Design of a mooring system for a Wave Energy Converter

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July 2015
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Summary

In the current scenario of exhaustion of fossil fuels, new forms of alternative energy must gain ground. Taking advantage of the great amount of water, the element that gives name to our planet (blue planet); this thesis is focused on one of the ways to extract energy from water: using the waves motion in order to generate electricity. The aim of this thesis is to collaborate in a project of a Wave Energy Converter developed by a group of researchers of Politecnico di Torino. Specifically, much of the study will focus on the design a mooring system for the Wave Energy Converter and integrate it to the linear model of the system.

First of all, in the first chapter an overview of the energy consumption problem is given. It is also commented the need of using renewable sources and some of them are presented.

In chapter two, it is presented the wave power as a possible clean energy source. It is also explained with numerical values the great viability of this source both globally and in Europe. Finally, some sea characteristics of the region where the Wave Energy Converter will be placed (Mediterranean Sea, Pantelleria) are also commented.

Then in chapter three, the concept of a Wave Energy Converter is described. Two different ways to classify these devices are given and some existent Wave Energy Converters projects are presented.

In chapter four it is presented the Wave Energy Converter studied in this thesis: the PEWEC (PEndulum Wave Energy Converter). Its working principle it is briefly remarked and some aspects of the mechanism that makes the energy conversion (the pendulum) are commented.

Chapter five is focused on one of the main goals of this thesis: the mooring system. Firstly, a presentation of the objectives, requirements, different configurations and components of a mooring system is given. Then, the PEWEC’s mooring system is described: its design, the integration of the mooring system into the PEWEC’s linear model and the obtaining of the mooring stiffness and parameters.

In chapter six, the 1:45 PEWEC prototype is fully described. Its components are named and are explained. This work is done by comparing the SolidWorks drawings with real images of the prototype.

Chapter seven is focused on the description of the PEWEC’s linear model. It will be commented its parameters and the linear model will be launched. The results of the simulation will be presented and analyzed. In order to facilitate the analysis, a graphic data analysis will be developed.

Then in chapter eight, the tests carried out in Rome with the 1:45 prototype are explained. The testing facilities and the different tests developed are described and the data collected is analyzed.

In chapter nine, it will be possible to develop a first comparison between the linear model simulation and the testing results.
Finally, the last part consists in a conclusion of all the work developed in this thesis. It will be possible to extract some conclusions about the comparison between the linear model and the testing results.

At the end of this document it is possible to find an annex with extra information of the work developed, such as parts of Matlab code.

It is important to comment that this thesis is focused in the 1:45 prototype. However, in parallel it has been developing the 1:12 prototype. The 1:12 prototype is expected to be tested during July 2015. All the study developed in the mooring system for the 1:45 prototype can also be useful for the 1:12 prototype, with a necessary adaptation of the parameters.
Nomenclature

\( a \) Amplitude
\( c \) PTO damping coefficient
\( A(\omega) \) Added mass matrix
\( B(\omega) \) Radiation damping matrix
\( c_p \) Speed of the wave profile
\( CG \) Distance between the center of gravity of the floating body and the mooring point
\( d \) Distance between the hull center of gravity and the pendulum hinge
\( D_{PTO} \) Damping matrix of the PTO (Lagrangian coordinates)
\( E_{p,c} \) Potential energy of the mooring point (concerning elastic forces)
\( f \) Wave frequency
\( F(j\omega) \) Vector of the forces caused by the waves
\( F_a \) Sinking mass downward force
\( F_g \) Jumper upward force
\( g \) Gravitational acceleration
\( h \) Water depth
\( H \) Wave height
\( H_T \) Total depth of the test tank
\( I \) Inertia of the pendulum
\( k \) PTO stiffness coefficient
\( k_x \) Mooring stiffness along x axis
\( k_z \) Mooring stiffness along z axis
\( K_{moor} \) Stiffness matrix related only to the mooring system (Lagrangian coordinates)
\( K_{lin} \) Linearized stiffness matrix (Lagrangian coordinates)
\( l \) Pendulum’s length
\( l_1 \) Length of chain 1
\( l_2 \) Length of chain 2
\( l_3 \) Length of chain 3
\( m_a \) Mass of the sinking mass
\( m_b \) Mass of the hull
\( m_g \) Mass of the jumper
\( m_t \) Linear mass density of the chains
\( m_p \) Mass of the pendulum
\( M_{lin} \) Linearized mass matrix (Lagrangian coordinates)
\( P_{PTO} \) Power of the PTO
\( q \) Vector of generalized coordinates
\( q \) Vector of generalized velocities
\( RAO(\omega) \) Response Amplitude Operator
\( RCW \) Relative Capture Width
\( t \) Time
\( T \) Period
\( T_{PTO} \) Torque of the PTO
\( V_g \) Volume of the jumper
\( x_C \) Displacement of the mooring point along surge direction
\( x_G \) Displacement of the floating body center of gravity along surge direction
\( \dot{x}_G \) Velocity of the floating body center of gravity along surge direction
\( \ddot{x}_G \) Acceleration of the floating body center of gravity along surge direction
\( x_m \) Horizontal coordinate from the mooring reference frame
\( z_C \) Displacement of the mooring point along heave motion
\( z_G \) Displacement of the floating body center of gravity along heave motion
\( \dot{z}_G \) Velocity of the floating body center of gravity along heave motion
\( \ddot{z}_G \) Acceleration of the floating body center of gravity along heave motion
\( z_m \) Vertical coordinate from the mooring reference frame
\( \delta \) Pitch motion
\( \dot{\delta} \) First time derivative of pitch motion
\( \ddot{\delta} \) Second time derivative of pitch motion
\( \varepsilon \) Angular coordinate of the pendulum
\( \dot{\varepsilon} \) Angular velocity of the pendulum
\( \ddot{\varepsilon} \) Angular acceleration of the pendulum
\( \theta_H \) Angular coordinate of the mooring point position respect to the floating body COG
\( \theta_1 \) Angle of chain 1 respect the horizontal axis
\( \theta_2 \) Angle of chain 2 respect the horizontal axis
\( \theta_3 \) Angle of chain 3 respect the horizontal axis
\( \lambda \) Wave length
\( \rho \) Fluid density
\( \omega \) Radian frequency
1. Energy consumption

Currently, mankind is overloading the use of the Earth’s natural resources. The exponential growth of the number of Earth’s inhabitants (more than 7 billion people) is obviously one of the main reasons.

However, the worst problem is not that fact. The real problem is the origin of the energy consumed. The world’s electricity consumption is estimated in 20.45 trillion kWh (2010), 66.6% is obtained from fossil fuels, 18.8% from hydroelectric plants, 7.4% from nuclear fuels and 7.1% from other renewable resources (such as wind energy, geothermal energy or wave energy) [2].

That amount of two thirds of the electricity consumption origin from fossil fuels, which are going to run out in less than a century and also are also highly polluting, is the real big problem.

It is said in many estimations that by the next century, most of the fossil fuel resources will run out. So the energy provided by that kind of resource will must be replaced by other natural resources. In that point, renewable resources charge a leading role. Investigations must be done in order to increase the amount of energy provided by renewable resources that will allow mankind to grow in a sustainable and green way.

Nowadays, different kind of renewable resources already exist and are used. A renewable energy is an energy which comes from a resource that is naturally replenished in a human timescale. The most common renewable energies are related to natural resources such as wind, sunlight, waves, tides, geothermal heat, biomass, biofuel, hydrogen...
The most commonly known renewable technologies are the following ones:

- **Wind power:** consists (generally) in using airflow in order to move a wind turbine. Wind turbines are estimated to produce (maximum) 6 MW, but they normally produce from 1.5 to 3 MW. Wind, as an energy resource, is considered to be one of the most important substituents when fossil fuels will run out. The main inconvenient for ecologists is the visual impact that produces a large area with enormous wind turbines in it.

- **Solar energy:** consists in using the radiation and the heat from the sun in order to produce electricity. The most commonly way to benefit from solar energy is using photovoltaic panels in order to produce direct current (DC). If the electricity produced must be delivered to the electrical grid, it is necessary to install a DC/AC converter (power inverter). This is an inconvenient because it involves an economic cost and, obviously, introducing an electrical circuit will involve a little loss in performance. As in the wind power case, solar energy usually involves a visual impact. To produce a significant amount of electricity, a large number of photovoltaic panels are needed.

- **Biomass:** consists in using biological material coming from living (or recently living) organisms as an energy resource. There are different ways to transform these materials into energy. The most common way is to transform biomass to biofuels (ethanol, butanol, methanol, biodiesel or hydrogen). This transformation can be done by thermal, chemical or biochemical methods. One of the biggest controversies of this technology is that some people criticize the use of cereals as biomass, because it involves an increase in the cereals price which can cause more poverty.

- **Geothermal energy:** consists in using the thermal energy generated and stored in the Earth. The extremely hot temperatures in the Earth’s core produces a heat flow from the core to the surface that can be used to produce energy. For example, this heat flow can be used in order to transform liquid water to vapor and use this vapor to move a turbine.

- **Hydropower:** consists in using water flows (or sea swells) in order to produce energy. There are several ways to benefit from hydropower. It is possible to distinct between energy extracted from the oceans and from other water sources (rivers and lakes commonly). In both cases, it is possible to produce energy by using the thermal gradient in the water. The other way, is to produce energy by using water flow from waves, tides or rivers in order to move turbines or similar to produce energy. For many people, the most common type of hydropower is dams that later produce a waterfall in order to increase the power of the water flow. However, considering the large amount of the Earth’s surface covered by oceans, lots of investigations and projects thinks of the ocean (and the waves, of course) as a potentially important source of energy in a near future because it is a kind of energy not fully explored yet and with lots of opportunities and innovations to do.
2. Wave power

A 71% of our planet extension is covered by water from oceans and seas. So there is a big amount of water available to be used in order to produce energy. Energy can be produced by using the wave energy, tidal turbines, ocean currents turbines or ocean thermal energy. All of these ways to produce energy are renewable sources and involve low environmental impacts.

The aim of study of this thesis is focused in wave energy. The energy of the waves has been studied since years. However, the first device known for converting wave energy to electricity was done by Bochaux-Praceique in 1910 in order to light and power his house near Bordeaux, France. The system was based in an oscillating water-column device.

Decades later (1940s), Yoshio Masuda did several studies of different kind of devices to convert wave energy to electricity. Masuda insisted in the idea of extracting power from the angular motion of an articulated raft.

During the 1970s, due to the oil crisis, the interest for wave energy increased. Researchers like Stephen Salter (Scotland), Kjell Budal and Johannes Falnes (Norway) or McCormick and Mei (USA) investigated and developed new ways to convert wave energy into mechanical energy, and of course, using this mechanical energy to produce electricity.

But as it is known by everybody, the oil price went down and since then fossil fuels have monopolized the energetic usage. However, due to increased concern for environmental issues, the studies of renewable energies (including wave energy) and the manufacturing of devices with this aim have increased in the last years.

First of all, it is important to define the concept of a wave. A wave is a disturbance that propagates through space and time, transferring energy. A regular wave (taking a sinusoidal model as the regular wave) is characterized by different parameters observed in the next figure:

![Figure 2. General parameters of a wave [3]](image)

The amplitude ($a$ [m]) is the distance between the crest and the zero level and also can be defined as the maximum distance measured during a wave period from the mean value. The wave height ($H$ [m]) is the difference between the wave crest and the wave trough, this can also be calculated as two times the amplitude.
The wavelength ($\lambda [m]$) is the distance between two points of the wave that are in the same state of oscillation.

The period ($T [s]$) is the time interval between two points in the same state of oscillation pass a fixed point. The inverse of the period is the frequency ($f [Hz]$) which is the number of wave cycles in one second.

There is another parameter that is not included in the Figure 2, the rate of propagation ($c_p [m/s]$). The $c_p$ is the speed of the wave profile.

A wave in the oceans or seas can be caused by different reasons: by the action of the wind, by the gravity of the Earth, of the Sun or of the Moon; by the Coriolis force due to the Earth’s rotation and by earthquakes and surface tensions. Depending on the reason that causes the wave, it has different characteristics and the energy that transfers varies. The next figure shows the relative energy and the period of the wave distinguishing between different kinds of waves:

![Figure 3. Different kinds of oceans waves by period [3]](image)

Capillary waves are mostly generated by the wind effect, while tidal waves are mostly generated by the gravity of the Sun and the Moon. Moreover, ordinary gravity waves are mostly generated by the own Earth’s gravity. As it seen in Figure 3, capillary waves have a short period (less than 0.1 seconds), while the period of the tidal waves can be of a day or half a day.

Wind waves are produced when wind hits the free surface of oceans and seas. The size of the resulting wave depends on the wind (its speed and duration), the fetch (the distance of open water affected by the wind), the width of area affected by fetch and the water depth. Wind waves can be generated by heavy storms in the middle of the ocean and travel thousands of kilometers to reach the shore. When reaching the shore, the water depth decreases, causing that wind waves lose energy while gaining height. This loose of energy is caused by the friction between the wave and the seabed.
2.1. Wave power as an energy resource

Several studies show that there is actually a large amount of wave power not used but in the future part of it could be used. A study presented during the 29th International Conference on Ocean, Offshore Mechanics and Arctic Engineering (Shanghai, 2010) from the data of the ECMWF (European Centre for Medium-Wage Weather Forecast) shows that fact [3]. In the following table, there is an estimation of the global and regional net power resource:

<table>
<thead>
<tr>
<th>Region</th>
<th>Net power (GW)</th>
</tr>
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<tbody>
<tr>
<td>Europe (N and W)</td>
<td>286</td>
</tr>
<tr>
<td>Baltic Sea</td>
<td>1</td>
</tr>
<tr>
<td>European Russia</td>
<td>3</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>37</td>
</tr>
<tr>
<td>North Atlantic Archipelagos</td>
<td>111</td>
</tr>
<tr>
<td>North America (E)</td>
<td>35</td>
</tr>
<tr>
<td>North America (W)</td>
<td>207</td>
</tr>
<tr>
<td>Greenland</td>
<td>3</td>
</tr>
<tr>
<td>Central America</td>
<td>171</td>
</tr>
<tr>
<td>South America (E)</td>
<td>202</td>
</tr>
<tr>
<td>South America (W)</td>
<td>324</td>
</tr>
<tr>
<td>North Africa</td>
<td>40</td>
</tr>
<tr>
<td>West and Middle Africa</td>
<td>77</td>
</tr>
<tr>
<td>Africa (S)</td>
<td>178</td>
</tr>
<tr>
<td>Africa (E)</td>
<td>127</td>
</tr>
<tr>
<td>Asia (E)</td>
<td>157</td>
</tr>
<tr>
<td>Asia (SE) and Melanesia</td>
<td>283</td>
</tr>
<tr>
<td>Asia (W and S)</td>
<td>84</td>
</tr>
<tr>
<td>Asiatic Russia</td>
<td>23</td>
</tr>
<tr>
<td>Australia and New Zealand</td>
<td>574</td>
</tr>
<tr>
<td>Polynesia</td>
<td>63</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2985</strong></td>
</tr>
</tbody>
</table>

Table 1. Table of the theoretical net power all over the world [4]

As it is seen in the table 1, there is a large amount of wave power possible to use. Only in Europe, there is an estimation of 327 GW. It is true that the Mediterranean Sea has a poor value of net power available comparing to Northern and Western Europe, but this is a normal fact, considering the Mediterranean Sea as a non-open sea and with less extension of other seas or oceans. However, with a good design of a Wave Energy Converter (WEC) specifically for these conditions, it is possible to extract and use all this power.

As it is seen in Table 1, the wave power resource is distributed all over the world. This fact is also possible to observe in the next figure. The next figure is a world map with the part that corresponds to seas and oceans is filled with different colors depending on the intensity of wave power per linear meter able in that part of the world.
As it can be observed in Figure 4, the equatorial zone has less wave power density than the tropics zone (30° to 60°). In the tropics zone is where the power density is maximum. It is also possible to see that in the southern hemisphere there is more amount of wave power density than in the northern hemisphere. It is important to comment that the southern hemisphere has a large amount of oceans and open water comparing to the northern hemisphere.

As this thesis will be focused in a Wave Energy Converter that will be located in the Mediterranean, it is important to take a look in detail of it. In the next figure, it is possible to observe in detail the average wave power density in the Mediterranean:

As it can be observed, the maximum wave power density (about 15 kW/m) is located in the gulf of Lion (between the Spanish/French shore and Corsica and Sardinia’s shore). Then, there are places located in the south part of the Mediterranean that have a wave power density between 8 and 10 kW/m. However, in a general way, it is possible to assure that the Mediterranean is not a powerful sea. But with a specifically design for this sea, a Wave Energy Converter may have a good performance.
3. Wave Energy Converter (WEC)

A Wave Energy Converter is defined as a device that transforms the energy of a wave into electricity. WECs can be classified in different types depending on two factors: its location and its power take-off system.

3.1. Wave Energy Converters classifications

Depending on the location, there are three different types:

- **Shoreline devices**: they are situated in the shore. No moorings and underwater electrical cables are required. However, they can involve a visual impact in the shoreline.
- **Near shore devices**: they are situated in places between 100-200 meters of the shoreline and with a water depth of 10-25 meters. These devices must be fixed to the seabed by moorings or ropes.
- **Offshore devices**: they are situated far away from the shore, where the wave power is higher. They must have a complex mooring system to keep the WEC in the same position and large distances of underwater electrical cables.

On the other hand, WECs can be classified depending on its power take-off system. Following the classification done by *European Marine Energy Centre* (EMEC) [5], 8 different types of WECs are described:

![Figure 6. The different types of WECs (note that number 7 and 8 are not in this figure)](image)

1) **Point absorber**: it is a floating structure that absorbs energy from all directions through transforming its buoyancy movement into electricity. The power take-off (PTO) system can vary depending on the point absorber considered.
2) **Attenuator**: An attenuator is a floating device which operates parallel to the wave direction and effectively rides the waves. These devices capture energy from the relative motion of the two arms as the wave passes them.

3) **Oscillating wave surge converter**: it extracts energy from wave surges and the movement of water particles within them. The arm oscillates as a pendulum mounted on a pivoted joint in response to the movement of water in the waves.
4) *Oscillating water column*: it is a partially submerged and hollow structure. It is open to the sea below the water line, enclosing a column of air on top of a column of water. Waves cause the water column to rise and fall, which in turn compresses and decompresses the air column. This trapped air forms an airflow that then moves a turbine, which usually has the ability to rotate in both directions of the airflow. The rotation of the turbine is used to generate electricity.

![Figure 10. Oscillating water column](image)

5) *Overtopping device*: it captures water when the wave breaks in the shore. When the water is returned to the sea, it passes through a turbine in order to produce electricity. It is common to see overtopping devices with collectors of water in order to concentrate the energy (such as the dams in the rivers).

![Figure 11. Overtopping device](image)

6) *Submerged pressure differential*: it is usually located near the shore line and attached to the seabed. The movement of the waves causes a differential of pressure above the device, which produces a vertical movement of it. This movement pumps a fluid through a system that then produces electricity.
7) **Bulge wave**: it is a rubber tube filled with water and it is moored to the seabed. The water enters through the stern and the passing wave causes pressure variations along the length of the tube, creating a “bulge”. This bulge can be used to move a turbine situated at the bow, where the water it is also returned to the sea. The turbine produces electricity.

8) **Rotating mass**: Two forms of rotation are used to capture energy by the movement of the device heaving and swaying in the waves. This motion drives either an eccentric weight or a gyroscope causes precession. In both cases the movement is attached to an electric generator inside the device.
Apart from these 8 types, there are some devices that cannot be classified in one of the groups above presented such as wave rotors or flexible structures.

### 3.2. Wave Energy Converters projects around the world

Nowadays, there are some projects of WECs all over the world that are interesting to take a look:

- *Anaconda Wave Energy Converter*: it is a bulge wave device. It is developed by *Checkmate SeaEnergy Ltd* and it has been tested at *QinetiQ’s Haslar Marine Technology Park at Gosport*, Hampshire. It is thought to be commercialized in the next five years [6].
- **AquaBuoy**: it is a point absorber device. It is developed by *Finavera*. During 2007, it had been tested near the coast of Newport, Oregon [7].

![Figure 16. AquaBuoy](image)

- **AWS**: it is a submerged pressure differential device developed by *AWS Ocean Energy*. A pilot plan was situated in front of the coast of Portugal in 2004. Then different evolutions of this prototype have been tested in UK waters [8].

![Figure 17. Several AWS devices installed in the seabed](image)
• **CETO**: it is a point absorber device developed by *Carnegie Wave Energy Limited*. It is a full submerged WEC. Apart from producing electricity, the high pressure sea water can also be used to supply a desalination plant, creating a zero emission desalination plant [9].

![CETO device](image1)

*Figure 18. CETO device. Power System (left) and desalination plant concept (right)*

• **Islay LIMPET**: it is a shoreline oscillating water column device developed by *Wavegen* (later called *VOITH*) in cooperation with *Queen’s University Belfast*. It was installed in the Scottish island of Islay in 2000 and it is connected to the UK electrical grid. It is the first wave energy converter commercialized in the entire world. In 2011, a similar device was installed in Mutriku, Basque Country, Spain [10].

![Islay LIMPET](image2)

*Figure 19. LIMPET device*

• **Oceanlinx**: it is a floating oscillating water column device developed in Australia. Different evolutions of the original design have been installed in Port Kembla, New South Wales, Australia: MK1 (2005), MK2 (2007) and MK3PC (2010) [11].
• **OE buoy**: it is an offshore oscillating water column device owned by *OceanEnergy*. It is located in Irish waters. It is based on Yoshio Masuda’s investigations. It has been able to resist powerful Atlantic storms [12].

![OE buoy resisting heavy waves](image)

• **Oyster**: it is a near shore oscillating wave surge device developed by *Aquamarine Power*. It was installed at the *European Marine Energy Centre* (EMEC) in Orkney, Scotland. It is attached to the seabed between 10 and 15 meters depth. As the *LIMPET* device, it is connected to the UK electrical grid since 2009 [13].
- **Wave Dragon**: it is an offshore floating overtopping device developed by Wave Dragon Aps. It is deployed in Nissum Bredning, Denmark. It has been tested since 2003, converting it in one of the first offshore wave energy converters. The project is developed in cooperation with the European Union [14].

- **Pelamis**: it is an offshore attenuator device developed by the Scottish company Pelamis Wave Power. At the beginning, it was tested in Scottish waters in 2004. Then another prototype was deployed in Portuguese waters in 2009. It is composed by four semi-submerged tube sections linked by three power conversion modules [15].
Lysekil Project: it is an offshore point absorber device developed by the Centre for Renewable Electric Energy Conversion at Uppsala University in Sweden. This project has the objective to test a wave energy converter during a long period of time and also to study the impact that the WEC produces to marine organisms [16].

Penguin: it is a rotating mass wave energy converter device developed by the company Wello Oy. It has been tested in the coast of Orkney, Scotland since 2012. Since that, it has resisted several powerful storms with waves of 12 meters height, so it has been shown that a minimum maintenance is required [17].
PowerBuoy: it is an offshore point absorber device developed by Ocean Power Technologies in Pennington, New Jersey. There are two different designs for this project, the APB-350 and the PB40, with different specifications each. It has been deployed (or it is projected to be deployed) in 9 different places of the world, but the first locations where in USA and Australia [18].

Wavepiston: it is a near shore oscillating wave surge device developed by the Danish company Wavepiston ApS. One of the important things of this project is that it is thought to need a modest mooring system due to the force cancelling effect. The system consists in attaching several plates in parallel on a single structure, causing that the forces applied on the structure by the plates will tend to neutralize each other. A prototype will be tested from summer 2015 [19].
- **ISWEC**: the ISWEC (Inertial Sea Wave Energy Converter) is a project developed by researchers of Politecnico di Torino. The conversion from wave power to electrical power is done due to gyroscopic effects. The gyroscope is contained into a floating hull. The hull is sealed with all the mechanical parts inside it, so the deterioration of the mechanical parts is minimum. This involves a low maintenance cost. The hull is slack moored in order to allow a self-orientation of the hull to the waves direction. The system is specially designed for Mediterranean waves. Mediterranean waves are less powerful than oceans waves, although Mediterranean waves have a high frequency. The ISWEC project is tested in Pantelleria, southern Italy [21].

![The ISWEC device](image)

**Figure 29. The ISWEC device**
4. Pendulum Wave Energy Converter (PEWEC)

The Pendulum Wave Energy Converter (PEWEC) is another project that it is being developed by a group of researchers of Politecnico di Torino. The concept is similar to the ISWEC project. The main idea is to cause an oscillation in a pendulum using the hull’s motion caused by the waves. The pendulum’s oscillation involves a mechanical energy that is converted into electrical energy by the power take-off (PTO). The PTO is located in where the pendulum is hinged to the hull structure. In Figure 30, it is possible to observe the basic structure of the PEWEC device:

The generator indicated in the figure above corresponds to the PTO. The PTO is composed by an electrical generator controlled by a servo drive. The PEWEC device, as the ISWEC device, will be slack moored in order to allow the free oscillation movement caused by the waves [21].

As the pendulum is one of the most important parts of this system, it is important to study some basic aspects about pendulums.

4.1. Pendulum history

First of all, it is important to know a brief definition about the pendulum. A pendulum is a physic system that can oscillate due to gravity or due to another physic characteristic. It is composed by a weight suspended from a pivot that can swing freely. However, the most common pendulum and the one studied in this part is the pendulum with an oscillation caused due to gravity. When the pendulum is displaced sideways from its resting equilibrium position, a restoring force due to gravity will accelerate it back toward the equilibrium position. Then the restoring force combined with the pendulum’s mass causes it to oscillate about the equilibrium position, swinging back and forward. The period of a pendulum is the time for a complete cycle, one right swing and one left swing. The length of the pendulum as well as the slight degree on the amplitude will define the period of the pendulum.
Through history, pendulums have had several uses. Until the middle of the XX century, pendulums were considered as the most accurate timekeeping technology. This fact came from Galileo Galilei’s studies about pendulums around 1602. Pendulums were also used as a length standard, as a way of avoid using a piece of metal as a length standard, because a piece of metal could be vulnerable to damage and destruction and also for the need of a length standard that could be recreated in every part of the world (not as a piece of metal located in a specific location). Pendulums have also been used as accelerometers (as well as gravimeters) and as seismometers.

The first known pendulum device was a 1st century seismometer created by the scientist Zhang Heng. The system consisted in a pendulum inside a closed recipient with eight small balls and eight possible exits; each exit was one possible position of a compass. When an earthquake was registered, the pendulum collided a small ball and pushed it through one of the eight different exits (thanks to a lever system). The exit through which the small ball came out indicated the direction where the earthquake was located [22].

Later, during the Renaissance, pendulums were used as a source of power for machines as saws or pumps. It was no until the XVII century when the Italian scientist Galileo Galilei started to study the properties of pendulums. Galileo discovered that the period of a pendulum is independent of the amplitude of the swing. This property, called isochronism, makes pendulums able to be timekeepers. Moreover, Galileo discovered that the period is also independent of the mass of the weight and proportional to the square root of the length of the pendulum. Galileo designed a pendulum clock, but neither Galileo nor his son were able to finish its construction [23].

The first pendulum clock was built by Christiaan Huygens in 1656. With pendulums, the accuracy of clocks at the end of a day raised to a deviation of only 15 seconds. However, in the same century, René Descartes and Marin Mersenne discovered that the pendulum depends on its amplitude in some way. So Huygens studied this problem and determined that the curve that follows a pendulum is a cycloid (not a circular arc as Galileo thought) [24].

In 1666, Robert Hooke studied the conical pendulum, which is a pendulum free to move in two dimensions. Some of his conclusions were suggested to Isaac Newton and were important in his formulation of the law of universal gravitation. Hooke also thought that pendulums would be good gravimeters [25].

Later, it was discovered that the temperature changes affect in the accuracy of pendulum clocks. So John Harrison invented the gridiron pendulum in 1726, which was a temperature-compensated clock pendulum [26].

One of the last important facts about pendulums is the use that Jean Bernard León Foucault gave to pendulums. Foucault used a pendulum free to swing in two dimensions in order to demonstrate the rotation of the Earth. That was the first demonstration of the Earth’s rotation that did not depend on astronomical observation. Nowadays, several science museums of the world have their own “Foucault pendulum” [27].
After this short historical summary of pendulums, it is important to explain in detail the mathematical expressions that define three cases of pendulums: simple gravity pendulum, compound pendulum and cycloidal pendulum.

### 4.2. Simple gravity pendulum

A simple gravity pendulum is an idealized mathematical model of a real pendulum. This idealization assumes that the cord or rod between the hinge and the weight is massless, that the weight is considered as point mass, that motion occurs in only two directions so the weight will trace a circular arc (the cord is not extensible) and that there is not an energy loss due to friction or air drag.

![Figure 31. Simple gravity pendulum](image)

Observing the scheme of Figure 31 and after some simplifications, the differential equation of the pendulum motion is obtained (1):

\[
\frac{\partial^2 \theta}{\partial t^2} + \frac{g}{l} \sin \theta = 0
\]

Where \( g \) is the Earth’s gravity, \( l \) is the length of the simple pendulum and \( \theta \) is the angular coordinate.
If the swings are quite small, it is possible to linearize the equation given above by applying Taylor series (2):

$$\frac{\partial^2 \theta}{\partial t^2} + \frac{g}{l} \theta = 0$$

(2)

This is the typical equation of the harmonic oscillator, that can be solved in the following way (3):

$$\theta(t) = \theta_0 \cos \left( \sqrt{\frac{g}{l}} t \right)$$

(3)

Where $\theta_0$ is the initial amplitude given to the pendulum that is as well the maximum one. From the equation above, it is also easily observed the natural radian frequency of the pendulum ($\omega_n$) (4):

$$\omega_n = \sqrt{\frac{g}{l}}$$

(4)

With the natural radian frequency, the natural period of the pendulum is easily calculated (5):

$$T_n = 2\pi \sqrt{\frac{l}{g}}$$

(5)

From this equation (5) is observed that the period does not depend on the amplitude. This is the property of isochronism observed by Galileo in the XVII century. However, this fact is only true taking into account small swings. This means a small initial amplitude ($\theta_0$), that in mathematical terms means $\theta_0 \ll 1$.

For larger amplitudes, another equation is needed to define the true natural period. There are several ways to find this equation, but are generally large methods with heavy integrals to solve.
4.3. Compound pendulum

A compound pendulum or physical pendulum is the general case of a simple gravity pendulum: the cord or rod between the hinge point and mass point is not massless. This means that every rigid body hinged and swinging from a fixed axis can be considered as a compound pendulum.

As it is seen in Figure 32, the distance $d$ refers to the distance between the hinge point and the centre of mass. In this case, a mechanical torque appears in the hinge point due to the act of the perpendicular component to the distance $d$ of the gravity force (6):

$$\tau = I_p \alpha$$  \hspace{1cm} (6)

Where $\tau$ is the mechanical torque, $I$ is the moment of inertia around the hinge point and $\alpha$ is the angular acceleration. The angular acceleration can also be expressed by $\frac{\partial^2 \theta}{\partial t^2}$.

Besides, the torque is generated by the force of gravity, so can be expressed in the following way (7):

$$\tau = -mgd \sin \theta$$  \hspace{1cm} (7)

With both equations and assuming an amplitude of the swings quite small, it is possible to obtain the following linearized equation of motion (8):
\[ \frac{\partial^2 \theta}{\partial t^2} + \frac{mgd}{I_p} \theta = 0 \]  \hspace{1cm} (8)

The equation is very similar to the simple gravity pendulum. So the period of the pendulum can be expressed in a similar way (9):

\[ T_n = 2\pi \sqrt{\frac{I_p}{mgd}} \]  \hspace{1cm} (9)

If the natural periods of the simple gravity pendulum and the compound pendulum are compared, it is possible to observe that the natural period of the compound pendulum is the same as the simple gravity pendulum but taking as the parameter \( l \) an equivalent length (10):

\[ l = \frac{I_p}{md} \]  \hspace{1cm} (10)

This equivalent length \( l \) refers to the distance between the hinge point and the center of oscillation. The center of oscillation (or center of percussion) is the point of a rigid body where a perpendicular impact to the imaginary line between the hinge point and the center of gravity will produce a cancellation of the force and the torque in the hinge point.

### 4.4. Cycloidal pendulum

A cycloidal pendulum is a simple pendulum suspended from the cusp of an inverted cycloid. This also involves a cycloidal trace of the pendulum’s weight. As Christiaan Huygens discovered (seen before in pendulum history), following a cycloidal trace involves that the period of the pendulum is independent on the amplitude, in other words, the cycloidal pendulum is isochronous.

![Cycloidal pendulum](Figure 33. Cycloidal pendulum)
The cycloidal pendulum has the following equations of motion (being $x$ the horizontal axis and $y$ the vertical axis) (11):

\begin{align}
  x &= a(\theta - \sin \theta) \\
  y &= a(1 + \cos \theta)
\end{align}

(11)

As seen in Figure 33, $a$ is the radius of the circumference that generates the cycloid.

It is possible to assure that the cycloidal pendulum is isochronous and harmonic. This allows defining its natural period in a similar way as the simple gravity pendulum and the compound pendulum (12):

\[ T_n = 2\pi \sqrt{\frac{4a}{g}} \]

(12)

### 4.5. PEWEC working principle

PEWEC (Pendulum Wave Energy Converter) is a passive system designed to produce electricity through the motion of a pendulum. The pendulum is located inside a hull which is attached to the seabed by a slack mooring system. In the next figure, it is possible to observe a scheme of the system with the reference frames and some of the system coordinates:

![Figure 34. Scheme of the PEWEC system [21]](image-url)
As it is seen in Figure 34, there are three reference frames:

- \( \text{O-xyz} \): the general reference frame with origin \( \text{O} \).
- \( \text{G-x}_{1}\text{y}_{1}\text{z}_{1} \): the hull reference frame with origin in its center of gravity (\( \text{G} \)).
- \( \text{A-x}_{2}\text{y}_{2}\text{z}_{2} \): the moving mass reference frame with origin in its hinge point (\( \text{A} \)).

The \( x \) axis refers to the waves direction, while \( z \) axis refers to the heave motion. The \( y \) axis is oriented by using the right hand rule. It is not an important axis due to the fact that the hull is able to self-orient itself thanks to a slack mooring system, so the motion in \( y \) axis is considered insignificant.

Point \( \text{O} \) is the origin of the general reference frame, point \( \text{A} \) is the hinge point, point \( \text{P} \) is the center of gravity of the pendulum’s mass and point \( \text{G} \) is the center of gravity of the hull.

The parameter \( l \) is the distance between the hinge point (\( \text{A} \)) and the center of gravity of the pendulum’s mass (\( \text{P} \)). Finally, the parameter \( d \) is the distance between the hinge point (\( \text{A} \)) and the center of gravity of the hull (\( \text{G} \)).

The motion of the hull is given by the surge motion \( x_{G} \), by the heave motion \( z_{G} \) and by the pitch motion \( \delta \). The pitch motion \( \delta \) is a rotation around \( y \) axis.

The angular coordinate \( \epsilon \) refers to the relative motion between the hull and the pendulum. This coordinate is of utmost importance because is the motion that will produce electricity. In fact, the power can be calculated, in a general way, as (13):

\[
P_{\epsilon} = T_{\epsilon} \dot{\epsilon}
\]

(13)

Where \( T_{\epsilon} \) is the mechanical torque in the hinge point and \( \dot{\epsilon} \) is the angular speed of the pendulum motion.

The oscillation of the waves will produce an oscillation of the hull. In consequence, the pendulum will start to swing. The pendulum is hinged to a generator that will do its function and will produce electricity. As it has been commented in the beginning of the chapter, a PTO will do the function of the generator, since the PTO also incorporates a servo drive controller.
5. Mooring system

In this chapter is going to be studied and developed the main goal of the thesis: the mooring system. A mooring is a system that keeps a floating body in the same position in the sea allowing it to move with waves and tides motion without involving forces punctually high. In the case of this study, the floating body is a WEC. This fact involves a list of requirements that the mooring system must accomplish [28]:

- Maintain the floating structure on station under normal operation conditions or extreme storm conditions.
- Avoid tension loads in the electrical transmission.
- Sufficiently compliant to the environmental loading to reduce the forces acting on anchors, mooring lines and the device itself; unless the stiffness of the mooring itself is an active element in the wave energy conversion principle used.
- The components must have an adequate strength, fatigue life and durability depending on the device lifetime.
- A degree of redundancy is highly recommended for individual devices and essential for several devices linked together.
- The system should be able to last 30 or more years, with replacement of some components in a period no shorter of 5 years.
- Able to accommodate the tidal range at installation location.
- Should allow the removal of single devices without affecting the mooring of adjacent devices.
- Should allow the removal of mooring lines for inspection and maintenance.
- The mooring must be sufficiently stiff to allow berthing for inspection and maintenance purposes.
- Contact between mooring lines must be avoided.
- The mooring system must develop its function without affecting the device performance. If the mooring system is an active part of the system, it must be designed in order to increase the efficiency of the global system.

5.1. Mooring configurations

In this section, it is going to be explained the classification of different mooring configurations existing. For this purpose it will be used an study developed by Robert E. Harris, Lars Johanning and Julian Wolfram [28], that classifies the possible mooring configurations in three different groups: spread moorings, single point moorings and dynamic positioning. The study also comments the suitability of each configuration for a WEC.

5.1.1. Spread moorings

- **Catenary mooring**: the mooring lines arrive horizontal to the seabed so it only appears a horizontal force in the anchor point. The weight of the mooring lines causes the restoring forces, returning the system to the equilibrium position. It is considered a good option for use in WECs’ mooring systems.
- **Multi-catenary mooring**: multi-catenary mooring incorporates buoys and weights. Buoys help to support the weight of the mooring system in order to reduce the restoring forces that the WEC has to resist. It is highly recommended for WECs’ mooring systems.

- **Taut spread mooring**: also called Tethered mooring, the mooring lines arrive with a certain angle to the seabed. However, the anchor is able to resist either horizontal or vertical forces. The restoring forces are caused by the buoyancy of the topside body, due to the fact that the mooring lines tend to be perpendicular to the seabed. It is not considered as a good option for WECs’ mooring systems.

![Taut & Catenary Mooring Scopes](image1)

**Figure 35. Comparison between catenary and taut mooring [29]**

### 5.1.2. Single point moorings

- **Turret mooring**: a catenary moored turret (internal or external) is attached to the floating body that is allowed to weathervane around the turret. It is not considered as one of the best options for WEC’s mooring system.

![Turret Mooring FPSO](image2)

**Figure 36. Internal turret mooring attached to a ship**

- **Catenary anchor leg mooring (CALM)**: The floating structure is moored to a catenary moored buoy and is able to weathervane around the moored buoy. It is highly recommended for WECs’ mooring systems.
• Single anchor leg mooring (SALM): The floating structure is moored to a single anchored taut buoy and is able to weathervane around the moored buoy. It is highly recommended for WECs’ mooring systems.

• Articulated Loading Column (ALC): a moored floating structure can weathervane around a bottom hinged column, which has a swivel above the water line. It is accepted for WEC’s mooring system but no as the best option.
- **Single point mooring and reservoir (SPAR):** it is composed by a catenary anchored buoy that allows the storage of hydrogen or oil and a floating body can weathervane aground the mooring point. It is also accepted for WEC’s mooring system but no as the best option.

- **Fixed tower mooring:** A fixed tower anchored into the seabed allows the moored floating structure to weathervane around the mooring point. It is also accepted for WEC’s mooring system but no as the best option.
5.1.3. Dynamic positioning

- **Active mooring**: several mooring lines are attached around the floating body. Every inboard end of each mooring line is controlled by a servo in order to regulate the tension applied to the mooring lines depending on the sea conditions. It is not considered as a good option for WECs’ mooring systems.

- **Propulsion**: it consists in using propellers or thrusters controlled by a central computer in order to maintain fixed a floating body in the sea respect a point of reference. It is not considered as a good option for WECs’ mooring systems.

5.2. Mooring components

A mooring system is commonly composed by two components: the mooring line and the anchor. These two components are used along with connecting elements, floats, jumpers or buoys. All components must be chosen as a result of an extensive study with key factors such as the mooring configuration, location and the expected duration of the mooring system. In the following lines, different types of mooring lines and anchors are presented [28].

5.2.1. Mooring lines

- **Chain**: chains provide a good catenary stiffness effect and have good abrasion and bending properties. Suitable for long term moorings but require regular inspections.

![Figure 42. Chain mooring line](image)

- **Wire rope**: wire ropes are elastic and able to be part of a tensioned mooring system. However, extreme bending must be avoided. Wire ropes involve a lower cost than other mooring lines considered.

![Figure 43. Wire rope mooring line](image)
- **Synthetic rope:** synthetic ropes are usually made of materials like polyester, aramid, HMPE or nylon. The weight of the ropes in water is around zero allowing them to be close to neutrally buoyant or buoyant. The weight and elasticity properties make them more common for very deep water tether applications. However, an expensive cost, the need of re-tensioning the rope and the need of avoiding axial compression, heating and fish bites could represent a serious problem.

![Synthetic rope mooring line](image)

**Figure 44. Synthetic rope mooring line**

5.2.2. **Anchors**

- **Gravity anchor:** a dead weight is placed on the seabed. The mooring line is attached to the anchor. This system provides a horizontal holding capacity due to the friction between the seabed and the anchor.

![Gravity anchor](image)

**Figure 45. Gravity anchor**

- **Drag-embedment anchor:** horizontal holding capacity is generated in the main instalment direction by the embedment of the anchor in the ground.

![Drag-embedment anchor](image)

**Figure 46. Drag-embedment anchor**
- **Driven Pile / Suction anchor:** horizontal and vertical holding capacity is generated by forcing a pile mechanically or from a pressure difference into the ground, providing friction along the pile and the ground.

![Figure 47. Suction anchor](image)

- **Vertical load anchor:** horizontal and vertical holding capacity is generated due to a specific embedment anchor allowing loads not only in the main instalment direction. Vertical load anchors are similar to drag-embedment but are anchored deeper.

![Figure 48. Vertical load anchor](image)

- **Drilled and grouted anchor:** horizontal and vertical holding capacity is generated by grouting a pile in a rock with a pre-drilled hole.

![Figure 49. Drilled and grouted anchor](image)
5.3. **PEWEC’s mooring system**

The PEWEC needs a mooring system which permits the self-orientation of the device to the waves direction, because it only works in an efficient way if the waves reach it in the \( x \) axis direction. In addition, the PEWEC needs a mooring system able to keep the device in a certain position in the ocean. However, this mooring system must not influence the performance of the system. In other words, the mooring system must avoid high stress forces. So one of the main goals of the design is to try to minimize the stiffness introduced by the mooring line to the PEWEC system.

For the reasons exposed in the paragraph above, it is thought as a possible solution a single point slack mooring. Being a single point mooring will allow the device a certain freedom to self-orientate to the waves direction. Moreover, being a slack mooring will allow the device to keep in a position in the ocean without affecting its optimum performance.

All the work developed in this chapter has been done taking the 1:45 PEWEC prototype as the reference one. However, the 1:8 PEWEC prototype is being developed at the same time. The mooring system will be the same for both prototypes so it is only necessary to scale the parameters of the 1:45 prototype.

5.3.1. **Mooring design**

Following the premises presented above, in the next figure it is possible to observe a representation of how the PEWEC’s mooring system will be:

![Mooring system scheme](image)

*Figure 50. Mooring system scheme*
The Figure 50 shows the basic scheme of the PEWEC’s mooring system. It is composed by three chains, a dead weight, a jumper and a mass. This configuration with three different chains instead of only one is done to provide the system a slack mooring. In the next lines, the items of the mooring system are described:

- **Chains**: there are three chains that are used as mooring lines. The chains have been selected as the mooring line because it is the most common material in the sector with a wide range of dimensions and types available. Chain 1 joins the dead weight with the jumper, chain 2 joins the jumper with the mass and chain 3 joins the mass with the PEWEC device. The chains have all the same linear mass density ($m_l$).

- **Dead weight**: the dead weight has the mission to provide the system a horizontal holding capacity, it is a gravity anchor (seen in 5.2.2.). It consists in a block of a heavy material, such as cement, which is placed on the seabed or the tank where the tests are done.

- **Jumper**: the jumper is the yellow circle drawn in Figure 50. It has the mission to generate an upward force, a buoyancy effect. As it will be seen later, the jumper in the 1:45 prototype is a plastic football ball.

- **Mass**: the mass is the red square drawn in Figure 50. It has the mission to generate a downward force, a sinking effect. In the 1:45 prototype, the mass is a cubic piece of metal.

In addition, in Figure 50 it is also possible to observe some parameters. The distances $l_1$, $l_2$ and $l_3$ correspond to the chains length. $H_T$ corresponds to the total depth that is covered by water, while $h$ is the real depth between the start of the first chain and the water surface. $h$ is the parameter that will be taken into account for the mooring design and it can be calculated by subtracting the dead weight’s height to the total depth $H_T$. $F_a$ and $F_g$ are the downward force generated by the mass and the upward force generated by the jumper respectively. Point $C$ corresponds to the mooring point in the PEWEC prototype. Point $C$ can be located in different positions, which are determined by the parameters $CG$ (the straight distance from the PEWEC’s center of gravity to point $C$) and $\theta$ (the angular distance in radians between the x axis of the reference frame and the mooring point $C$). Furthermore, it is easily deductible that point $G$ corresponds to the PEWEC’s center of gravity. Finally, the scheme includes a reference frame with origin point $G$, that will be used later to develop some equations.

Thus at this point appears the main goal of this thesis: to incorporate the mooring design to the PEWEC whole system. The challenge resides in the fact that it is a linear system. This challenge will be dealt with in detail in the following points. To achieve this objective, a concrete strategy will be applied. The strategy consists in divide the system into two different subsystems: the first subsystem will be the PEWEC device and the other one will be only the mooring system. From the mooring subsystem, its stiffness will be obtained ($k_x$ and $k_z$) and its parameters (such as the chains length, the chains’ linear mass density or the mass and the jumper that will determine the forces $F_a$ and $F_g$) will be discussed in order to find the most adequate ones. On the other hand, the PEWEC subsystem will be used to...
integrate the mooring stiffness into the linear system and also to study the most proper position for the mooring point $C$. In the next figure, it is possible to observe a conceptual diagram of this strategy:

Figure 51. Conceptual diagram of the strategy
5.3.2. Integration of the mooring stiffness into the PEWEC’s linear system

The goal of this point is to add the mooring stiffness parameters \( k_x \) and \( k_z \) to the system stiffness matrix \( K_{lin} \). First of all, it is important to show how the linearized dynamic equations are (in matrix form) [21]:

\[
\mathbf{M}_{lin} \begin{bmatrix} \ddot{x}_G \\ \ddot{z}_G \\ \ddot{\delta} \\ \ddot{\varepsilon} \end{bmatrix} + \mathbf{D}_{PTO} \begin{bmatrix} \dot{x}_G \\ \dot{z}_G \\ \dot{\delta} \\ \dot{\varepsilon} \end{bmatrix} + \mathbf{K}_{lin} \begin{bmatrix} x_G \\ z_G \\ \delta \\ \varepsilon \end{bmatrix} = \mathbf{0} \tag{1}
\]

If equation (1) is observed, there are three different matrixes: \( \mathbf{M}_{lin} \) (linearized mass matrix associated to Lagrangian coordinates), \( \mathbf{D}_{PTO} \) (damping matrix of the Power Take Off in Lagrangian coordinates) and \( \mathbf{K}_{lin} \) (linearized restoring matrix or stiffness matrix associated to Lagrangian coordinates). The Lagrangian coordinates are the generalized coordinates that allow describing the motion of the system. Observing Figure 34, it is possible to identify the vector of generalized coordinates:

\[
\mathbf{q} = \begin{bmatrix} x_G \\ z_G \\ \delta \\ \varepsilon \end{bmatrix} \tag{2}
\]

It is important to comment that \( x_G \) and \( z_G \) correspond to the horizontal and vertical displacement of the center of gravity \( G \).

Thus to achieve the objective of this point, it is going to be used the expression of the potential energy for point \( C \). However, this expression is going to be limited to the elastic forces caused by the mooring stiffness. In the next figure, it is possible to observe a simplified scheme of how will the system be analyzed:

As seen in Figure 52, the mooring system is simplified in two springs with a mooring stiffness value each one. Following this scheme, it is possible to find the expression of the potential energy:
\[ E_{p,C} = \frac{1}{2} (k_x x_C^2 + k_z z_C^2) \] (3)

Where \( k_x \) and \( k_z \) correspond to the mooring stiffness, while \( x_C \) and \( z_C \) correspond to the horizontal and vertical displacement of point \( C \) respectively. So the next logical step is to find the motion of point \( C \). This motion can be described as the composition of the point \( G \) displacement \( (x_G, z_G) \) and the contribution of \( \delta \) rotation (pitch motion). Observing Figure 34 and Figure 50:

\[
\begin{bmatrix}
  x_C \\
  z_C
\end{bmatrix} = \begin{bmatrix}
  x_G + CG \cos (\delta + \theta_H) \\
  z_G - CG \sin (\delta + \theta_H)
\end{bmatrix}
\] (4)

With the expression (4), it is possible redefine the expression of the potential energy:

\[ E_{p,C} = \frac{1}{2} (k_x (x_G + CG \cos (\delta + \theta_H))^2 + k_z (z_G - CG \sin (\delta + \theta_H))^2) \] (5)

The expression of the potential energy can also be expressed as its quadratic form:

\[ E_{p,C} = \frac{1}{2} q^T K_{moor} q \] (6)

Where \( K_{moor} \) is the mooring stiffness matrix and \( q \) is the vector of generalized coordinates of the mooring system, which can be defined as:

\[ q = \begin{bmatrix}
  x_G \\
  z_G \\
  \delta
\end{bmatrix} \] (7)

The objective is to develop equation (5) so as to reach a quadratic form that could be compared with equation (6). After using trigonometric identities and quadratic approximations for \( \text{sines} \) and \( \text{cosines} \), the quadratic approximation for the potential energy is:

\[ E_{p,C} = \frac{1}{2} (k_x x_G^2 + k_z z_G^2 - 8 k_x CG \theta_H x_G \delta - 2 k_x CG z_G \delta + CG^2 (k_z - k_x) \delta^2) \] (8)
From a quadratic form such as equation (8), it is easy to obtain its quadratic matrix:

\[
K_{\text{moor}} = \begin{bmatrix}
    k_x & 0 & -4k_xCG \theta_H \\
    0 & k_z & -k_z CG \\
    -4k_xCG \theta_H & -k_z CG & CG^2(k_z - k_x)
\end{bmatrix}
\]  

(9)

Observing \( K_{\text{moor}} \) (9), it is interesting to comment the existence of a coupling between coordinates \( x_G, \delta \) and between coordinates \( z_G, \delta \) in terms of mooring stiffness. This fact suggests that the quadratic approximation could be valid, because those couplings make sense mechanically and physically.

At this point, it is possible to add the mooring linear system to the entire system, ergo this means to add \( K_{\text{moor}} \) matrix to \( K_{\text{lin}} \) matrix. Therefore, the new \( K_{\text{lin}} \) matrix is the following one:

\[
K_{\text{lin}} = \begin{bmatrix}
    k_x & 0 & -4k_xCG \theta_H & 0 \\
    0 & k_z & -k_z CG & 0 \\
    -4k_xCG \theta_H & -k_z CG & CG^2(k_z - k_x) - m_p g(d - l) & m_p g l \\
    0 & 0 & m_p g l & k + m_p g l
\end{bmatrix}
\]  

(10)

Matrix (10) is composed by the stiffness terms of the mooring system plus stiffness terms associated to the gravity force and to the PTO \( (k) \). The new parameters introduced in this matrix \( (m_p, d, l \text{ and } k) \) can be consulted at the beginning of this document in the section Nomenclature.

With \( K_{\text{lin}} \) matrix, the mooring system has been integrated into the PEWEC’s linear system. It remains to develop a study of possible values for the mooring stiffness. This study will be done in the next section of this chapter.

Finally, this possible solution allows modifying the mooring point \( C \) in order to find the most suitable position. The most suitable position is the one that minimizes the negative influence of the mooring system to the PEWEC’s performance. This task could be easily done by modifying the parameters that define the mooring point \( (CG \text{ and } \theta_H) \). On the one hand, \( CG \) is a constant distance that cannot be modified. On the other hand, \( \theta_H \) is an angle that can be modified depending on the mooring point selected. Different angles \( \theta_H \) can be tested in the linear model in order to find the most suitable one, which will determine the mooring point position.
5.3.3. Obtaining of the mooring stiffness and discussion of mooring parameters

Obtaining of the mooring stiffness is not an easy task. Thus to achieve this objective, some simplifications are going to be applied:

- The study will be carried out as a static case.
- The chains are going to be considered as rigid bodies with the mass concentrated in their center of gravity.
- The depth $h$ (distance between the sea surface and the dead weight) is going to be considered as the vertical distance between the mooring point $C$ and the dead weight due to the fact that applying this simplification allows the obtaining of the stiffness (the stiffness is not going to vary for a few centimeters and it would be enough significant) and also allows an easier calculus.
- It is only going to be considered the case $F_g > F_a$ because it is the one that permits the mooring design proposed in point 5.3.1.

The mooring scheme (applying the simplifications) will be the following one:

![Simplified mooring scheme](image)

**Figure 53. Simplified mooring scheme**

In the scheme presented in Figure 53 compared with other mooring schemes presented previously, there are two important news: the presence of three angles ($\theta_1$, $\theta_2$ and $\theta_3$) that represents the inclination of the chains respect the horizontal axis; and the presence of a new reference frame ($x_m$, $z_m$ and $y_m$ following the right hand rule). This new reference frame is the mooring reference frame.
With these premises, a *Matlab* script has been developed. This script is attached in the Appendix A.1. The script needs as input data the following parameters:

- **h**: depth covered by water. Taking into account the towing tank used for testing, this depth will be of 3,1 meters ($H_T$ is about 3,5 meters and the dead weight measures 40 centimeters of height).
- **Chains length**: lengths $l_1$ and $l_2$ need to be input, while $l_3$ is calculated by the *Matlab* code depending on the other two lengths and the depth $h$. So as to assess different lengths, $l_1$ and $l_2$ are going to be vectors of lengths:
  
  $l_1 = [0.75, 1, 1.25, 1.5, 2] m$
  
  $l_2 = [0.2, 0.4, 0.5] m$
- **Linear mass density of the chains** ($m_l$): all the chains have the same linear mass density. This value will be a commercial chain value of $\frac{32860}{800}$ kg/m.
- **Forces**: forces $F_g$ and $F_a$ (upward and downward force respectively) are calculated with the following formulas:

  $$F_g = (V_g \rho - m_g) g$$

  $$F_a = m_a g$$

Where $V_g$ the volume of a sphere that acts as a jumper, $\rho$ is the water density ($1000$ kg/m$^3$), $m_g$ is the mass of the jumper, $m_a$ is the mass of the sinking mass and $g$ is the Earth’s gravitational acceleration ($9.81$ m/s$^2$).

In order to test different values of these forces, a vector of 10 values for each force is going to be input. These values are going to be input taking into account the scale of magnitude of the prototype.

The objective of this study is to get an idea of the scale of magnitude of the mooring stiffness due to the need of inputting $k_x$ and $k_z$ values to the linear system. For each configuration of mooring parameters, it is obtained a contour plot as the following one:

![Figure 54. Contour plot for mooring stiffness](image-url)
Figure 54 shows how the contour plots for mooring stiffness are. In each plot there are two different graphics: one corresponds to \( k_x \) and the other one corresponds to \( k_z \). \( k_x \) and \( k_z \) values are represented in different colors, as it is seen in the right hand color scale. In addition, it is easily seen that the condition \( F_g > F_a \) is fulfilled, because the area not colored belongs to points that not fulfill this condition. There are a total of 24 contour plots as the example one above in Figure 54. Each contour plot represents one configuration of the mooring parameters \((l_1, l_2, m_l)\).

As a way of understanding the maximum influence that the mooring system can cause in the PEWEC’s performance, it will be considered the worst case scenario. This means considering the maximum stiffness values for each parameters configuration. It is important to comment that maximum values occur when forces \( F_g \) and \( F_a \) are maximum, in the upper right corner of each plot. In the following table, it is possible to observe these values (note that parameter \( m_l \) is not included in the table because is a constant value):

<table>
<thead>
<tr>
<th>( l_1 (m) )</th>
<th>( l_2 (m) )</th>
<th>( k_x ) (N/m)</th>
<th>( k_z ) (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,75</td>
<td>0,2</td>
<td>2,5909</td>
<td>0,46428</td>
</tr>
<tr>
<td>0,75</td>
<td>0,3</td>
<td>2,3145</td>
<td>0,31971</td>
</tr>
<tr>
<td>0,75</td>
<td>0,4</td>
<td>2,1118</td>
<td>0,24765</td>
</tr>
<tr>
<td>0,75</td>
<td>0,5</td>
<td>1,9512</td>
<td>0,20519</td>
</tr>
<tr>
<td>1</td>
<td>0,2</td>
<td>2,5820</td>
<td>0,45655</td>
</tr>
<tr>
<td>1</td>
<td>0,3</td>
<td>2,3084</td>
<td>0,33163</td>
</tr>
<tr>
<td>1</td>
<td>0,4</td>
<td>2,1038</td>
<td>0,24677</td>
</tr>
<tr>
<td>1</td>
<td>0,5</td>
<td>1,9427</td>
<td>0,19626</td>
</tr>
<tr>
<td>1,25</td>
<td>0,2</td>
<td>2,5752</td>
<td>0,46284</td>
</tr>
<tr>
<td>1,25</td>
<td>0,3</td>
<td>2,2992</td>
<td>0,32554</td>
</tr>
<tr>
<td>1,25</td>
<td>0,4</td>
<td>2,0956</td>
<td>0,24589</td>
</tr>
<tr>
<td>1,25</td>
<td>0,5</td>
<td>1,9353</td>
<td>0,19921</td>
</tr>
<tr>
<td>1,5</td>
<td>0,2</td>
<td>2,5656</td>
<td>0,45515</td>
</tr>
<tr>
<td>1,5</td>
<td>0,3</td>
<td>2,2918</td>
<td>0,32438</td>
</tr>
<tr>
<td>1,5</td>
<td>0,4</td>
<td>2,0873</td>
<td>0,24501</td>
</tr>
<tr>
<td>1,5</td>
<td>0,5</td>
<td>1,9280</td>
<td>0,20214</td>
</tr>
<tr>
<td>1,75</td>
<td>0,2</td>
<td>2,5551</td>
<td>0,44749</td>
</tr>
<tr>
<td>1,75</td>
<td>0,3</td>
<td>2,2846</td>
<td>0,32321</td>
</tr>
<tr>
<td>1,75</td>
<td>0,4</td>
<td>2,0813</td>
<td>0,24827</td>
</tr>
<tr>
<td>1,75</td>
<td>0,5</td>
<td>1,9189</td>
<td>0,19330</td>
</tr>
<tr>
<td>2</td>
<td>0,2</td>
<td>2,5489</td>
<td>0,45373</td>
</tr>
<tr>
<td>2</td>
<td>0,3</td>
<td>2,2737</td>
<td>0,31719</td>
</tr>
<tr>
<td>2</td>
<td>0,4</td>
<td>2,0731</td>
<td>0,24737</td>
</tr>
<tr>
<td>2</td>
<td>0,5</td>
<td>1,9117</td>
<td>0,19622</td>
</tr>
</tbody>
</table>

Table 2. Maximum stiffness values for each configuration
With the stiffness values contained in Table 2, it is possible to perform several data analysis in order to understand the behavior of the stiffness while the mooring parameters change, thus it will be possible to find the most suitable parameters for the mooring system.

First of all, a graphic with the stiffness in the y axis and a relation \( l_1/l_2 \) in the x axis (a division between \( l_1 \) and \( l_2 \)) is represented:

![Figure 55. Stiffness of the mooring depending on \( l_1/l_2 \)](image)

As seen in Figure 55, it seems to exist a tendency in the mooring stiffness when the chains lengths change. It is a growth tendency: when \( l_1/l_2 \) increases, the stiffness of the mooring increases. This relation \( l_1/l_2 \) can increase by two reasons: \( l_1 \) increases more than \( l_2 \) or \( l_2 \) decreases more than \( l_1 \). These cases will be studied in the following lines. This is of great importance because it will be possible to recognize which length parameter is more sensitive to variations of its value. In other words, it will be possible to know which parameter is freer to be changed without affecting overly the stiffness of the mooring.

Firstly, it is going to be studied what happens with the length \( l_1 \). A graphic with the stiffness in the y axis and \( l_1 \) in the x axis is represented in order to evaluate the influence of \( l_1 \) in the mooring tendency observed in Figure 55:
As seen in Figure 56, for both $k_x$ and $k_z$ there is not a clear tendency when $l_1$ increases. It seems that the influence of parameter $l_1$ is not significant, or at least it is not the parameter that affects more to the variability of the stiffness. So it will be interesting to focus on parameter $l_2$.

Finally, it is going to be studied the influence of $l_2$. For this purpose, a graphic with the stiffness in the y axis and $l_2$ in the x axis is represented in Figure 57:
As observed in Figure 57, for both $k_x$ and $k_z$ there is a clear tendency in the stiffness. Specifically, when $l_2$ increases, the stiffness (both $k_x$ and $k_z$) decreases. It is also important to observe that for each value of $l_2$, all the stiffness values seem to be the same point in the graphic (they are superposed). This could mean that parameter $l_1$ does not involve a significant change in the stiffness, which reaffirms the conclusions observed in Figure 56.

So as a conclusion of this data analysis, changing parameter $l_1$ does not affect significantly to the behavior of the mooring stiffness. However, changing parameter $l_2$ affects significantly to the mooring stiffness. Concretely, if $l_2$ increases, then the stiffness values decreases. This means that $l_2$ will be as long as possible (respecting the original mooring design and within reasonable values) and $l_1$ may be as a joker to fulfill the specifications of the original mooring design due to not affecting considerably to the mooring stiffness.

Taking into account this analysis, the mooring parameters will be set in order to find the most suitable configuration, that one that allows a perfect performance of the prototype.
6. PEWEC prototype

In this chapter it is going to be introduced and explained the 1:45 PEWEC prototype. The 1:45 prototype is the first step on a long road to reach the final stage for every project: the marketing of the prototype. The scaled prototype is based on the device that will fit to Pantelleria’s sea condition. Taking into account that future “1:1 device”, the 1:45 prototype has been designed. With the aim of introducing and showing this prototype, SolidWorks images and drawings as well as real prototype images are going to be used.

This chapter is going to be divided into two points: one point will refer to the floating body and the other point will refer to the mooring system.

6.1. Floating body

The floating body is composed by three main parts: a hull, which is the part in direct contact with water; a pendulum, which is one of the most important parts of the system due to the fact that its motion is what generates energy; and a frame, which supports the pendulum and the electric generator. These basic parts assure a correct performance from a mechanical point of view. However, there are more items and components that need to be placed in the prototype in order to assure a correct performance from all points of view, such as an electric generator, a pack of batteries, wire connections, Wi-Fi connection and other items to assure a correct control of the system and an accurate data collection. Thus in this point it will be observed an overview of these elements.

First of all, in the following figures a general view of the floating body is shown. The first figure corresponds to the SolidWorks design, while the other one corresponds to the real prototype:
Both in Figure 58 and in Figure 59, it is possible to observe the floating body. In the case of Figure 59, the floating body is half submerged in water in the test tank. As it is indicated in both figures, it is easily observed the hull and the frame. It is also possible to observe the resemblance between the SolidWorks design and the real prototype. This is of great help in all the designing process of the prototype.

Taking the SolidWorks design and applying a transparency on one side of the hull, it is possible to observe the different components of the device:

Figure 59. General view of the PEWEC prototype

Figure 60. Design of the PEWEC with its components inside it.
As it is observed in Figure 60, all the components are placed in its real position in order to evaluate the mass of the system and also if the mass is distributed in a balanced way throughout the floating body. In the next pictures it is possible to observe how the real distribution of the components throughout the floating body is:

![Image of floating body with components identified]

As seen above in Figure 61, all the components are placed in the same position as the PEWEC SolidWorks design. In addition, it is possible to observe two components that do not appear in the design’s view of Figure 60: the MTi (a movement sensor) and the braking resistor. Moreover, under the frame and the batteries, there are placed some added masses in order to equalize the mass distribution of the floating body.

After this brief presentation of the floating body’s parts and components, in the following pages it is going to be done a brief description of each.
6.1.1. Hull

The hull is made of three stainless steel sheets of 1 mm thick. The sheets are welded together in order to take the shape designed. There is one central section with radius $R$ of 0.52 m and width $W$ of 0.505 m. There are also two wings to stabilize the system. This allows the device to orientate to the waves direction and also allows avoiding a motion outside the hypothesis of planar motion considered. In the wings, it is possible to observe four different holes (Figure 58). These holes are part of the mooring system (concretely, one of them will be selected as the mooring point) and it will be explained in the mooring system section.

6.1.2. Frame

The frame is bolted on the hull and supports a plate where are placed a bearing and a flange where the PTO is settled. The frame is made of ionized aluminum components joined by screws. As seen in the different figures where the frame is viewed, there are several holes through its height; this allows different configurations where the PTO could be settled. There are exactly 16 possible positions. Each one of these positions allows modifying the floating body's center of gravity and this ends up affecting the performance of the system (in a positive or negative way depending on the adopted position).

6.1.3. Pendulum

The motion of the pendulum allows obtaining electrical energy through the PTO. The pendulum is composed by a rod of 0.334 m length and by a weight. The weight is composed by a center mass of 1 kg and it can be added two pair of masses to reach a total mass of 2 kg or 3 kg. The length of the rod is also variable: it can be configured from 0,094 m to 0,334 m.

6.1.4. Power Take Off (PTO)

An electric generator with the stator fixed on the frame carries out the energy conversion (from mechanical energy to electrical energy). The shaft of the generator also acts as the pendulum’s hinge. The generator used is a synchronous brushless motor with permanent magnets. It is developed by MOTOR POWER COMPANY and its commercial name is SKA DDR 090.30. More details can be found in the link [30].

![Figure 62. SKA DDR 090.30 developed by MOTOR POWER COMPANY [30]](image-url)
6.1.5. Digital servo drive and control

The digital servo drive (driver in Figure 60) is connected to the PTO. The digital servo drive receives a command signal from a control system (typically a desired speed, desired position or desired torque), amplifies that signal and transmits electric current to the servo motor (the electric generator is also considered as a motor due to the reversibility of these devices). In addition, the servo drive receives from the motor its actual status, allowing the servo drive to compare the actual status to the commanded motor status. It is also possible to configure some parameters, such as the proportional gain (or stiffness $k$), the derivative gain (or damping $c$) and the feedback gain.

Due to the fact that the electric generator is not connected to any electrical grid or similar, it is necessary to dissipate the power produced by the system. For this purpose, an external braking resistor connected to the digital servo drive is implemented. In the next figure it is possible to observe that configuration:

![Digital servo drive](image)

Figure 63. Digital servo drive MOTOR POWER FLEXI PRO 1DS and breaking resistance

To control the servo motor, a CompactRio (cRio) is used. A cRio is a real-time embedded industrial controller made by National Instrument for industrial control systems. A cRio is composed by four different parts: a real-time controller (a processor with several clock frequencies available), reconfigurable I/O modules (which provide high quality measurement and advanced monitoring), a FPGA module (used for implementing low level logic on the data acquired using the I/O modules) and an Ethernet expansion chassis (used for connecting the cRio to a host computer).

6.1.6. Load cell

The load cell detects the mechanical deformation through an extensometer applied to the load cell body. The signal obtained is conditioned in the load cell box or conditioner. The load cell is placed one end in the frame and the other end in the PTO. In this way it is possible to measure the force along its axis and the torque along the PTO axis. In the next figure it is possible to observe the load cell system:
6.1.7. Movement sensor (MTi)

The movement sensor allows measuring the translation and rotation of the hull. It is important to place the MTi in a position close to the center of gravity of the system, because of the need to compare the testing results to the numerical simulation results, which takes the center of gravity as the origin of one the reference frames. For this purpose, as it has been seen before in Figure 61, the MTi is placed over the batteries. This position is more or less the exact position of the center of gravity in x and z axis; and a really near position in the y axis.

6.2. Mooring system

The mooring system is composed (making an overview) of three parts: the mooring point, the mooring line (that includes the chains, the jumper and the mass) and the dead weight. In this section, it is going to be explained the characteristics of the prototype’s mooring system and the different mooring parameters seen in section 5.3.1. are going to be defined with its numerical values.
In Figure 65 it is possible to observe how looks like the real mooring system. The black lines correspond to the three segments of chain (the chains were not able to be seen due to the poor water visibility). It is also impossible to observe the dead weight in this picture. However, it is possible to observe the cubic mass and the spherical jumper (the yellow ball).

**6.2.1. Mooring point**

First of all is important to comment that the denomination of “point” comes from the planar motion point of view. It is true that, from a mechanical point of view, considering a single joint between the mooring line and the PEWEC device is completely valid. However, from the wear of materials point of view, the action point of the mooring system should be distributed in two different points. In this way, it will be possible to diminish the force that supports each point. This is the reason why the mooring point looks like this:

![Figure 66. Mooring point with two acting points](image)

In Figure 66 it is observed this “triangle” of chains in order to avoid a quick wear of the materials. It is also possible to observe that the chains are attached to the hull through a metal bar. This metal bar can be placed in one of the four different holes made for the mooring system. Depending on the mooring point selected, in other words, depending on the hole selected to place the metal bar, the mooring parameters \( CG \) and \( \theta_H \) will be defined.

In the next page it is possible to observe a scheme of the different possible mooring configuration for the first position of the PTO, and also a chart with the numerical values of \( CG \) and \( \theta_H \).
As it is observed in Figure 67, there are four possible mooring points. Each point has a pair of values \( CG \) and \( \theta_H \). In the following table there is a brief resume of these values. It is important to note that these values are not in the units needed to launch the linear model: \( CG \) must be in meters and \( \theta_H \) in radians.

<table>
<thead>
<tr>
<th>Point</th>
<th>( CG ) (mm)</th>
<th>( \theta_H ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>363.80</td>
<td>20.25</td>
</tr>
<tr>
<td>2</td>
<td>326.75</td>
<td>31.69</td>
</tr>
<tr>
<td>3</td>
<td>294.39</td>
<td>44.85</td>
</tr>
<tr>
<td>4</td>
<td>269.21</td>
<td>59.94</td>
</tr>
</tbody>
</table>

Table 3. Numerical values of the mooring point

Moreover, the position of the center of gravity (G) is taken when the PTO is in its upper position. When the position of the PTO changes, the position of G changes a little and parameters \( CG \) and \( \theta_H \) change. So for each possible PTO position, the position of the center of gravity and the parameters \( CG \) and \( \theta_H \) have been determined.
6.2.2. Mooring line

The mooring line is composed by three segments of a metal chain, a plastic ball that acts as the jumper and a cubic metal mass that acts as a sinking mass. In the following figure it is possible to observe these components:

![Figure 68. Mooring line](image)

Taking the parameters names proposed in section 5.3.1., the numerical values of the parameters in this prototype are:

<table>
<thead>
<tr>
<th>Chains</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>0.03721 kg/m</td>
</tr>
<tr>
<td>$l_1$</td>
<td>2 m</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.4 m</td>
</tr>
<tr>
<td>$l_3$</td>
<td>2 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jumper (yellow ball)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_g$</td>
<td>15,3737 N</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_a$</td>
<td>8,3679 N</td>
</tr>
</tbody>
</table>

Table 4. Numerical values of the mooring system

6.2.3. Dead weight

The dead weight is a block of cement placed in the test tank bottom. It allows a horizontal holding capacity because of the friction between its bottom side and the test tank ground. The block height of the tests is 0.4 m. Taking into account the test tank depth (3.5 m), it is possible to calculate the numerical value of $h$.

$$h = 3.5 - 0.4 = 3.1 \text{ m}$$
7. Linear model

In this chapter it is going to be explained the numerical simulation used for this prototype, as well as its characteristic parameters. It will also be possible to evaluate the system when some parameters change. In this way, it will be possible to analyze the behavior of the system when the control parameters \( c \) and \( k \) change and, a very important section in this thesis, it will be possible to analyze the effect of the mooring system in the global performance.

7.1. Explanation of the linear model

The linear model was chosen as the simulation model because of its ease and its quickness of execution. In addition, in previous studies developed on this prototype, it had been seen that the level of precision of the linear model compared with the nonlinear model was quite correct to consider it as a valid numerical solution.

As seen in section 5.3.2, the linear model can be simplified in the following way:

\[
M_{\text{lin}} \begin{bmatrix} \dot{x}_G \\ \dot{z}_G \\ \dot{\delta} \end{bmatrix} + D_{\text{PTO}} \begin{bmatrix} \dot{x}_G \\ \dot{z}_G \\ \dot{\delta} \end{bmatrix} + K_{\text{lin}} \begin{bmatrix} x_G \\ z_G \\ \delta \end{bmatrix} = 0
\]  

However, in order to include the hydrodynamic effects and to adapt the system to be studied in the frequency domain, the expression considered to run the linear model is the following one:

\[
[-\omega^2 M_{\text{lin}} + A(\omega)] + j\omega[D_{\text{PTO}} + B(\omega)] + K_{\text{lin}} X(j\omega) = F(j\omega)
\]  

Expression (2) is the one prepared to be implemented into the Matlab script. Matrices \( M_{\text{lin}} \), \( D_{\text{PTO}} \) and \( K_{\text{lin}} \) have been explained in detail in section 5.3.2. Moreover, \( \omega \) is the radian frequency of the waves. However, matrices \( A(\omega) \) and \( B(\omega) \), as well as the vector \( F(j\omega) \), must be explained in the following lines:

- \( A(\omega) \): added mass matrix due to the fluid near the floating body. Depends on the frequency
- \( B(\omega) \): linear radiation damping matrix. Depends on the frequency.
- \( F(j\omega) \): vector of the forces caused by the waves. Depends on the frequency.

In order to evaluate the different parameters of the system, the Matlab script developed for the linear model allows the possibility to test a range of values for each parameter. These values have been chosen according with the possibilities that the real prototype offers to
change these parameters. Coming up next, the parameters are explained and their values are presented.

7.1.1. Wave parameters

Wave parameters are the two characteristic values that describe a regular wave: period and height.

- **Period (T)**: it has been set from 0,9 seconds to 1,5 seconds, with a difference of 0,05 seconds between each period. For all the periods a simulation will be done so it will be possible to analyze a frequency sweep.
- **Height (H)**: two possible values have been set: 0,05 meters and 0,1 meters. However, as it will be seen in testing section, 0,05 meters will be of most interest because of is the one chosen for the testing campaign.

7.1.2. Pendulum parameters

These parameters comprise up to four characteristic values that describe the pendulum:

- **Distance between the hull center of gravity and the pendulum hinge (d)**: three different values have been set (0,4761 meters, 0,3717 meters and 0,2152 meters), that correspond to the upper, middle and lower position of the PTO respectively. These values have been calculated with SolidWorks. It will take an important relevance the upper PTO position value, because of is the position chosen for the testing campaign.
- **Mass of the pendulum (mₚ)**: three different values have been set (1, 2 or 3 kg). It will be of most interest the 3 kg one because of the testing configuration.
- **Inertia of the pendulum (Iₚ)**: three different values have been set that correspond to each different mass of the pendulum. The values set are 0,00122 kgm², 0,00246 kgm² and 0,00347 kgm².
- **Length of the pendulum (L)**: there are four possible values (0,094 meters, 0,174 meters, 0,274 meters and 0,334 meters). The last one possible value corresponds to the maximum length of the pendulum and it will be the most relevant one.

7.1.3. PTO parameters

These parameters are also called control parameters and comprise up two parameters. All their values are useful in order to discover the most suitable configuration of the PTO:

- **PTO damping coefficient (c)**: ten different values have been set from 0,1 Nms/rad to 1 Nms/rad, with a difference of 0,1 Nms/rad between each value.
- **PTO stiffness coefficient (k)**: nine possible values have been considered (-5, -2, -1, -0,5, 0, 0,5, 1, 2 and 5 Nm/rad).

7.1.4. Mooring parameters

Mooring parameters are composed by the ones related with the mooring stiffness and the ones related to the mooring point.
- **Mooring stiffness** \((k_x \text{ and } k_z)\): three different couple of values have been set for the mooring stiffness. The first case corresponds to the absence of mooring, which means 0 N/m for each parameter. The second case corresponds to the most realistic one with values of \(k_x\) and \(k_z\) of 1.25 and 0.22 N/m respectively. The last case corresponds to an extreme case of a maximum mooring effect with values of \(k_x\) and \(k_z\) of 2.7 and 0.5 N/m respectively.

- **Mooring point** \((CG \text{ and } \theta_H)\): these values correspond to the positions of the mooring point explained in section 6.2.1. Each value of \(CG\) has a \(\theta_H\) value linked. There are four possible mooring points whose values can be observed in Table 3.

### 7.2. Linear model simulations

In this part are going to be developed and analyzed the simulations with the linear model. It is going to be divided in two subsections: the first one will be focused on the analysis of the performance of the system with the parameters set in a similar configuration to the prototype tests; while the second one will be focused on a comparison of the performance of the system with or without the application of a mooring, in order to validate the model presented in section 5.3.

#### 7.2.1. Evaluation of the performance of the system

So as to develop this simulation, the following table shows the values of the parameters considered:

<table>
<thead>
<tr>
<th>Wave parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(T) (s)</td>
<td>0,9(\pm)1,5 (with a difference of 0,1)</td>
</tr>
<tr>
<td>(H) (m)</td>
<td>0,05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pendulum parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(d) (m)</td>
<td>0,4761</td>
</tr>
<tr>
<td>(m_p) (kg)</td>
<td>3</td>
</tr>
<tr>
<td>(I_y) (kgm(^2))</td>
<td>0,00347</td>
</tr>
<tr>
<td>(L) (m)</td>
<td>0,334</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PTO parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(c) (Nms/rad)</td>
<td>0,1-0,2-0,3-0,4-0,5-0,6-0,7-0,8-0,9-1</td>
</tr>
<tr>
<td>(k) (Nm/rad)</td>
<td>-5--2--1--0,5--0--0,5--1--2--5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mooring parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_x) (N/m)</td>
<td>1,25</td>
</tr>
<tr>
<td>(k_z) (N/m)</td>
<td>0,22</td>
</tr>
<tr>
<td>(CG) (m)</td>
<td>0,3638</td>
</tr>
<tr>
<td>(\theta_H) (deg)</td>
<td>20,25</td>
</tr>
</tbody>
</table>

**Table 5. Model parameters configuration**

As it can be seen in Table 5, the pendulum has been set in the upper position of the PTO and with the maximum length and height. The PTO has been set free in order to find the suitable configuration. While the mooring parameters have been set in the first possible position for the mooring point and with the realistic value of the mooring stiffness.
With the parameters set, it is possible to study and analyze some important variables of the system when the wave period changes. First of all, it is important to observe the behavior of the pitch motion ($\delta$):

As it is seen in Figure 69, each graphic corresponds to a value of PTO stiffness coefficient while its line color corresponds to a PTO damping coefficient. It is important to observe that when the value of $c$ is minimum, the pitch motion tends to maximize its value (blue line, $c=0.1 \text{ Nm/\text{rad}}$). On the other hand, when the PTO stiffness value is changed, it seems to have an influence in when the maximum value occurs. For example, when $k=5 \text{ Nm/\text{rad}}$ or $k=-5 \text{ Nm/\text{rad}}$, the maximum value occurs when the period is 1.05 and 1.15 seconds respectively; while when the PTO stiffness is null, the maximum value occurs around a period of 1.3 seconds.

Secondly, it is important to analyze what happens with the motion of the pendulum ($\varepsilon$). This variable is of much interest because it will determine the power produced by the device.
As it can be observed in Figure 70, the behavior of $\epsilon$ is quite similar to the behavior of $\delta$. This fact is totally normal, because the pendulum motion is induced by the hull motion. Moreover, with the naked eye it is possible to assure that $\epsilon$ values are comprised between 30 and 50 degrees, while there are some peaks that reach approximately 70 degrees. These peaks coincide with blue lines, the minimum $c$ value. Regarding the period when the maximum $\epsilon$ value occurs, it is exactly the same behavior as the one commented for the pitch motion.

Finally, two of the most important variables to analyze are the PTO variables, which will determine if the performance of the device is or is not optimum. These variables are the torque and the power. The torque and the angular speed of the pendulum determine the power produced by the device. The next two figures show their evolution through a frequency sweep in order to find the configuration and the wave period that allow a better performance.
Figure 71. Evolution of the PTO torque through a frequency sweep

Figure 72. Evolution of the PTO power through a frequency sweep
Figure 71 refers to the PTO torque while Figure 72 refers to the PTO power. In both figures, it is possible to observe that if the stiffness value (in absolute terms) moves away from 0, regardless the damping coefficient \(c\) considered, the shapes for each \(c\) value in the graphics are quite similar as well as the numerical values. However, if the stiffness value is around 0, the influence of the damping coefficient is quite remarkable, provoking difference with each \(c\) value and a peak of power with \(c=0,1\) Nms/rad.

Focusing on Figure 72, it is possible to observe that the maximum power produced is around 2 W and 5 W approximately, except some peaks that surpass 5 W and in some cases 10-15 W. As it has become redundant in this analysis, these peaks are produced when the damping coefficient is 0,1 Nms/rad. If other \(c\) values are considered, their peaks are not as extreme as the 0,1 Nms/rad peaks. For example, if the case with \(k=1\) Nm/rad is studied, it is possible to see that the peak for the blue line \((c=0,1\) Nms/rad\) is about two times greater than the other peaks located in this period of 1,25 seconds (the blue line is almost 10 W while other lines only reach 5 W as maximum).

So as to conclude this first analysis, it is important to understand that the device has a better performance around a range from 1 second to 1,3 seconds of period. Moreover, it seems that increasing the PTO stiffness value increases the numerical value of the power produced. However, observing the PTO damping coefficient is not as clear. It is true that selecting the lower value \((0,1\) Nms/rad\) provokes an increase in the power produced, but there is not a clear tendency with other values of the PTO damping coefficient, which have values of power produced quite similar between them.

### 7.2.2. Evaluation of the mooring system model proposed

In order to simplify this analysis and with aim of focusing on the effects of incorporating a mooring system into the global system, the following table shows the new parameters configuration considered:

| Wave parameters |  
|-----------------|--- |
| \(T\) (s)       | 0,9÷1,5 (with a difference of 0,1) |
| \(H\) (m)       | 0,05 |

| Pendulum parameters |  
|--------------------|--- |
| \(d\) (m)          | 0,4761 |
| \(m_p\) (kg)       | 3 |
| \(I_y\) (kgm\(^2\)) | 0,00347 |
| \(L\) (m)          | 0,334 |

| PTO parameters |  
|----------------|--- |
| \(c\) (Nms/rad) | 0,1-0,2-0,3-0,4-0,5-0,6-0,7-0,8-0,9-1 |
| \(k\) (Nm/rad)  | 0 |

| Mooring parameters |  
|-------------------|--- |
| \(k_x\) (N/m)     | 0 or 1,25 |
| \(k_z\) (N/m)     | 0 or 0,22 |
| \(CG\) (m)        | 0,3638 |
| \(\theta_{in}\) (deg) | 20,25 |

Table 6. Model parameters new configuration
The most important fact of Table 6 is the presence of two possible couple of values for $k_x$ and $k_z$. This will determine two possible cases to study: without mooring ($k_x$ and $k_z$ null) and with mooring ($k_x=1.25$ N/m and $k_z=0.22$ N/m).

In the following figure it is possible to observe the difference in the variables of the system between the two cases before presented:

In Figure 73 it is possible to observe two columns of graphics. The first column is referred to without mooring case and the second one is referred to with mooring case. The four different rows refer to the different variables analyzed ($\delta$, $\epsilon$, PTO torque and PTO power in this order).

The differences between both cases are almost insignificant. With the naked eye, it is impossible to detect any difference between the two cases. This could mean that the effect of the mooring system is almost imperceptible. This fact is great news because one of the aims of a mooring system for this type of wave energy converter is to keep the device in a position in the sea without affecting its optimum performance. Obviously, there will be small numerical differences between the two cases but within a range of magnitude that allows considering them almost insignificant.

There is a possible answer for this fact. The mooring parameters are quite small compared to other parameters and stiffness of the system (it is important to remember that $\theta_m$ must be expressed in radians (it will be a small value) but in Table 5 and 6 is in degrees, a higher value). This provokes that their effect in the variables of the system is less important than other parameters of the system.
8. Testing

In this chapter it is going to be explained the tests carried out in Rome, in the INSEAN (Istituto Nazionale per Studi ed Esperienze di Architettura Navale) facilities. The tests were performed in May 2015, between May 18th and May 22nd. These five days were enough to develop all work planned. This includes tests to check the prototype setup and also the operation of probes, tests to evaluate the RAO, tests to evaluate a sweep in frequency fixing the control parameters (c and k), tests to evaluate the influence of the control parameters fixing the wave considered and tests to evaluate the behavior of the prototype when its faced to irregular waves. In addition, as the 1:12 prototype tests are planned for the end of July, when the 1:45 prototype tests were finished it was a great opportunity to test how the waves for the 1:12 prototype will be. In this way, some interesting data were possible to be registered.

8.1. Testing facilities

The tests were developed in the INSEAN facilities, located in the surroundings of Rome. The INSEAN is a research institute within the frame of the National Research Council of Italy (CNR). The INSEAN facilities include several water tanks to test different aspects related with marine devices, such as towing tests, self-propulsion tests or tests to evaluate the influence of waves to a moored device (which is the case of interest of this project).

The 1:45 PEWEC prototype was tested in the INSEAN tank number 2. This tank is equipped with a wave maker in an extreme and with an artificial beach in the other extreme in order to absorb the waves motion. The tank is 220 meters length, 9 meters width and 3,5 meters depth. In the next figure it is possible to observe some of the tank technical specifications:

![Figure 74. Technical specifications of the tests tank](image)
In Figure 74, it is possible to observe the tank dimensions as commented before. Moreover, it is also possible to observe that the tank is equipped with a carriage. This carriage allows displacing the prototype to every position desired through the tank and controlling the tests from a preferential place and also allows installing the probes therein.

The wave maker is a one-side flap-type, 9 m wide, electro-hydraulically powered with 3 pumps of 38.5 kW total power, controlled by a 100 harmonic components electronic programming device, each harmonic modulated both in amplitude and frequency. It allows generating waves regular waves from 1 to 10 meters in length, with corresponding height of 100 to 450 millimeters. It also allows generating irregular waves according to any desired sea spectrum condition in appropriate scale.

The carriage is also equipped with a group of 12 fans in order to generate a wind effect in case the device tested required them (such as testing marine wind turbines).

Around the place where the prototype was located during the tests, some probes were placed. Specifically, a total of eleven probes were used (4 probes were property of Politecnico di Torino/ENEA and the rest were property of CNR). The 4 Politecnico/ENEA probes and 6 CNR probes were placed around the prototype, forming a square. The other CNR probe was placed in the front side of the carriage. In the following scheme it is shown the distribution of the probes during the tests:
The probes were used to record the real wave spectrum at any time. With this data is possible to know the exact wave height and period and to know the real power wave density of the waves. Politecnico/ENEA probes were ultrasound type, while CNR probes were capacitive type. However, not all the probes functioned correctly. As it will be seen below, there was an ultrasound probe that did not record in a correct way. Moreover, CNR probes seemed to function correctly, but studying them in detail was possible to observe that this fact was not exactly true.
In Figure 77, it is possible to observe all the acquisitions that the probes did in a random test (concretely, the RAO test number 8). At the top of the figure, it is shown the test name and the theoretical waves parameters selected (height and period). In addition, this distribution of the different graphics inside Figure 76 is not random. It corresponds to the distribution of the probes seen before, considering CNR probe 2 as the first that the waves reach. Politecnico/ENEA probes are colored in green, while CNR probes mostly in blue or in red.

With the naked eye, it is possible to assure that Politecnico/ENEA probe number 2 is not functioning well. It is not recording the typical sinusoidal shape of a regular wave. For this reason, Politecnico/ENEA probe 2 was discarded as an indicative probe.

Furthermore, if a zoom is done in the left-lower graphic, it is possible to observe the following particularity:

Figure 77. Example of the wave elevation that the probes acquired

Figure 78. Left-lower graphic zoom
In Figure 78 is possible to perceive that both CNR probes (blue and red lines) do not finish the sinusoidal shape, they are cut abruptly. However, the green line (Politecnico/ENEA probe) ends the typical wave sinusoidal shape, so makes it more reliable that CNR probes. For this reason, CNR probes are going to be discarded for data analysis.

In addition, it is possible to see that there is a phase difference between all the probes. This is due to the fact that the probes are not exactly located in the same place; there is a little distance even though they are considered to be in the same position.

Finally, to end this section, it is important to comment that the first tests (basically the setup tests and some RAOs) were developed at a distance of 122 meters from the wave maker. Then, in order to avoid the beach influence, the tests were developed at a distance of 60 meters from the wave maker.

### 8.2. Experimental tests

In this section, the different tests performed will be explained and its results will be presented and analyzed. First of all, it is important to know that not all the tests carried out will be explained in this section. During all the tests days, several type of tests were performed: setup tests, RAO tests, frequency sweep tests, control parameters tests, irregular waves tests and testing of the wave for the 1:12 PEWEC prototype. Except the cases of irregular waves tests, all the other cases were carried out with regular waves. From all the list of tests presented, only RAO tests, frequency sweep tests and control parameters tests will be explained in this section. This is because that these are the really important ones to contrast with the linear model results and graphics obtained.

For this purpose, a *Matlab* program has been developed. This program allows selecting the data analysis desired through the *Matlab* interface. There are four different options that represent four possible cases:

1. RAO test 1 @ 122 m with locked pendulum
2. RAO test 2 @ 60 m with locked pendulum
3. Frequency sweep \( H = 0.05 \) (m), \( T = [1:0.1:1.5] \) (s), \( k = 0 \) (Nm/rad), \( c = 0.5 \) (Nm/rad)
4. Regular wave test \( H = 0.05 \) (m), \( T = 1.3 \) (s), \( k = [-5 -2 -1 -0.5 -0.25 0 0.25 0.5 1 2] \) (Nm/rad), \( c = 0.5 \) (Nm/rad)

Select the experimental analysis: |

---

**Figure 79. Matlab interface asking for the desired data analysis**

If a 1 or a 2 is introduced, a RAO analysis is selected. Number 1 refers to RAOs developed at 122 meters from the wave maker and number 2 to RAOs developed at 60 meters. On the other hand, if a 3 or a 4 is introduced, a regular wave analysis is selected. However, case 3 refers to a frequency sweep testing and case 4 to a control parameters testing. To perform the data analysis selected, this program contains two external functions (*FUNC_RAO_Analysis* and *FUNC_regular_waves*) that allow it. For further information, the program is available in the section *Appendix A.2*.
8.2.1. RAO tests

The RAO (Response Amplitude Operator) is a transfer function used to determine the effect that a sea state causes in the motion of a floating body through water. The RAO can generally be described as the ratio between the response amplitude and the wave amplitude (1):

$$RAO(\omega) = \frac{\text{response amplitude}}{\text{wave amplitude}}$$ (1)

The RAO tests were developed with the pendulum locked in order to avoid its influence in the hull’s motion. In the next table it is possible to observe the different RAO tests that were developed and its waves parameters:

<table>
<thead>
<tr>
<th>Test</th>
<th>$f$ (Hz)</th>
<th>$T$ (s)</th>
<th>$H_{\text{theoretical}}$ (m)</th>
<th>$P_{\text{theoretical}}$ (W/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAO_1</td>
<td>1</td>
<td>1</td>
<td>0,05</td>
<td>2,5</td>
</tr>
<tr>
<td>RAO_2</td>
<td>1</td>
<td>1</td>
<td>0,05</td>
<td>2,5</td>
</tr>
<tr>
<td>RAO_3</td>
<td>0,91</td>
<td>1,1</td>
<td>0,05</td>
<td>2,5</td>
</tr>
<tr>
<td>RAO_4</td>
<td>0,83</td>
<td>1,2</td>
<td>0,05</td>
<td>3,012</td>
</tr>
<tr>
<td>RAO_5</td>
<td>0,77</td>
<td>1,3</td>
<td>0,05</td>
<td>3,247</td>
</tr>
<tr>
<td>RAO_6</td>
<td>0,71</td>
<td>1,4</td>
<td>0,05</td>
<td>3,521</td>
</tr>
<tr>
<td>RAO_7</td>
<td>1</td>
<td>1</td>
<td>0,05</td>
<td>2,5</td>
</tr>
<tr>
<td>RAO_8</td>
<td>0,91</td>
<td>1,1</td>
<td>0,05</td>
<td>2,747</td>
</tr>
<tr>
<td>RAO_9</td>
<td>0,83</td>
<td>1,2</td>
<td>0,05</td>
<td>3,012</td>
</tr>
<tr>
<td>RAO_10</td>
<td>0,77</td>
<td>1,3</td>
<td>0,05</td>
<td>3,247</td>
</tr>
<tr>
<td>RAO_11</td>
<td>0,71</td>
<td>1,4</td>
<td>0,05</td>
<td>3,521</td>
</tr>
<tr>
<td>RAO_12</td>
<td>0,67</td>
<td>1,5</td>
<td>0,05</td>
<td>3,731</td>
</tr>
</tbody>
</table>

Table 7. RAO tests

As seen in Table 7, the principal variation in the parameters of the waves is in the frequency or in the period. The difference between each RAO lies in this frequency sweep observed in Table 7. In addition, the first two RAOs of the table are not going to be considered in the data analysis due to testing problems: in RAO_1 the prototype was not balanced, it was inclined; and in RAO_2 there were problems with the waves’ height. Moreover, the seven first RAOs were performed at a distance of 122 meters from the wave maker, while the other ones were performed at a distance of 60 meters from the wave maker.

In the case of PEWEC, as it is considered as a planar motion device, the only relevant RAO is the one associated with the pitch motion ($\delta$):

$$RAO(\omega) = \frac{\delta}{a_w}$$ (2)

74
Where \( \delta \) is the pitch motion and \( a_w \) is the amplitude of the waves. Thus the general idea is to take \( \delta \) signal from the cRIO acquisition and \( a_w \) from the Politecnico/ENEA probes in order to calculate the RAO.

In order to facilitate this analysis, it is going to be used the FFT analysis, an algorithm to compute the discrete Fourier transform. The discrete Fourier transform converts a finite list of equally spaced samples of a function into the list of coefficients of a finite combination of complex sinusoids, ordered by their frequencies. In the next figure, it is possible to observe an example of its application in the RAO data analysis:

In the figure above it is possible to observe the variables \( \delta \) and \( \dot{\delta} \). Between them there is the discrete Fourier transform of the pitch motion. However, it is indicated as NFFT \( \delta \). This means that is applied the non equispaced discrete Fourier transform, a variation of the classical FFT that Matlab includes.

Observing the NFFT \( \delta \) graphic in Figure 80, it can be divided in five different parts: an initial part where its value it is near zero degrees, a second part of abrupt increasing, an intermediate generally constant part, a decreasing part and finally a part with a low value as the initial part. Comparing to the upper \( \delta \) graphic, the initial part corresponds to the moment while waves are reaching the device, increasing and decreasing parts correspond to the transitory parts; and the constant part corresponds to the part where the device is oscillating in a stationary way. So as to perform a RAO analysis, it is only required this central section, obviating transitory sections.

For this purpose, it is taken the wave elevation profile (concretely, its amplitude) recorded by the probes and it is applied the discrete Fourier transform. Of this transform, it is only selected the section with a value equal or greater than the 50\% of the maximum value of all the discrete Fourier transform for the wave elevation profile. However, this procedure
does not avoid the inclusion of two peaks observed in the graphic NFFT δ of Figure 80 (located about 90 seconds and 350 seconds in this example). To avoid these peaks, the section limited before is divided into three different subsections and the central one is which will be taken into account for RAO analysis. In the next figure, it can be seen the whole procedure:

![Figure 80. NFFT of the amplitude of wave profile with the procedure explained above](image1)

The horizontal red line corresponds to the limitation of 50% or greater than the maximum value. The two vertical red lines denote the section where the limitation is fulfilled. The two green lines denote the section that avoids including the peaks; and the yellow trace is the section considered to perform RAO analysis. It is possible to extract the start time and the end time of this section in order to cut the discrete Fourier transform of the pitch motion within the same interval. Then, RAO is easily calculated as:

\[
RAO(\omega) = \frac{\text{mean}(\text{NFFT } \delta_{\text{cut}})}{\text{mean}(\text{NFFT } a_{w_{\text{cut}}})} \left[ \frac{\text{deg}}{\text{m}} \right]
\]

(3)

If this procedure is repeated for different waves period, it is possible to plot a graphic of the RAO depending on the period:

![Figure 82. RAO depending on the wave period selected](image2)
In Figure 82, there are four different RAOs plotted, one for each Politecnico/ENEA probe. However, as commented before, probe 2 does not take an important relevance, despite it seems to have a normal RAO value. As it is observed, there seems to be a RAO peak when the period is about 1.3 seconds. This peak is of great importance since it is the period value that amplifies more the pitch motion response. Moreover, from this 1.3 seconds wave period, if the period is reduced (increasing frequency) or if the period is increased (reducing frequency) the RAO seems to tend to zero. A low period near zero means an infinite frequency, which also means that the floating body acts as a filter. While a high period means a low frequency, which also means that the floating body moves together with the wave due to the null wave steepness.

### 8.2.2. Frequency sweep tests

Frequency sweep tests consist in evaluate the behavior of some variables of the system when between each test it is only modified the wave frequency. For this purpose, the control parameters (c and k) as well as the wave height are fixed. This kind of tests will give an idea of what frequency/period allows a better performance of the device, both in mechanical and power generation terms. The next table shows the tests considered for the frequency sweep, and also its relevant parameters:

<table>
<thead>
<tr>
<th>Test</th>
<th>f (Hz)</th>
<th>T (s)</th>
<th>$H_{\text{theoretical}}$ (m)</th>
<th>$P_{\text{theoretical}}$ (W/m)</th>
<th>k (Nm/rad)</th>
<th>c (Nms/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test_21052015-2</td>
<td>1</td>
<td>1</td>
<td>0.05</td>
<td>2.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-3</td>
<td>0.91</td>
<td>1.1</td>
<td>0.05</td>
<td>2.747</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-4</td>
<td>0.83</td>
<td>1.2</td>
<td>0.05</td>
<td>3.012</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-5</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-6</td>
<td>0.71</td>
<td>1.4</td>
<td>0.05</td>
<td>3.521</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-8</td>
<td>0.67</td>
<td>1.5</td>
<td>0.05</td>
<td>3.731</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 8. Frequency sweep tests

As it is seen in Table 8, there is a sweep from a period of 1 second to 1.5 seconds. The wave height is fixed in 0.05 m, while the wave power density varies in each test because it also depends on the wave period. The control parameters k and c are fixed with a values of 0 Nm/rad and 0.5 Nms/rad respectively.

All of these tests were performed at a distance of 60 meters from the wave maker to avoid the influence of the beach at the end of the tank, as commented in section 8.1. Obviously, in contrast to RAO tests, the pendulum was unlocked.

In order to analyze all the data collected, the procedure is similar to the one used for RAO tests. First of all, it is important to verify that the values of the wave parameters (amplitude or height, period and power density) are close to the theoretical ones thought for these tests. To compare the experimental data to the target value of the parameter, the FFT algorithm for the discrete Fourier transform is going to be used.
As seen in this figure, for each Politecnico probe there are four graphics of waves parameters plotted (from top to bottom): amplitude, period, power density and the wave elevation profile. The first three parameters graphics are the important ones to analyze and the ones where the FFT has been applied. The wave elevation profile is added in order to observe the real data that the probe recorded through time. In the graphics, the blue lines represent the experimental data while the discontinuous black lines represent the theoretical value of the parameter considered. Again, as commented in section 8.1, it is easily seen that probe 2 does not work correctly because of the strange wave elevation profile and that the experimental data for the several parameters are too much distant from the target value. However, observing the other probes, it is possible to assure that the experimental waves are close to the theoretical target values expected. After the transitory part, parameters amplitude, period and power density stabilize in a value close to the theoretical ones.

The other important thing to analyze in these tests is the evolution of the PEWEC’s variables through time. These variables are the ones related to the hull motion (pitch motion and pitch speed) and the ones related to the pendulum motion ($\epsilon$, $\dot{\epsilon}$, torque of the PTO and power of the PTO). The variables related to the hull motion and also the angular variables of the pendulum motion can be obtained directly from the cRio acquisitions. However, the torque and the power of the PTO have to be calculated. The next equations show these facts:

\[ T_{PTO} = c \dot{\epsilon} + k \epsilon \]  \hspace{1cm} (4)

\[ P_{PTO} = T_{PTO} \cdot \dot{\epsilon} \]  \hspace{1cm} (5)
Expression (4) shows the equation to calculate $T_{PTO}$, while expression (5) shows the equation to calculate power through the torque and the angular speed. Expression (4) has been obtained using the control parameters ($c$ and $k$).

At this time, it is possible to plot through time all the PEWEC’s variables desired:

From left to right and from top to bottom in Figure 84, it is possible to observe the next variables: $\delta$, $\epsilon$, torque of the PTO (first row), $\dot{\delta}$, $\dot{\epsilon}$ and power of the PTO (second row). For each variable, there are three lines plotted: blue lines correspond to the experimental data, red lines represent the interval of interest to analyze (determined in the same way as in section 8.2.1) and discontinuous black lines correspond to a statistical measure of the variables. This statistical measure is done in order to obtain a reference constant value to later compare the different tests. However, as it can be seen in the legends of Figure 84, most of the variables (except $P_{PTO}$) use a root square mean ($rms$) as statistical measure, while $P_{PTO}$ uses a standard mean. There is an easy explanation for this fact: if $P_{PTO}$ is observed, it is the only variable that does not oscillate around 0. As all the other variables oscillate around 0, doing a standard mean would involve a value near 0. However, doing an $rms$ allows having a significant value, because is a mean of the squares of the sample. On the other hand, if the standard mean is calculated for $P_{PTO}$, a significant value is obtained because all its values are positive and it is not necessary doing a root square mean. With these mean values, it is possible to do graphics showing the behavior of the variables through the different tests, through the frequency sweep:
In Figure 85, it is possible to observe the PEWEC’s variables (following the same order as Figure 84) through different periods that compose the frequency sweep. Moreover, the graphic of the last row corresponds to the Relative Capture Width (RCW), which is an index to evaluate the performance of a WEC that follows the next equation (6):

\[ RCW = \frac{T_{PTO} \dot{\varepsilon}}{H^2 T L} \]  

(6)

Where \( H \) and \( T \) are the wave height and period respectively, and \( L \) is the PEWEC’s width.

In addition, the other graphics (except the power one) represent the maximum values of the variables, which are calculated by multiplying their rms value by \( \sqrt{2} \).

As it can be observed in Figure 85, it seems to exist a peak in the variables when the period is of 1.4 seconds, while in the RCW graphic the peak occurs when the period is of 1.3 seconds.

8.2.3. **Control parameters tests**

Control parameters tests consist in the modification of such parameters while waves characteristics are fixed. The goal of these tests is to understand which configuration causes a better performance of the device. Coming up next, it is shown a table with the tests done as well as their characteristics:
Table 9. Control parameters tests

<table>
<thead>
<tr>
<th>Test</th>
<th>f (Hz)</th>
<th>T (s)</th>
<th>H_{theoretical} (m)</th>
<th>P_{theoretical} (W/m)</th>
<th>k (Nm/rad)</th>
<th>c (Nms/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test_21052015-5</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-8</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>-0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-9</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-10</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>-0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-11</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-12</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>-2</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-13</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-14</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-15</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Test_21052015-16</td>
<td>0.77</td>
<td>1.3</td>
<td>0.05</td>
<td>3.247</td>
<td>-5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In the table above, the different tests developed are shown. It is important to note that from column two to column five, the values are the same for each test. These columns correspond to waves characteristics and as it was commented, they are the same for each test. The last two columns correspond to the control parameters (k and c). The PTO damping coefficient (c) is fixed in all the tests with a value of 0.5 Nms/rad, while the important parameter to analyze, the PTO stiffness coefficient (k), is modified in each test. The PTO stiffness coefficient is included within a range of values from -5 Nm/rad to 2 Nm/rad. The wave period chosen for these tests is 1.3 seconds, because is one of the periods that has a better performance of the device.

The procedure to analyze all the data is exactly the same as the one commented in section 8.2.2. So in this section it is only going to be commented the most relevant aspects. First of all, it is important to plot the PEWEC’s variables depending on time for each test, in order to observe possible anomalies.

Figure 86. PEWEC’s variables plotted through time (Test_21052015-10)
The figure above is very similar to the one shown in the frequency sweep section. There are no anomalies present in these graphics. However, during the test in Rome, there was a test (Test_21052015-13) that clearly showed an unstable behavior. When the wave motion ceased, the pendulum did not stop its movement and the system had to be disconnected. This parameter \( k \) in this test was of 2 Nm/rad, so values greater than this were not considered because they would be unstable. This fact is easily observable in the following figure:

![Figure 87. Comparison between the wave elevation and the variables of the system (Test_21052015-13)](image)

Comparing the wave elevation profile and the evolution of the variables through time, it is easily seen the aspect commented before: around the 250 seconds, the wave motion ceases but the pendulum is still moving until the system is disconnected around the 450 seconds. In addition, in the wave elevation profile it is possible to see a small waves generation. These waves are generated by the hull motion provoked by the unstable pendulum motion.

Finally, storing the average values of each variable for each test (root square mean or standard mean depending on what variable, as in section 8.2.2), allows doing a graphic for each variable depending on the parameter modified (the stiffness of the PTO, \( k \)) in order to evaluate its influence in the global performance of the system.
The red circles correspond to the experimental data of each test, while the blue lines represent a linear regression in order to appreciate the tendency of the data. There is a clear increasing tendency in most of the graphics, which means that increasing the stiffness of the PTO also increases the value of these variables. However, if $T_{PTO}$ is observed, it is possible to appreciate that its value seems to increase when the stiffness of the PTO moves away from 0. This effect can also be appreciated in $P_{PTO}$ but without the same evidence of the torque case due to the influence of the pendulum’s angular speed.
9. Experimental and numerical results comparison

The last chapter of this thesis is dedicated to compare the numerical results seen in chapter 7 with the experimental results seen in chapter 8. The goal is to understand if the linear model developed is sufficiently faithful to reality. Obviously, it is almost impossible to obtain a perfect level of accuracy due to different facts, such as experimental errors (some probes were not as accurate as they should, the impossibility to generate a perfect regular wave or not considering the air effect) or simplifications in the linear model (for example, the mooring system presented in this thesis).

9.1. Comparison of RAO

In this section the numerical RAO is going to be compared with the experimental RAO. The numerical RAO is obtained by using the program ANSYS AQWA, while the experimental RAO is the one obtained in section 8.2.1. The comparison is done by plotting the graphics of both RAOs when the wave period changes.

As seen in Figure 89, RAO comparison seems to present little differences. First of all, there is a little difference of the maximum value. Secondly, there is a difference in the period of the maximum, in the numerical case occurs when the period is about 1.2 seconds while in the experimental case occurs when the period is of 1.3 seconds.

However, it can be said that the numerical model represent in a satisfactory way the real prototype behavior. These differences can be caused by that mass and inertial characteristics are estimated on the prototype design using a tool that provides the program SolidWorks, so it is possible that these values are not exactly the real ones.

Figure 89. RAO comparison
9.2. Comparison of the frequency sweep

In this section the frequency sweep will be compared. The cases compared will be with $k=0$ Nm/rad and $c=0.5$ Nms/rad. In the following figure it is possible to observe the comparison between the numerical case and the experimental case of the different variables of the system:

![Graph comparison](image)

Figure 90. Comparison of the frequency sweep

As seen in Figure 90, the graphics that correspond to the pitch motion (both angle and angular speed) show a close similarity between the numerical case (red line) and the experimental case (blue line). This proves that the linear model represents quite well the hydrodynamic effects that are responsible for the pitch motion.

In addition, the graphics showing the pendulum motion, the torque and the power suggest that the numerical model tends to overestimate the real behavior of the prototype (the experimental case). This means that there are probably some effects on the system that linearizing them involves an overestimation of the prototype real behavior. However, considering this is a linear model, it is possible to assure that the results of this comparison are highly satisfactory because of the small difference in results achieved.
9.3. **Comparison of the effect of the control parameters**

In this section it is going to be tried to understand what occurs to the global performance when the control parameters are changed. This comparison is done with the PTO damping coefficient fixed (c=0.5 Nms/rad) and only changing the PTO stiffness coefficient.

![Comparison of the effect of the control parameters](image)

In the figure above it is possible to observe different graphics with the variables that are going to be compared. In each graphic there are three different data plotted: red circles correspond to experimental data, black circles correspond to numerical data and the blue line corresponds to a linear regression of the experimental data in order to appreciate the tendency of the prototype’s behavior. As it can be seen, the horizontal axis refers to the different possible values of the PTO stiffness, which its variation is the target of this comparison.

In the same way as the comparison of the frequency sweep, the variables related with the pitch motion present a graphic with no significant differences between the numerical case and the experimental case. Once more, this fact is due to a good estimation of the hydrodynamic effects by the linear model.

On the other hand, the variables related with the motion of the pendulum as well as the RCW present a small difference between the numerical case and the experimental case. The numerical case overestimates the response of the system. However, it is worth to be mentioned that the numerical case also follows a tendency similar to the experimental one.
plotted in the graphics. This is of much interest because it proves that the linear model gives a good idea of how the system behaves.
Conclusions

The development of new ways to produce electricity has become a key factor of the energy engineering field. Moreover, it is not only demanded this, it is also demanded a source of energy clean, respectful and sustainable with the environment. Taking into account that water covers more than 70% of the Earth’s surface, using this resource is not foolishness. One of the ways to benefit from this resource is a Wave Energy Converter. This is why a Wave Energy Converter is studied in this thesis. Specifically, the aim of study is the PEWEC, a Wave Energy Converter whose energy conversion is done by the motion of a pendulum.

The principal goal of this thesis is to integrate a mooring system into the PEWEC’s linear system and to evaluate if it accomplishes its mission: to keep the device in a position in the sea without affecting its optimum performance. The first part (the integration into the linear model) is performed simplifying the mooring system to two springs (one vertical and one horizontal) so in this way it is possible to find an expression for the potential energy and extract the mooring stiffness matrix that can be added to the PEWEC’s linear model. The second part (the validation of the mooring system proposed) is done by comparing a simulation without the mooring and a simulation with the mooring. As it has been seen, the effect of the mooring system is almost insignificant. Moreover, during the tests developed in Rome, it was possible to observe that the mooring developed its function perfectly. Thus for this part, one of the main goals of the thesis has been accomplished.

Furthermore, as a testing campaign has been developed, it has been possible to perform a comparison between the numerical simulation (linear model) and the experimental tests. Broadly speaking, it could be said that the comparison has allowed checking the validity of the linear model regarding the experimental results. It is mainly important to mention that the reliability of the linear model for the pitch motion is incredibly high, due to the good estimation of the hydrodynamic effects. Furthermore, for the pendulum motion as well as for the torque and power of the PTO, it has been possible to observe that the linear model overestimates slightly the behavior of the system. However, the general conclusion is that the comparison results were highly satisfactory, proving that the linear model simulates in a correct way the behavior of the system.

This thesis has been carried out focusing mainly on the 1:45 prototype. However, at the same time, the 1:12 prototype has been developed. All the mooring system proposed in chapter 5 is perfectly valid for the 1:12 prototype. In fact, during the modeling of the mooring system it was not thought for any particular prototype. It was developed in generic terms and the adequacy of the mooring parameters will determine its suitability for each prototype. Therefore, the development of the 1:12 prototype and the tests in Rome planned for July 2015 are a great opportunity to continue validating the mooring system proposed or, in the worst case, to introduce improvements with the aim of finding a model that simulates the motion of the PEWEC in a highly reliable way.

During the tests, the device was able to produce around 1,2 W, which was coherent with the value expected. It is thought that the 1:1 device will be able to produce 400 kW of nominal
power. With only ten of these 1:1 PEWEC devices it will be possible to electrically supply a city of 3000 inhabitants. Considering that it is a low cost and renewable energy device, it can be predicted a bright future for it, proof of this are the many WEC prototypes that are being developed lately. So the next years will be crucial and extremely innovative in the race for a place in the market of WEC devices.
Appendix

A.1. Obtaining of the mooring stiffness (*Matlab* file)

clear all
close all
clic

%% Cartella functions
addpath Subroutines

%% Cartella risultati
if ~exist('Risultati','dir')
    mkdir('Risultati');
end

%% Dati di input (1:45)
% Ambiente
h = 3.1;
% profondità fondale [m]
l1_vett = [0.75 1.25 1.5 1.75 2];
% vettore lunghezza tratto l1 [m]
l2_vett = [0.2 0.3 0.4 0.5];
% vettore lunghezza tratto l2 [m]

% Mooring
ml_vett = [32/860];
% vettore massa degli ormeggi [kg/m]
Fg_vett = linspace(5,70,10)*9.81*(12/45)^3;
% vettore forza di galleggiamento jumper nel punto A [N]
Fa_vett = linspace(0.25,1.25,10)*9.81;
% vettore forza di affondamento massa nel punto B [N]

%% Ciclo per la realizzazione delle varie combinazioni
% Inizializzazione
kk_mat = zeros(length(Fg_vett),length(Fa_vett));
kl_mat = zeros(length(Fg_vett),length(Fa_vett));

%% Waitbar
hh = waitbar(0);
len = length(l1_vett)*length(l2_vett)*length(ml_vett)*length(Fg_vett)*length(Fa_vett);
index_ = 0;
index_fig = 0;

for ii = 1:length(l1_vett)
    for jj = 1:length(l2_vett)
        for kk = 1:length(ml_vett)
            for mm = 1:length(Fg_vett)
                for nn = 1:length(Fa_vett)

                    index_ = index_+1;
                    l1 = l1_vett(ii);
                    l2 = l2_vett(jj);
                    ml = ml_vett(kk);
                    Fg = Fg_vett(mm);
                    Fa = Fa_vett(nn);

                    if Fg > Fa

                        %% Pre-processing dei dati
                        % calcolo del tratto l3 [m]
                        l3 = h+12-l1;

                        % calcolo della massima x [m]
                        xmax = (sqrt((l1+l2+l3)^2-h^2))*(1-0.2);
                        x = (linspace(0.2,xmax,50));

                    end

                    kk_mat(index_,mm) = kl/length(Fg_vett);
                    kl_mat(index_,mm) = kl/length(Fg_vett);
                end
            end
        end
    end
end
m3 = m1*l3;

% posizione iniziale del corpo morto [m]
x0=0;y0=0;
% posizione verticale del punto di ormeggio [m]
yC=h;

%% Calcolo delle tensioni degli ormeggi
% inizializzazione matrice e vettori
teta1 = zeros(length(x),1);
teta2 = zeros(length(x),1);
teta3 = zeros(length(x),1);
gamma = zeros(length(x),1);
psi = zeros(length(x),1);
alfa = zeros(length(x),1);
T1 = zeros(length(x),1);
T2 = zeros(length(x),1);
T3 = zeros(length(x),1);
Fx = zeros(length(x),1);
Fz = zeros(length(x),1);

% calcolo di forze, tensioni e angoli
for i=1:length(x) % ciclo su x
    % calcolo teta
    if i == 1,
        teta(i) = calcola_teta(Fg,Fa,x(i),h,l1,l2,l3,m1,m2,m3,pi/2);
    else
        teta(i) = calcola_teta(Fg,Fa,x(i),h,l1,l2,l3,m1,m2,m3,teta(i-1));
    end
    % calcolo tensioni, forze e angoli
    [T1(i),T2(i),T3(i),Fx(i),Fz(i),gamma(i),psi(i),alfa(i),teta2(i),teta3(i)] = calcola_tensioni(Fg,Fa,x(i),h,l1,l2,l3,m1,m2,m3,teta(i));
end

%% Interpolazione delle forze
% interpolazione polinomiale
N = 4;
Fx = polyfit(x,Fx,N);
Fz = polyfit(x,Fz,N);
x_pol = linspace(0,xmax*1.5,100)';
Fx_pol = polyval(Fx,x_pol);
Fz_pol = polyval(Fz,x_pol);
% simmetria per x negative
x_pol = [-flipud(x_pol); x_pol(2:end)];
Fx_pol = [-flipud(Fx_pol); Fx_pol(2:end)];
Fz_pol = [flipud(Fz_pol); Fz_pol(2:end)];

%% Valutazione rigidezza
% Totale
dFx = diff(Fx);
dx = diff(x);
.kx = dFx./dx;
dFz = diff(Fz);
kz = dFz./dx;
% Lineariizzata
.kx_lin = mean(kx_);
kz_lin = mean(kz_);
%% Costruzione della matrice dei risultati
kx_mat(mm,nn) = kx_lin;
kz_mat(mm,nn) = kz_lin;

else
kx_mat(mm,nn) = NaN;
kz_mat(mm,nn) = NaN;
end

% Waitbar
waitbar(index_fig/len, hh);

end

index_fig = index_fig+1;
figure(index_fig)
subplot(211)
contourf(Fg_vett, Fa_vett, kx_mat')
colorbar
xlabel('F_g (N)')
ylabel('F_a (N)')
str = ['k_x (N/m) = ', num2str(l1_vett(ii)), ' (m), l_2 = ' , num2str(l2_vett(jj)), ' (m), m_l = ' , num2str(ml_vett(kk)), ' (kg/m)'];
title(str)

subplot(212)
contourf(Fg_vett, Fa_vett, kz_mat')
colorbar
xlabel('F_g (N)')
ylabel('F_a (N)')
str = ['k_z (N/m) = ', num2str(l1_vett(ii)), ' (m), l_2 = ' , num2str(l2_vett(jj)), ' (m), m_l = ' , num2str(ml_vett(kk)), ' (kg/m)'];
title(str)
saveas(figure(index_fig), [cd, '\Risultati\l1 ', num2str(l1_vett(ii)), '\l2 ', num2str(l1_vett(jj)), '\ml ', num2str(ml_vett(kk)), '.fig'])

end

end

end
A.2. Rome tests data analysis (Matlab file)

```matlab
clear
close all
clc

%% Folder with acquisition files
addpath Acq

%% Folder with NFFT subroutines
addpath Subroutines

%% Loading logbook xls file
logbook = importdata('PEWEC 1to45 Roma 18-22 maggio 2015.xlsx');

%% Set-up parameters

% Politecnico probes parameters
Gain_probe(:,1) = 58.477;  % Gain Politecnico probe 1 [mm/V]
Gain_probe(:,2) = 58.307;  % Gain Politecnico probe 2 [mm/V]
Gain_probe(:,3) = 58.295;  % Gain Politecnico probe 3 [mm/V]
Gain_probe(:,4) = 58.012;  % Gain Politecnico probe 4 [mm/V]

Offset_probe(:,1) = -52.241;  % Offset Politecnico probe 1 [mm]
Offset_probe(:,2) = -52.920;  % Offset Politecnico probe 2 [mm]
Offset_probe(:,3) = -53.747;  % Offset Politecnico probe 3 [mm]
Offset_probe(:,4) = -51.702;  % Offset Politecnico probe 4 [mm]

Poli_freq = 50;  % Acquisition frequency Politecnico probes [Hz]

% CNR probes parameters
CNR_freq = 50;  % Acquisition frequency CNR probes [Hz]

% cRio parameters
cRio_fraq = 10;  % cRio acquisition frequency [Hz]

% NFFT parameters
NFFT_step = 10;
NFFT = 2048;

%% Building structure
setup.Poli_probes.gain = Gain_probe;
setup.Poli_probes.offset = Offset_probe;
setup.Poli_probes.freq = Poli_freq;
setup.CNR_probes.freq = CNR_freq;
setup.cRio_freq = cRio_freq;
setup.NFFT.step = NFFT_step;
setup.NFFT.value = NFFT;

%% Test selection

% Legend
% - Index from 5 to 15 ---> RAO @ 122 (m) and first probes layout, RAO @ 60 (m) and second probes layout
% - Index from 16 to 21 ---> Frequency sweep H = 0.05 (m), T = [1:0.1:1.5] (s), k = 0 (Nm/rad), c = 0.5 (Nms/rad)
% - Index from 21 to 30 ---> Regular wave test H = 0.05 (m), T = 1.3 (s), k = [-5 -2 -1 -0.5 -0.25 0 0.25 0.5 1 2] (Nm/rad), c = 0.5 (Nms/rad)

disp('1) RAO test 1 @ 122 m with locked pendulum')
disp('2) RAO test 2 @ 60 m with locked pendulum')
disp('3) Frequency sweep H = 0.05 (m), T = [1:0.1:1.5] (s), k = 0 (Nm/rad), c = 0.5 (Nms/rad)')
```
disp('4) Regular wave test H = 0.05 (m), T = 1.3 (s), k = [-5 -2 -1 -0.5 -0.25 0 0.25 0.5 1 2] (Nm/rad), c = 0.5 (Nms/rad)')
disp(' ')
index_test = input('Select the experimental analysis: ');
disp(' ')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RAO test @ 122 m with locked pendulum
if index_test == 1
  for i = 5:9
    test_name = logbook.textdata.DiarioDiBordo{i,2}; % Test name
    wave(1,1) = logbook.data.DiarioDiBordo(i-2,2); % Wave period (s)
    wave(1,2) = logbook.data.DiarioDiBordo(i-2,3); % Wave height (m)

    % CNR probes
    CNR = importdata([test_name, '.dat'], '');

    % Politecnico probes
    Poli = xlsread([test_name, '.xlsx'], 'Untitled');

    % cRio data
    cRio = importdata([test_name, '.txt']);
    [RAO] = FUNC_RAO_Analysis(test_name, setup, wave, CNR, Poli, cRio);
    RAO_mat(i-4,:) = RAO;
  end

figure(100)
T_vett = logbook.data.DiarioDiBordo(5:2:9-2,2);

% Legend
leg{1} = 'RAO Poli probe 1';
leg{2} = 'RAO Poli probe 2';
leg{3} = 'RAO Poli probe 3';
leg{4} = 'RAO Poli probe 4';
leg{5} = 'RAO numeric';

for j = 1:4
  plot(T_vett,RAO_mat(:,j))
  grid on
  hold all
end

addpath Scafo_1a45
hull = load('hull_roma_config1_PTO.mat');

figure(100)
hold on
plot(2*pi./hull.w,abs(hull.RAO(:,5)),'--k')
xlabel('Wave period - T (s)')
ylabel('RAO_{pitch} (deg/m)')
legend(leg)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% RAO test @ 60 m with locked pendulum
elseif index_test == 2
  for i = 10:15

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```matlab
% Test name

test_name = logbook.textdata.DiarioDiBordo{i,2}; % Test name

wave(1,1) = logbook.data.DiarioDiBordo{i-2,2}; % Wave period (s)
wave(1,2) = logbook.data.DiarioDiBordo{i-2,3}; % Wave height (m)

% CNR probes
CNR = importdata([test_name,'.dat'],'+',9);

% Politecnico probes
Poli = xlsread([test_name,'.xlsx'],'Untitled');

% cRio data
cRio = importdata([test_name,'.txt']);

[RAO] = FUNC_RAO_Analysis(test_name,setup,wave,CNR,Poli,cRio);

RAO_mat(i-9,:) = RAO;

end

figure(100)
T_vett = logbook.data.DiarioDiBordo{10-2:15-2,2};

% Legend
leg{1} = 'RAO Poli probe 1';
leg{2} = 'RAO Poli probe 2';
leg{3} = 'RAO Poli probe 3';
leg{4} = 'RAO Poli probe 4';
leg{5} = 'RAO numeric';

for j = 1:4
    plot(T_vett,RAO_mat(:,j))
    grid on
    hold all
end

addpath Scafo_1a45
hull = load('hull_roma_config1_PTO.mat');

figure(100)
hold on
plot(2*pi./hull.w,abs(hull.RAO(:,5)),'--k')
xlabel('Wave period - T (s)')
ylabel('RAO_{pitch} (deg/m)')
legend(leg)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Frequency sweep test
elseif index_test == 3

ii = 0;

% Numerical results linear model with c = 0.5 and k = 0
sweep_res = load('Risultati_c05_k0.mat');

for i = 18:23 % Tests 16 and 17 (excel row numeration) are not considered (16 durata tropo corta and 17 equal to 18)
    ii = ii+1;

    test_name = logbook.textdata.DiarioDiBordo{i,2}; % Test name
    wave(1,1) = logbook.data.DiarioDiBordo{i-2,2}; % Wave period (s)
    wave(1,2) = logbook.data.DiarioDiBordo{i-2,3}; % Wave height (m)

    % CNR probes
```
CNR = importdata([test_name,'.dat'],',9);

% Politecnico probes
Poli = xlsread([test_name,'.xlsx'],'Untitled');

% cRio data
cRio = importdata([test_name,'.txt']);

[delta_rms,delta_dot_rms,epsilon_rms,epsilon_dot_rms,T_PTO_rms,P_PTO_mean,RCW_] = FUNC_regular_waves(test_name,setup,wave,CNR,Poli,cRio);

deltarms(i-17,:) = delta_rms;
deltadotrms(i-17,:) = delta_dot_rms;
epsilonrns(i-17,:) = epsilon_rms*180/pi;
epsilondotrms(i-17,:) = epsilon_dot_rms;
TPTOrms(i-17,:) = T_PTO_rms;
PPTOmean(i-17,:) = P_PTO_mean;
RCW(i-17,:) = RCW_;

Matrextr = squeeze(sweep_res.RES(ii,1,1,1,1,1,1,1,:))';

delta_num(1,ii) = abs(Matrextr(1,5))*180/pi;
deltadot_num(1,ii) = abs(Matrextr(1,6));
epsilon_num(1,ii) = abs(Matrextr(1,7))*180/pi;
epsilondot_num(1,ii) = abs(Matrextr(1,8));
TPTO_num(1,ii) = sqrt(2)*abs(Matrextr(1,14));
Pmed_num(1,ii) = abs(Matrextr(1,16));
RCW_num(1,ii) = abs(Matrextr(1,18));

end

figure('Name','Frequency sweep testing','NumberTitle','off')
T_freq = logbook.data.DiarioDiBordo(18-2:23-2,2);

subplot(3,3,1)
plot(T_freq, sqrt(2)*deltarms, '-b')
hold on
plot(T_freq, delta_num, '-r')
grid on
xlabel('T (s)')
ylabel('\delta max (deg)')
legend('Exp', 'Num')

subplot(3,3,4)
plot(T_freq, sqrt(2)*deltadotrms, '-b')
hold on
plot(T_freq, deltadot_num, '-r')
grid on
xlabel('T (s)')
ylabel('\delta \prime max (rad/s)')
legend('Exp', 'Num')

subplot(3,3,2)
plot(T_freq, sqrt(2)*epsilonrns, '-b')
hold on
plot(T_freq, epsilon_num, '-r')
grid on
xlabel('T (s)')
ylabel('\epsilon max (deg)')
legend('Exp', 'Num')

subplot(3,3,5)
plot(T_freq, sqrt(2)*epsilondotrms, '-b')
hold on
plot(T_freq, epsilondot_num, '-r')
grid on
xlabel('T (s)')
ylabel('\epsilon \prime max (rad/s)')
legend('Exp', 'Num')
subplot(3,3,3)
plot(T_freq,sqrt(2)*TPTOrms,'-b')
hold on
plot(T_freq,TPTO_num,'-r')
grid on
xlabel('T (s)')
ylabel('T_{PTO} max (Nm)')
legend('Exp','Num')

subplot(3,3,6)
plot(T_freq,PPTOmean,'-b')
hold on
plot(T_freq,Pmed_num,'-r')
grid on
xlabel('T (s)')
ylabel('P_{PTO} mean (W)')
legend('Exp','Num')

subplot(3,3,7:9)
plot(T_freq,RCW,'-b')
hold on
plot(T_freq,RCW_num,'-r')
grid on
xlabel('T (s)')
ylabel('RCW (\ldots)')
legend('Exp','Num')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

special control parameters
elseif index_test == 4
    ii = 0;

    % Numerical results linear model with c = 0.5 and k = 0
    sweep_res = load('Risultati_c05_kvar.mat');

    for i = 21:32 % Tests 22 and 23 correspond to sweep frequency, included in order to
                % include test 21
        ii = ii+1;

        test_name = logbook.textdata.DiarioDiBordo{i,2}; % Test name
        wave(1,1) = logbook.data.DiarioDiBordo(i-2,2); % Wave period (s)
        wave(1,2) = logbook.data.DiarioDiBordo(i-2,3); % Wave height (m)

        % CNR probes
        CNR = importdata([test_name,'.dat'],',',9);

        % Politecnico probes
        Poli = xlsread([test_name,'.xlsx'],'Untitled');

        % cRio data
        cRio = importdata([test_name,'.txt']);

        [deltarsm,deltadotsm,epsilon_rms,epsilon_dot_rms,T_PTO_rms,P_PTO_mean,RCW] = FUNC_regular_waves(test_name,setup,wave,CNR,Poli,cRio);

        deltarms(i-20,:) = delta_rms;
        deltadotrms(i-20,:) = delta_dot_rms;
        epsilonrms(i-20,:) = epsilon_rms*180/pi;
        epsilondotrms(i-20,:) = epsilon_dot_rms;
        TPTOrms(i-20,:) = T_PTO_rms;
        PPTOmean(i-20,:) = P_PTO_mean;
        RCW(i-20,:) = RCW_;
for ii = 1:11

    Matrextr = squeeze(sweep_res.RES(1,1,1,1,1,1,1,1,1,1,:))';

    delta_num(1,ii) = abs(Matrextr(1,5))*180/pi;
    deltadot_num(1,ii) = abs(Matrextr(1,6));
    epsilon_num(1,ii) = abs(Matrextr(1,7))*180/pi;
    epsilondot_num(1,ii) = abs(Matrextr(1,8));
    TPTO_num(1,ii) = sqrt(2)*abs(Matrextr(1,14));
    Pmed_num(1,ii) = abs(Matrextr(1,16));
    RCW_num(1,ii) = abs(Matrextr(1,18));

end

figure( 
    'Name', 'Control parameter testing', 
    'NumberTitle', 'off' 
)

k_vet = logbook.data.DiarioDiBordo(21-2:32-2,6);

k_vet(2:3) = []; % to eliminate tests 22 and 23
k_vet_lin = [-5 -2 -1 -0.5 -0.25 0 0.25 0.5 1 2 5];
deltarms(2:3) = [];
deltadotrms(2:3) = [];
epsilonrms(2:3) = [];
epsilondotrms(2:3) = [];
TPTOrms(2:3) = [];
PPTOmean(2:3) = [];
RCW(2:3) = [];

subplot(3,3,1)
grid on
plot(k_vet,sqrt(2)*deltarms,'or')
hold on
plot(k_vet_lin,delta_num,'ok')
xlabel('k (Nm/rad)')
ylabel('
delta max (deg)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,sqrt(2)*deltarms,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,4)
grid on
plot(k_vet,sqrt(2)*deltadotrms,'or')
hold on
plot(k_vet_lin,deltadot_num,'ok')
xlabel('k (Nm/rad)')
ylabel('
delta prime max (rad/s)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,sqrt(2)*deltadotrms,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,2)
grid on
plot(k_vet,sqrt(2)*epsilonrms,'or')
hold on
plot(k_vet_lin,epsilon_num,'ok')
xlabel('k (Nm/rad)')
ylabel('
epsilon max (deg)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,sqrt(2)*epsilonrms,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,5)
grid on
plot(k_vet,sqrt(2)*epsilondotrms,'or')
hold on
plot(k_vet_lin,epsilondot_num,'ok')
xlabel('k (Nm/rad)')
ylabel('
epsilon prime max (rad/s)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,sqrt(2)*epsilondotrms,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,3)
grid on
plot(k_vet,sqrt(2)*TPTOrms,'or')
hold on
plot(k_vet_lin,TPTO_num,'ok')
xlabel('k (Nm/rad)')
ylabel('T_{PTO} max (Nm)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,sqrt(2)*TPTOrms,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,6)
grid on
plot(k_vet,PPTOmean,'or')
hold on
plot(k_vet_lin,Pmed_num,'ok')
xlabel('k (Nm/rad)')
ylabel('P_{PTO} mean (W)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,PPTOmean,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')

subplot(3,3,7:9)
grid on
plot(k_vet,RCW,'or')
hold on
plot(k_vet_lin,RCW_num,'ok')
xlabel('k (Nm/rad)')
ylabel('RCW (-)')
hold on
plot(-5:1:2,polyval(polyfit(k_vet,PPTOmean,1),-5:1:2),'-b')
legend('EXP', 'NUM', 'linear regression')
end
References


[16] **Seabased**: Lysekil Project (Accessed 16/04/2015)

http://www.wello.eu/

http://www.oceanpowertechnologies.com/powerbuoy/

http://www.wavepiston.dk/

http://www.waveforenergy.com/


[23] **Drake, Stillman**, 2003, “Galileo at Work: His scientific biography”, Courier Dover, USA

http://www.17centurymaths.com/contents/huygenscontents.html


http://www.julianrubin.com/bigten/foucaultpendulum.html


[29] **Tension Technology International**: Catenary and taut mooring (Accessed 27/05/2015)
http://www.tensiontech.com/services/mooring.html


[31] **CNR, INSEAN**: Testing facilities (Accessed 21/06/2015)
http://www.insean.cnr.it/
Acknowledgments

In this final part, I would like to thank everyone who has had an important role in the development of this thesis, either helping me or supporting me. If I am allowed, I would like to write this part in different languages depending on whom it is addressed.

First of all, I want to thank Professor Giuliana Mattiazzo for allowing me to work on this project as well as allowing me to go to Rome tests, that was a truly enriching experience. I would like to especially thank Nicola due to his constant help and patience. I have learned a lot from Nicola and he has always made me feel involved in the project. I would also like to thank all the group of researchers (Giovanni, Vito, Giacomo, Enzo...) and all the other students preparing a thesis (Alfonso) for making me feel comfortable in the workplace.

També m’agradaria agrair a la meva família el suport constant que m’han donat des de Barcelona. Als meus pares, a la meva germana triatleta, als meus avis, als meus tiets, als meus cosins... I també a la meva besàvia, que encara que no ho ha pogut veure, de ben segur que estaria contenta.

Et à toi ma Juju, pour ton soutien.