter bank, matched to the last part [T_{cp},T] of the transmitter waveforms $\varphi_k(t)$, i.e.[23],

$$\varphi_k(t) = \begin{cases} \emptyset_k^*(T-t) & if\ t \in \left[0, T-T_{cp}\right] \\ 0 & otherwise \end{cases} . \tag{2.4}$$

Effectively this means that the cyclic prefix is removed in the receiver. Since the cyclic prefix contains all ISI from the previous symbol, the sampled output from the receiver filter bank contains no ISI. Hence we can ignore the time index l when calculating the sampled output at the kth matched filter. By using (2.2), (2.3) and (2.4), we get

$$y_k = (r * \varphi_k)(t) = \int_{-\infty}^{\infty} r(t)\, \varphi_k(T-t)dt =$$

$$= \int_{T_{cp}}^{T} \left(\int_{0}^{T_{cp}} g(\tau;t) \left[\sum_{k=0}^{N-1} x_{k'} \cdot \emptyset_k(t-\tau) \right] \emptyset_k^* d\tau \right)(t) \ + \int_{T_{cp}}^{T} \tilde{n}\,(T-t)\emptyset_k^*(t)dt$$

We consider the channel to be fixed over the OFDM symbol interval and denote it by $g(\tau)$, which gives [23]

$$y_k = \sum_{k}^{N-1} x_k \cdot \int_{T_{cp}}^{T} \left(\int_{0}^{T_{cp}} g(\tau)\emptyset_{k'}(t-\tau) \right) \emptyset_k^*\,(t)dt + \int_{T_{cp}}^{T} \tilde{n}\,(T-t)\emptyset_k^*(t)dt$$

The integration intervals are $T_{cp} < t < T$ and $0 < \tau < T_{cp}$ which implies that $0 < t\text{-}\tau < T$ and the inner integral can be written as [23]

$$\int_{0}^{T_{cp}} g(\tau)\emptyset_{k'}(t-\tau)d\tau = \int_{0}^{T_{cp}} g(\tau) \frac{e^{j2\pi k'(t-\tau-T_{cp})W/N}}{\sqrt{T-T_{cp}}} d\tau =$$

$$= \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} \int_{0}^{T_{cp}} g(\tau)\, e^{-j2\pi k'\tau W/N} d\tau, \qquad T_{cp} < t < T$$

The latter part of this expression is the sampled frequency response of the channel at frequency $f = k^{'}W/N$, i.e., at the k'th subcarrier frequency:

$$h_{k'} = G\left(k^{'}\frac{W}{N}\right) = \int_{0}^{T_{cp}} g(\tau)e^{-\frac{j2\pi k^{'}\tau W}{N}} d\tau, \tag{2.5}$$

where G(f) is the Fourier transform of g(τ). Using this notation the output from the receiver filter bank can be simplified to [23]

$$y_k = \sum_{k'=0}^{N-1} x_{k'} \int_{T_{cp}}^{T} \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} h_{k'} \emptyset_k^*(t)dt + \int_{T_{cp}}^{T} \tilde{n}\,(T-t)\emptyset_k^*(t)dt =$$

$$= \sum_{k'=0}^{N-1} x_{k'} h_{k'} \int_{T_{cp}}^{T} \emptyset_{k'}(t)\,\emptyset_k^*(t)dt + n_k \quad (2.6)$$

where $n_k = \int_{T_{cp}}^{T} \tilde{n}\,(T-t)\emptyset_k^*(t)dt$. Since the transmitter filters $\emptyset_k(t)$ are orthogonal [23]

$$\int_{T_{cp}}^{T} \emptyset_{k'}(t)\emptyset_k^*(t)dt = \int_{T_{cp}}^{T} \frac{e^{j2\pi k'(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} \frac{e^{-j2\pi k(t-T_{cp})W/N}}{\sqrt{T-T_{cp}}} dt = \delta[k-k'],$$

where $\delta[k]$ is the Kronecker delta function [19], we can simplify (2.6) and obtain

$$y_k = x_k h_k + n_k\,, \quad (2.7)$$

where n_k is the additive white Gaussian noise (AWGN).

The benefit of a cyclic prefix is twofold: it avoids ISI (since it acts as a guard space) and ICI (since it maintains the orthogonality of the subcarriers): By re-introducing the time index *l*, we may now view the OFDM system as a set of parallel Gaussian channels, according to Figure 4.4.

An effect to consider at this stage is that the transmitted energy increases with the length of the cyclic prefix, while the expressions for the received and sampled signals stay the same. The transmitted energy per subcarrier is $\int |\emptyset_k(t)|^2\, dt = T/(T-T_{cp})$, and the SNR loss, because of the discarded cyclic prefix in the receiver, becomes

$$SNR_{loss} = -10\log_{10}(1-\gamma),$$

where $\gamma = T_{cp}/T$ is the relative length of the cyclic prefix. The longer the cyclic prefix, the larger the SNR loss. Typically, the relative length of the cyclic prefix is small and the ICI-and ISI-free transmission motivates the SNR loss (less than 1 dB for $\gamma < 0.2$).

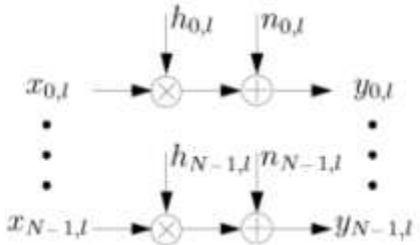


Figure 4.4. The continuous-time OFDM system interpreted as parallel Gaussian channels [*23*].

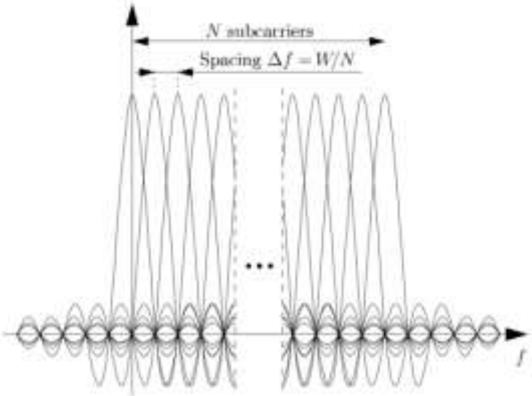


Figure 4.5.: A symbolic picture of the individual sub channels for an OFDM system with N tones over a bandwidth W [*23*].

Figure 4.5 displays a schematic picture of the frequency response of the individual sub channels in an OFDM symbol. In this figure the individual sub channels of the system are separated. The rectangular windowing of the transmitted pulses results in a sinc-shaped frequency response for each channel. Thus, the power spectrum of the OFDM system decays as f^{-2}. In some cases this is not sufficient and methods have been proposed to shape the spectrum. In [3], a raised cosine pulse is used where the roll-off region also acts as a guard space, see Figure 4.6. If the flat part is the OFDM symbol, including the cyclic prefix, both ICI

Figure 4.6.: Pulse shaping using raised-cosine function. The gray parts of the signal indicate the extensions

and ISI are avoided. The spectrum with this kind of pulse shaping is shown in Figure 4.7., where it is compared with a rectangular pulse. The overhead introduced by an extra guard space with a graceful roll-off can be a good investment, since the spectrum falls much more quickly and reduces the interference to adjacent frequency bands.

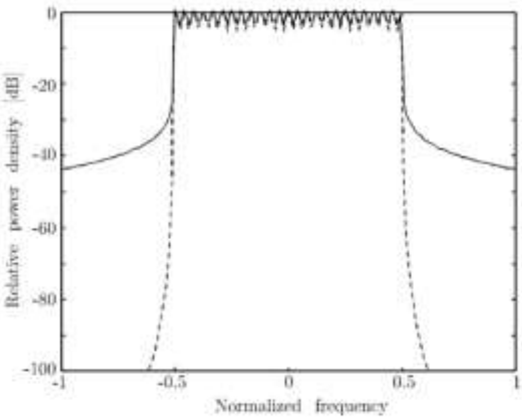


Figure 4.7.: Spectrum with rectangular pulse (solid) and raised-cosine (dashed) [23].

4.2 DISCRETE-TIME MODEL

A entirely discrete-time model of an OFDM system is displayed in Figure 4.8. Compared to the continuous-time model, the modulation and demodulation are replaced by an inverse DFT (IDFT) and a DFT, respectively, and the channel is a discrete-time convolution. The cyclic prefix operates in the same fashion in this system and the calculations can be performed in essentially the same way. The main difference is that all integrals are replaced by sums.

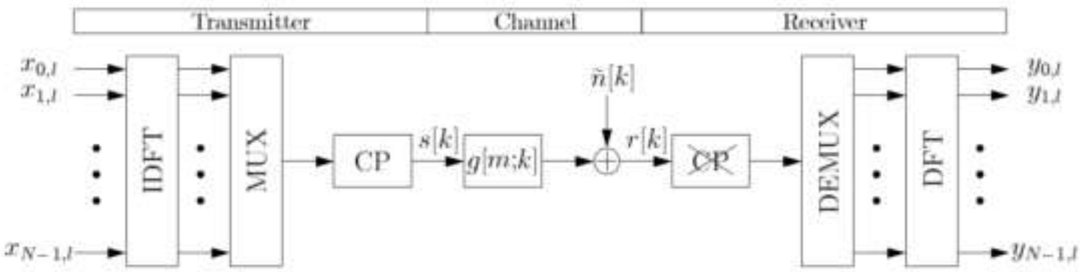


Figure 4.8: Discrete-time OFDM system [23].

From the receiver's point of view, the use of a cyclic prefix longer than the channel will transform the linear convolution in the channel to a cyclic convolution. Denoting cyclic convolution by '⊗' , we can write the whole OFDM system as [23]

$$y_l = DFT(IDFT(x_l) \otimes \mathrm{g_l} + \tilde{\mathrm{n}}_\mathrm{l}) = DFT(IDFT(x_l) \otimes \mathrm{g_l}) + n_l \ ,$$

where y_l contains the N received data points, x_l the N transmitted constellation points, g_l the impulse response of the channel (padded with zeros to obtain a length of N), and $\tilde{n}_l$ the channel noise.

Notice that

$$X(k) = DFT\{x[n]\} = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk} \ , \qquad 0 \le k \le N-1$$

$$x[n] = IDFT\{X(k)\} = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}nk} \ , \qquad 0 \le n \le N-1$$

Since the channel noise is assumed white and Gaussian, the term $n_l = DFT(\tilde{n}_l)$ represents uncorrelated Gaussian noise. Further, we use that the DFT of two cyclically convolved signals is equivalent to the product of their individual DFTs. Denoting element-by-element multiplication by '.', the above expression can be written [23]

$$y_l = x_l \cdot DFT(g_l) + n_l = x_l \cdot h_l + n_l,$$

where $h_l = DFT(g_l)$ is the frequency response of the channel. Thus we have obtained the same type of parallel Gaussian channels as for the continuous-time model. The only difference is that the channel attenuations h_l are given by de *N-point DFT* of the discrete-time channel, instead of the sampled frequency response as in (2.5).

4.3. EQUALIZATION

The equalizer is a linear filter that provides an approximate inverse of the channel response. Since it is common for the channel characteristics to be unknown or to change over time, the preferred embodiment of the equalizer is a structure that is adaptive in nature. In digital communications, purpose of equalizer is to reduce intersymbol interference to allow recovery of the transmit symbols [20]. It may be a simple linear filter or a complex algorithm. One of the practical problems in digital communications is inter-symbol interference (ISI), which causes a given transmitted symbol to be distorted by other transmitted symbols. The ISI is imposed on the transmitted signal due to the band limiting effect of the practical channel and also due to the multi-path effects (echo) of the channel. One of the most commonly used techniques to counter the channel distortion (ISI) is linear channel equalization. The equalizer is a linear filter that provides an approximate inverse of the channel response. Since it is common for the channel characteristics to be unknown or to change over time, the preferred embodiment of the equalizer is a structure that is adaptive in nature.

Conventional equalization techniques employ a pre-assigned time slot (periodic for the time-varying situation) during which training signal, known in advance by the receiver, is transmitted. In the receiver the equalizer coefficients are then changed or adapted by using some adaptive algorithm (e.g. ZF, MMSE,…) so that the output of the equalizer closely matches the training sequence.

4.3.1 Zero Forcing Equalizer

Zero Forcing Equalizer refers to a form of linear equalization algorithm used in communication systems which applies the inverse of the frequency response of the channel.

The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel [21]. The name Zero Forcing corresponds to bringing down the intersymbol interference (ISI) to zero in a noise free case. This will be useful when ISI is significant compared to noise.

For a channel with frequency response $F(f)$ the zero forcing equalizer $C(f)$ is constructed by:

$$C(f) = \frac{1}{F(f)}$$

Thus the combination of channel and equalizer gives a flat frequency response and linear phase $C(f)F(f) = 1$.

In reality, zero-forcing equalization does not work in some applications, for the following reasons:

- Even though the channel impulse response has finite length, the impulse response of the equalizer needs to be infinitely long

- At some frequencies the received signal may be weak. To compensate, the magnitude of the zero-forcing filter ("gain") grows very large. As a consequence, any noise added after the channel gets boosted by a large factor and destroys the overall signal-to-noise ratio. Furthermore, the channel may have zeroes in its frequency response that cannot be inverted at all.

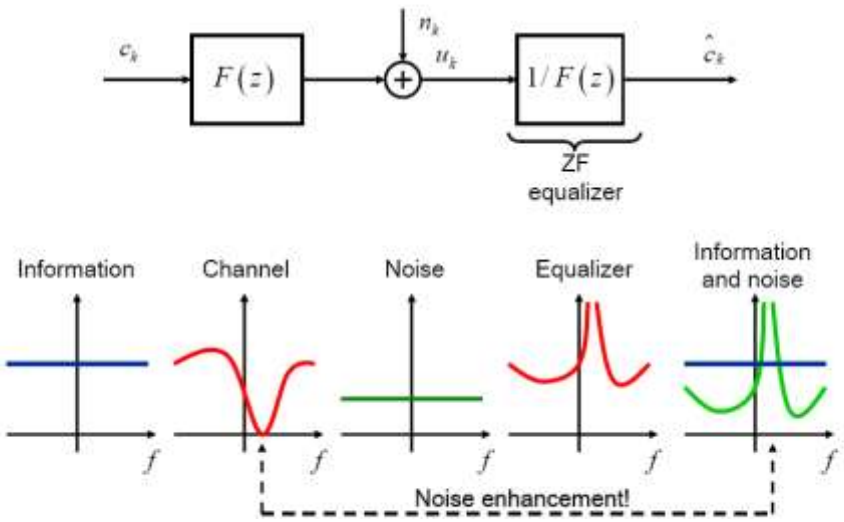


Figure 4.9. Zero Forcing Equalize [*21*].

These problems are addressed in the linear MMSE equalizer.

4.3.2 MMSE Equalizer

To minimize the intersymbol interference and additive noise effects, the equalizer coefficients can be optimized using the minimum mean squared error (MMSE) criterion. When the SNR has elevated values the MMSE equalizer works as Zero Forcing does, but when the SNR has lower values, the fact that MMSE equalizer takes into account the noise and signal variance, makes to not amplify the noise as Zero Forcing does.

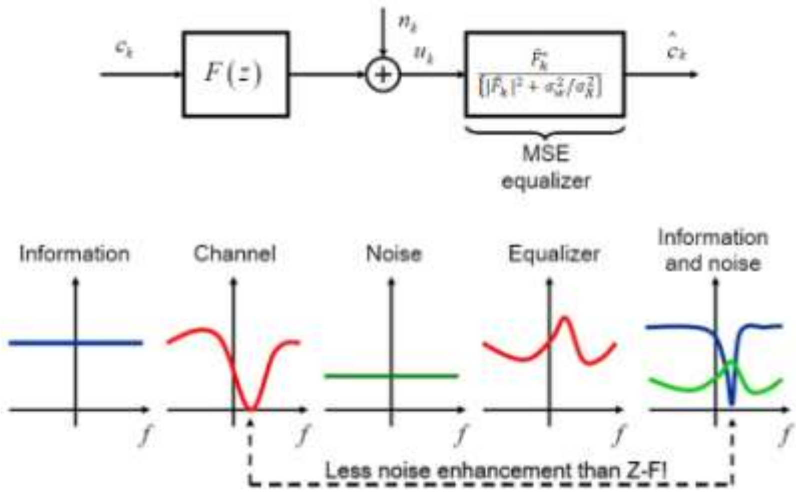


Figure 4.10: MMSE equalizer [*21*].

As it can be seen in the Figure 4.10., when the Signal to Noise Ratio (SNR) has high values, the MMSE equalizer works as the Zero Forcing does, but for the rest of values that SNR can take, the MMSE equalizer works better in terms of distortion.

That is got for the term σ_w^2/σ_x^2 , because when a deep null appears in the frequency response of the channel, this equalizer does not amplifies the received signal just multiplying for the inverse of the channel, it takes into account the SNR in order to not amplify so much the noise term.

This is the big difference about the two equalizers and it can be clearly appreciated in the deep nulls of the channel. While Zero Forcing equalizer increases significantly the noise term in order to recover the original signal, MMSE does not do.

5. DESIGN OF THE IMPLEMENTED SYSTEM

As we told before, we are going to work with a discrete-time OFDM system, with the following characteristics:

- 4-QAM signal with an input data of k = 159744 bits and 64 subcarriers.
- Modulator and Demodulator.
- Cyclic Prefix.
- IFFT and FFT blocks.
- Discrete Channel.
- AWGN.
- Equalization.

We show all of this in the following diagram block, in the same order as we will work with.

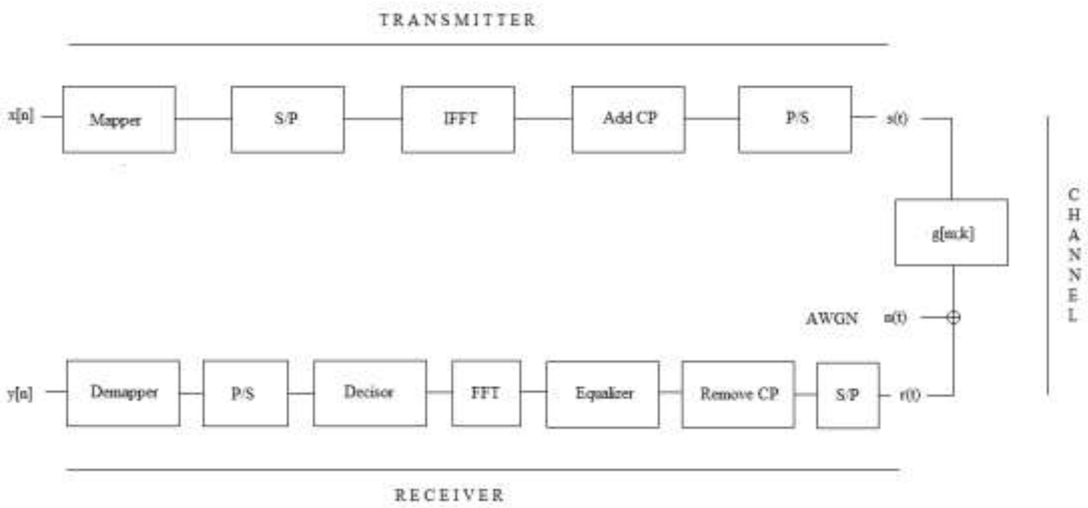


Figure 5.1.: OFDM system diagram block

We will proceed with an accurate explanation of each element used in the design, starting from the transmitter to the receiver.

5.1. TRANSMITTER

As we can see in Figure 5.1. the transmitter is composed for the next blocks:

- Convert input bits to symbols
- IFFT (Inverse Fast Fourier Transformation)
- CP (Cyclic Prefix)

- **Convert input bits to symbols**

The first thing we have to do in the system is convert the input data into symbols. As we told before, our data is a chain of 159744 bits. Our system 4-QAM needs 4 symbols so what we have done is get pairs of bits ant convert into symbols, as we can see:

x=[*00 01 10 11 00 ... ...*] *#159744* bits

For example, bits *00* are the symbol *1+j*, *01* are *-1+j*, *10* are *1-j* and *11* are *-1-j*.

So we get the four symbols needed: ±*1*±*j*. After doing the conversion we have a vector with *k/2* symbols, where k is the total number of bits.

- **IFFT (Inverse Fast Fourier Transformation)**

Before implementing the IFFT we need to convert our serial vector of symbols into a matrix of *SC* rows and $\frac{k/2}{N}$ columns, where SC is the number of subcarriers, in our case *N = 16*.

It is needed because the IFFT block takes SC symbols at a time as a subsymbol, where each one is orthogonal from the others.

The SC subsymbols are then sent to an IFFT block that performs an N-point inverse fast fourier transform. Recall that an inverse fast fourier transform converts frequency-domain data into the time-domain. Hence, the user's original input data is a treated by OFDM as though it is in the frequency-domain. The output of the inverse fast fourier transform block is an N-time domain samples.

- **Cyclic Prefix**

After computing the N inverse fast fourier transform output samples, a cyclic prefix of length L_p samples is pre-pended to the N samples to form a cyclically extended OFDM symbol. The cyclic prefix is simply the last L_p samples of the N inverse fast fourier transform, output samples.

For example, suppose $N=4$ and $L_p=2$. If the output of the 4-point inverse fast fourier transform was [*1 2 3 4*], the cyclic prefix would be [*3 4*]. The cyclically extended symbol would be [*3 4 1 2 3 4*]. Therefore, after pre-pending the cyclic prefix, the length of the transmitted OFDM symbol is $N+L_p$.

Pre-pending the cyclic prefix exploits a concept from discrete-time linear system theory to aid in removing the effects of the channel at the receiver. In continuous time, the convolution of two time-domain signals is equivalent to multiplying the Fourier transforms of two signals.

This property is not true in discrete-time unless N is infinite or at least one convolved signal is periodic (in which case the signals are circularly convolved). Although we are limited to using a finite N, the cyclic prefix makes the OFDM symbol appear periodic over the time spam of interest.

In other case, if $N=16$ and $L_p=4$, and the output of the 16-point inverse fast fourier transform was [*1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16*], the cyclic prefix would be [*13 14 15 16*], and the cyclically extended symbol would be [*13 14 15 16 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16*].

After that, the $N + L_p$ samples of the cyclically extended symbol are passed through a parallel-to-serial converter to be transmitted in series across the channel.

5.2. CHANNEL

Testing the system, we will use with three different channels in order to get more results and information about the equalization. The three channels with their respectively frequencies responses are:

- Channel 1: *g(k)= [0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07]*

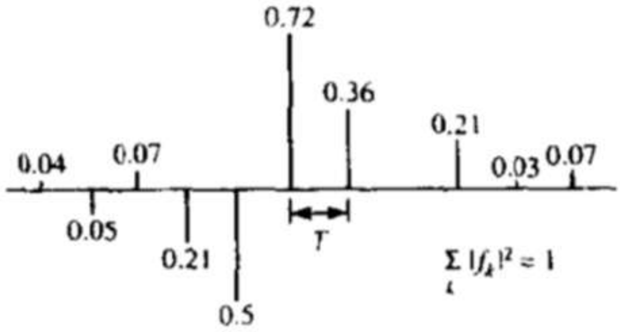


Figure 5.2.: Channel 1 [*20*].

- Channel 2: $g_2(k)$=*[0.407 0.815 0.407]*

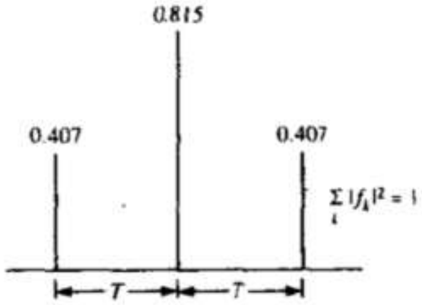


Figure 5.3.: Channel 2 [*20*].

- Channel 3: $g_3(k)$= *[0.227 0.460 0.688 0.460 0.227]*

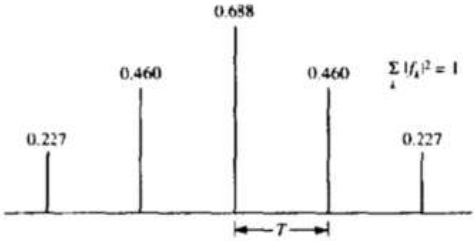


Figure 5.4: Channel 3 [*20*].

After the channel, the AWGN noise is added.

5.3. RECEIVER

The OFDM receiver basically does the reverse of what was done at the transmitter.

The following blocks, seen in Figure 3.1, composes the receiver system:

- Remove CP
- FFT (Fast Fourier Transformation)
- Equalizer
- Convert symbols to bits

The received signal $r_{1 \ldots N+Lp \ldots k}$ is in the time-domain. It is distorted due to the effects of the channel. The received signal first goes through a serial-to-parallel converter in order to remove the cyclic prefix.

- **Remove CP**

Removing the cyclic prefix does not get rid of any information since the data in it is redundant. The advantage of using the cyclic prefix was that it allowed the previously mentioned convolution property to hold.

There is another advantage if the value of L_p is greater than or equal to the size of the channel memory. Intersymbol interference can occur when the multipath channel causes delayed versions of precious OFDM symbols to corrupt the current received symbol. However, if L_p is large enough, the intersymbol interference will only affect the cyclic prefix. The actual OFDM symbol will arrive unharmed. The value of L_p should be greater than or equal to the size of the channel memory.

Since the cyclic prefix is discarded, the result is to also discard the effects of intersymbol interference. Hence, the use of a property sized cyclic prefix allows OFDM to remove the effects of intersymbol interference.

- **FFT (Fast Fourier Transformation)**

After removing the cyclic prefix, the signal is passed through an N-point fast fourier transform to convert the signal back to the frequency domain.

The signal is distorted due to the effects of the channel. Again, by introducing a cyclic prefix, the circular convolution of the time domain transmitted signal with the impulse response of the channel is equivalent to multiplying the frequency response of the transmitted signal with the channel's frequency response. Thus, the received signal (with subscripts removed for clarity) can be written as

$$Y = HX + n$$

where Y is the received signal's frequency response of the channel, X is the transmitted signal's frequency response and n is an additive noise term.

The channel's effect is to multiply each bin of the transmitted signal's discrete fourier transform by a complex number. The complex number's value is the value of the corresponding bin in the discrete fourier transform of the channel response. After that, takes place the channel equalization.

- **Equalization**

In order to analyse the system, two different equalizers are used: The Zero Forcing and MMSE.

The first one, Zero Forcing channel equalization can be performed by dividing the received signal by the channel's discrete fourier transform. This operation can be written as:

$$Y_{equalized} = \frac{Y}{C} = X + \frac{n}{C}$$

Thus, the equalized symbol will ideally equal the transmitted symbol plus as noise term.

Although the channel response is initially unknown, it can be determined with the use of a training sequence. However, this equation has problems where there is a deep null in the frequency response of the channel. Then, one or more of the bins of the channel's discrete fourier transform will have very small values. When the receiver equalizes the received signal, the term n/C will take on large values for the bins where the nulls occurred. Instead of equalizing that bin, this process will amplify the noise and produce a highly distorted output.

To minimize the intersymbol interference and additive noise effects, the equalizer coefficients can be optimized using the minimum mean squared error (MMSE) criterion, the MMSE equalizer told before.

This optimization yields [21]

$$C_{k,MMSE} = \frac{\widehat{H}_k^*}{\{|\widehat{H}_k|^2 + \sigma_w^2/\sigma_X^2\}}$$

where σ_w^2 is the variance of the additive noise, and σ_X^2 is the variance of the transmitted data symbols.

After equalization, the signal is passed through a parallel-to-series converter. The output is the receiver's estimate of the original OFDM symbol.

- **Convert the output symbols into bits**

At the end of the system we just need to the opposite that we did in the beginning. After the estimation of the original OFDM symbol, they have to be converted into bits. The output symbols are in a vector with a *k/2* length, where k is the total number of bits.

This symbols will be converted into bits following the same structure used at the beginning.

For example, symbol *1+j* are bits *00, -1+j* are *01, 1-j* are 10 and *-1-j* are *11*.

Then a vector of *k* bits is obtained, which is the output data.

Finally, to get the BER (Bit Error Rate), the input data must be compared with the output data.

6. RESULTS

In this section we are going to analyse the equalization of the system designed, using the three channels and two equalizers described in the section 3.

As we told before, our time-discrete OFDM system has the following characteristics:

- 159744 bits of input data.
- 4-QAM.
- 64 subcarriers.
- Cyclic prefix with 16 unit length.
- Channel: One of the three channels described in section 3.
- AWGN noise.
- Equalization: Zero-Forcing or MMSE.

The first thing that we must do is test the system with an ideal channel, to be sure that we have designed correctly.

The channel will be: *g(k)*=h[1]. This channel does not distort the transmitted signal, so we do not need to use any equalizer to test it.

After the code simulation, the plot obtained of the BER respect the SNR is:

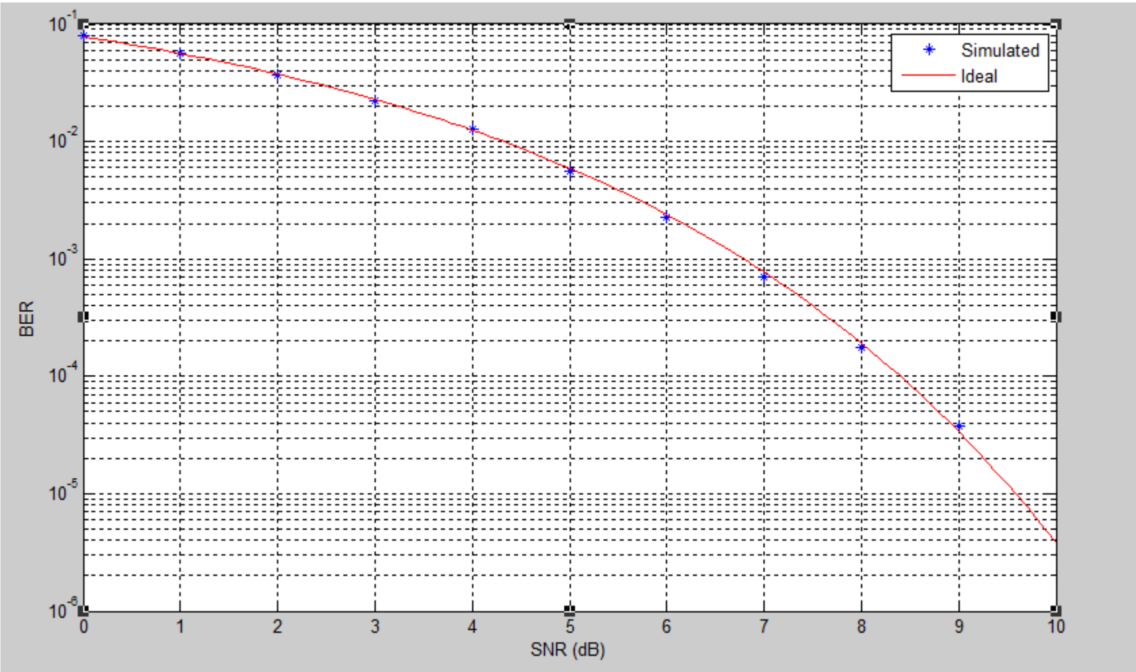


Figure 6.1.: BER of an ideal channel

The red wire is the plot of the ideal BER for an OFDM system. As it can be observed in the Figure 6.1., the code works correctly, so we will proceed to do the analysis with the three channels and using the two equalizers described.

6.1 CHANNEL 1

As it is told in section 3, the channel is: *h*=[*0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07*], shown in the Figure 6.2. and his frequency response is shown in the Figure 6.3.:

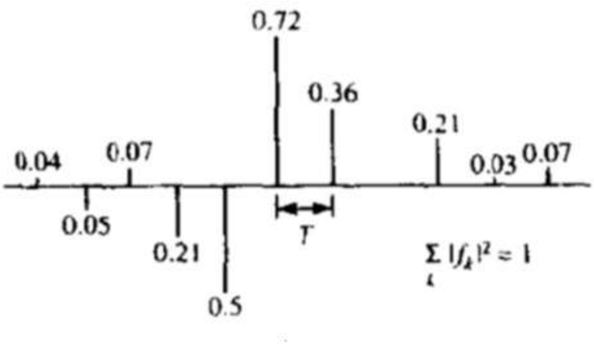


Figure 6.2.: Channel 2 [20].

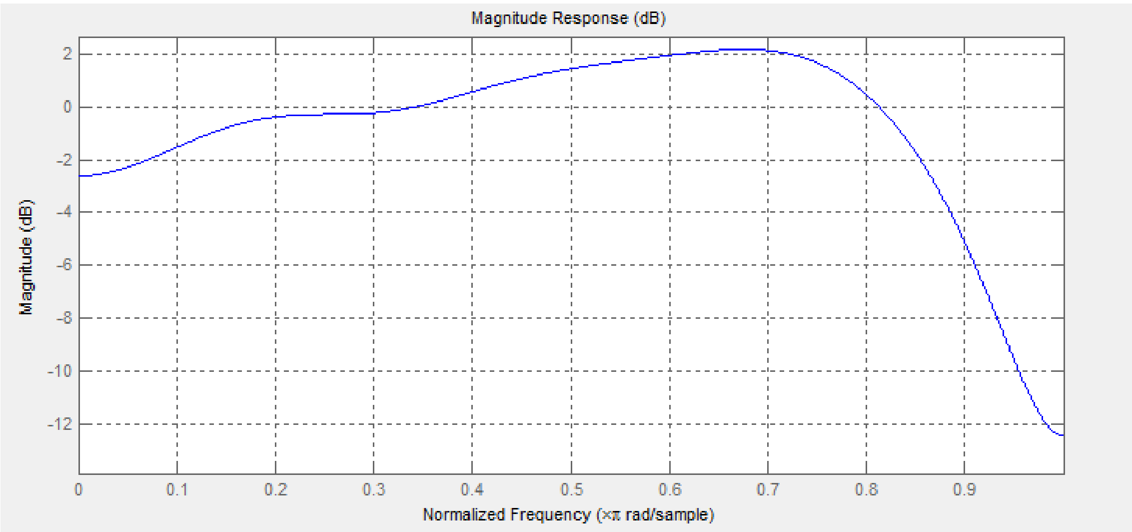


Figure 6.3.: Frequency response of the channel 1.

Firstly, we will start analysing the effects of the channel in the transmitted signal. As we know, we are transmitting a 4-QAM signal as it can be seen in the Figure 6.4.:

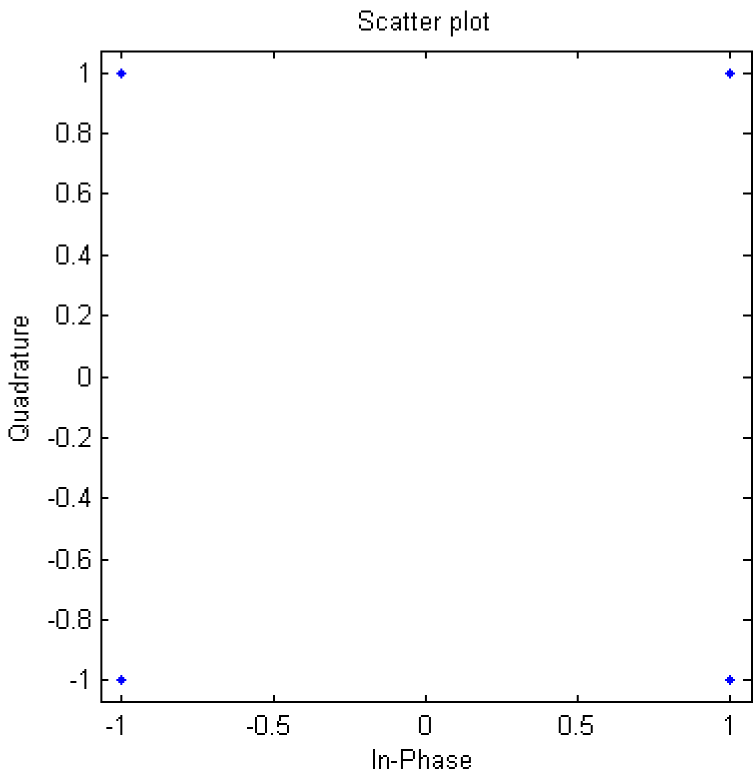


Figure 6.4.: 4-QAM constellation

After that, the signal passes through the following steps:

- 64-Point Inverse Fast Fourier Transformation
- Addition of cyclic prefix.
- Channel.
- AWGN.
- Cyclic prefix removal.
- 64-Point Fast Fourier Transformation.

Then, we are just before the equalization. The signal is affected in phase and amplitude due to the effects of the channel and noise. In the next figures can be seen the received signal in the constellation of different subcarriers at SNR=10dB.

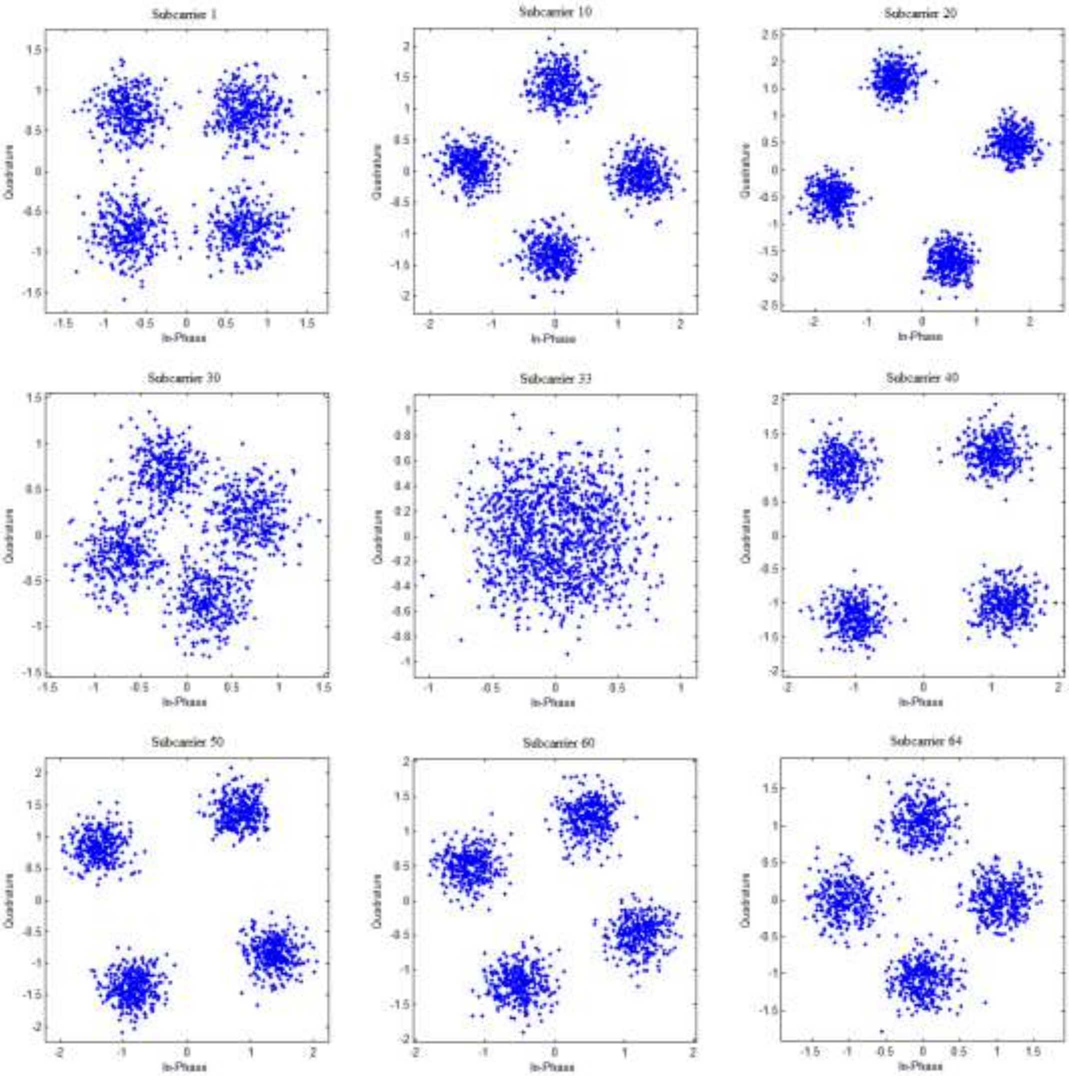


Figure 6.5.: Constellation of different subcarriers before the equalization.

To know which frequency response of the channel affects which subcarrier, $2\pi\frac{\mathrm{k}}{\mathrm{N}} = 0.66\pi$, where N is the total number of subcarriers and k is an specific subcarrier.

It can be observed that each subcarrier is affected for different points of the frequency response of the channel. The frequency response of the channel in the case of the subcarrier 30 is very close to -10dB, so for this reason a lot of dispersion is seen. In the constellation plot of the subcarrier 33 we can appreciate that all we have is distortion, this is caused due to the frequency response of the channel is much lower than the SNR. Oppositely, in the case of subcarrier 20, the frequency response of the channel is very close to 2dB, so few dispersion appears.

After that, what it is studied is how each equalizer works with the different frequencies responses of the channel. We will analyse how Zero forcing and MMSE equalizers work in the subcarriers 1, 24 and 33.

These subcarriers are chosen because:

- Frequency response of the channel for subcarrier 24 is close to 2 dB, which is the higher point.
- Frequency response of the channel for subcarrier 33 is close to -12 dB, which is the moss attenuated point.
- And frequency response of the channel for subcarrier 1 is close to -2 dB, which is an intermediate point.

As done before, will be compared the constellations of the different subcarriers at SNR=10dB.

Firstly, in the Figure 6.6. are shown the constellations of these subcarriers before the equalization, where we can see the effects of the channel and the noise in the distorted received signal.

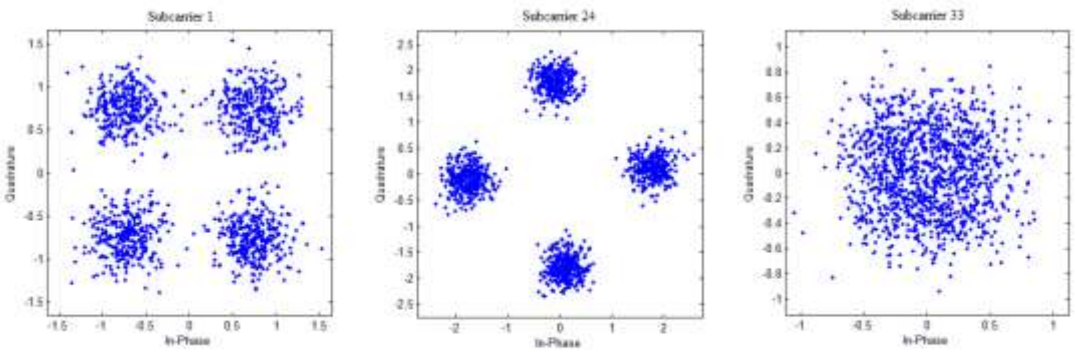


Figure 6.6.: Constellation of subcarriers 1, 24 and 33 before the equalization.

Secondly, in the Figure 6.7. are shown the constellations of the same subcarriers after doing the Zero Forcing equalization.

Remember that Zero Forcing equalizer does the inverse that the channel does:

$$Y_{equalized} = \frac{Y}{C} = X + \frac{n}{C}$$

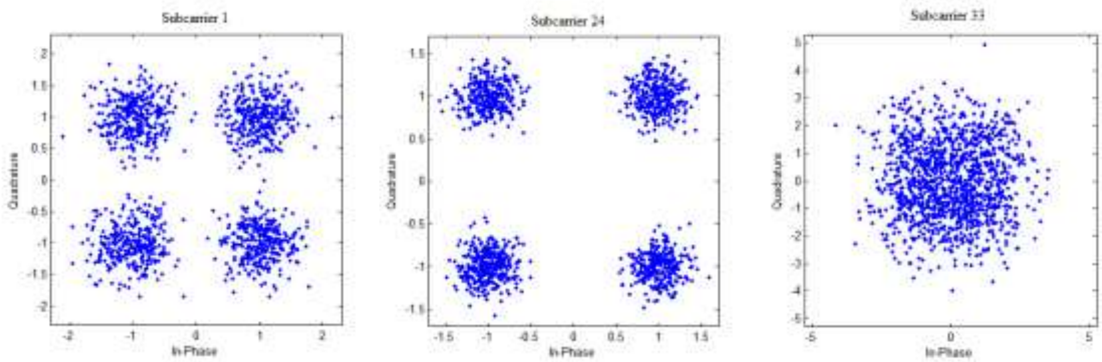


Figure 6.7.: Constellation of the subcarriers 1, 24 and 33 at SNR = 10dB after the Zero Forcing equalization.

It can be clearly seen that the phase and amplitude has been rectified. However, this equalizer has problems where there is a deep null in the frequency response of the channel. Then, one or more of the bins of the channel's discrete fourier transform will have very small values. When the receiver equalizes the received signal, the term *n/C* will take on large values for the bins where the nulls occurred. Instead of equalizing that bin, this process will amplify the noise and produce a highly distorted output.

Finally, in the Figure 6.8. are shown the constellations of the same subcarriers after doing the MMSE equalization.

To minimize the intersymbol interference and additive noise effects, the MMSE equalizer coefficients can be optimized using the minimum mean squared error (MMSE) criterion:

$$C_{k,MMSE} = \frac{\widehat{H}_k^*}{\left\{|\widehat{H}_k|^2 + \sigma_w^2/\sigma_X^2\right\}}$$

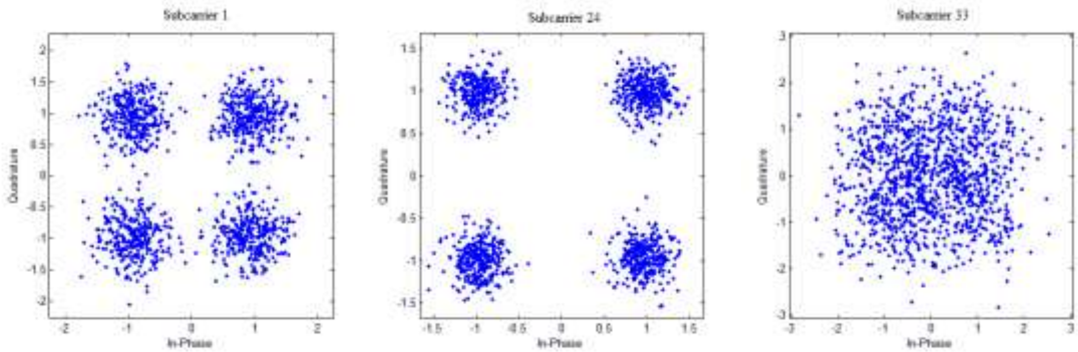


Figure 6.8.: Constellation of the subcarriers 1, 24 and 33 at SNR = 10dB after the MMSE equalization.

The problem of the phase and amplitude has been corrected also. But in this case, it can be seen in the subcarrier 33 that less distortion is obtained. In the constellation can be appreciate that the amplitude of the symbols more distorted ups to 3, when in the case of the Zero Forcing Equalizers ups to 5. The reason is because, as told in section 2, the MMSE equalizer does not amplify the noise as much as zero forcing does when the frequency response of the channel has a small value.

In the case of subcarrier 33, the received signal cannot be well equalized because for the reason told before; the frequency response of the channel in this point is -12 dB in front of the SNR of 10 dB.

What we have done is analyse the constellation of the subcarrier 33 with an SNR of 15 dB. The results can be seen in the Figure 4.9.

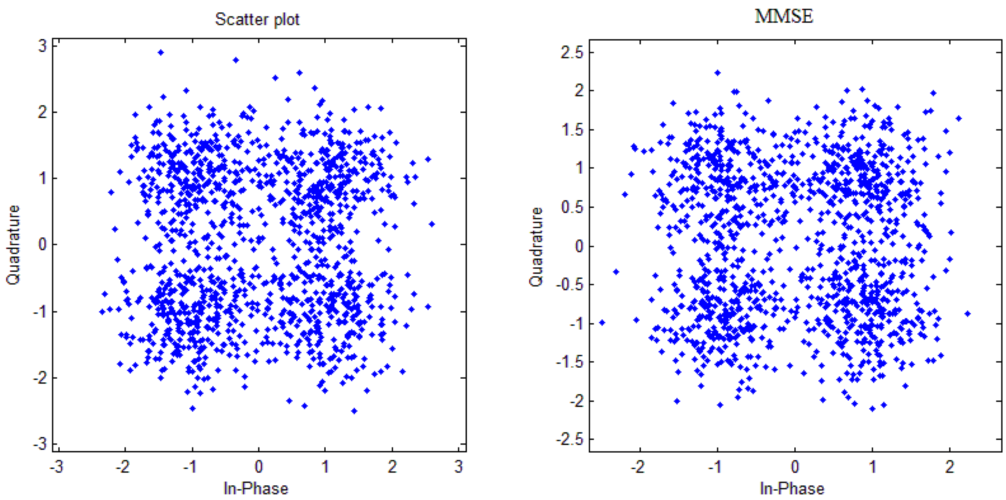


Figure 6.9.: Constellation of the subcarrier 33 at SNR = 15 dB after Zero Forcing and MMSE equalization.

In the figure above, the frequency response of the channel is close to -12 dB and the SNR is 15 dB. It can be seen that the MMSE equalizer has les distortion when the frequency response of the channel has lower values. It is caused because the Zero Forcing Equalizer is multiplying the noise term by the inverse of the channel, and when lower is the frequency response, higher becomes the noise term.

In the Figure 4.10. can be seen the comparison plot of the BER of the system using Zero Forcing and MMSE equalizers.

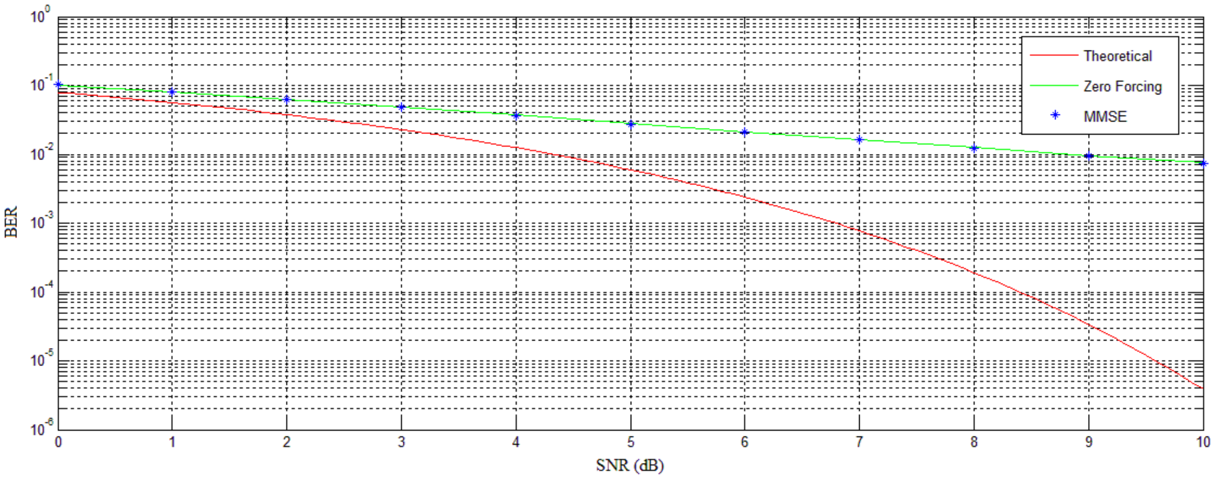


Figure 6.10.: BER of Zero Forcing and MMSE comparison.

What can be seen in the Figure is that the Zero-Forcing equalizer has the same BER than the MMSE. It is strange because, the MMSE equalizer, as is demonstrated before, works better and has less distortion in the output than Zero Forcing equalizer.

But, has less distortion not implicates that the BER has to be better. As is told in the Section 5, the coefficients of the two equalizers are:

- Zero Forcing: $C_k = \frac{1}{H_k}$

- MMSE: $C_{k,MMSE} = \frac{\hat{H}_k^*}{\{|\hat{H}_k|^2 + \sigma_W^2/\sigma_X^2\}}$

where H_k is the k-FFT point of the channel.

Then, we will explain why the BER of Zero Forcing and MMSE equalizers is the same.

We have seen that due to the channel, the output symbols are more amplitude distorted using the Zero Forcing equalizer than the MMSE. It can be more clearly appreciated when the frequency response of the channel has a null or a low value, because the Zero Forcing equalizer amplifies more the noise term. The MMSE distorts less the output symbols because the coefficients are optimized with the Minimum Mean Squared Error and takes care of the noise amplification with the factor σ_w^2/σ_X^2 . It explains why the output symbols are less distorted using MMSE equalizer.

But, in the case of the phase distortion, both equalizers have the same results. It is caused because:

Suppose $H(k) = a + jb$, the phase of the k term using Zero Forcing equalizer is $-arc\ tg(\frac{b}{a})$.

In the MMSE, the denominator is a real number, so it only affects to the amplitude distortion. We have $H^*(k) = a - jb$, so the phase is $arc\ tg\left(\frac{-b}{a}\right) = -arc\ tg(\frac{b}{a})$.

It has been proved that the phase term of the output symbols is the same using the Zero Forcing equalizer or the MMSE. That explains why the BER is the same; the output symbols are equally phase equalized, the difference is the amplitude distortion, where MMSE works better than Zero Forcing. The fact that the differences are only in the amplitude distortion, makes not possible to appreciate differences after the decision, where only is observed the sign of the real and imaginary part of the symbols.

For example, if one output symbol using the Zero Forcing is *0.5 – 0.5j* and the same symbol using the MMSE is *0.25 – 0.25j*, where can be observed that have the same phase but different amplitude distortion, both will be transformed into *1 – 1j* to do the comparison with the input symbols and then get the BER.

6.2 CHANNEL 2

The second channel that is used in order to analyse the system is: *h*=[*0.407 0.815 0.407*]

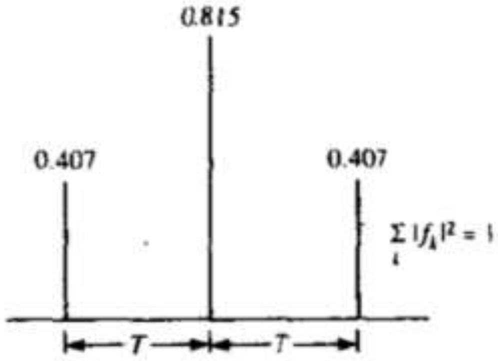


Figure 6.11.: Channel 2 [*20*].

Before analysing the system using the equalizers, what must be known is the frequency response of the channel.

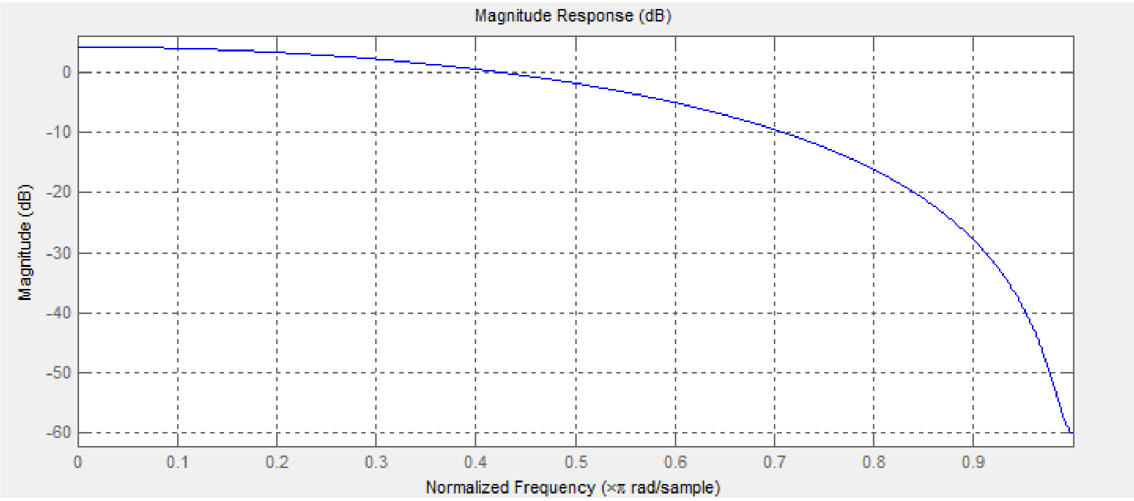


Figure 6.12.: Frequency response of the channel 2.

Remind that the OFDM system is working with 64 subcarriers. So, it can be appreciate in the Figure that the first 16 and the last 16 subcarriers will not be very affected for the channel because its frequency response is close to 0dB. The real problem of this channel appears from the subcarrier 25 to the subcarrier 33, where the frequency response is lower than -10dB. In the case of the subcarrier 33, the frequency response is -60dB, what makes very difficult equalize this symbol.

In the Figure 6.13., can be seen the constellation of different subcarriers at the receiver with an SNR of 10dB, just before the equalization with an SNR of 10dB.

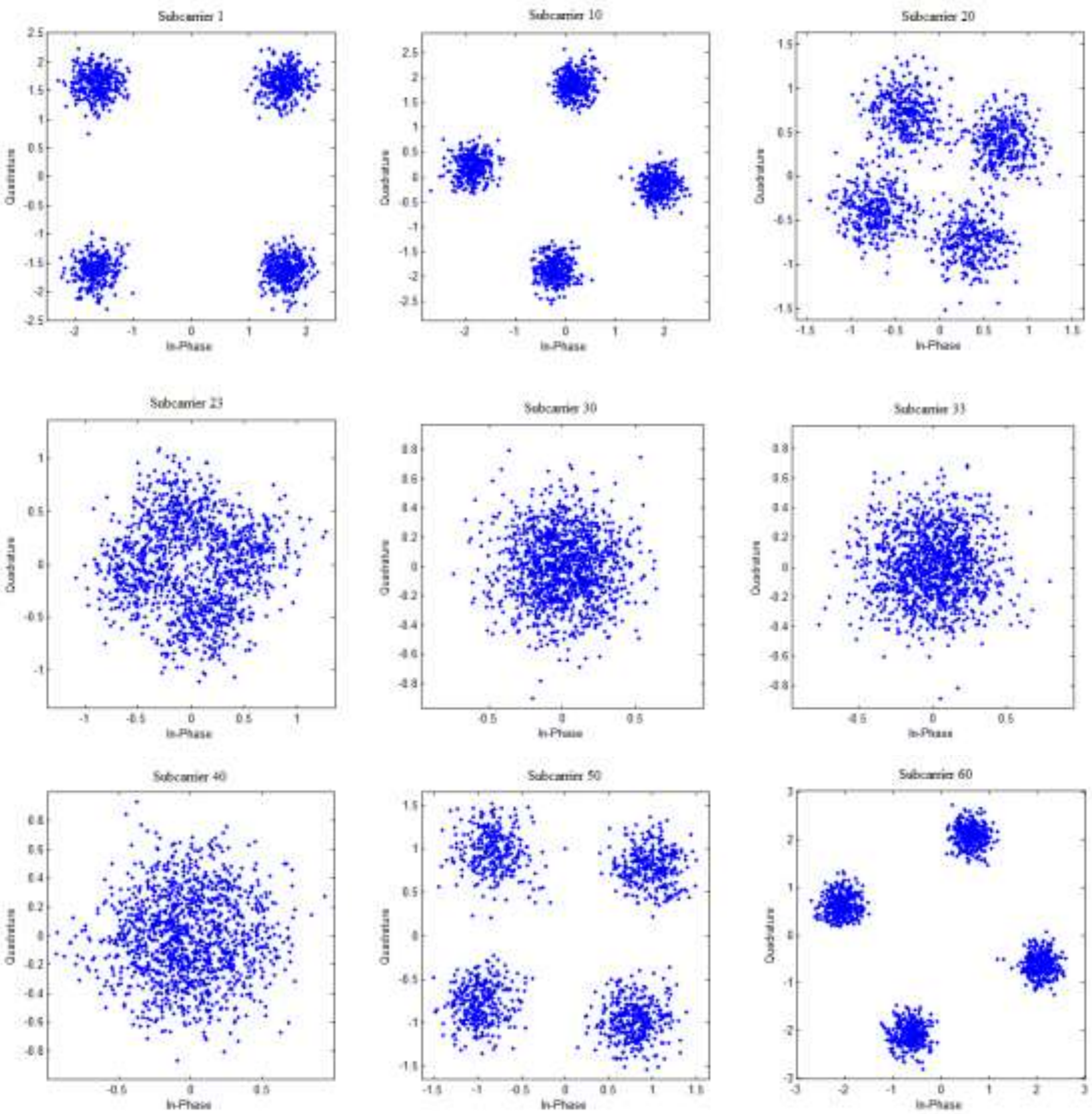


Figure 6.13.: Constellation of different subcarriers before the equalization with SNR = 10dB.

As told before, until the subcarrier 20, the signal at the receiver does not have too much distortion so it can be good equalized. In the constellation of the subcarrier 23 can be appreciated that the frequency response of the channel begins to decrease. Then from subcarrier

25 to 33, only distortion can be appreciated. It is because the frequency response of the channel is much lower than the SNR.

With an increase of the SNR to 30dB an improvement in the received signal will be noticed.

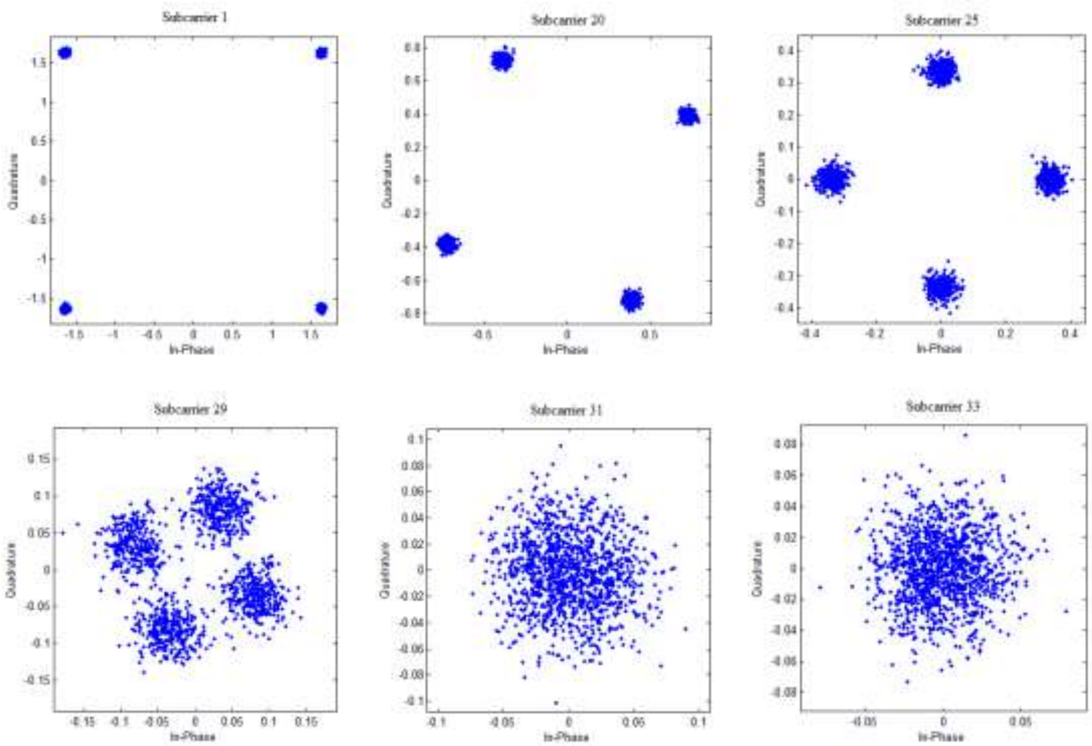


Figure 6.14.: Constellation of different subcarriers before the equalization with SNR=30dB.

The distortion of the first subcarriers has been significantly improved. Now in the subcarrier 29, can be appreciate that the SNR compensates the frequency response of the channel, but the subcarriers 31 to 33 cannot be well received because the SNR does not compensate the frequency response of the channel, which is from -35dB to -60dB in the worst case. In order to receive the subcarrier 33 more clear a SNR of 60 dB is needed to compensate the frequency response of the channel.

As we did with the channel 1, the system will be analysed after the equalization, starting for Zero Forcing, then MMSE and finishing with the BER comparison.

In the next figure can be seen the Zero Forcing equalization of the subcarriers 25, 29, 31 and 33, using an SNR of 30dB.

This subcarriers are chosen because is where the effects of the equalization can be more clearly observed. From the 1st subcarrier until the 25th the SNR compensates the effects of the channel and the differences between Zero Forcing and MMSE cannot be easily appreciated.

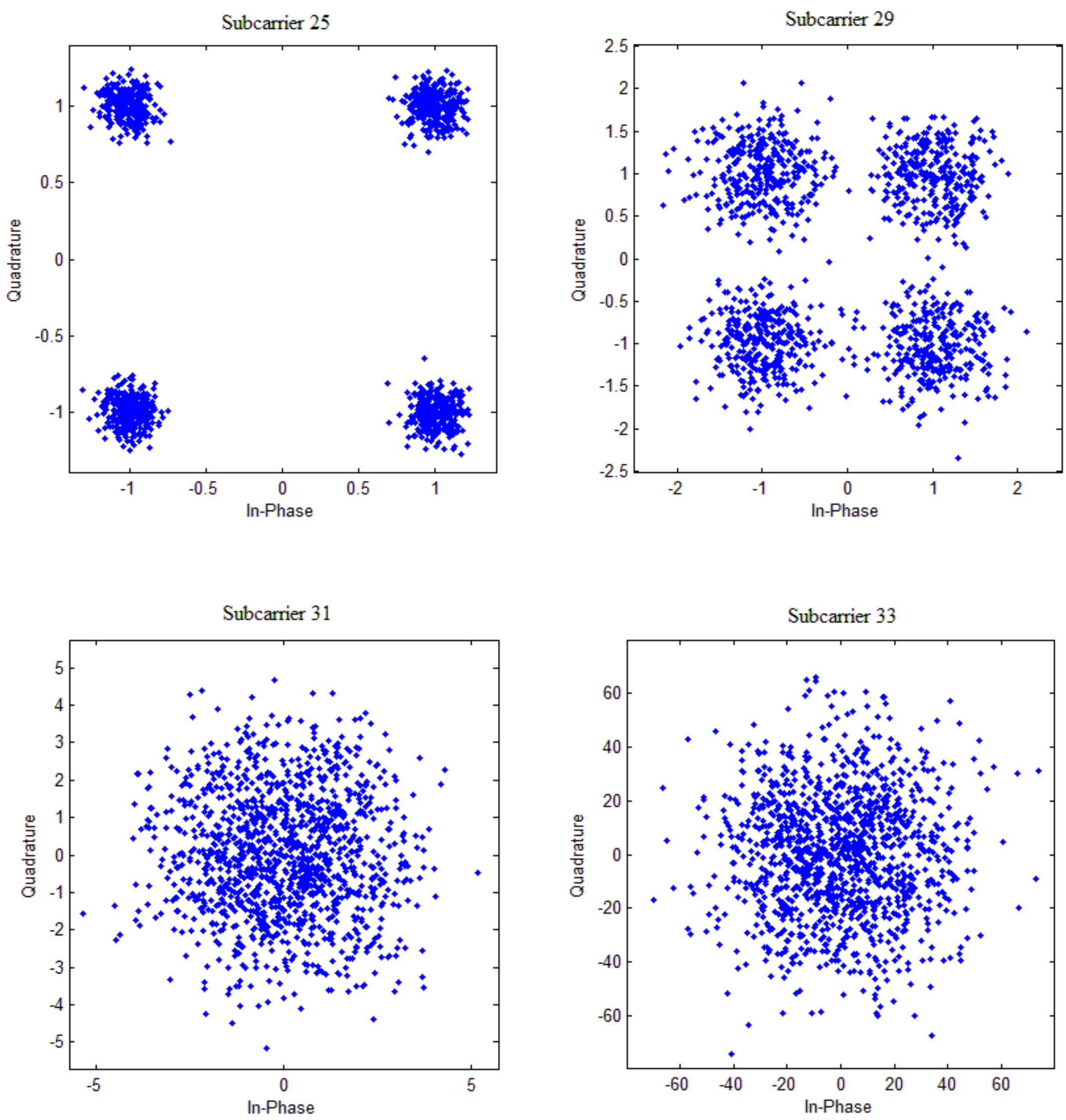


Figure 6.15. Constellation of the subcarriers 25, 29, 31 and 33 at SNR=30dB after the Zero Forcing equalization.

It can be appreciated in the first two constellations that the equalizer has worked. The phase has been corrected and the amplitude of the symbols is around 1, which is the transmitted amplitude. In the last two constellations can be observed that the SNR is not compensating the effects of the channel, and that the low values in the frequency response of the channel has made a high increase to the amplitude distortion.

As it was told in the case of the channel 1, equalizing with the Zero Forcing, when a null appears in the frequency response of the channel, makes to increase heavily the noise term. For this reason a high amplitude distortion is observed.

Using the MMSE equalizer and plotting the same subcarriers, the following constellations can be observed:

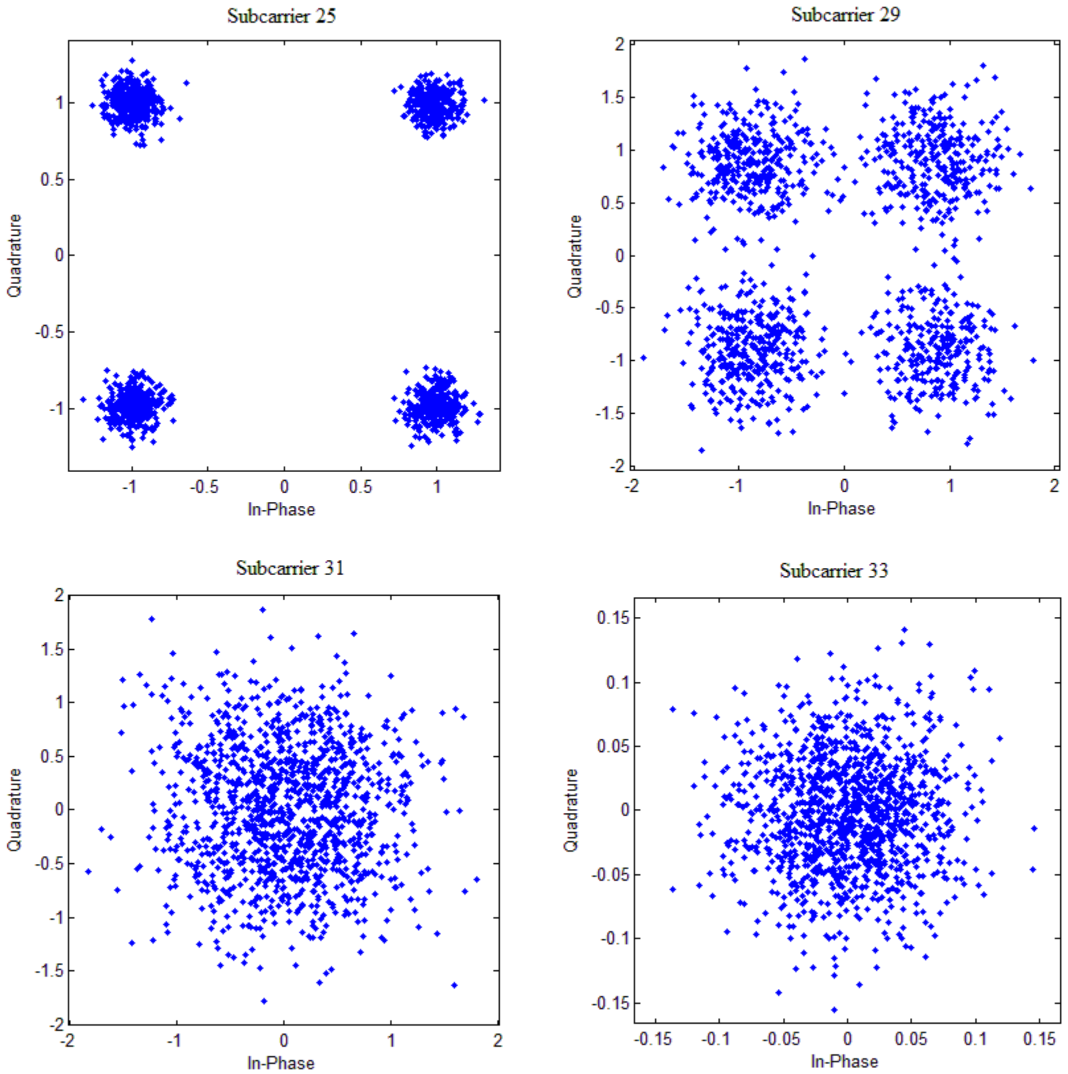


Figure 6.16. Constellation of the subcarriers 25, 29, 31 and 33 at SNR=30dB after the MMSE equalization.

In the first picture, can be observed the constellation of the subcarrier 25, where is practically the same than the obtained with the Zero Forcing equalizer. It is because the SNR compensates well the effects of the channel.

In the second picture, where the constellation of the subcarrier 29 is observed, the SNR just compensates the channel’s effects, so can be observed that the amplitude distortion is a little bit lower than the obtained with the Zero Forcing.

And finally, in the last two constellations, where the SNR does not compensate the effects of the channel, the distortion amplitude of the subcarriers 31 and 33 is much lower than the obtained with the Zero Forcing equalizer.

It confirms that MMSE works equalizes much better than the Zero Forcing when the SNR cannot compensate the effects of the channel.

To finish the analyse of the channel 2, in the following figure a plot of the BER comparing the Zero Forcing and MMSE equalizers can be observed.

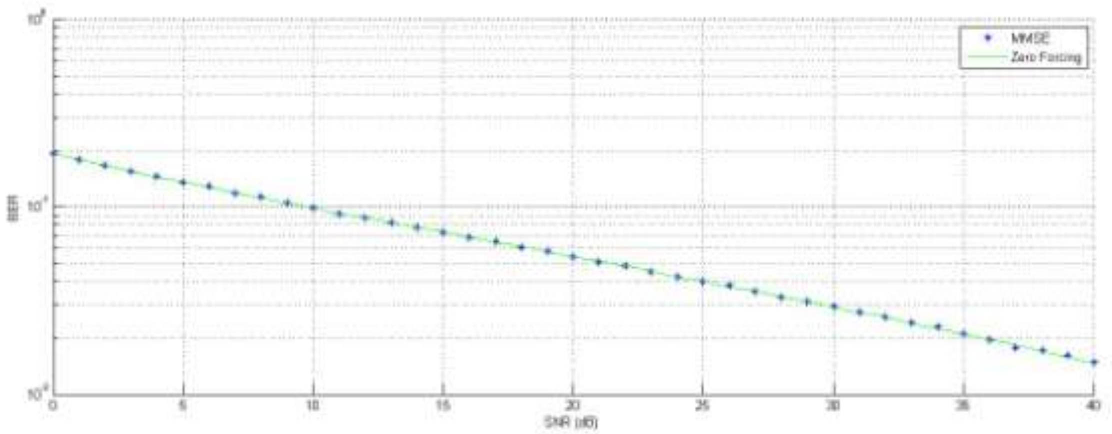


Figure 6.17.: BER of Zero Forcing and MMSE comparison.

6.3 CHANNEL 3

The third and the last channel that will be used to compare the Zero Forcing and MMSE equalizers is: h = [*0.227 0.460 0.688 0.460 0.227*].

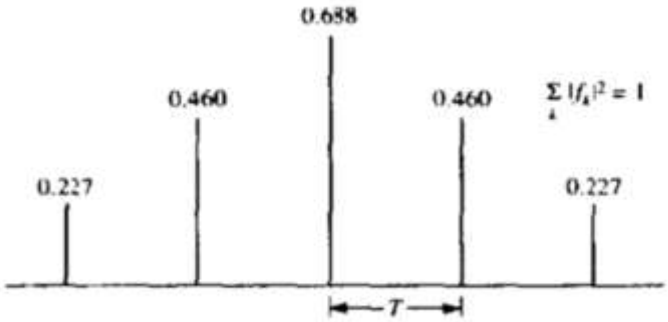


Figure 6.18: Channel 3[*20*].

And in the following figure is shown its frequency response:

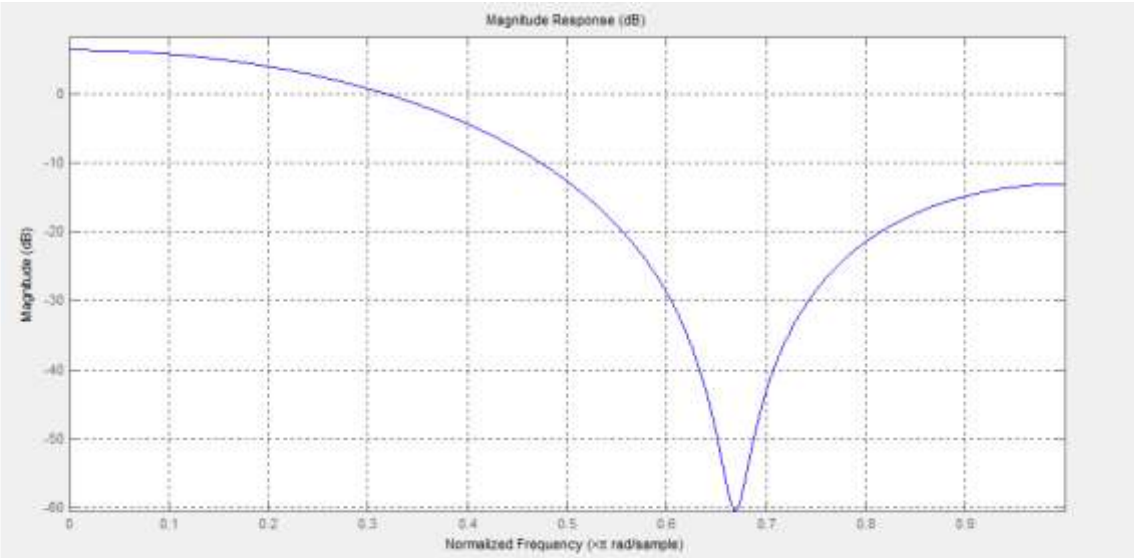


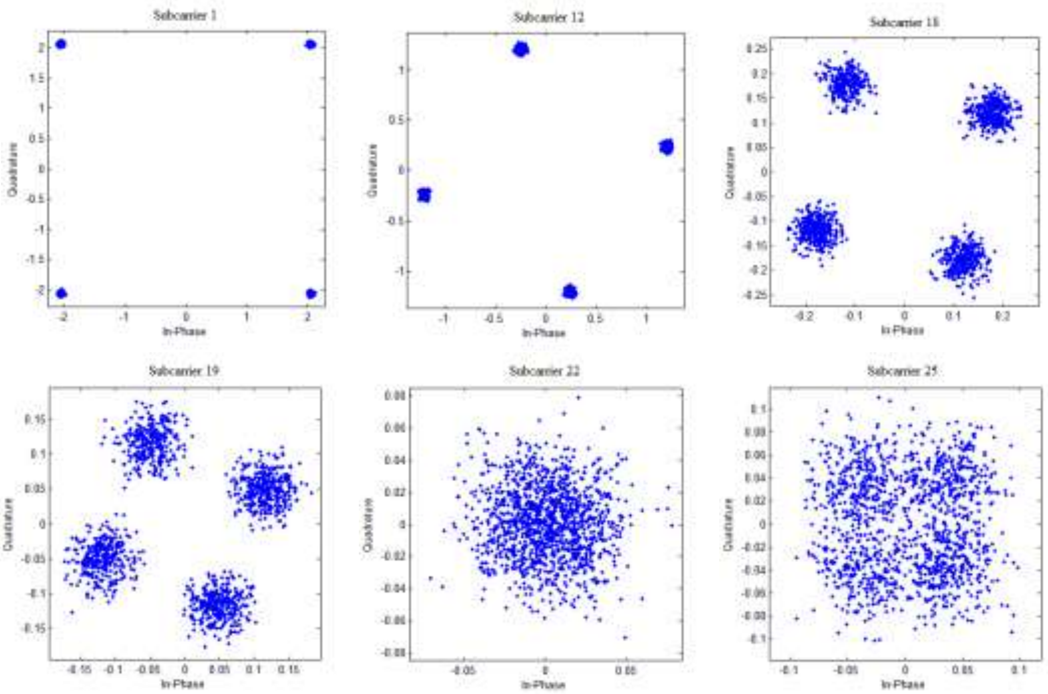
Figure 6.19. Frequency response of the channel 3.

It can be clearly appreciate a deep null. It is in the point 0.66π rad/sample, which corresponds to the subcarrier 21.

To know which frequency response of the channel affects which subcarrier, $2\pi\frac{k}{N} = 0.66\pi$, where N is the total number of subcarriers and k is an specific subcarrier. When k = 21, is the point of the deep null, so the subcarrier 31 is the most affected by the channel. Can be noticed that the channel is symmetric, so the subcarrier 42 will be equally affected as the subcarrier 21.

Also can be seen, that from the subcarrier 1 to the subcarrier 15 the channel is no affecting so much, is between the subcarriers 18 to 24 that the signal will be highly distorted. Then, the last subcarriers will be a little bit more affected than the first's ones.

What is explained before, is shown in the next figure, where the constellation of different subcarriers is analysed using an SNR = 30dB.



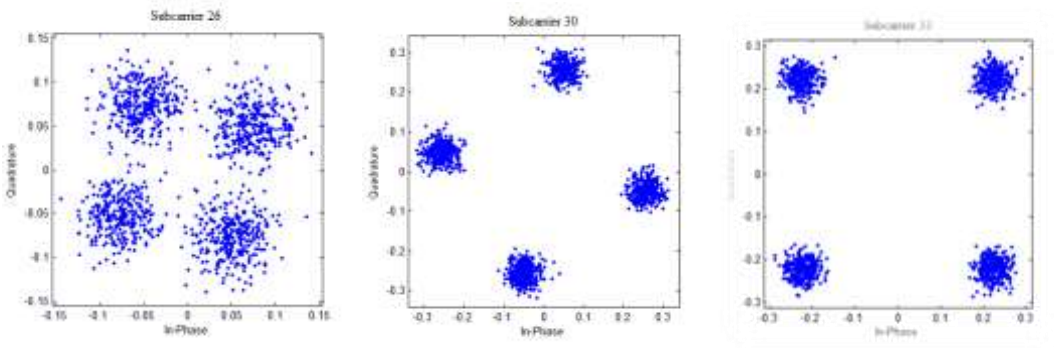


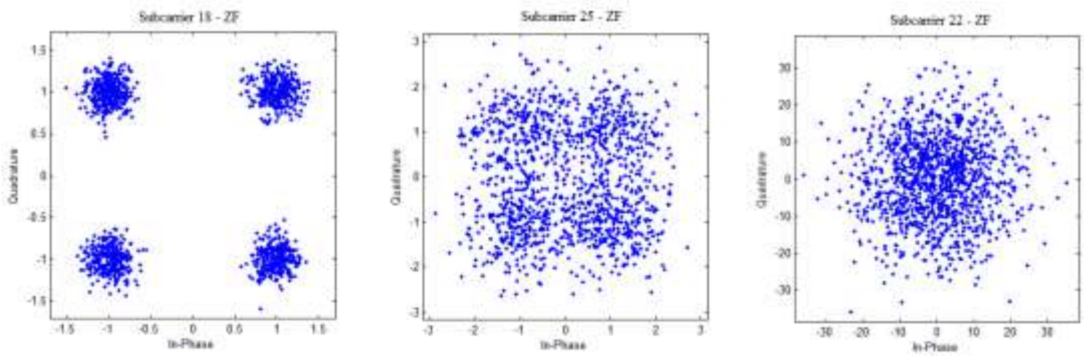
Figure 6.20. Constellation of different subcarriers before the equalization with SNR=30dB.

In the first three constellations and in the last two it can be appreciated that the SNR of 30 dB compensates the attenuation of the channel and the output symbols are received with a few distortion.

The constellation of the subcarriers 19, 35 and 26 shows a little bit more distortion than the others told before because the frequency response of the channel is lower.

Finally, in the constellation of the subcarrier 22 can be appreciated that a deep null is in the frequency response of the channel. To compensate this, an SNR of at least 60 dB is needed, which makes very difficult to recuperate this part of information.

Then, we will compare the Zero Forcing equalizer and MMSE using the constellation of the subcarriers 18, 22 and 25, using an SNR of 30 dB. These subcarriers are chosen because one is with less attenuation than 30dB, another with a 30 dB attenuation and another in the deep null, so big different results will be obtained.



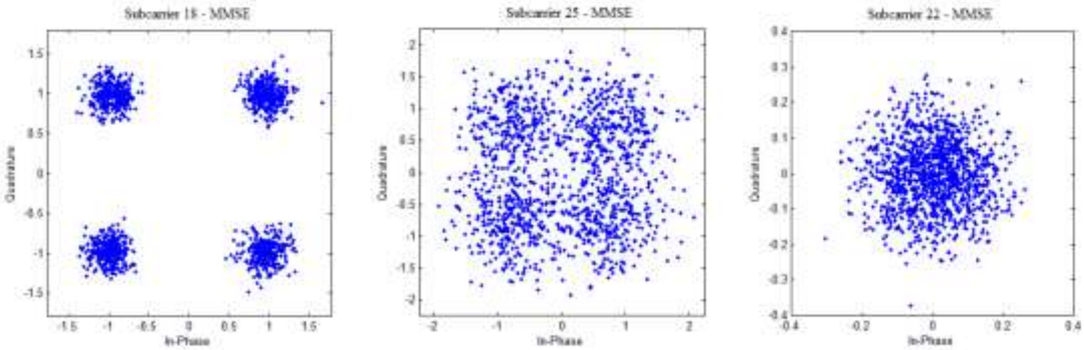


Figure 6.21: Comparison constellations using Zero Forcing and MMSE.

In the constellation of the subcarriers 18, where the SNR compensates the attenuation introduced by the channel, there is no difference between the plot using MMSE and the plot using Zero Forcing. Both equalize the phase and the amplitude distortion well.

Then, in the constellation of the subcarriers 25, where the attenuation introduced by the channel is higher, can be observed that the MMSE equalizer produce less amplitude distortion to the output symbols than the Zero Forcing does. But also, can be seen that both equalizers equalize well the phase distortion, because, as told at the beginning of this section, both equalizers works equal with the phase distortion.

Finally, in the constellation plot of the subcarriers 22, can be clearly seen that MMSE equalizer is by far, better than the Zero Forcing when a deep null appears in the frequency response of the channel. The distortion amplitude of the output symbols equalized by the MMSE is much lower than the equalized by the Zero Forcing.

To conclude the analysis of this channel and of the system, the next figure shows a comparison BER of the system equalized by Zero Forcing and MMSE.

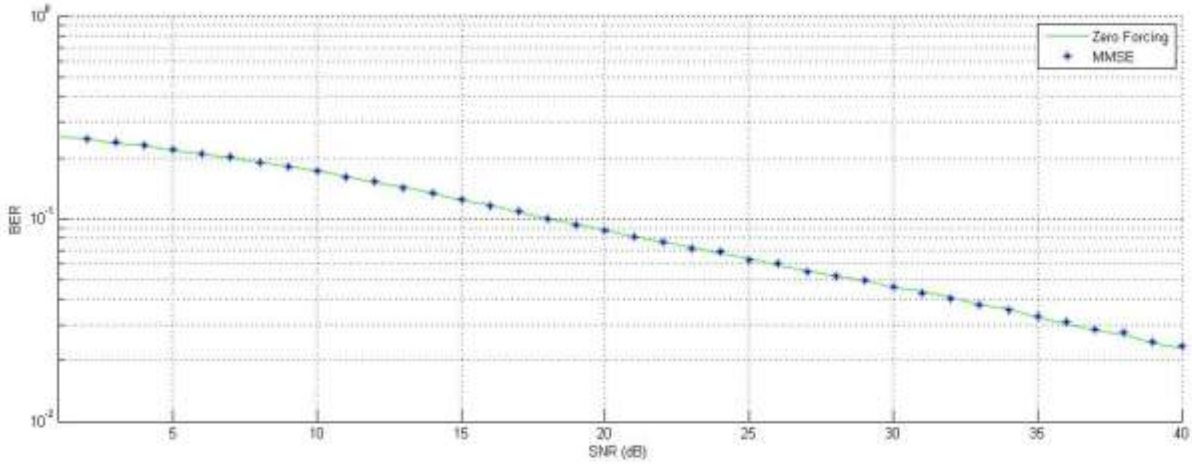


Figure 6.22.: BER comparing Zero Forcing and MMSE with the channel 3.

As told at the end of the analysis of the channel 1, we check that the BER is the same using Zero Forcing or MMSE.

7. CONCLUSION

This work presents a simulation study on the performance comparison of Zero Forcing and MMSE equalizers in a discrete time OFDM system using three different linear channels.

The OFDM system is performed with:

- 159744 bits of input data.
- 4-QAM symbols.
- 64 subcarriers.
- Use of Cyclic Prefix.
- AWGN

After all the simulations, has been observed that the Zero Forcing equalizer removes all the ISI and is ideal only when the channel is noiseless. However, this equalizer has problems where there is a deep null in the frequency response of the channel. Then, one or more of the bins of the channel's Discrete Fourier Transform will have very small values. When the receiver equalizes the received signal, the noise term will take on large values for the bins where the nulls occurred. Instead of equalizing that bin, this process will amplify the noise and produce a highly distorted output.

To minimize the intersymbol interference and additive noise effects, the equalizer coefficients can be optimized using the minimum mean squared error (MMSE) criterion. When the SNR has elevated values the MMSE equalizer works as Zero Forcing does, but when the SNR has lower values, the fact that MMSE equalizer takes into account the noise and signal variance, will not amplify the noise as Zero Forcing does.

Finally, analysing the BER of the both equalizers, we have noticed that is the same. It happens because both equalizers has the same phase, the unique difference is that MMSE gets less amplitude distortion of the signal when the frequency response of the channel has small values.

All the results given have been simulated and analysed during the project. For this reason there are no references in the results section. Getting a good contribution to the study of OFDM equalization.

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9. ANNEXES

In this section the code implemented to analyse the OFDM discrete time system is shown.

```
format long
N=159744;% #bits
SC=64; % #subcarriers
CP=16;  % #cycle prefix
SNR=30;
z=zeros(1,SNR+1);
%h=zeros(1,SC);
h=1; %ideal channel
h=[0.04 -0.05 0.07 -0.21 -0.5 0.72 0.36 0 0.21 0.03 0.07 ]; %channel 1
h=[0.407 0.815 0.407]; %channel 2
h=[0.227 0.460 0.688 0.460 0.227]; %cannel 3

smat=zeros(SC,(N/2)/SC);
smatifft=zeros(SC,(N/2)/SC);
xt=zeros(1,SC);
xr=zeros(1,SC);
t=zeros(SC,(N/2)/SC);
xscat=zeros(SC,(N/2)/SC);
xrfftscat=zeros(SC,(N/2)/SC);
rmat=zeros(SC,(N/2)/SC);
xtx=zeros(1,N/2);
ts=zeros(1,N/2);

vectcp=zeros(1,SC+CP);
smatcp=zeros(SC+CP,(N/2)/SC);
vectrcp=zeros(SC,1);
rmatcp=zeros(SC+CP,(N/2)/SC);

I=eye(SC);
W=zeros(1,SC);

%create input chain of bits

x=randi(2,1,N);
x=x-1;

%convert bits into symbols

s=zeros(1,N/2);
b=1;
c=2;
for d=1:N/2
if x(b)==0 && x(c)==0
s(d)=1+1j;
elseif x(b)==0 && x(c)==1
s(d)=-1+1j;
elseif x(b)==1 && x(c)==1
```

```
s(d)=-1-1j;
elseif x(b)==1 && x(c)==0
s(d)=1-1j;
end
b=b+2;
c=c+2;
end

for d=1:N/2
    smat(d)=s(d);
end

for k=1:(SNR+1)

%ifft

for e=1:((N/2)/SC)
    xt=smat(:,e);
    xtifft=ifft(xt);
    smatifft(:,e)=xtifft;
end

%cyclic prefix

 for e=1:((N/2)/SC)
    smatcp(1:CP,e)=smatifft(SC-CP+1:SC,e);
    smatcp(1+CP:SC+CP,e)=smatifft(1:SC,e);
end

%paralel to serial

for d=1:((N/2)+CP*((N/2)/SC))
    xtx(d)=smatcp(d);
end

%variance tx

potx=0;
for m=1:((N/2)+CP*((N/2)/SC))
    potx=potx + (abs(xtx(1,m)))^2;
end
px=potx/((N/2)+CP*((N/2)/SC));

%channel

xtxch=conv(xtx,h);
trch=zeros(1,(N/2)+CP*((N/2)/SC));
for d=1:((N/2)+CP*((N/2)/SC))
  trch(d)=xtxch(d);
end
```

```
%variance after the channel to perform the AWGN

pot=0;
for m=1:((N/2)+CP*((N/2)/SC))
    pot=pot + (abs(trch(1,m)))^2;
end
ps=pot/((N/2)+CP*((N/2)/SC));
var=ps/(4*10^((k-1)/10));


%noise

nr=randn(1,((N/2)+CP*((N/2)/SC)));
nr=sqrt(var)*nr;

ni=randn(1,((N/2)+CP*((N/2)/SC)));
ni=sqrt(var)*ni;
ni= 1i*ni;

n=nr+ni;


%variance of the noise

potn=0;
for m=1:((N/2)+CP*((N/2)/SC))
    potn=potn + (abs(n(1,m))^2);
end
pn=potn/((N/2)+CP*((N/2)/SC));


r = trch + n ;


%serial to paralel

for d=1:((N/2)+CP*((N/2)/SC))
    rmatcp(d)=r(d);
end


%remove cyclic prefix

for e=1:((N/2)/SC)
    rmat(:,e)=rmatcp(1+CP:SC+CP,e);
end
```

```
%coeficients mmse equalizer

H=fft(h,64);
Ckmmse=zeros(1,SC);
for d=1:SC
   Ckmmse(d)= H(d)'/(abs(H(d))^2 + (pn/ps) );
end


%coeficients zero forcing equalizer

H=fft(h,64);
Ckzf=zeros(1,SC);
for d=1:SC
   Ckzf(d)= H(d)'/(abs(H(d))^2 + (pn/ps) );
End


%fft

for e=1:((N/2)/SC)
    xr=rmat(:,e);

    xrfft=fft(xr);
    xrfftscat(:,e)=xrfft;

    for d=1:SC
        xrfft(d)=xrfft(d)*(Ckmmse(d) or Ckzf(d)); %equalization
    end
    xscat(:,e)=xrfft;
    t(:,e)=xrfft;
end

for d=1:N/2
    ts(d)=t(d);
end

f=zeros(1,N);

fr=real(ts);
fi=imag(ts);

%decisor

for d=1:N/2
    if fr(d)>0
        fr(d)=1;
    elseif fr(d)<0
        fr(d)=-1;
    end
end

for d=1:N/2
```

```
   if fi(d)>0
      fi(d)=1;
   elseif fi(d)<0
      fi(d)=-1;
   end
end

b=1;
c=2;

%convert symbols into bits

for d=1:N/2
   if fr(d)==1 && fi(d)==1
      f(b)=0;
      f(c)=0;
   elseif fr(d)==1 && fi(d)==-1
      f(b)=1;
      f(c)=0;
   elseif fr(d)==-1 && fi(d)==-1
      f(b)=1;
      f(c)=1;
   elseif fr(d)==-1 && fi(d)==1
      f(b)=0;
      f(c)=1;
   end
   b=b+2;
   c=c+2;
end

%BER

d=1;
count=0;
for d=1:N
   if x(d)==f(d)
   else count=count+1;
   end
end
BER=count/N;
z(k)=BER;
end

%theoretical BER

yid=0:0.1:SNR;
z_id=qfunc(sqrt(2*10.^(yid/10)));

%BER plots

y=0:1:SNR;
semilogy(yid,z_id, 'r-')
hold on
semilogy(y,z, 'b*')
```