

Study for the numerical resolution of conservation equations of mass, momentum and energy to be applied to solar thermal collectors

**Bachelor's thesis
Annex D: Code of the program**

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1 Code of the program

1.1 Main program

```

1 #include <iostream>
2 #include <sstream>
3 #include <fstream>
4 #include <vector>
5 #include <cmath>
6 #include <cassert>
7 #include <cstdlib>
8 #include <string>
9
10 #define PI 3.14159265
11
12 using namespace std;
13
14 enum R_u_BccValues {
15     Wall,
16     Airflow
17 };
18
19 enum T_BccValues {
20     Neumann,
21     Dirichlet
22 };
23
24 enum R_u_BccValues str2R_u_BccValues(string str);
25 enum T_BccValues str2T_BccValues(string str);
26 double delta_inc(const vector<vector<double>> &A, const vector<vector<double>> &B);
27 double A_scheme (double P, string scheme="CDS");
28 double det_coord_Mesh ( double A , unsigned int N , double g , unsigned int i );
29 vector<vector<double>> conjugate_gradient ( const vector<vector<double>> &a_p , const
30         vector<vector<double>> &a_e , const vector<vector<double>> &a_w , const vector<
31         vector<vector<double>> &a_n , const vector<vector<double>> &a_s
32         ,const vector<vector<double>> &phi_0 , const vector<vector<double>>
33             > &b , int max_iter , double epsilon , int N_x , int N_y );
34 vector<vector<double>> biconjugate_gradient_stb ( const vector<vector<double>> &a_p ,
35         const vector<vector<double>> &a_e , const vector<vector<double>> &a_w , const
36         vector<vector<double>> &a_n , const vector<vector<double>> &a_s ,
37         const vector<vector<double>> &phi_0 , const vector<vector<double>>
38             &b , int max_iter , double epsilon , int N_x , int N_y );
39
40 class MatrixCoef
41 {
42     public:
43         vector <vector<double>> a_p , a_n , a_s , a_e , a_w , b_p;
44         MatrixCoef(unsigned int N_i , unsigned int N_j ) :
45             a_p(N_i , vector<double>(N_j, 1.0) ), a_n(N_i , vector<double>(N_j, 0.0) ),
46             a_s(N_i , vector<double>(N_j, 0.0) ), a_e(N_i , vector<double>(N_j, 0.0) ),
47             a_w(N_i , vector<double>(N_j, 0.0) ), b_p(N_i , vector<double>(N_j, 0.0) ) {};
48     };
49
50 class velocity_x
51 {
52     public:
53         vector <vector<double>> u_n , u_s , u_e , u_w ;
54         velocity_x (unsigned int N_i , unsigned int N_j ) :
55             u_n (N_i , vector<double>(N_j, 0.0) ) ,

```

```

51     u_s (N_i , vector<double>(N_j, 0.0) ) ,
52     u_e (N_i , vector<double>(N_j, 0.0) ) ,
53     u_w (N_i , vector<double>(N_j, 0.0) ) {};
54 };
55
56 class velocity_y
57 {
58     public:
59         vector <vector<double> > v_n , v_s , v_e , v_w ;
60         velocity_y (unsigned int N_i , unsigned int N_j) :
61             v_n (N_i , vector<double>(N_j, 0.0) ) ,
62             v_s (N_i , vector<double>(N_j, 0.0) ) ,
63             v_e (N_i , vector<double>(N_j, 0.0) ) ,
64             v_w (N_i , vector<double>(N_j, 0.0) ) {};
65 };
66
67 class Massflow
68 {
69     public:
70         vector <vector<double> > F_n , F_s , F_e , F_w;
71         Massflow (unsigned int N_i , unsigned int N_j):
72             F_n (N_i , vector<double>(N_j, 0.0) ) ,
73             F_s (N_i , vector<double>(N_j, 0.0) ) ,
74             F_e (N_i , vector<double>(N_j, 0.0) ) ,
75             F_w (N_i , vector<double>(N_j, 0.0) ) {};
76 };
77
78 class gradiente
79 {
80     public:
81         vector <vector<double> > grad_n , grad_s , grad_e , grad_w;
82         gradiente (unsigned int N_i , unsigned int N_j):
83             grad_n (N_i , vector<double>(N_j, 0.0) ) ,
84             grad_s (N_i , vector<double>(N_j, 0.0) ) ,
85             grad_e (N_i , vector<double>(N_j, 0.0) ) ,
86             grad_w (N_i , vector<double>(N_j, 0.0) ) {};
87 };
88
89 class Diffusion
90 {
91     public:
92         vector <vector<double> > D_n , D_s , D_e , D_w;
93         Diffusion (unsigned int N_i , unsigned int N_j):
94             D_n (N_i , vector<double>(N_j, 0.0) ) ,
95             D_s (N_i , vector<double>(N_j, 0.0) ) ,
96             D_e (N_i , vector<double>(N_j, 0.0) ) ,
97             D_w (N_i , vector<double>(N_j, 0.0) ) {};
98 };
99
100 class Peclet
101 {
102     public:
103         vector <vector<double> > Pe_n , Pe_s , Pe_e , Pe_w;
104         Peclet (unsigned int N_i , unsigned int N_j):
105             Pe_n (N_i , vector<double>(N_j, 0.0) ) ,
106             Pe_s (N_i , vector<double>(N_j, 0.0) ) ,
107             Pe_e (N_i , vector<double>(N_j, 0.0) ) ,
108             Pe_w (N_i , vector<double>(N_j, 0.0) ) {};
109 };
110
111 class caseConv2D
112 {
113     private:
114         double L_x , L_y , Ax , Ay , e , Pr , Ra , gx , gy , angle ;
115         unsigned int N_i , N_j , max_iter ;
116         vector <vector<double> > v_0 , v_1 , v_2 ;
117         vector <vector<double> > u_0 , u_1 , u_2 ;
118         vector <vector<double> > u_p2 , v_p2 ;
119         vector <vector<double> > S_w , S_e , S_n , S_s , Vol ;
120         vector <vector<double> > x , y , x_w , y_w , x_e , y_e , x_n , y_n , x_s , y_s ;
121         vector <vector<double> > R_u0 , R_v0 , R_ul , R_vl ;
122         vector <vector<double> > mu , rho ;
123         velocity_x velx ;

```

```

124     velocity_y vely;
125     Massflow mass_x;
126     Massflow mass_y;
127     gradiente gradient_u;
128     gradiente gradient_v;
129     vector <vector<double>> v_out , u_out ;
130     Diffusion diff_T ;
131     Massflow mass_T ;
132     Peclet pe_T ;
133
134
135
136 public:
137     double At , t_2 , epsilon_vT , epsilon_pT ;
138     double t_end ;
139     MatrixCoef coef;
140     vector <vector<double>> p_0 , p_1 , p_lsup ;
141     vector <vector<double>> T_0 , T_1 , T_lsup;
142     MatrixCoef coef_T ;
143     caseConv2D(double Lx , double Ly , double A_x , double A_y , double E , double
144                 Epsilon_vT , double Epsilon_pT , double Pr_vol , double Ra_vol , double Mu ,
145                 double Rho , unsigned int Nx , unsigned int Ny , unsigned int Max_iter ,
146                 double t_End , double Gx , double Gy , double Angle);
147     void SetUniformMesh();
148     void SetNonUniformMesh();
149     void Initialvalues();
150     void Eval_boundary_velocity(enum R_u_BccValues bcc[4] , const vector<double> &value);
151     void interfacevelocity_u();
152     void interfacevelocity_v();
153     void massflow_x();
154     void massflow_y();
155     void grad_u();
156     void grad_v();
157     void EvalR_u();
158     void EvalR_v();
159     void EvalVelocity_p();
160     void EvalCoef();
161     void EvalCoefbc();
162     void Eval_coef_constants_P();
163     void Eval_coef_variables_P();
164     void solveGS();
165     void solveTDMA();
166     void EvalVelocityfield();
167     void Eval_boundary_temperature(enum T_BccValues bcc[4] , const vector<double> &value);
168     void Evaldiffusion_T();
169     void Evalmassflow_T();
170     void Evalpeclet_T();
171     void EvalCoef_T();
172     void Eval_boundary_CoefbcT_temperature(enum T_BccValues bcc[4] , const vector<double>
173                                             &value);
174     void solveTDMA_Temperature();
175     void solveGS_Temp();
176     double GetConvergence();
177     void NewStep();
178     void saveValues(double conv , unsigned int iter);
179     void EvalTime();
180 };
181
182 int main()
183 {
184
185     ifstream infile ("input.txt");
186     if (infile.fail()){
187         cout << "Error opening file" << "input.txt" << endl;
188         exit(0);
189     }
190
191     string s;
192     double Lx , Ly , A_x , A_y , E = 1.0 , Epsilon_vT , Epsilon_pT , Pr_vol , Ra_vol , Mu ,

```

```

193     Rho ,t_End , convergence;
194     double Gx , Gy , Angle ;
195     unsigned int Nx , Ny , Max_iter , comp = 0 ;
196     vector<string> s_R_u_bcc(4) , s_T_bcc(4) ;
197     vector<double> v_R_u_bcc(8,0.0) , v_T_bcc(4,0.0) ;
198     unsigned int count = 1;
199     unsigned int i , j , k , l;
200
201     infile >> s >> Lx;
202     infile >> s >> Ly;
203     infile >> s >> Nx;
204     infile >> s >> Ny;
205
206     infile >> s >> s ;
207     if (s.compare("NonUniform") == 0)
208     {
209         infile >> Gx >> Gy ;
210     }
211
212     infile >> s >> Max_iter;
213     infile >> s >> Epsilon_vT;
214     infile >> s >> Epsilon_pT;
215     infile >> s >> t_End;
216     infile >> s >> Pr_vol;
217     infile >> s >> Ra_vol;
218     infile >> s >> Angle;
219
220     Mu = 1.0 ;
221     Rho = Mu / sqrt(9.81 * Lx * Lx * Lx * Pr_vol / Ra_vol ) ;
222
223     infile >> s >> s_R_u_bcc[0] ;
224     if(str2R_u_BccValues(s_R_u_bcc[0]) == Airflow) infile >> v_R_u_bcc[0] >> v_R_u_bcc[1] ;
225     infile >> s >> s_R_u_bcc[1] ;
226     if(str2R_u_BccValues(s_R_u_bcc[1]) == Airflow) infile >> v_R_u_bcc[2] >> v_R_u_bcc[3] ;
227     infile >> s >> s_R_u_bcc[2] ;
228     if(str2R_u_BccValues(s_R_u_bcc[2]) == Airflow) infile >> v_R_u_bcc[4] >> v_R_u_bcc[5] ;
229     infile >> s >> s_R_u_bcc[3] ;
230     if(str2R_u_BccValues(s_R_u_bcc[3]) == Airflow) infile >> v_R_u_bcc[6] >> v_R_u_bcc[7] ;
231
232     infile >> s >> s_T_bcc[0] ;
233     if(str2T_BccValues(s_T_bcc[0]) == Dirichlet) infile >> v_T_bcc[0] ;
234     infile >> s >> s_T_bcc[1] ;
235     if(str2T_BccValues(s_T_bcc[1]) == Dirichlet) infile >> v_T_bcc[1] ;
236     infile >> s >> s_T_bcc[2] ;
237     if(str2T_BccValues(s_T_bcc[2]) == Dirichlet) infile >> v_T_bcc[2] ;
238     infile >> s >> s_T_bcc[3] ;
239     if(str2T_BccValues(s_T_bcc[3]) == Dirichlet) infile >> v_T_bcc[3] ;
240
241     A_y = Ly / Ny;
242     A_x = Lx / Nx;
243
244     enum R_u_BccValues R_u_bcc[] = {str2R_u_BccValues(s_R_u_bcc[0]), str2R_u_BccValues(
245         s_R_u_bcc[1]), str2R_u_BccValues(s_R_u_bcc[2]), str2R_u_BccValues(s_R_u_bcc[3])};
246
247     enum T_BccValues T_bcc[] = {str2T_BccValues(s_T_bcc[0]), str2T_BccValues(s_T_bcc[1]),
248         str2T_BccValues(s_T_bcc[2]), str2T_BccValues(s_T_bcc[3])};
249
250     caseConv2D cas( Lx , Ly , A_x , A_y , E , Epsilon_vT , Epsilon_pT , Pr_vol , Ra_vol ,
251         Mu , Rho , Nx , Ny , Max_iter , t_End , Gx , Gy , Angle);
252     cas.SetNonUniformMesh();
253     cas.Initialvalues();
254
255     //first time step
256     //find predictor velocity field
257     cas.Eval_boundary_velocity(R_u_bcc , v_R_u_bcc);
258     cas.interfacevelocity_u();
259     cas.interfacevelocity_v();
260     cas.massflow_x();
261     cas.massflow_y();
262     cas.grad_u();

```

```

262     cas.grad_v();
263     cas.EvalR_u();
264     cas.EvalR_v();
265     cas.EvalVelocity_p();
266
267     //find pressure field
268     cas.Eval_coef_constants_P();
269     cas.Eval_coef_variables_P();
270     cas.p_1 = conjugate_gradient( cas.coef.a_p , cas.coef.a_e , cas.coef.a_w , cas.coef.a_n
271         , cas.coef.a_s , cas.p_0 , cas.coef.b_p , Max_iter , Epsilon_pT , Nx , Ny);
272     cas.p_1sup = cas.p_1 ;
273
274     //find velocity field and check mass conservation
275     cas.EvalVelocityfield();
276     cas.Massconservation();
277
278     //Find temperature field
279     cas.Evaldiffusion_T();
280     cas.Evalmassflow_T();
281     cas.Evalpeclet_T();
282     cas.EvalCoef_T();
283     cas.Eval_boundary_CoefbcT_temperature( T_bcc , v_T_bcc );
284     cas.T_1 = biconjugate_gradient_stb ( cas.coef_T.a_p , cas.coef_T.a_e , cas.coef_T.a_w ,
285         cas.coef_T.a_n , cas.coef_T.a_s , cas.T_1sup , cas.coef_T.b_p , Max_iter ,
286         Epsilon_pT , Nx , Ny );
287     cas.T_1sup = cas.T_1 ;
288
289     //new time step parameters
290     cas.EvalTime();
291     cas.NewStep();
292     cout << cas.t_2 << endl;
293
294     ofstream outfile("conv.m");
295     if(outfile.fail()){
296         cerr << "Error creating conv.m file." << endl;
297         exit(-1);
298     }
299
300     outfile.setf(ios::floatfield,ios::scientific);
301     outfile.precision(10);
302     outfile << "convs=[";
303     outfile << cas.t_2 << " " << cas.GetConvergence() << " " << cas.Nu_value() << endl;
304
305     while (cas.t_2 < cas.t_end && comp==0)
306     {
307         //Find predictor-velocity field
308         cas.Eval_boundary_velocity(R_u_bcc , v_R_u_bcc );
309         cas.interfacevelocity_u();
310         cas.interfacevelocity_v();
311         cas.massflow_x();
312         cas.massflow_y();
313         cas.grad_u();
314         cas.grad_v();
315         cas.EvalR_u();
316         cas.EvalR_v();
317         cas.EvalVelocity_p();
318
319         cas.p_1 = conjugate_gradient( cas.coef.a_p , cas.coef.a_e , cas.coef.a_w , cas.coef.
320             a_n , cas.coef.a_s , cas.p_0 , cas.coef.b_p , Max_iter , Epsilon_pT , Nx , Ny);
321         cas.p_1sup = cas.p_1 ;
322
323         //Find velocity field
324         cas.EvalVelocityfield();
325         cas.Massconservation();
326
327         //Find temperature field
328         cas.Evalmassflow_T();
329         cas.Evalpeclet_T();
330         cas.EvalCoef_T();
331         cas.Eval_boundary_CoefbcT_temperature( T_bcc , v_T_bcc );
332         cas.T_1 = biconjugate_gradient_stb ( cas.coef_T.a_p , cas.coef_T.a_e , cas.coef_T.a_w

```

```

            , cas.coef_T.a_n , cas.coef_T.a_s , cas.T_lsup , cas.coef_T.b_p , Max_iter ,
            Epsilon_pT , Nx , Ny);
331 cas.T_lsup = cas.T_1 ;
332
333 convergence = cas.GetConvergence();
334 outfile << cas.t_2 << "uu" << convergence << "u" << cas.Nu_value() << endl;
335 if( convergence < cas.epsilon_vT && cas.t_2 > 1.0 )
336 {
337     comp=1;
338 }
339 else
340 {
341     if ( count % 1000 == 0 )
342     {
343         cas.saveValues(convergence , count);
344     }
345     count++;
346
347     cas.EvalTime();
348     if (cas.t_2 < cas.t_end)
349     {
350         cas.NewStep();
351     }
352     cout << "time=" << cas.t_2 << "uiter=" << count << "uerror=" << convergence << "Nu="
            << cas.Nu_value() << endl;
353     }
354 }
355
356 cas.saveValues(convergence , count);
357 outfile << "];" ;
358 outfile.close();
359
360 return 0;
361 }
362
363 caseConv2D::caseConv2D(double Lx , double Ly , double Ax_x , double Ay_y , double E ,
            double Epsilon_vT , double Epsilon_pT , double Pr_vol ,
364             double Ra_vol , double Mu , double Rho , unsigned int Nx , unsigned int Ny ,
            unsigned int Max_iter , double t_End , double Gx , double Gy , double
            Angle):
365     L_x(Lx) , L_y(Ly) , Ax(A_x) , Ay(A_y) , e(E) , epsilon_vT(Epsilon_vT) , epsilon_pT(
            Epsilon_pT) , Pr(Pr_vol) , Ra(Ra_vol) , At(1E-15) , t_2(1E-15) , N_i(Ny) , N_j(
            Nx) , max_iter(Max_iter) , t_end(t_End) , gx(Gx) , gy(Gy) , angle(Angle) , v_0(
            N_i+1, vector<double>(N_j+2,0.0)) , v_1(N_i+1, vector<double>(N_j+2,0.0)) ,
            v_2(N_i+1, vector<double>(N_j+2,0.0)) , u_0(N_i+2, vector<double>(N_j+1,0.0))
            , u_1(N_i+2, vector<double>(N_j+1,0.0)) , u_2(N_i+2, vector<double>(N_j+1,0.0))
            , u_p2(N_i+2, vector<double>(N_j+1,0.0)) , v_p2(N_i+1, vector<double>(N_j
            +2,0.0)) , S_w(N_i+1, vector<double>(N_j+1,0.0)) , S_e(N_i+1, vector<double>(N_j
            +1,0.0)) , S_n(N_i+1, vector<double>(N_j+1,0.0)) , S_s(N_i+1, vector<double>(N_j
            +1,0.0)) , Vol(N_i+1, vector<double>(N_j+1,0.0)) , x(N_i+2, vector<double>(N_j
            +2,0.0)) , y(N_i+2, vector<double>(N_j+2,0.0)) , x_w(N_i+2, vector<double>(N_j
            +2,0.0)) , y_w(N_i+2, vector<double>(N_j+2,0.0)) , x_e(N_i+2, vector<double>(N_j
            +2,0.0)) , y_e(N_i+2, vector<double>(N_j+2,0.0)) , x_n(N_i+2,
            vector<double>(N_j+2,0.0)) , y_n(N_i+2, vector<double>(N_j+2,0.0)) , x_s(N_i
            +2, vector<double>(N_j+2,0.0)) , y_s(N_i+2, vector<double>(N_j+2,0.0)) , R_u0(
            N_i+2, vector<double>(N_j+1,0.0)) , R_v0(N_i+1, vector<double>(N_j+2,0.0)) ,
            R_u1(N_i+2, vector<double>(N_j+1,0.0)) , R_v1(N_i+1, vector<double>(N_j+2,0.0))
            , p_0(N_i+2, vector<double>(N_j+2,1.0)) , p_1(N_i+2, vector<double>(N_j
            +2,1.0)) , p_lsup(N_i+2, vector<double>(N_j+2,1.0)) , mu(N_i+1, vector<double>(N_j
            +1,Mu)) , rho(N_i+1, vector<double>(N_j+1,Rho)) , T_0(N_i+2, vector<double>(N_j
            +2,0.5)) , T_1(N_i+2, vector<double>(N_j+2,0.5)) , T_lsup(N_i+2,
            vector<double>(N_j+2,0.5)) , coef(N_i+1, N_j+1) , velx(N_i+1, N_j) , vely(N_i,
            N_j+1) , mass_x(N_i+1, N_j+1) , mass_y(N_i+1, N_j+1) , gradient_u(N_i+1, N_j) ,
            gradient_v(N_i, N_j+1) , v_out(N_i+1, vector<double>(N_j+1,0.0)) , u_out(N_i+1,
            vector<double>(N_j+1,0.0)) , diff_T(N_i+1, N_j+1) , mass_T(N_i+1, N_j+1) ,
            pe_T(N_i+1, N_j+1) , coef_T(N_i+1, N_j+1) {}};

366
367 void caseConv2D::SetUniformMesh()
368 {
369     unsigned int i , j;
370
371     for ( i=1;i<=N_i; i++)
372     {

```

```

373     for (j=1;j<=N_j;j++)
374     {
375         x[i][j] = (j - 0.5) * Ax ;
376         y[i][j] = (i - 0.5) * Ay ;
377
378         x_w[i][j] = x[i][j] - 0.5 * Ax ;
379         x_e[i][j] = x[i][j] + 0.5 * Ax ;
380         x_n[i][j] = x[i][j] ;
381         x_s[i][j] = x[i][j] ;
382         y_w[i][j] = y[i][j] ;
383         y_e[i][j] = y[i][j] ;
384         y_n[i][j] = y[i][j] + 0.5 * Ay ;
385         y_s[i][j] = y[i][j] - 0.5 * Ay ;
386
387         S_w[i][j] = (y_n[i][j] - y_s[i][j]) * e;
388         S_e[i][j] = (y_n[i][j] - y_s[i][j]) * e;
389         S_n[i][j] = (x_e[i][j] - x_w[i][j]) * e;
390         S_s[i][j] = (x_e[i][j] - x_w[i][j]) * e;
391         Vol[i][j] = (y_n[i][j] - y_s[i][j]) * (x_e[i][j] - x_w[i][j]) * e;
392     }
393 }
394
395 for (j=1;j<=N_j;j++)
396 {
397     x[0][j] = (j - 0.5) * Ax ;
398     x[N_i+1][j] = (j - 0.5) * Ax ;
399     y[0][j] = 0.0 ;
400     y[N_i+1][j] = L_y ;
401
402     x_w[0][j] = x[0][j] - 0.5 * Ax ;
403     y_w[0][j] = y[0][j] ;
404     x_e[0][j] = x[0][j] + 0.5 * Ax ;
405     y_e[0][j] = y[0][j] ;
406     x_n[0][j] = x[0][j] ;
407     y_n[0][j] = y[0][j] ;
408     x_s[0][j] = x[0][j] ;
409     y_s[0][j] = y[0][j] ;
410
411     x_w[N_i+1][j] = x[N_i+1][j] - 0.5 * Ax ;
412     y_w[N_i+1][j] = y[N_i+1][j] ;
413     x_e[N_i+1][j] = x[N_i+1][j] + 0.5 * Ax ;
414     y_e[N_i+1][j] = y[N_i+1][j] ;
415     x_n[N_i+1][j] = x[N_i+1][j];
416     y_n[N_i+1][j] = y[N_i+1][j] ;
417     x_s[N_i+1][j] = x[N_i+1][j];
418     y_s[N_i+1][j] = y[N_i+1][j] ;
419 }
420
421 for (i=1;i<=N_i;i++)
422 {
423     x[i][0] = 0.0 ;
424     x[i][N_j+1] = L_x ;
425     y[i][0] = (i - 0.5) * Ay ;
426     y[i][N_j+1] = (i - 0.5) * Ay ;
427
428     x_n[i][0] = x[i][0] ;
429     y_n[i][0] = y[i][0] + 0.5 * Ay ;
430     x_s[i][0] = x[i][0] ;
431     y_s[i][0] = y[i][0] - 0.5 * Ay ;
432     x_e[i][0] = x[i][0] ;
433     y_e[i][0] = y[i][0] ;
434     x_w[i][0] = x[i][0] ;
435     y_w[i][0] = y[i][0] ;
436
437
438     x_n[i][N_j+1] = x[i][N_j+1] ;
439     y_n[i][N_j+1] = y[i][N_j+1] + 0.5 * Ay ;
440     x_s[i][N_j+1] = x[i][N_j+1] ;
441     y_s[i][N_j+1] = y[i][N_j+1] - 0.5 * Ay ;
442     x_e[i][N_j+1] = x[i][N_j+1] ;
443     y_e[i][N_j+1] = y[i][N_j+1] ;
444     x_w[i][N_j+1] = x[i][N_j+1] ;
445     y_w[i][N_j+1] = y[i][N_j+1] ;

```

```

446    }
447    y[N_i+1][0] = L_y ;
448    y_n[N_i+1][0] = L_y ;
449    y_s[N_i+1][0] = L_y ;
450    y_e[N_i+1][0] = L_y ;
451    y_w[N_i+1][0] = L_y ;
452
453    x[0][N_j+1] = L_x ;
454    x_n[0][N_j+1] = L_x;
455    x_s[0][N_j+1] = L_x;
456    x_e[0][N_j+1] = L_x;
457    x_w[0][N_j+1] = L_x;
458
459    x[N_i+1][N_j+1] = L_x ;
460    y[N_i+1][N_j+1] = L_y ;
461    x_n[N_i+1][N_j+1] = L_x ;
462    y_n[N_i+1][N_j+1] = L_y ;
463    x_s[N_i+1][N_j+1] = L_x ;
464    y_s[N_i+1][N_j+1] = L_y ;
465    x_e[N_i+1][N_j+1] = L_x ;
466    y_e[N_i+1][N_j+1] = L_y ;
467    x_w[N_i+1][N_j+1] = L_x ;
468    y_w[N_i+1][N_j+1] = L_y ;
469 }
470
471 void caseConv2D::SetNonUniformMesh()
472 {
473
474     unsigned int i , j;
475     unsigned int N_x = N_j , N_y = N_i ;
476
477     for ( i=1;i<=N_i; i++)
478     {
479         for ( j=1;j<=N_j; j++)
480         {
481
482             x[i][j] = (det_coord_Mesh ( L_x , N_x , gx , j ) + det_coord_Mesh ( L_x , N_x , gx ,
483                                         j+1 ))/2.0 ;
484             y[i][j] = (det_coord_Mesh ( L_y , N_y , gy , i ) + det_coord_Mesh ( L_y , N_y , gy ,
485                                         i+1 ))/2.0 ;
486
487             x_w[i][j] = det_coord_Mesh ( L_x , N_x , gx , j ) ;
488             x_e[i][j] = det_coord_Mesh ( L_x , N_x , gx , j+1 ) ;
489             x_n[i][j] = x[i][j] ;
490             x_s[i][j] = x[i][j] ;
491             y_w[i][j] = y[i][j] ;
492             y_e[i][j] = y[i][j] ;
493             y_n[i][j] = det_coord_Mesh ( L_y , N_y , gy , i+1 ) ;
494             y_s[i][j] = det_coord_Mesh ( L_y , N_y , gy , i ) ;
495
496             S_w[i][j] = (y_n[i][j] - y_s[i][j]) * e;
497             S_e[i][j] = (y_n[i][j] - y_s[i][j]) * e;
498             S_n[i][j] = (x_e[i][j] - x_w[i][j]) * e;
499             S_s[i][j] = (x_e[i][j] - x_w[i][j]) * e;
500             Vol[i][j] = (y_n[i][j] - y_s[i][j]) * (x_e[i][j] - x_w[i][j]) * e;
501
502         }
503     }
504
505     for ( j=1;j<=N_j; j++)
506     {
507         x[0][j] = (det_coord_Mesh ( L_x , N_x , gx , j ) + det_coord_Mesh ( L_x , N_x , gx ,
508                                     j+1 ))/2.0 ;
509         x[N_i+1][j] = (det_coord_Mesh ( L_x , N_x , gx , j ) + det_coord_Mesh ( L_x , N_x , gx
510                                     , j+1 ))/2.0 ;
511         y[0][j] = 0.0 ;
512         y[N_i+1][j] = L_y ;
513
514         x_w[0][j] = det_coord_Mesh ( L_x , N_x , gx , j ) ;
515         y_w[0][j] = y[0][j] ;
516         x_e[0][j] = det_coord_Mesh ( L_x , N_x , gx , j+1 ) ;
517         y_e[0][j] = y[0][j] ;
518         x_n[0][j] = x[0][j] ;

```

```

515     y_n[0][j] = y[0][j] ;
516     x_s[0][j] = x[0][j] ;
517     y_s[0][j] = y[0][j] ;
518
519     x_w[N_i+1][j] = det_coord_Mesh ( L_x , N_x , gx , j ) ;
520     y_w[N_i+1][j] = y[N_i+1][j] ;
521     x_e[N_i+1][j] = det_coord_Mesh ( L_x , N_x , gx , j+1 ) ;
522     y_e[N_i+1][j] = y[N_i+1][j] ;
523     x_n[N_i+1][j] = x[N_i+1][j];
524     y_n[N_i+1][j] = y[N_i+1][j] ;
525     x_s[N_i+1][j] = x[N_i+1][j];
526     y_s[N_i+1][j] = y[N_i+1][j] ;
527 }
528
529 for ( i=1;i<=N_i; i++)
530 {
531     x[i][0] = 0.0 ;
532     x[i][N_j+1] = L_x ;
533     y[i][0] = (det_coord_Mesh ( L_y , N_y , gy , i ) + det_coord_Mesh ( L_y , N_y , gy , i
534             +1 ))/2.0 ;
535     y[i][N_j+1] = (det_coord_Mesh ( L_y , N_y , gy , i ) + det_coord_Mesh ( L_y , N_y , gy
536             , i+1 ))/2.0 ;
537
538     x_n[i][0] = x[i][0] ;
539     y_n[i][0] = det_coord_Mesh ( L_y , N_y , gy , i+1 ) ;
540     x_s[i][0] = x[i][0] ;
541     y_s[i][0] = det_coord_Mesh ( L_y , N_y , gy , i ) ;
542     x_e[i][0] = x[i][0] ;
543     y_e[i][0] = y[i][0] ;
544     x_w[i][0] = x[i][0] ;
545     y_w[i][0] = y[i][0] ;
546
547     x_n[i][N_j+1] = x[i][N_j+1] ;
548     y_n[i][N_j+1] = det_coord_Mesh ( L_y , N_y , gy , i+1 ) ;
549     x_s[i][N_j+1] = x[i][N_j+1] ;
550     y_s[i][N_j+1] = det_coord_Mesh ( L_y , N_y , gy , i ) ;
551     x_e[i][N_j+1] = x[i][N_j+1] ;
552     y_e[i][N_j+1] = y[i][N_j+1] ;
553     x_w[i][N_j+1] = x[i][N_j+1] ;
554     y_w[i][N_j+1] = y[i][N_j+1] ;
555 }
556     y[N_i+1][0] = L_y ;
557     y_n[N_i+1][0] = L_y ;
558     y_s[N_i+1][0] = L_y ;
559     y_e[N_i+1][0] = L_y ;
560     y_w[N_i+1][0] = L_y ;
561
562     x[0][N_j+1] = L_x ;
563     x_n[0][N_j+1] = L_x;
564     x_s[0][N_j+1] = L_x;
565     x_e[0][N_j+1] = L_x;
566     x_w[0][N_j+1] = L_x;
567
568     x[N_i+1][N_j+1] = L_x ;
569     y[N_i+1][N_j+1] = L_y ;
570     x_n[N_i+1][N_j+1] = L_x ;
571     y_n[N_i+1][N_j+1] = L_y ;
572     x_s[N_i+1][N_j+1] = L_x ;
573     y_s[N_i+1][N_j+1] = L_y ;
574     x_e[N_i+1][N_j+1] = L_x ;
575     y_e[N_i+1][N_j+1] = L_y ;
576     x_w[N_i+1][N_j+1] = L_x ;
577     y_w[N_i+1][N_j+1] = L_y ;
578
579     ofstream outfile( "mesh.m" );
580
581     if( outfile . fail() )
582     {
583         cerr << "Error creating mesh.m file." << endl;
584         exit(-1);
585     }

```

```

586     outfile << "x=[ " << endl;
587     for ( i=1;i<N_i+1;i++)
588     {
589         for ( j=1;j<N_j+1;j++)
590         {
591             outfile << x[ i ][ j ] << " " ;
592         }
593         outfile << endl ;
594     }
595     outfile << " ];" << endl << endl;
596
597     outfile << "y=[ " << endl;
598     for ( i=1;i<N_i+1;i++)
599     {
600         for ( j=1;j<N_j+1;j++)
601         {
602             outfile << y[ i ][ j ] << " " ;
603         }
604         outfile << endl ;
605     }
606     outfile << " ];" << endl << endl;
607
608     outfile .close () ;
609
610 }
611
612 void caseConv2D :: Initialvalues ()
613 {
614     u_1 = u_0;
615     v_1 = v_0;
616     p_1sup = p_0;
617     T_1sup = T_0 ;
618
619     unsigned int i , j ;
620     double temp = 0.0 ;
621     for ( i=1;i<N_i; i++)
622     {
623         for ( j=1;j<N_j+1;j++)
624         {
625             temp = (y_n[ i ][ j ]-y[ i ][ j ]) / (y[ i+1 ][ j ]-y[ i ][ j ]) ;
626             R_v0[ i ][ j ] = Pr * ( T_0[ i ][ j ] * (1.0-temp) + temp * T_0[ i+1 ][ j ] ) ;
627         }
628     }
629 }
630
631 enum R_u_BccValues str2R_u_BccValues(string str)
632 {
633     if (str=="Wall")
634     {
635         return (Wall);
636     }
637     else if (str=="Airflow")
638     {
639         return (Airflow);
640     }
641     else
642     {
643         cerr << "Error in " << __FUNCTION__ << ":" << str << " condition doesn't exist ." <<
644             endl;
645         exit(0);
646     }
647 }
648 enum T_BccValues str2T_BccValues(string str)
649 {
650     if (str=="Neumann")
651     {
652         return (Neumann);
653     }
654     else if (str=="Dirichlet")
655     {
656         return (Dirichlet);
657     }

```

```

658     else
659     {
660         cerr << "Error in " << __FUNCTION__ << ":" << str << " condition doesn't exist." <<
661             endl;
662         exit(0);
663     }
664 }
665 void caseConv2D :: Eval_boundary_velocity(enum R_u_BccValues bcc[4] , const vector<double>
666     &value)
667 {
668     unsigned int i , j ;
669     switch (bcc[0])
670 {
671     case Wall:
672         j=0;
673         for ( i=0;i<N_i+2;i++)
674         {
675             u_1[i][j] = 0.0;
676             u_p2[i][j] = 0.0;
677         }
678         for ( i=0;i<N_i+1;i++)
679         {
680             v_1[i][j] = 0.0;
681             v_p2[i][j] = 0.0;
682         }
683         break;
684     case Airflow:
685         j=0;
686         for ( i=0;i<N_i+2;i++)
687         {
688             u_1[i][j] = value[0];
689             u_p2[i][j] = value[0];
690         }
691         for ( i=0;i<N_i+1;i++)
692         {
693             v_1[i][j] = value[1];
694             v_p2[i][j] = value[1];
695         }
696         break;
697     default:
698         assert(false);
699         break;
700     }
701     switch (bcc[1]){
702     case Wall:
703         i=0;
704         for ( j=0;j<N_j+1;j++)
705         {
706             u_1[i][j] = 0.0;
707             u_p2[i][j] = 0.0;
708         }
709         for ( j=0;j<N_j+2;j++)
710         {
711             v_1[i][j] = 0.0;
712             v_p2[i][j] = 0.0;
713         }
714         break;
715     case Airflow:
716         i=0;
717         for ( j=0;j<N_j+1;j++)
718         {
719             u_1[i][j] = value[2];
720             u_p2[i][j] = value[2];
721         }
722         for ( j=0;j<N_j+2;j++)
723         {
724             v_1[i][j] = value[3];
725             v_p2[i][j] = value[3];
726         }
727         break;
728     default:

```

```

729         assert( false );
730         break;
731     }
732     switch (bcc[2]) {
733     case Wall:
734         j=N_j;
735         for (i=0;i<N_i+2;i++)
736         {
737             u_1[i][j] = 0.0;
738             u_p2[i][j] = 0.0;
739         }
740         j=N_j+1;
741         for (i=0;i<N_i+1;i++)
742         {
743             v_1[i][j] = 0.0;
744             v_p2[i][j] = 0.0;
745         }
746         break;
747     case Airflow:
748         j=N_j;
749         for (i=0;i<N_i+2;i++)
750         {
751             u_1[i][j] = value[4];
752             u_p2[i][j] = value[4];
753         }
754         j=N_j+1;
755         for (i=0;i<N_i+1;i++)
756         {
757             v_1[i][j] = value[5];
758             v_p2[i][j] = value[5];
759         }
760         break;
761     default:
762         assert( false );
763         break;
764     }
765     switch (bcc[3]) {
766     case Wall:
767         i=N_i+1;
768         for (j=0;j<N_j+1;j++)
769         {
770             u_1[i][j] = 0.0;
771             u_p2[i][j] = 0.0;
772         }
773         i=N_i;
774         for (j=0;j<N_j+2;j++)
775         {
776             v_1[i][j] = 0.0;
777             v_p2[i][j] = 0.0;
778         }
779         break;
780     case Airflow:
781         i=N_i+1;
782         for (j=0;j<N_j+1;j++)
783         {
784             u_1[i][j] = value[6];
785             u_p2[i][j] = value[6];
786         }
787         i=N_i;
788         for (j=0;j<N_j+2;j++)
789         {
790             v_1[i][j] = value[7];
791             v_p2[i][j] = value[7];
792         }
793         break;
794     default:
795         assert( false );
796         break;
797     }
798 }
799
800 double caseConv2D::CDSscheme_u1(int i , int j , int i1 , int j1)
801 {

```

```

802     double temp=0.0;
803
804     if ( i==0 && i!=i1 )
805     {
806         temp = 0.0 ;
807     }
808     else if( i1==N_i+1 && i!=i1 )
809     {
810         temp = 1.0 ;
811     }
812     else if( i==i1 && j!=j1 )
813     {
814         temp = (x[i1][j1]-x_e[i][j]) / (x_e[i1][j1]-x_e[i][j]) ;
815     }
816     else
817     {
818         temp = (y_n[i][j]-y[i][j]) / (y[i1][j1]-y[i][j]) ;
819     }
820
821     return u_1[i][j] * (1.0-temp) + temp * u_1[i1][j1];
822 }
823
824 double caseConv2D :: CDSscheme_v1( int i , int j , int i1 , int j1 )
825 {
826     double temp=0.0;
827
828     if ( j==0 && j!=j1 )
829     {
830         temp = 0.0 ;
831     }
832     else if( j1==N_j+1 && j!=j1 )
833     {
834         temp = 1.0 ;
835     }
836     else if( j==j1 && i!=i1 )
837     {
838         temp = (y[i1][j1]-y_n[i][j]) / (y_n[i1][j1]-y_n[i][j]) ;
839     }
840     else
841     {
842         temp = (x_e[i][j]-x[i][j]) / (x[i1][j1]-x[i][j]) ;
843     }
844
845     return v_1[i][j] * (1.0-temp) + temp * v_1[i1][j1];
846 }
847
848 void caseConv2D :: interfacevelocity_u()
849 {
850     unsigned int i , j ;
851
852     for ( i=1;i<N_i+1;i++)
853     {
854         for ( j=1;j<N_j;j++)
855         {
856             velx.u_e[i][j] = CDSscheme_u1(i,j,i,j+1) ;
857             velx.u_w[i][j] = CDSscheme_u1(i,j-1,i,j) ;
858             velx.u_n[i][j] = CDSscheme_u1(i,j,i+1,j) ;
859             velx.u_s[i][j] = CDSscheme_u1(i-1,j,i,j) ;
860         }
861     }
862 }
863
864 void caseConv2D :: interfacevelocity_v()
865 {
866     unsigned int i , j ;
867
868     for ( i=1;i<N_i;i++)
869     {
870         for ( j=1;j<N_j+1;j++)
871         {
872             vely.v_e[i][j] = CDSscheme_v1(i,j,i,j+1) ;
873             vely.v_w[i][j] = CDSscheme_v1(i,j-1,i,j) ;
874             vely.v_n[i][j] = CDSscheme_v1(i,j,i+1,j) ;

```

```

875         vely.v_s[i][j] = CDSscheme_v1(i-1,j,i,j) ;
876     }
877 }
878 }
879
880 void caseConv2D::massflow_x()
881 {
882     unsigned int i , j;
883
884     for (i=1;i<N_i+1;i++)
885     {
886         for (j=1;j<N_j;j++)
887         {
888             mass_x.F_e[i][j] = ( u_1[i][j] + u_1[i][j+1] ) / 2.0 * ( y_n[i][j] - y_s[i][j] ) ;
889             mass_x.F_w[i][j] = ( u_1[i][j] + u_1[i][j-1] ) / 2.0 * ( y_n[i][j] - y_s[i][j] ) ;
890             mass_x.F_n[i][j] = ( v_1[i][j] * ( x_e[i][j] - x[i][j] ) + v_1[i][j+1] * ( x[i][j+1]
891                                         - x_e[i][j] ) ) ;
892             mass_x.F_s[i][j] = ( v_1[i-1][j] * ( x_e[i][j] - x[i][j] ) + v_1[i-1][j+1] * ( x[i][j
893                                         +1] - x_e[i][j] ) ) ;
894         }
895     }
896     void caseConv2D::massflow_y()
897     {
898         unsigned int i , j;
899
900         for (i=1;i<N_i;i++)
901     {
902         for (j=1;j<N_j+1;j++)
903         {
904             mass_y.F_e[i][j] = ( u_1[i][j] * ( y_n[i][j] - y[i][j] ) + u_1[i+1][j] * ( y[i+1][j]
905                                         - y_n[i][j] ) ) ;
906             mass_y.F_w[i][j] = ( u_1[i][j-1] * ( y_n[i][j] - y[i][j] ) + u_1[i+1][j-1] * ( y[i
907                                         +1][j] - y_n[i][j] ) ) ;
908             mass_y.F_n[i][j] = ( v_1[i+1][j] + v_1[i][j] ) / 2.0 * ( x_e[i][j] - x_w[i][j] ) ;
909             mass_y.F_s[i][j] = ( v_1[i-1][j] + v_1[i][j] ) / 2.0 * ( x_e[i][j] - x_w[i][j] ) ;
910         }
911     }
912     void caseConv2D::grad_u()
913     {
914         unsigned int i , j ;
915
916         for (i=1;i<N_i+1;i++)
917     {
918         for (j=1;j<N_j;j++)
919         {
920             gradient_u.grad_e[i][j] = (u_1[i][j+1] - u_1[i][j]) / (x_e[i][j+1] - x_e[i][j]) * (
921                                         y_n[i][j] - y_s[i][j]) ;
922             gradient_u.grad_w[i][j] = (u_1[i][j] - u_1[i][j-1]) / (x_e[i][j] - x_e[i][j-1]) * (
923                                         y_n[i][j] - y_s[i][j]) ;
924             gradient_u.grad_n[i][j] = (u_1[i+1][j] - u_1[i][j]) / (y[i+1][j] - y[i][j]) * (x[i][j
925                                         +1] - x[i][j]) ;
926             gradient_u.grad_s[i][j] = (u_1[i][j] - u_1[i-1][j]) / (y[i][j] - y[i-1][j]) * (x[i][j
927                                         +1] - x[i][j]) ;
928     }
929     }
930     void caseConv2D::grad_v()
931     {
932         unsigned int i , j ;
933
934         for (i=1;i<N_i;i++)
935     {
936         for (j=1;j<N_j+1;j++)
937         {
938             gradient_v.grad_e[i][j] = (v_1[i][j+1] - v_1[i][j]) / (x[i][j+1] - x[i][j]) * (y[i
939                                         +1][j] - y[i][j]);
940             gradient_v.grad_w[i][j] = (v_1[i][j] - v_1[i][j-1]) / (x[i][j] - x[i][j-1]) * (y[i
941                                         +1][j] - y[i][j]);
942         }
943     }
944 }
```

```

938     gradient_v.grad_n[i][j] = (v_1[i+1][j] - v_1[i][j]) / (y_n[i+1][j] - y_n[i][j]) * (
939         x_e[i][j] - x_w[i][j]) ;
940     gradient_v.grad_s[i][j] = (v_1[i][j] - v_1[i-1][j]) / (y_n[i][j] - y_n[i-1][j]) * (
941         x_e[i][j] - x_w[i][j]) ;
942 }
943 }
944 void caseConv2D::EvalR_u()
945 {
946     unsigned int i, j;
947     double temp = 0.0, temp1 = 0.0, temp2 = 0.0, temp3 = 0.0;
948
949     for (i=1;i<N_i+1;i++)
950     {
951         for (j=1;j<N_j;j++)
952         {
953             temp1 = gradient_u.grad_e[i][j] + gradient_u.grad_n[i][j] - gradient_u.grad_w[i][j] -
954                 gradient_u.grad_s[i][j];
955             temp2 = mass_x.F_e[i][j] * velx.u_e[i][j] + mass_x.F_n[i][j] * velx.u_n[i][j] -
956                 mass_x.F_w[i][j] * velx.u_w[i][j] - mass_x.F_s[i][j] * velx.u_s[i][j];
957
958             temp = (x_e[i][j]-x[i][j]) / (x[i][j+1]-x[i][j]);
959             temp3 = Pr * (T_0[i][j] * (1.0-temp) + temp * T_0[i][j+1]) * cos (angle*PI/180);
960
961             R_u1[i][j] = (Pr*temp1/sqrt(Ra) - temp2) / ((x[i][j+1]-x[i][j])*(y_n[i][j]-y_s[i][j])) + temp3;
962         }
963     }
964 void caseConv2D::EvalR_v()
965 {
966     unsigned int i, j;
967     double temp = 0.0, temp1 = 0.0, temp2 = 0.0, temp3 = 0.0;
968
969     for (i=1;i<N_i;i++)
970     {
971         for (j=1;j<N_j+1;j++)
972         {
973             temp1 = gradient_v.grad_e[i][j] + gradient_v.grad_n[i][j] - gradient_v.grad_w[i][j] -
974                 gradient_v.grad_s[i][j];
975             temp2 = mass_y.F_e[i][j] * vely.v_e[i][j] + mass_y.F_n[i][j] * vely.v_n[i][j] -
976                 mass_y.F_w[i][j] * vely.v_w[i][j] - mass_y.F_s[i][j] * vely.v_s[i][j];
977
978             temp = (y_n[i][j]-y[i][j]) / (y[i+1][j]-y[i][j]);
979             temp3 = Pr * (T_0[i][j] * (1.0-temp) + temp * T_0[i+1][j]) * sin (angle*PI/180);
980
981             R_v1[i][j] = (Pr*temp1/sqrt(Ra) - temp2) / ((x_e[i][j]-x_w[i][j])*(y[i+1][j]-y[i][j])) + temp3;
982         }
983     }
984 void caseConv2D::EvalVelocity_p()
985 {
986     unsigned int i, j;
987
988     for (i=1;i<N_i+1;i++)
989     {
990         for (j=1;j<N_j;j++)
991         {
992             u_p2[i][j] = u_1[i][j] + At * (3.0/2.0 * R_u1[i][j] - 1.0/2.0 * R_u0[i][j]);
993         }
994     }
995     for (i=1;i<N_i;i++)
996     {
997         for (j=1;j<N_j+1;j++)
998         {
999             v_p2[i][j] = v_1[i][j] + At * (3.0/2.0 * R_v1[i][j] - 1.0/2.0 * R_v0[i][j]);
1000         }
1001     }
1002 }

```

```

1003
1004 void caseConv2D :: EvalCoef()
1005 {
1006     unsigned int i , j;
1007
1008     for (i=2;i<N_i; i++)
1009     {
1010         for (j=2;j<N_j; j++)
1011         {
1012             coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1013             coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1014             coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1015             coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1016             coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1017             coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1018                               + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1019         }
1020     }
1021 }
1022
1023 void caseConv2D :: EvalCoefbc()
1024 {
1025     unsigned int i , j;
1026
1027     {
1028         i=1; j=1;
1029         coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1030         coef.a_s[i][j] = 0.0 ;
1031         coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1032         coef.a_w[i][j] = 0.0 ;
1033         coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1034         coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1035                               + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1036     }
1037
1038     {
1039         i=1; j=N_j;
1040         coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1041         coef.a_s[i][j] = 0.0 ;
1042         coef.a_e[i][j] = 0.0 ;
1043         coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1044         coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1045         coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1046                               + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1047     }
1048
1049     {
1050         i=N_i; j=1;
1051         coef.a_n[i][j] = 0.0 ;
1052         coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1053         coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1054         coef.a_w[i][j] = 0.0 ;
1055         coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1056         coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1057                               + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1058     }
1059
1060     {
1061         i=N_i; j=N_j;
1062         coef.a_n[i][j] = 0.0 ;
1063         coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1064         coef.a_e[i][j] = 0.0 ;
1065         coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1066         coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1067         coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1068                               + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1069     }
1070
1071     i=1;
1072     for (j=2;j<N_j; j++)
1073     {
1074         coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1075         coef.a_s[i][j] = 0.0 ;

```

```

1076     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1077     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1078     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1079     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1080                     + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1081 }
1082
1083 i=N_i;
1084 for (j=2;j<N_j;j++)
1085 {
1086     coef.a_n[i][j] = 0.0 ;
1087     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1088     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1089     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1090     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1091     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1092                     + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1093 }
1094
1095 j=1;
1096 for (i=2;i<N_i;i++)
1097 {
1098     coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1099     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1100     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1101     coef.a_w[i][j] = 0.0 ;
1102     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1103     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1104                     + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1105 }
1106
1107 j=N_j;
1108 for (i=2;i<N_i;i++)
1109 {
1110     coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1111     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1112     coef.a_e[i][j] = 0.0 ;
1113     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1114     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1115     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1116                     + u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1117 }
1118 }
1119
1120 void caseConv2D :: Eval_coef_constants_P()
1121 {
1122     unsigned int i , j;
1123
1124     for (i=2;i<N_i;i++)
1125     {
1126         for(j=2;j<N_j;j++)
1127         {
1128             coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1129             coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1130             coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1131             coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1132             coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1133         }
1134     }
1135
1136     {
1137         i=1; j=1;
1138         coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1139         coef.a_s[i][j] = 0.0 ;
1140         coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1141         coef.a_w[i][j] = 0.0 ;
1142         coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1143     }
1144
1145     {
1146         i=1; j=N_j;
1147         coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1148         coef.a_s[i][j] = 0.0 ;

```

```

1149     coef.a_e[i][j] = 0.0 ;
1150     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1151     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1152 }
1153 {
1154     i=N_i; j=1;
1155     coef.a_n[i][j] = 0.0 ;
1156     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1157     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1158     coef.a_w[i][j] = 0.0 ;
1159     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1160 }
1161 }
1162 {
1163     i=N_i; j=N_j;
1164     coef.a_n[i][j] = 0.0 ;
1165     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1166     coef.a_e[i][j] = 0.0 ;
1167     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1168     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1169 }
1170 }
1171 i=1;
1172 for (j=2;j<N_j;j++)
1173 {
1174     coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1175     coef.a_s[i][j] = 0.0 ;
1176     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1177     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1178     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1179 }
1180 }
1181 i=N_i;
1182 for (j=2;j<N_j;j++)
1183 {
1184     coef.a_n[i][j] = 0.0 ;
1185     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1186     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1187     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1188     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1189 }
1190 }
1191 j=1;
1192 for (i=2;i<N_i;i++)
1193 {
1194     coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1195     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1196     coef.a_e[i][j] = S_e[i][j] / (x[i][j+1] - x[i][j]) ;
1197     coef.a_w[i][j] = 0.0 ;
1198     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1199 }
1200 }
1201 j=N_j;
1202 for (i=2;i<N_i;i++)
1203 {
1204     coef.a_n[i][j] = S_n[i][j] / (y[i+1][j] - y[i][j]) ;
1205     coef.a_s[i][j] = S_s[i][j] / (y[i][j] - y[i-1][j]) ;
1206     coef.a_e[i][j] = 0.0 ;
1207     coef.a_w[i][j] = S_w[i][j] / (x[i][j] - x[i][j-1]) ;
1208     coef.a_p[i][j] = +coef.a_n[i][j] + coef.a_s[i][j] + coef.a_e[i][j] + coef.a_w[i][j] ;
1209 }
1210 }
1211 }
1212 void caseConv2D :: Eval_coef_variables_P()
1213 {
1214     unsigned int i , j;
1215     for (i=2;i<N_i;i++)
1216     {
1217         for (j=2;j<N_j;j++)
1218         {
1219             coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]

```

```

1222         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1223     }
1224 }
1225 {
1226     i=1; j=1;
1227     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1228                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1229 }
1230 }
1231 {
1232     i=1; j=N_j;
1233     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1234                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1235 }
1236 }
1237 {
1238     i=N_i; j=1;
1239     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1240                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1241 }
1242 }
1243 {
1244     i=N_i; j=N_j;
1245     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1246                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1247 }
1248 }
1249 i=1;
1250 for (j=2;j<N_j; j++)
1251 {
1252     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1253                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1254 }
1255 }
1256 i=N_i;
1257 for (j=2;j<N_j; j++)
1258 {
1259     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1260                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1261 }
1262 }
1263 j=1;
1264 for (i=2;i<N_i; i++)
1265 {
1266     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1267                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1268 }
1269 }
1270 j=N_j;
1271 for (i=2;i<N_i; i++)
1272 {
1273     coef.b_p[i][j] = - ( v_p2[i][j] * S_n[i][j] - v_p2[i-1][j] * S_s[i][j]
1274                         +u_p2[i][j] * S_e[i][j] - u_p2[i][j-1] * S_w[i][j] ) ;
1275 }
1276 }
1277 }
1278 void caseConv2D :: solveGS()
1279 {
1280     unsigned int i , j;
1281     unsigned int iter;
1282     double deltap_ = epsilon_pT + 1.0;
1283
1284     for (iter=1; deltap_> epsilon_pT && iter<max_iter ; iter++)
1285     {
1286         for (i=1;i<N_i+1;i++)
1287         {
1288             for (j=1;j<N_j+1;j++)
1289             {
1290                 p_1[i][j] = 1.0/coef.a_p[i][j] * ( coef.a_n[i][j] * p_1sup[i+1][j] + coef.a_s[i][j]
1291                         * p_1[i-1][j] + coef.a_e[i][j] * p_1sup[i][j+1] + coef.a_w[i][j] * p_1[i][j-1]
1292                         + coef.b_p[i][j]);
1293             }
1294         }
1295     }
1296 }

```

```

1293     }
1294     deltap_ = delta_inc( p_1 , p_1sup );
1295     p_1sup = p_1 ;
1296 }
1297 if (iter>=max_iter) cerr << "Warning: max number of iterations achieved: " << deltap_ <<
1298     endl;
1299 assert (deltap_ < epsilon_pT);
1300 }
1301 void caseConv2D :: solveTDMA()
1302 {
1303     unsigned int i , j;
1304     unsigned int iter;
1305     double deltap_ = epsilon_pT + 1.0;
1306     vector<double> P_x(N_j+1,0.0) , R_x(N_j+1,0.0) , b_x(N_j+1,0.0);
1307     vector<double> P_y(N_i+1,0.0) , R_y(N_i+1,0.0) , b_y(N_i+1,0.0);
1308
1309     for (iter=1; deltap_>epsilon_pT && iter<max_iter ; iter++)
1310     {
1311
1312         for (i=1;i<N_i+1;i++)
1313         {
1314             for (j=1;j<N_j+1;j++)
1315             {
1316                 b_x[j] = coef.b_p[i][j] + coef.a_n[i][j] * p_1sup[i+1][j] + coef.a_s[i][j] * p_1[i-1][j];
1317             }
1318
1319             j=1;
1320             P_x[j] = coef.a_e[i][j] / coef.a_p[i][j];
1321             R_x[j] = b_x[j] / coef.a_p[i][j];
1322
1323             for (j=2;j<N_j+1;j++)
1324             {
1325                 P_x[j] = coef.a_e[i][j] / (coef.a_p[i][j] - coef.a_w[i][j]*P_x[j-1]);
1326                 R_x[j] = (b_x[j] + coef.a_w[i][j]*R_x[j-1]) / (coef.a_p[i][j] - coef.a_w[i][j]*P_x[j-1]);
1327             }
1328
1329             for (j=N_j;j>=1;j--)
1330             {
1331                 p_1[i][j]=P_x[j]*p_1[i][j+1]+R_x[j];
1332             }
1333     }
1334
1335     p_1sup = p_1 ;
1336
1337     for (j=1;j<N_j+1;j++)
1338     {
1339         for (i=1;i<N_i+1;i++)
1340         {
1341             b_y[i] = coef.b_p[i][j] + coef.a_e[i][j] * p_1sup[i][j+1] + coef.a_w[i][j] * p_1[i-1][j];
1342         }
1343
1344         i=1;
1345         P_y[i] = coef.a_n[i][j] / coef.a_p[i][j];
1346         R_y[i] = b_y[i] / coef.a_p[i][j];
1347
1348         for (i=2;i<N_i+1;i++)
1349         {
1350             P_y[i] = coef.a_n[i][j] / (coef.a_p[i][j] - coef.a_s[i][j]*P_y[i-1]);
1351             R_y[i] = (b_y[i] + coef.a_s[i][j]*R_y[i-1]) / (coef.a_p[i][j] - coef.a_s[i][j]*P_y[i-1]);
1352         }
1353
1354         for (i=N_i;i>=1;i--)
1355         {
1356             p_1[i][j]=P_y[i]*p_1[i+1][j]+R_y[i];
1357         }
1358     }
1359
1360     deltap_ = delta_inc( p_1 , p_1sup );

```

```

1361     p_lsup = p_1 ;
1362 }
1363 if (iter>=max_iter) cerr << "Warning: max number of iterations achieved: " << deltap_ <<
1364     endl;
1365 assert (deltap_ < epsilon_pT);
1366 }
1367
1368 void caseConv2D :: Evaldiffusion_T()
1369 {
1370     unsigned int i , j ;
1371
1372     for (i=1;i<N_i+1;i++)
1373     {
1374         for (j=1;j<N_j+1;j++)
1375         {
1376             diff_T.D_e[i][j] = S_e[i][j] / ( ( x[i][j+1]-x[i][j] ) * ( sqrt(Ra) ) ) ;
1377             diff_T.D_w[i][j] = S_w[i][j] / ( ( x[i][j]-x[i][j-1] ) * ( sqrt(Ra) ) ) ;
1378             diff_T.D_n[i][j] = S_n[i][j] / ( ( y[i+1][j]-y[i][j] ) * ( sqrt(Ra) ) ) ;
1379             diff_T.D_s[i][j] = S_s[i][j] / ( ( y[i][j]-y[i-1][j] ) * ( sqrt(Ra) ) ) ;
1380         }
1381     }
1382 }
1383
1384 void caseConv2D :: Evalmassflow_T()
1385 {
1386     unsigned int i , j ;
1387
1388     for (i=1;i<N_i+1;i++)
1389     {
1390         for (j=1;j<N_j+1;j++)
1391         {
1392             mass_T.F_e[i][j] = u_2[i][j] * S_e[i][j] ;
1393             mass_T.F_w[i][j] = u_2[i][j-1] * S_w[i][j] ;
1394             mass_T.F_n[i][j] = v_2[i][j] * S_n[i][j] ;
1395             mass_T.F_s[i][j] = v_2[i-1][j] * S_s[i][j] ;
1396         }
1397     }
1398 }
1399
1400 void caseConv2D :: Evalpeclet_T()
1401 {
1402     unsigned int i , j ;
1403
1404     for (i=1;i<N_i+1;i++)
1405     {
1406         for (j=1;j<N_j+1;j++)
1407         {
1408             pe_T.Pe_e[i][j] = mass_T.F_e[i][j] / diff_T.D_e[i][j] ;
1409             pe_T.Pe_w[i][j] = mass_T.F_w[i][j] / diff_T.D_w[i][j] ;
1410             pe_T.Pe_n[i][j] = mass_T.F_n[i][j] / diff_T.D_n[i][j] ;
1411             pe_T.Pe_s[i][j] = mass_T.F_s[i][j] / diff_T.D_s[i][j] ;
1412         }
1413     }
1414 }
1415
1416 void caseConv2D :: EvalCoef_T()
1417 {
1418     unsigned int i , j ;
1419
1420     for (i=2;i<N_i; i++)
1421     {
1422         for (j=2;j<N_j; j++)
1423         {
1424             coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0>(-mass_T.F_n[i][j]))?0.0:(-mass_T.F_n[i][j])) ;
1425             coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][j]))?0.0:(mass_T.F_s[i][j])) ;
1426             coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i][j]))?0.0:(-mass_T.F_e[i][j])) ;
1427             coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][j]))?0.0:(mass_T.F_w[i][j])) ;
1428             coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.

```

```

1429     a_w[i][j] + Vol[i][j] / At ;
1430     coef_T.b_P[i][j] = T_0[i][j] * Vol[i][j] / At ;
1431 }
1432 }
1433
1434 void caseConv2D::Eval_boundary_CoefbcT_temperature(enum T_BccValues bcc[4], const vector
1435 <double> &value)
1436 {
1437     unsigned int i, j;
1438
1439     switch (bcc[0]) //left
1440     {
1441         case Neumann:
1442             j=0;
1443             for (i=0;i<N_i+2;i++)
1444             {
1445                 T_1sup[i][j] = 0.0;
1446                 T_1[i][j] = 0.0;
1447             }
1448             i=1; j=1; coef_T.a_w[i][j] = 0.0 ;
1449         }
1450         j=1;
1451         for (i=2;i<N_i;i++)
1452         {
1453             coef_T.a_w[i][j] = 0.0 ;
1454         }
1455         {
1456             i=N_i; j=1; coef_T.a_w[i][j] = 0.0 ;
1457         }
1458
1459         break;
1460     case Dirichlet:
1461         j=0;
1462         for (i=0;i<N_i+2;i++)
1463         {
1464             T_1sup[i][j] = value[0];
1465             T_1[i][j] = value[0];
1466         }
1467         {
1468             i=1; j=1; coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][j]))?0.0:(mass_T.F_w[i][j]));
1469         }
1470         j=1;
1471         for (i=2;i<N_i;i++)
1472         {
1473             coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][j]))?0.0:(mass_T.F_w[i][j]));
1474         }
1475         {
1476             i=N_i; j=1; coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) +
1477             ((0.0>(mass_T.F_w[i][j]))?0.0:(mass_T.F_w[i][j]));
1478         }
1479
1480         break;
1481     default:
1482         assert(false);
1483     }
1484     switch (bcc[1]) //bottom
1485     {
1486         case Neumann:
1487             i=0;
1488             for (j=0;j<N_j+2;j++)
1489             {
1490                 T_1sup[i][j] = 0.0;
1491                 T_1[i][j] = 0.0;
1492             }
1493             {
1494                 i=1; j=1; coef_T.a_s[i][j] = 0.0 ;
1495             }
1496         }

```

```

1497     i=1; j=N_j; coef_T.a_s[i][j] = 0.0 ;
1498 }
1499 i=1;
1500 for (j=2;j<N_j;j++)
1501 {
1502     coef_T.a_s[i][j] = 0.0 ;
1503 }
1504
1505 break;
1506 case Dirichlet:
1507     i=0;
1508     for (j=0;j<N_j+2;j++)
1509     {
1510         T_1sup[i][j] = value[1];
1511         T_1[i][j] = value[1];
1512     }
1513     {
1514         i=1; j=1; coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][j]))?0.0:(mass_T.F_s[i][j]));
1515     }
1516     {
1517         i=1; j=N_j; coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][j]))?0.0:(mass_T.F_s[i][j]));
1518     }
1519     i=1;
1520     for (j=2;j<N_j;j++)
1521     {
1522         coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][j]))?0.0:(mass_T.F_s[i][j]));
1523     }
1524
1525 break;
1526 default:
1527     assert(false);
1528     break;
1529 }
1530 switch (bcc[2])//right
1531 {
1532     case Neumann:
1533         j=N_j+1;
1534         for (i=0;i<N_i+2;i++)
1535         {
1536             T_1sup[i][j] = 0.0;
1537             T_1[i][j] = 0.0;
1538         }
1539         {
1540             i=N_i; j=N_j; coef_T.a_e[i][j] = 0.0 ;
1541         }
1542         j=N_j;
1543         for (i=2;i<N_i;i++)
1544         {
1545             coef_T.a_e[i][j] = 0.0 ;
1546         }
1547         {
1548             i=1; j=N_j; coef_T.a_e[i][j] = 0.0 ;
1549         }
1550
1551     break;
1552     case Dirichlet:
1553         j=N_j+1;
1554         for (i=0;i<N_i+2;i++)
1555         {
1556             T_1sup[i][j] = value[2];
1557             T_1[i][j] = value[2];
1558         }
1559         {
1560             i=N_i; j=N_j; coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i][j]))?0.0:(-mass_T.F_e[i][j]));
1561         }
1562         j=N_j;
1563         for (i=2;i<N_i;i++)
1564         {
1565             coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i][j]))?0.0:(-mass_T.F_e[i][j]));
1566         }

```

```

1566     F_e[i][j])) ? 0.0 : (-mass_T.F_e[i][j])) ;
1567   }
1568   {
1569     i=1; j=N_j; coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) +
1570       ((0.0 > (-mass_T.F_e[i][j])) ? 0.0 : (-mass_T.F_e[i][j])) ;
1571   }
1572   break;
1573 default:
1574   assert(false);
1575   break;
1576 switch (bcc[3]) //top
1577 {
1578   case Neumann:
1579     i=N_i+1;
1580     for (j=0;j<N_j+2;j++)
1581     {
1582       T_1sup[i][j] = 0.0;
1583       T_1[i][j] = 0.0;
1584     }
1585     {
1586       i=N_i; j=1; coef_T.a_n[i][j] = 0.0 ;
1587     }
1588     i=N_i;
1589     for (j=2;j<N_j;j++)
1590     {
1591       coef_T.a_n[i][j] = 0.0 ;
1592     }
1593     {
1594       i=N_i; j=N_j; coef_T.a_n[i][j] = 0.0 ;
1595     }
1596   }
1597   break;
1598 case Dirichlet:
1599   i=N_i+1;
1600   for (j=0;j<N_j+2;j++)
1601   {
1602     T_1sup[i][j] = value[3];
1603     T_1[i][j] = value[3];
1604   }
1605   {
1606     i=N_i; j=1; coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) +
1607       ((0.0 > (-mass_T.F_n[i][j])) ? 0.0 : (-mass_T.F_n[i][j])) ;
1608   }
1609   i=N_i;
1610   for (j=2;j<N_j;j++)
1611   {
1612     coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0 > (-mass_T.
1613       F_n[i][j])) ? 0.0 : (-mass_T.F_n[i][j])) ;
1614   }
1615   {
1616     i=N_i; j=N_j; coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) +
1617       ((0.0 > (-mass_T.F_n[i][j])) ? 0.0 : (-mass_T.F_n[i][j])) ;
1618   }
1619   break;
1620 default:
1621   assert(false);
1622   break;
1623   {
1624     i=1; j=1;
1625     coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0 > (-mass_T.F_n[i]
1626       [j])) ? 0.0 : (-mass_T.F_n[i][j])) ;
1627     coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0 > (-mass_T.F_e[i]
1628       [j])) ? 0.0 : (-mass_T.F_e[i][j])) ;
1629     coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1630       i][j] + Vol[i][j] / At ;
1631     coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1632   }
1633 }
```

```

1631 j=1;
1632 for ( i=2;i<N_i; i++)
1633 {
1634   coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0>(-mass_T.F_n[i]
1635   ][j]))?0.0:(-mass_T.F_n[i][j))) ;
1636   coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][
1637   j]))?0.0:(mass_T.F_s[i][j])) ;
1638   coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i]
1639   ][j]))?0.0:(-mass_T.F_e[i][j))) ;
1640   coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1641   i][j] + Vol[i][j] / At ;
1642   coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1643 }
1644 {
1645   i=N_i; j=1;
1646   coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][
1647   j]))?0.0:(mass_T.F_s[i][j])) ;
1648   coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i]
1649   ][j]))?0.0:(-mass_T.F_e[i][j))) ;
1650   coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1651   i][j] + Vol[i][j] / At ;
1652   coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1653 }
1654 i=N_i;
1655 for ( j=2;j<N_j; j++)
1656 {
1657   coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][
1658   j]))?0.0:(mass_T.F_s[i][j])) ;
1659   coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i]
1660   ][j]))?0.0:(-mass_T.F_e[i][j))) ;
1661   coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][
1662   j]))?0.0:(mass_T.F_w[i][j])) ;
1663   coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1664   i][j] + Vol[i][j] / At ;
1665   coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1666 }
1667 i=N_j;
1668 for ( i=2;i<N_i; i++)
1669 {
1670   coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0>(-mass_T.F_n[i]
1671   ][j]))?0.0:(-mass_T.F_n[i][j))) ;
1672   coef_T.a_s[i][j] = diff_T.D_s[i][j] * A_scheme(pe_T.Pe_s[i][j]) + ((0.0>(mass_T.F_s[i][
1673   j]))?0.0:(mass_T.F_s[i][j])) ;
1674   coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][
1675   j]))?0.0:(mass_T.F_w[i][j])) ;
1676   coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1677   i][j] + Vol[i][j] / At ;
1678   coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1679 }
1680 i=1; j=N_j;
1681 coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0>(-mass_T.F_n[i]
1682   ][j]))?0.0:(-mass_T.F_n[i][j))) ;
1683 coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][
1684   j]))?0.0:(mass_T.F_w[i][j])) ;
1685 coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[
1686   i][j] + Vol[i][j] / At ;
1687 coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;

```

```

1683 }
1684
1685 i=1;
1686 for (j=2;j<N_j;j++)
1687 {
1688     coef_T.a_n[i][j] = diff_T.D_n[i][j] * A_scheme(pe_T.Pe_n[i][j]) + ((0.0>(-mass_T.F_n[i][j]))?0.0:(-mass_T.F_n[i][j])) ;
1689     coef_T.a_e[i][j] = diff_T.D_e[i][j] * A_scheme(pe_T.Pe_e[i][j]) + ((0.0>(-mass_T.F_e[i][j]))?0.0:(-mass_T.F_e[i][j])) ;
1690     coef_T.a_w[i][j] = diff_T.D_w[i][j] * A_scheme(pe_T.Pe_w[i][j]) + ((0.0>(mass_T.F_w[i][j]))?0.0:(mass_T.F_w[i][j])) ;
1691     coef_T.a_p[i][j] = coef_T.a_n[i][j] + coef_T.a_s[i][j] + coef_T.a_e[i][j] + coef_T.a_w[i][j] + Vol[i][j] / At ;
1692     coef_T.b_p[i][j] = T_0[i][j] * Vol[i][j] / At ;
1693 }
1694 }
1695
1696 void caseConv2D :: solveTDMA_Temperature()
1697 {
1698     unsigned int i , j;
1699     unsigned int iter;
1700     double deltaT = epsilon_pT + 1.0;
1701     vector<double> P_x(N_j+1,0.0) , R_x(N_j+1,0.0) , b_x(N_j+1,0.0);
1702     vector<double> P_y(N_i+1,0.0) , R_y(N_i+1,0.0) , b_y(N_i+1,0.0);
1703
1704     for (iter=1; deltaT>epsilon_pT && iter<max_iter ; iter++)
1705     {
1706
1707         for (i=1;i<N_i+1;i++)
1708         {
1709             for (j=1;j<N_j+1;j++)
1710             {
1711                 b_x[j] = coef_T.b_p[i][j] + coef_T.a_n[i][j] * T_1sup[i+1][j] + coef_T.a_s[i][j] *
1712                     T_1sup[i-1][j] ;
1713             }
1714             j=1;
1715             P_x[j] = coef_T.a_e[i][j] / coef_T.a_p[i][j];
1716             R_x[j] = b_x[j] / coef_T.a_p[i][j];
1717
1718             for (j=2;j<N_j+1;j++)
1719             {
1720                 P_x[j] = coef_T.a_e[i][j] / (coef_T.a_p[i][j] - coef_T.a_w[i][j]*P_x[j-1]);
1721                 R_x[j] = (b_x[j] + coef_T.a_w[i][j]*R_x[j-1]) / (coef_T.a_p[i][j] - coef_T.a_w[i][j]
1722                     *P_x[j-1]);
1723             }
1724             for (j=N_j;j>=1;j--)
1725             {
1726                 T_1[i][j] = P_x[j] * T_1[i][j+1] + R_x[j];
1727             }
1728         }
1729
1730         T_1sup = T_1 ;
1731
1732         for (j=1;j<N_j+1;j++)
1733         {
1734             for (i=1;i<N_i+1;i++)
1735             {
1736                 b_y[i] = coef_T.b_p[i][j] + coef_T.a_e[i][j] * T_1sup[i][j+1] + coef_T.a_w[i][j] *
1737                     T_1sup[i][j-1] ;
1738             }
1739             i=1;
1740             P_y[i] = coef_T.a_n[i][j] / coef_T.a_p[i][j];
1741             R_y[i] = b_y[i] / coef_T.a_p[i][j];
1742
1743             for (i=2;i<N_i+1;i++)
1744             {
1745                 P_y[i] = coef_T.a_n[i][j] / (coef_T.a_p[i][j] - coef_T.a_s[i][j]*P_y[i-1]);
1746                 R_y[i] = (b_y[i] + coef_T.a_s[i][j]*R_y[i-1]) / (coef_T.a_p[i][j] - coef_T.a_s[i][j]
1747                     *P_y[i-1]);
1748             }
1749     }

```

```

1748
1749     for ( i=N_i; i>=1; i-- )
1750     {
1751         T_1[ i ][ j ] = P_y[ i ] * T_1[ i+1 ][ j ] + R_y[ i ];
1752     }
1753 }
1754
1755 deltaT = delta_inc( T_1 , T_1sup );
1756 T_1sup = T_1 ;
1757 }
1758 if ( iter>=max_iter ) cerr << "Warning: max number of iterations achieved: " << deltaT <<
1759     endl;
1760 assert ( deltaT < epsilon_pT );
1761 }
1762 void caseConv2D :: solveGS_Temp()
1763 {
1764     unsigned int i , j ;
1765     unsigned int iter ;
1766     double deltaT = epsilon_pT + 1.0;
1767
1768     for ( iter=1; deltaT > epsilon_pT && iter<max_iter ; iter++ )
1769     {
1770         for ( i=1;i<N_i+1; i++ )
1771         {
1772             for ( j=1;j<N_j+1; j++ )
1773             {
1774                 T_1[ i ][ j ] = 1.0 / coef_T.a_p[ i ][ j ] * ( coef_T.a_n[ i ][ j ] * T_1sup[ i+1 ][ j ] + coef_T.a_s
1775                     [ i ][ j ] * T_1[ i-1 ][ j ]
1776                     + coef_T.a_e[ i ][ j ] * T_1sup[ i ][ j+1 ] + coef_T.a_w[ i ][ j ] * T_1[ i ][ j-1 ] +
1777                     coef_T.b_p[ i ][ j ] );
1778             }
1779         }
1780         deltaT = delta_inc( T_1 , T_1sup );
1781         T_1sup = T_1 ;
1782     }
1783     if ( iter>=max_iter ) cerr << "Warning: max number of iterations achieved: " << deltaT <<
1784         endl;
1785     assert ( deltaT < epsilon_pT );
1786 }
1787
1788 void caseConv2D :: EvalTime()
1789 {
1790     double v_condx_max=0.0 , v_condy_max=0.0 , At_conv=0.0;
1791     double mu_condx_max=0.0 , mu_condy_max=0.0 , At_visc=0.0;
1792     unsigned int i,j ;
1793
1794     for ( i=1;i<N_i+1; i++ )
1795     {
1796         for ( j=1;j<N_j; j++ )
1797         {
1798             v_condx_max = ( v_condx_max > fabs(u_1[ i ][ j ]/(x[ i ][ j+1 ] - x[ i ][ j ])) )
1799             ? v_condx_max : fabs(u_1[ i ][ j ]/(x[ i ][ j+1 ] - x[ i ][ j ]));
1800         }
1801     }
1802
1803     for ( i=1;i<N_i; i++ )
1804     {
1805         for ( j=1;j<N_j+1; j++ )
1806         {
1807             v_condy_max = ( v_condy_max > fabs(v_1[ i ][ j ]/(y[ i+1 ][ j ] - y[ i ][ j ])) )
1808             ? v_condy_max : fabs(v_1[ i ][ j ]/(y[ i+1 ][ j ] - y[ i ][ j ]));
1809         }
1810     }
1811
1812     for ( i=1;i<N_i+1; i++ )
1813     {
1814         for ( j=1;j<N_j+1; j++ )
1815         {
1816             mu_condx_max = ( mu_condx_max > fabs(mu[ i ][ j ]/rho[ i ][ j ]/(x_e[ i ][ j ] - x_w[ i ][ j ]) / (x_e[ i ][ j ] - x_w[ i ][ j ]))
1817             ? mu_condx_max : fabs(mu[ i ][ j ]/rho[ i ][ j ]/(x_e[ i ][ j ] - x_w[ i ][ j ]) / (x_e[ i ][ j ] - x_w[ i ][ j ]));
1818         }
1819     }
1820 }
```

```

1816     j])) ;
1817     mu_condy_max = (mu_condy_max > fabs(mu[i][j]/rho[i][j]/(y_n[i][j] - y_s[i][j])/(y_n[i][j] - y_s[i][j])) )
1818     ? mu_condy_max : fabs(mu[i][j]/rho[i][j]/(y_n[i][j] - y_s[i][j])/(y_n[i][j] - y_s[i][j]));
1819   }
1820 }
1821 At_conv = (v_condx_max>v_condy_max) ? 0.35/v_condx_max : 0.35/v_condy_max;
1822 At_visc = (mu_condx_max>mu_condy_max) ? 0.2/mu_condx_max : 0.2/mu_condy_max;
1823
1824 At = (At_conv<At_visc) ? At_conv : At_visc ;
1825
1826 t_2 += At;
1827 }
1828
1829
1830 void caseConv2D :: EvalVelocityfield()
1831 {
1832   unsigned int i , j;
1833
1834   for (i=1;i<N_i+1;i++)
1835   {
1836     for (j=1;j<N_j;j++)
1837     {
1838
1839       u_2[i][j] = u_p2[i][j] - ( p_1[i][j+1] - p_1[i][j] )/(x[i][j+1] - x[i][j]);
1840     }
1841   }
1842
1843   for (i=1;i<N_i;i++)
1844   {
1845     for (j=1;j<N_j+1;j++)
1846     {
1847       v_2[i][j] = v_p2[i][j] - ( p_1[i+1][j] - p_1[i][j] )/(y[i+1][j] - y[i][j]);
1848     }
1849   }
1850 }
1851
1852 void caseConv2D :: Massconservation()
1853 {
1854   unsigned int i , j;
1855
1856   for (i=1;i<N_i+1;i++)
1857   {
1858     for (j=1;j<N_j+1;j++)
1859     {
1860       assert ( fabs(u_2[i][j] * S_e[i][j] + v_2[i][j] * S_n[i][j] - u_2[i][j-1] * S_w[i][j]
1861             - v_2[i-1][j] * S_s[i][j])/Vol[i][j] < 1E-1);
1862     }
1863   }
1864
1865 double caseConv2D :: GetConvergence()
1866 {
1867   unsigned int i,j;
1868   double maximum=0.0;
1869
1870   maximum = delta_inc ( u_1 , u_2 );
1871   maximum = (maximum > delta_inc ( v_1 , v_2 ) ) ? maximum : delta_inc ( v_1 , v_2 ) ;
1872   maximum = (maximum > delta_inc ( T_0 , T_1 ) ) ? maximum : delta_inc ( T_0 , T_1 ) ;
1873
1874   return maximum/At;
1875 }
1876
1877 void caseConv2D :: NewStep()
1878 {
1879
1880   u_0 = u_1 ;
1881   v_0 = v_1 ;
1882
1883   u_1 = u_2 ;
1884   v_1 = v_2 ;

```

```

1885
1886     p_0 = p_1 ;
1887
1888     R_u0 = R_u1 ;
1889     R_v0 = R_v1 ;
1890
1891     T_0 = T_1 ;
1892 }
1893
1894 void caseConv2D::saveValues(double conv , unsigned int iter)
1895 {
1896     unsigned int i,j;
1897     string outputname;
1898     string temp;
1899
1900     stringstream ss(stringstream::in | stringstream::out);
1901     ss.setf(ios::fixed);
1902     ss.precision(0);
1903     ss << iter ;
1904     ss >> temp ;
1905
1906     outputname = "output_" + temp + ".m";
1907     ofstream outfile(outputname.c_str());
1908
1909     if(outfile.fail())
1910     {
1911         cerr << "Error creating output.m file." << endl;
1912         exit(-1);
1913     }
1914
1915     outfile.setf(ios::floatfield ,ios::scientific );
1916     outfile.precision(10);
1917
1918     for (i=1;i<N_i+1;i++)
1919     {
1920         for (j=1;j<N_j+1;j++)
1921         {
1922             u_out[i][j] = (u_2[i][j]+u_2[i][j-1])/2 * sqrt(Ra);
1923         }
1924     }
1925
1926     for (i=1;i<N_i+1;i++)
1927     {
1928         for (j=1;j<N_j+1;j++)
1929         {
1930             v_out[i][j] = (v_2[i][j]+v_2[i-1][j])/2 * sqrt(Ra);
1931         }
1932     }
1933
1934     double Nu_average = 0.0;
1935     j = 0;
1936     for (i=1;i<N_i+1;i++)
1937     {
1938         Nu_average += - (T_1[i][j+1]-T_1[i][j])/(x[i][j+1]-x[i][j]) * S_e[i][j+1] / L_y ;
1939     }
1940
1941 //Nusselt number expected
1942 double Nu_expected = 0.0 , Nu_60 = 0.0 , Nu_90 = 0.0 ;
1943
1944 if (angle<=60.0)
1945 {
1946     Nu_expected = 1.0 + 1.44 * max(0.0 , 1.0 - 1708.0/(Ra * cos (angle * PI /180))) * (1.0 -
1947             - 1708.0 * pow(sin(1.8*angle*PI/180.0) ,1.6)/(Ra * cos (angle * PI /180.0)))
1948             + max(0.0 ,pow((Ra * cos(angle*PI/180.0) / 5830.0 ) ,1.0/3.0)-1.0);
1949 }
1950 else
1951 {
1952     Nu_60 = 1.0 + 1.44 * max(0.0 , 1.0 - 1708.0/(Ra * cos (60.0 * PI /180))) * (1.0 -
1953             - 1708.0 * pow(sin(1.8*60.*PI/180.0) ,1.6)/(Ra * cos (60.0 * PI /180.0)))
1954             + max(0.0 ,pow((Ra * cos(60.0*PI/180.0) / 5830.0 ) ,1.0/3.0)-1.0);
1955
1956     Nu_90 = sqrt( 1.0 + pow( (0.066*pow(Ra,1.0/3.0))/(1.0+pow(9000.0/Ra,1.0/4.0) ) , 2.0 ) );

```

```

1955     Nu_expected = (90.0-angle)/30.0 * Nu_60 + (angle-60.0)/30.0 * Nu_90;
1956 }
1957
1958     outfile << "%Time:" << t_2 << "s" << endl;
1959     outfile << "%Derivative value:" << conv << endl << endl;
1960     outfile << "%Nusselt number at x=0.0:" << Nu_average << endl;
1961     outfile << "%Nusselt number expected" << Nu_expected << endl;
1962
1963     outfile << "u=[ " << endl;
1964     for (i=1;i<N_i+1;i++)
1965     {
1966         for (j=1;j<N_j+1;j++)
1967         {
1968             outfile << u_out[i][j] << " ";
1969         }
1970         outfile << endl ;
1971     }
1972     outfile << " ];" << endl;
1973
1974     outfile << endl << endl;
1975     outfile << "v=[ " << endl;
1976     for (i=1;i<N_i+1;i++)
1977     {
1978         for (j=1;j<N_j+1;j++)
1979         {
1980             outfile << v_out[i][j] << " ";
1981         }
1982         outfile << endl ;
1983     }
1984     outfile << " ];" << endl;
1985
1986     outfile << endl << endl;
1987     outfile << "p=[ " << endl;
1988     for (i=1;i<N_i+1;i++)
1989     {
1990         for (j=1;j<N_j+1;j++)
1991         {
1992             outfile << p_1[i][j] / At << " ";
1993         }
1994         outfile << endl ;
1995     }
1996     outfile << " ];" << endl;
1997
1998     outfile << endl << endl;
1999     outfile << "T=[ " << endl;
2000     for (i=1;i<N_i+1;i++)
2001     {
2002         for (j=1;j<N_j+1;j++)
2003         {
2004             outfile << T_1[i][j] << " ";
2005         }
2006         outfile << endl ;
2007     }
2008     outfile << " ];" << endl;
2009     outfile.close();
2010 }
2011 }
2012
2013 double caseConv2D::Nu_value()
2014 {
2015     unsigned int i , j;
2016     double Nu_average = 0.0;
2017     j = 0;
2018     for (i=1;i<N_i+1;i++)
2019     {
2020         Nu_average += - (T_1[i][j+1]-T_1[i][j])/((x[i][j+1]-x[i][j]) * S_e[i][j+1] / L_y ;
2021     }
2022
2023     return Nu_average;
2024 }
2025
2026 double delta_inc (const vector<vector<double> > &A , const vector<vector<double> > &B )
2027 {

```

```

2028     unsigned int i , j;
2029     double temporal = 0.0 , delta = 0.0;
2030
2031     assert(A. size ()==B. size () && A[1]. size ()==B[1]. size ());
2032
2033     for (i=1;i<A. size () -1;i++)
2034     {
2035         for (j=1;j<A[1]. size () -1;j++)
2036         {
2037             temporal = fabs(A[i][j] - B[i][j]) ;
2038             delta = ( (delta > temporal) ? delta : temporal );
2039         }
2040     }
2041     return delta;
2042 }
```

1.2 Numerical scheme selection program

```

1  #include <iostream>
2  #include <cmath>
3
4  using namespace std;
5
6  enum schValues {
7      CDS,
8      UDS,
9      HDS,
10     PLDS,
11     EDS
12 };
13
14 enum schValues str2schValues(string str){
15     if (str=="CDS") return (CDS);
16     else if (str=="UDS") return (UDS);
17     else if (str=="HDS") return (HDS);
18     else if (str=="PLDS") return (PLDS);
19     else if (str=="EDS") return (EDS);
20     else {
21         cerr<<" Error in " << __FUNCTION__ << ":" << str << " condition doesn't exist." << endl;
22     }
23 }
24
25 double A_scheme (double P, string scheme="CDS"){
26     schValues sch;
27
28     sch=str2schValues(scheme);
29
30     switch (sch){
31         case CDS: //Central difference
32             return 1-0.5*fabs(P);
33         case UDS: //Upwind
34             return 1;
35         case HDS: //Hybrid
36             return (0 > (1 - 0.5*fabs(P)) )? 0 : (1 - 0.5*fabs(P));
37         case PLDS: //power law
38             return ( 0 > pow((1 - 0.1*fabs(P)) , 5) )? 0 : pow((1 - 0.1*fabs(P)) , 5);
39         case EDS: //exponential
40             return fabs(P) / ( exp(fabs(P)) - 1 );
41         default:
42             cerr << "Wrong scheme" << endl;
43             break;
44     }
45 }
```

1.3 Conjugate gradient solver

```

1  #include <iostream>
2  #include <vector>
```

```

3  #include <cmath>
4  #include <cassert>
5
6  using namespace std;
7
8  double max_residu (const vector<vector<double> > r);
9
10 vector<vector<double> > conjugate_gradient ( const vector<vector<double> > &a_p , const
11   vector<vector<double> > &a_e , const vector<vector<double> > &a_w , const vector<
12   vector<double> > &a_n , const vector<vector<double> > &a_s , const vector<vector<
13   double> > &phi_0 , const vector<vector<double> > &b , int max_iter , double epsilon
14   , int N_x , int N_y)
15 {
16
17   unsigned int i , j;
18   unsigned int iter;
19   unsigned int N=N_x*N_y;
20
21   vector<vector<double> > r_0(N_y+2,vector<double>(N_x+2,0.0)) , r_1(N_y+2,vector<double>
22   >(N_x+2,0.0));
23   vector<vector<double> > p_0(N_y+2,vector<double>(N_x+2,0.0)) , p_1(N_y+2,vector<double>
24   >(N_x+2,0.0));
25   vector<vector<double> > x_0 = phi_0 , x(N_y+2,vector<double>(N_x+2,0.0));
26
27   for (i=1;i<N_y+1;i++)
28   {
29     for (j=1;j<N_x+1;j++)
30     {
31       r_0[i][j] = r_0[i][j] + a_p[i][j]*x_0[i][j] - a_e[i][j]*x_0[i][j+1] - a_w[i][j]*x_0[i]
32       ][j-1] - a_n[i][j]*x_0[i+1][j] - a_s[i][j]*x_0[i-1][j];
33       r_0[i][j] = r_0[i][j] - b[i][j];
34       p_0[i][j] = - r_0[i][j];
35     }
36   }
37
38   for (iter=1; max_residu(r_0) > epsilon && iter<max_iter ; iter++)
39   {
40     double a_0=0.0 ;
41     double temp1 = 0.0 , temp2 = 0.0;
42     for (i=1;i<N_y+1;i++)
43     {
44       for (j=1;j<N_x+1;j++)
45       {
46         temp1 = temp1 + r_0[i][j]*r_0[i][j];
47         temp2 = temp2 + p_0[i][j]*(a_p[i][j]*p_0[i][j] - a_e[i][j]*p_0[i][j+1] - a_w[i][j]*
48           p_0[i][j-1] - a_n[i][j]*p_0[i+1][j] - a_s[i][j]*p_0[i-1][j]);
49       }
50     }
51     a_0 = temp1 / temp2 ;
52
53     for (i=1;i<N_y+1;i++)
54     {
55       for (j=1;j<N_x+1;j++)
56       {
57         x[i][j] = x_0[i][j] + a_0 * p_0[i][j];
58       }
59     }
60
61     for (i=1;i<N_y+1;i++)
62     {
63       for (j=1;j<N_x+1;j++)
64       {
65         r_1[i][j] = r_0[i][j] + a_0 *(a_p[i][j]*p_0[i][j] - a_e[i][j]*p_0[i][j+1] - a_w[i][j]
66           ][j-1] - a_n[i][j]*p_0[i+1][j] - a_s[i][j]*p_0[i-1][j]);
67       }
68     }
69
70     double b_1 = 0.0 ;
71     temp1 = 0.0 , temp2=0.0 ;
72     for (i=1;i<N_y+1;i++)
73     {
74       for (j=1;j<N_x+1;j++)
75       {
76         r_0[i][j] = r_1[i][j];
77       }
78     }
79   }
80
81   cout << "Residu: " << max_residu(r_0) << endl;
82
83   return 0;
84 }
```

```

67     {
68         temp1 = temp1 + r_1[i][j] * r_1[i][j];
69         temp2 = temp2 + r_0[i][j] * r_0[i][j];
70     }
71 }
72 b_1 = temp1 / temp2 ;
73
74 for (i=1;i<N_y+1;i++)
75 {
76     for (j=1;j<N_x+1;j++)
77     {
78         p_1[i][j] = -r_1[i][j] + b_1 * p_0[i][j];
79     }
80 }
81
82 p_0 = p_1 ;
83 r_0 = r_1 ;
84 x_0 = x ;
85
86 }
87 if (iter>=max_iter) cerr << "Warning: max number of iterations achieved: " << max_residu(
88     r_0) << endl;
89 assert (max_residu(r_0) < epsilon);
90
91 return x;
92 }
93
94 double max_residu (const vector<vector<double> > r)
95 {
96     double temp = 0.0;
97     unsigned int i , j;
98
99
100    for (i=0;i<r.size ();i++)
101    {
102        for (j=0;j<r[1].size ();j++)
103        {
104            temp=(temp<fabs (r [i ][j]))?fabs (r [i ][j]):temp;
105        }
106    }
107
108    return temp;
109 }
110 }
```

1.4 Biconjugate gradient stabilized solver

```

1 #include <iostream>
2 #include <vector>
3 #include <cmath>
4 #include <cassert>
5
6 using namespace std;
7
8 double max_residu (const vector<vector<double> > r);
9
10 vector<vector<double> > biconjugate_gradient_stb ( const vector<vector<double> > &a_p ,
11     const vector<vector<double> > &a_e , const vector<vector<double> > &a_w , const
12     vector<vector<double> > &a_n , const vector<vector<double> > &a_s , const vector<
13     vector<double> > &phi_0 , const vector<vector<double> > &b , int max_iter , double
14     epsilon , int N_x , int N_y)
15 {
16
17     unsigned int i , j;
18     unsigned int iter;
19     unsigned int N=N_x*N_y;
20
21     vector<vector<double> > r_0tilde(N_y+2,vector<double>(N_x+2,0.0)) , r_0(N_y+2,vector<
22     double>(N_x+2,0.0)) , r_1(N_y+2,vector<double>(N_x+2,0.0));
23     vector<vector<double> > p_0(N_y+2,vector<double>(N_x+2,0.0)) , p_1(N_y+2,vector<double
24     >(N_x+2,0.0));
```

```

18     vector<vector<double>> v_0(N_y+2,vector<double>(N_x+2,0.0)) , v_1(N_y+2,vector<double>
19     >(N_x+2,0.0));
20     vector<vector<double>> t(N_y+2,vector<double>(N_x+2,0.0)) , s(N_y+2,vector<double>(N_x
21     +2,0.0));
22     vector<vector<double>> x_0 = phi_0 , x = phi_0;
23     double rho_0 = 1.0 , rho_1 = 0.0 , alfa = 1.0 , beta = 0.0 , w_0 = 1.0 , w_1 = 0.0 ;
24
25     for ( i=1;i<N_y+1;i++)
26     {
27         for ( j=1;j<N_x+1;j++)
28         {
29             x [ i ] [ j ] = 0.0;
30         }
31     for ( i=1;i<N_y+1;i++)
32     {
33         for ( j=1;j<N_x+1;j++)
34         {
35             r_0 [ i ] [ j ] = r_0 [ i ] [ j ] + a_p [ i ] [ j ] * x_0 [ i ] [ j ] - a_e [ i ] [ j ] * x_0 [ i ] [ j+1 ] - a_w [ i ] [ j ] * x_0 [ i
36             ] [ j-1 ] - a_n [ i ] [ j ] * x_0 [ i+1 ] [ j ] - a_s [ i ] [ j ] * x_0 [ i-1 ] [ j ] ;
37             r_0 [ i ] [ j ] = - r_0 [ i ] [ j ] + b [ i ] [ j ];
38         }
39     }
40     r_0tilde = r_0;
41
42     for ( iter=1; max_residu(r_0) > epsilon && iter<max_iter ; iter++)
43     {
44         rho_1 = 0.0 ;
45         for ( i=1;i<N_y+1;i++)
46         {
47             for ( j=1;j<N_x+1;j++)
48             {
49                 rho_1 = rho_1 + r_0tilde [ i ] [ j ] * r_0 [ i ] [ j ];
50             }
51         beta = rho_1 / rho_0 * alfa / w_0 ;
52
53         for ( i=1;i<N_y+1;i++)
54         {
55             for ( j=1;j<N_x+1;j++)
56             {
57                 p_1 [ i ] [ j ] = r_0 [ i ] [ j ] + beta * (p_0 [ i ] [ j ] - w_0*v_0 [ i ] [ j ]) ;
58             }
59         }
60
61         double temp2 = 0.0 ;
62         for ( i=1;i<N_y+1;i++)
63         {
64             for ( j=1;j<N_x+1;j++)
65             {
66                 v_1 [ i ] [ j ] = (a_p [ i ] [ j ] * p_1 [ i ] [ j ] - a_e [ i ] [ j ] * p_1 [ i ] [ j+1 ] - a_w [ i ] [ j ] * p_1 [ i ] [ j-1 ]
67                 - a_n [ i ] [ j ] * p_1 [ i+1 ] [ j ] - a_s [ i ] [ j ] * p_1 [ i-1 ] [ j ]) ;
68                 temp2 = temp2 + r_0tilde [ i ] [ j ] * v_1 [ i ] [ j ] ;
69             }
70         }
71         alfa = rho_1 / temp2 ;
72
73         for ( i=1;i<N_y+1;i++)
74         {
75             for ( j=1;j<N_x+1;j++)
76             {
77                 s [ i ] [ j ] = r_0 [ i ] [ j ] - alfa * v_1 [ i ] [ j ];
78             }
79         }
80
81         for ( i=1;i<N_y+1;i++)
82         {
83             for ( j=1;j<N_x+1;j++)
84             {
85                 t [ i ] [ j ] = (a_p [ i ] [ j ] * s [ i ] [ j ] - a_e [ i ] [ j ] * s [ i ] [ j+1 ] - a_w [ i ] [ j ] * s [ i ] [ j-1 ]
86                 - a_n [ i ] [ j ] * s [ i+1 ] [ j ] - a_s [ i ] [ j ] * s [ i-1 ] [ j ]) ;
87             }
}

```

```

88     }
89
90     double temp3 = 0.0 , temp4 = 0.0;
91
92     for ( i=1;i<N_y+1; i++)
93     {
94         for ( j=1;j<N_x+1; j++)
95         {
96             temp3 = temp3 + t[ i ][ j ] * s[ i ][ j ] ;
97             temp4 = temp4 + t[ i ][ j ] * t[ i ][ j ] ;
98         }
99     }
100    w_1 = temp3/temp4;
101
102   for ( i=1;i<N_y+1; i++)
103   {
104       for ( j=1;j<N_x+1; j++)
105       {
106           x[ i ][ j ] = x_0[ i ][ j ] + alfa * p_1[ i ][ j ] + w_1 * s[ i ][ j ];
107           r_1[ i ][ j ] = s[ i ][ j ] - w_1 * t[ i ][ j ] ;
108       }
109   }
110
111   p_0 = p_1 ;
112   v_0 = v_1 ;
113   r_0 = r_1 ;
114   x_0 = x ;
115   rho_0 = rho_1 ;
116   w_0 = w_1 ;
117
118 }
119 if ( iter>=max_iter) cerr << "Warning: max number of iterations achieved: " << max_residu(
120     r_0) << endl;
121 assert ( max_residu(r_0) < epsilon );
122
123 return x;
124 }
```

1.5 Mesh coordinates program

```

1 #include <iostream>
2 #include <cmath>
3
4 using namespace std;
5
6 double det_coord_Mesh ( double A , unsigned int N , double g , unsigned int i )
7 {
8
9     return A/2.0 * ( 1.0 + tanh( g*( 2.0*(i-1.0)/N - 1.0 ) ) / tanh(g) );
10
11 }
```

1.6 Input file

X_distance 1.0

Y_distance 50.0

Number_of_control_volumes_x 50

Number_of_control_volumes_y 500

Mesh_Type NonUniform 1.2 1.2

Maximum_iterations 100000

Epsilon_vT 1E-5

Epsilon_pT 1E-6

Time_to_end_(s) 500

Prandtl_number 0.71

Rayleigh_number 2.47e5

Angle_deg 90

left_velocity Wall

bottom_velocity Wall

right_velocity Wall

top_velocity Wall

left_temperature Dirichlet 1

bottom_temperature Neumann

right_temperature Dirichlet 0

top_temperature Neumann