

# Simultaneous energy supply and demand management in microgrids: A comparative study of MILP formulations

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## Abstract

This paper analyses the operational decision making procedures required to address the simultaneous management of energy supplies and requests in a microgrid scenario, in order to best accommodate arbitrary energy availability profiles resulting from an intensive use of renewable energy sources, and to extensively exploit the eventual flexibility of the energy requirements to be fulfilled. The optimization of the resulting short term scheduling problem in deterministic scenarios is addressed through a Mixed-Integer Linear Programming (MILP) mathematical model, which includes a new hybrid time formulation developed to take profit of the advantages of the procedures based on discrete time representations, while maintaining the ability to identify solutions requiring a continuous time representation, which might be qualitatively different to the ones constrained to consider a fixed time grid for decision-making. The performance of this new time representation has been studied, taking into account the granularity of the model and analyzing the associated trade-offs in front of other alternatives. The promising results obtained with this new formulation encourage further research regarding the development of decision-making tools for the enhanced operation of microgrids.

**Keywords:** Microgrid, Management, Energy storage, Scheduling, Flexibility, MILP.

## Nomenclature

### Indexes and sets

$i \in I$	Energy production source
$j \in J$	Energy consumer
$f \in F$	Energy demands
$jf \in F_j$	Subset of energy demands associated to energy consumer $j$
$k \in K$	Energy storage systems
$r \in R$	Power grid
$t \in T$	Time intervals included in the overall scheduling horizon

### Parameters

$Cons_{j,f}$	Individual power requirement $jf$ [kW]
$cpen_{j,f,t}$	Penalty cost [m.u./time]
$cpro_{i,t}$	Production energy cost [m.u./kWh]
$csto_{k,t}$	Storage energy cost [m.u./kWh]
$DT$	Span of the time interval [h]
$Dur_{j,f}$	Duration of consumption $jf$ [h]
$p_{i,t}^{min}$	Minimum power supply of source $i$ at interval $t$ [kW]
$p_{i,t}^{max}$	Maximum power supply of source $i$ at interval $t$ [kW]
$Price_{r,t}$	Energy price to be sold to power grid $r$ at interval $t$ [m.u./kWh]
$SE_{k,t}^{min}$	Minimum electricity storage of system $k$ at interval $t$ [kWh]
$SE_{k,t}^{max}$	Maximum electricity storage of system $k$ at interval $t$ [kWh]

$SE_{0k,t}$	Initial storage level of system $k$ at interval $t$ [kWh]
$XDem_{j,f,t}$	Period of time in which consumption $jf$ is active at interval $t$ [h]
$TS_{j,f}^{max}$	Maximum initial time of consumption $jf$ [h]
$TS_{j,f}^{min}$	Target initial time of consumption $jf$ [h]
$\eta_k^{in}$	Charging efficiency of energy storage system $k$
$\eta_k^{out}$	Discharging efficiency of energy storage system $k$

### Variables

Benefit	Microgrid benefit [m.u.]
CostPen	Total penalty cost [m.u.]
CostPro	Total production cost [m.u.]
CostSto	Total storage cost [m.u.]
$Dem_t$	Total energy consumption at interval $t$ [kWh]
Incomes	Microgrid incomes [m.u.]
$Ld_{k,t}$	Energy supplied to load system $k$ during interval $t$ [kWh]
$P_{i,t}$	Power supply of source $i$ at interval $t$ [kW]
$Pg_{r,t}$	Power supplied to power grid $r$ at interval $t$ [kW]
Profit	Total profit along the time horizon (objective function) [m.u.]
$PT_t$	Total power supply at interval $t$ [kW]
$SE_{k,t}$	Electricity storage level of system $k$ at the end of the interval $t$ [kWh]
$SP_{k,t}$	Energy supplied by storage system $k$ during interval $t$ [kWh]
$Tf_{j,f}$	Final time each consumption $jf$ [h]
$TS_{j,f}$	Initial time each consumption $jf$ [h]
$T_t$	Time [h]

### Binary variables

$X_{i,t}$	Binary variable indicating whether or not supply $i$ is used at interval $t$
$Y_{j,f,t}$	Binary variable indicating if consumption $jf$ starts at interval $t$
$Z_{j,f,t}$	Binary variable indicating if consumption $jf$ finishes at interval $t$
$W_{j,f,t}$	Binary variable indicating if consumption $jf$ is active at interval $t$

## 1. Introduction

Energy production and management is receiving growing attention in recent years. The interest in energy from renewable sources also increased due to the volatile price of fossil fuels, environmental concerns and the energy security. In this field, Energy Systems Engineering involves all the decision-making procedures associated to energy supply chain from the primary energy source to the final delivery to the customer. Its main objectives are to reduce costs, to reduce the environmental impact and to satisfy the market imposed energy demand.

Traditional power grids are based on static networks where large power plants generate electricity to be used at industrial or domestic level [1]. The optimization of the associated large-scale centralized production management problem is complicated by the need to include in the model the elements required to solve the transmission problems arising from the physical distance between energy production and energy demand, although usually, the flexibility in this classical energy supply chain is very limited, due to the need to match energy production and demand in the framework of an uncertain scenario.

On the other hand, most of renewable energy producers (i.e., photovoltaic panels, wind turbines) have relatively less capacity but are installed in a more distributed manner at different locations, potentially near the energy consumers, which reduces energy transmission losses in comparison with traditional power grids. These infrastructures may be locally interconnected in order to achieve the higher degree of flexibility required to match generation and demand. In this sense, the resulting microgrids usually include an extensive number of measuring devices as energy meters [2], to obtain prompt and reliable information on energy consumptions, since the access to real-time information becomes essential to exploit the above mentioned flexibility, to improve the efficiency and reliability of the grid (reducing the incidence of adverse events, as blackouts), the proactive maintenance schedule, and finally the customer savings [3].

The simultaneous management of energy production and demand would introduce an additional degree of freedom to the grid management problem, which would permit achieving further benefits. In the case of a system incorporating renewable energy sources, these benefits include the integration of such intermittent systems at the distribution level [4], and so the reduction of the dependence of non-renewable sources. This additional flexibility can be greatly potentiated by energy storage systems, which may be used to decouple production and demand peaks, and to cope with the uncertain (fluctuating) availability of renewable resources.

For these previous reasons, the development of an efficient energy planning and scheduling model is necessary to coordinate generation, storage and use of energy to maximize efficiency by optimally adjusting production and demand. This paper proposes a general model to solve the operational decision making problem for a microgrid, considering the simultaneous management of energy production and consumption. Two particular models using discrete and hybrid time representations have been implemented to study the trade-off between both approaches, taking into account the granularity of the problem. The assessment of their performance is presented through a case study addressing the optimal management of the energy generation, storage and consumption of several appliances within a single household served by a simple microgrid.

## **2. State of the art**

The growing interest in energy microgrids has led to the development of several mathematical models and representation schemes related to their management, as well as to their design, including the energy production management, the energy demand side management and the coordinated management of energy production and demand.

One of the recurrent issues to be discussed when addressing management or scheduling problems refers to the way how the time dimension should be represented. Different alternatives can be used including discrete, continuous and hybrid time representations.

The discrete time representation is based on dividing the scheduling horizon into a finite number of time intervals, forcing all activities to start/finish at the boundaries of these time intervals. Discrete time representation is widely applied in industrial processes [5]. Although it leads to a simplified version of the original problem, this time formulation is efficient in cases in which a reasonable number of time intervals is sufficient to represent appropriately the problem under study. The size of the mathematical model and the computational time required to solve it depend, among other factors, on the length of the time interval (i.e., the number of time intervals), and the appropriate length of the time interval depends on the characteristics of each problem, such as the duration of the events involved in the problem and the accuracy needed in the solution. Furthermore, an inappropriate selection of the length of the time interval may cause the solution to be suboptimal or even unfeasible, since the discrete formulation is based on an adaptation of the real times to the selected discrete modelling framework. This drawback can be addressed by using a variable interval size within the discrete time formulation, in order to obtain a better approximation and to improve the accuracy of the mathematical model [6].

On the other hand, in continuous time formulations, variables representing the initial and final times of all tasks are really representing the exact times in which each event is expected to take place [7], leading to decisions more accurate and sensitive when the durations of the different tasks to be managed are distributed in a wide range of values. However, the use of this formulation for large size problems becomes unaffordable in terms of computational time [5]. Further discussion on the advantages and drawbacks of discrete and continuous time representations to solve scheduling problems can be found in several review papers, most of them focused to cases related to the process industry domain [8],[9],[10].

The above mentioned limitations have led to the development of intermediate formulations between discrete and continuous time representations, classified as hybrid formulations, which combine the presence of discrete and continuous variables under the presence of time intervals. The hybrid time representations take advantage of the flexibility of the continuous time representation and the management of events that can take place in fixed points, although these formulations require more computational effort than the discrete time formulations. In this field, Neiro et al. [11] proposed a hybrid time formulation to minimize the total cost for diesel and distribution blending and distribution scheduling. This formulation involves time slots of variable length, in which the number of events within a period of time (i.e., a day), must be prespecified, since the number of time slots must be established. The aim of this work is to minimize the production cost of the production plant, satisfying the product demand of each period of time, which must be exactly fulfilled. This fact involves that events involved in each period of time must start and finish within this time window.

Regarding to the scheduling problem itself, different mathematical models have been developed in order to minimize the operational cost of a given network from the energy production management side. In this field, Zamarripa et al. [12] have developed a mathematical model to determine the production and storage levels to satisfy a deterministic energy demand, by minimizing the operational cost. More recently, a mixed integer programming model for the energy production planning related to an energy supply chain network based on a residential microgrid and considering combined heat and power systems under the objective of minimizing the total operational cost, was presented [13].

These approaches are useful for steady-state representations. However, the process dynamics should be formulated through other mathematical models to take into account generation constraints, such as production limits, ramping limits and minimum up and down times, which involve the presence of non-linear constraints. The aim of these approaches is to find how a given set of energy generators should satisfy a given demand in order to minimize the total operational costs. This is a very challenging optimization problem in the area of Energy Systems Engineering because of the huge number of possible combinations of the status of the generating units in the network [14]. In this field, several works were focused on the management of energy production of a fixed and given demand. Carrión and Arroyo [15] presented a discrete-time MILP formulation which minimizes the energy cost of a given network; this model was linearized through a piecewise linear approximation. More recently, Zondervan et al. [16] developed a Mixed Integer Non-Linear Programming (MINLP) formulation for process industries, in which the objective was to minimize the operational cost according to the availability and price of energy, by determining the optimal schedule of process tasks.

The management of the energy demand side in industrial processes represents an emerging challenge. In this area, Della Vedova and Facchinetti [17] developed a real-time scheduling approach to model and control the electrical availability. The aim was to reduce the presence of peaks of power consumption, which are negative for energy providers, for the grid and for the users, due to the fact that the presence of peaks reduces the grid efficiency (since the power grid must be designed for the peaks) and also because energy prices are based in peaks. Moreover, Kato et al. [18] presented a discrete-time energy model to manage the energy generation and storage in a demand-based power supply approach, in order to reduce the energy consumption.

Furthermore, the management of both energy production and demand has been studied in a sequential way, by adapting the process schedule to the energy availability from an external power grid. Regarding industrial processes as well, Nolde and Morari [19] have developed a continuous-time MILP model, useful to minimize the total energy cost by managing the energy consumption; the aim of the proposed formulation was to adapt the schedule of a steel plant, introducing a penalization for any variation from the contracted energy consumption from the plant to the energy supplier. Other works related to the simultaneous management of production and demand were focused on adapting the scheduling of the industrial process to the price of the energy in each period of time. In the area of industrial processes, Mitra et al. [20] developed a discrete-time MILP in order to adjust production planning according to time-dependent electricity pricing schemes for a continuous process. Also, the demand response has been studied by Hadera et al. [21] for a continuous-time scheduling of a steel plant in order to reduce energy costs. Moreover, Mohsenian-Rad and León-García [22] developed an offline residential energy consumption scheduling approach based in electricity pricing models.

But, although energy production management has been studied in the last years, the overall scheduling of a microgrid taking into account the simultaneous management of energy production and energy demand (including the possibility to shift energy consumptions) has not been reported and still represents an open challenge to the research community. Along this line, a discrete-time representation for the integrated management of energy production and demand was developed by Silvente et al. [23]. Also, a first MILP hybrid approach was presented by Silvente et al. [24] with the aim of minimizing the operational cost of the overall system.

This work is an extension of these previous works and compares the performance of a discrete time and a hybrid time representations, considering the energy production management in the presence of flexible energy requirements. One of the features of the proposed hybrid time model is the fact that this formulation do not required the prespecification of the total number of events that can take place, and all events can take place in different time intervals, not only during a particular time interval. Furthermore, the final objective is to fully exploit the flexibility offered by a complete microgrid system incorporating renewable energy sources, offering solutions which might be virtually continuously updated when an unexpected event appears in either production or demand sides, since they will be neither subjected to a prestablished time grid (model) nor to significant time delays (computing time).

### **3. Problem formulation**

The system under study consists of a set of interconnected elements (i.e., power generators, energy storages, energy consumptions) as well as a set of decisions (when, where, who, how much) that define a typical managerial problem (resource allocation and timing).

The specific problem in this case is to determine the production and storage levels to be established in the microgrid along a given time horizon, as well as to manage the consumption profiles in order to maximize some economic indicators, considering incomes associated to energy sales to the power grid as well as the costs related to production, storage and deviation in the energy consumptions from the initial targets. In the discrete time representation, decisions in terms of production, storage and consumptions are taken every given time interval. However, in order to represent a continuous energy demand profile, a hybrid time formulation has been developed, which incorporates the possibility of starting any consumption at any time. This time formulation was chosen in order to represent the different time scales related to energy production and consumption.

The problem under study is described in the following terms:

- (i) A scheduling horizon which is divided into a set of time intervals  $t \in T$ .
- (ii) A set of energy production sources  $i \in I$ , which are characterized by a minimum and maximum energy generation capacity and a given operational cost.
- (iii) A set of energy storage systems  $k \in K$  in order to accumulate energy, featuring a minimum and maximum energy storage capacity and a storage cost.
- (iv) A set of energy loads which define the energy demand. A desirable starting time of each consumption and its duration are established, but every consumption is bounded within a time window delimited by a starting time and a maximum completion time, which cannot be exceeded. Then, each consumption can be moved within this time window assuming a certain penalty cost. The power required by each consumer is assumed to be constant during each consumption.

The mathematical model presented contemplates two main aspects: the energy balances describing the energy flows, generation, storage, consumption and losses, and the capacity constraints associated to the equipment and technologies involved in the microgrid.

The decisions to be made, so as to maximize the profit of the microgrid, are related to:

- (i) The energy to be produced/purchased from source  $i$  at time interval  $t$ .
- (ii) The energy storage level to be maintained at the end of each time interval  $t$ .
- (iii) The specific time to execute an energy consumption.
- (iv) The energy to be sold to the power grid  $r$  at time interval  $t$ .

All energy consumptions incorporate a penalty cost, applicable in the case that a deviation from the initial target is decided. The value of this penalization ( $pen_{j,f}$ ) will depend on the priority and characteristics of each consumption.

The need to introduce these elements, when applied to real size cases using a continuous time representation, lead to mathematical model sizes which are out of the capacities of currently available optimization solvers [5],[7], so only discrete and hybrid time based formulations are considered in this paper. In the discrete time based model next presented, decisions related to energy production and consumption are taken at the beginning of each time interval  $t$ . However, when the hybrid time representation is used, decisions related to energy production are taken at the beginning of each time interval  $t$ , whereas decisions related to energy consumption can be taken continuously. This means that both models differ only in the equations related to energy consumption.

Eq. (1) restricts the minimum and maximum power that each energy source  $i$  can supply. This value will be zero in case it is not used and it is bounded in case it is switched on. Thus, the power produced at each interval  $t$  is calculated in eq. (2) as the summation of the production in each of the active sources. Notice that these equations can be applied for both energy production and purchases, since they consider the amount of energy to be produced (or purchased) and its production or acquisition cost.

$$P_{i,t}^{min} \cdot X_{i,t} \leq P_{i,t} \leq P_{i,t}^{max} \cdot X_{i,t} \quad \forall i, \forall t \quad (1)$$

$$PT_t = \sum_{i \in I} P_{i,t} \quad \forall t \quad (2)$$

The energy in each energy storage  $k$  at each time interval  $t$  is bounded within a minimum value and a maximum value. In addition, the energy balance for each storage system  $k$  at each interval  $t$  is given by the variation of storage level and the energy losses in eq. (4), assuming a maximum level of load, given by eq. (5):

$$SE_{k,t}^{min} \leq SE_{k,t} \leq SE_{k,t}^{max} \quad k, \forall t \quad (3)$$

$$SE_{k,t} = SE_{k,t-1} + \eta_k^{in} \cdot Ld_{k,t} - \frac{SP_{k,t}}{\eta_k^{out}} \quad \forall k, \forall t \quad (4)$$

$$0 \leq \sum_{k \in K} Ld_{k,t} \leq PT_t \cdot DT \quad \forall t \quad (5)$$

Constraints are also required for the energy demand side management, in order to determine the initial time of each energy consumption  $jf$ . The starting time,  $TS_{j,f}$ , bounded by eq. (6), is required to be greater or equal than minimum starting time  $TS_{j,f}^{min}$ , and less or equal to the time allowing due completion. Moreover, eq. (7) determines the final time of each energy consumption  $jf$ , which is given by the starting time and its duration. For the hybrid time representation model,  $TS_{j,f}$  corresponds to the real time in which consumption  $jf$  begins, whereas for the discrete time model representation, this term is related to the time interval in which each consumption starts. The same concept is applied to  $Tf_{j,f}$ .

$$TS_{j,f}^{min} \leq TS_{j,f} \leq Tf_{j,f}^{max} - Dur_{j,f} \quad \forall j, \forall f \in F_j \quad (6)$$

$$Tf_{j,f} = TS_{j,f} + Dur_{j,f} \quad \forall j, \forall f \in F_j \quad (7)$$

In order to locate the initial and final time of each consumption  $jf$  ( $TS_{j,f}$  and  $Tf_{j,f}$ ), the binary variables  $Y_{j,f,t}$  and  $Z_{j,f,t}$  are defined as active when energy consumptions starts and finishes, respectively, at time period  $t$ . These logical restrictions can be reformulated as a set of big-M constraints, where  $T_t$  corresponds to time:

$$TS_{j,f} \geq T_t - M \cdot (1 - Y_{j,f,t}) \quad \forall j, \forall f \in F_j, t \quad (8a)$$

$$TS_{j,f} \leq T_{t+1} + M \cdot (1 - Y_{j,f,t}) \quad \forall j, \forall f \in F_j, t \quad (8b)$$

$$Tf_{j,f} \geq T_t - M \cdot (1 - Z_{j,f,t}) \quad \forall j, \forall f \in F_j, t \quad (9a)$$

$$Tf_{j,f} \leq T_{t+1} + M \cdot (1 - Z_{j,f,t}) \quad \forall j, \forall f \in F_j, t \quad (9b)$$

Furthermore, eq. (10) forces that an energy consumption  $jf$  cannot start since the previous one in the same consumer unit  $j$  has finished, not allowing neither an overlap nor a change in the established consumption sequence in consumer  $j$ .

$$Tf_{j,f} \leq TS_{j,f'} \quad \forall j, \forall f, \forall f' \in F_j, \forall f < f' \quad (10)$$

Moreover, assignment constraints are implemented to enforce unique starting and finishing times for each energy consumption  $jf$ , according to eq. (11) and (12), respectively. Since values of  $Y_{j,f,t}$  and  $Z_{j,f,t}$  are equal to zero outside the time window, the summation of these binary variables is only considered within such window, in order to improve the efficiency in terms of computational effort. Eq. (13) determines when each energy consumption  $jf$  is active. Finally, eq. (14) is used to enforce the condition that all consumptions that start must finish.

$$\sum_{\substack{t \in T \\ T_t \leq Tf_{j,f}^{max} - Dur_{j,f}}} Y_{j,f,t} = 1 \quad \forall j, \forall f \in F_j \quad (11)$$

$$\sum_{\substack{t \in T \\ T_t \leq Tf_{j,f}^{max}}} Z_{j,f,t} = 1 \quad \forall j, \forall f \in F_j \quad (12)$$

$$W_{j,f,t} = \sum_{\substack{t' \in T \\ t' \leq t}} Y_{j,f,t'} - \sum_{\substack{t' \in T \\ t' < t}} Z_{j,f,t'} \quad \forall j, \forall f \in F_j, \forall t \quad (13)$$

$$\sum_{t \in T} Y_{j,f,t} = \sum_{t \in T} Z_{j,f,t} \quad \forall j, \forall f \in F_j \quad (14)$$

Using the discrete time representation, the total energy demand at time interval  $t$  is given by all the active energy consumptions at this time and determined by eq. (15a). The second term of this equation determines the exact energy consumption for those time intervals in which the energy consumption of each consumer will not take place in the overall time interval. Also, this allows to enforce the overall energy balance of the microgrid.

$$Dem_t = \sum_{j \in J} \sum_{f \in F_j} Cons_{j,f} \cdot DT \cdot W_{j,f,t} - \sum_{j \in J} \sum_{f \in F_j} Cons_{j,f} \cdot DT \cdot Z_{j,f,t} \cdot (T_{t+1} - Tf_{j,f}) \quad \forall t \quad (15a)$$

However, eq. (15a) is not valid for the hybrid time representation, since it does not take into account that the energy consumption can start at any time during the time interval, not only at the beginning. Also, one of the characteristics of this hybrid time formulation is that any energy consumption can take place in more than one time interval. Therefore, the energy consumption in the hybrid time representation requires a more complex mathematical model, as the one composed by eq. (15b), (16a), (16b), (16c) and (16d). The energy consumption at each period of time is given by eq. (15b), where  $XDem_{j,f,t}$  indicates the time period in which consumption  $jf$  is active at time interval  $t$ . In order to calculate this term, four different situations have to be taken into account (Figure 1):

- the consumption is active in the whole interval  $t$ , eq. (16a),
- the consumption starts during this interval  $t$ , eq. (16b),
- the consumption finishes during interval  $t$ , eq. (16c), and
- the consumption starts and finishes during interval  $t$ , eq. (16d).

$$Dem_t = \sum_{j \in J} \sum_{f \in F_j} Cons_{j,f} \cdot DT \cdot XDem_{j,f,t} \quad \forall t \quad (15b)$$

$$XDem_{j,f,t} \geq (T_{t+1} - T_t) - M \cdot (1 - W_{j,f,t} + Y_{j,f,t} + Z_{j,f,t}) \quad \forall j, \forall f \in F_j, \forall t \quad (16a)$$

$$XDem_{j,f,t} \geq (T_{t+1} - Ts_{j,f}) - M \cdot (2 - W_{j,f,t} - Y_{j,f,t} + Z_{j,f,t}) \quad \forall j, \forall f \in F_j, \forall t \quad (16b)$$

$$XDem_{j,f,t} \geq (Tf_{j,f} - T_t) - M \cdot (2 - W_{j,f,t} + Y_{j,f,t} - Z_{j,f,t}) \quad \forall j, \forall f \in F_j, \forall t \quad (16c)$$

$$XDem_{j,f,t} \geq (Tf_{j,f} - Ts_{j,f}) - M \cdot (3 - W_{j,f,t} - Y_{j,f,t} - Z_{j,f,t}) \quad \forall j, \forall f \in F_j, \forall t \quad (16d)$$

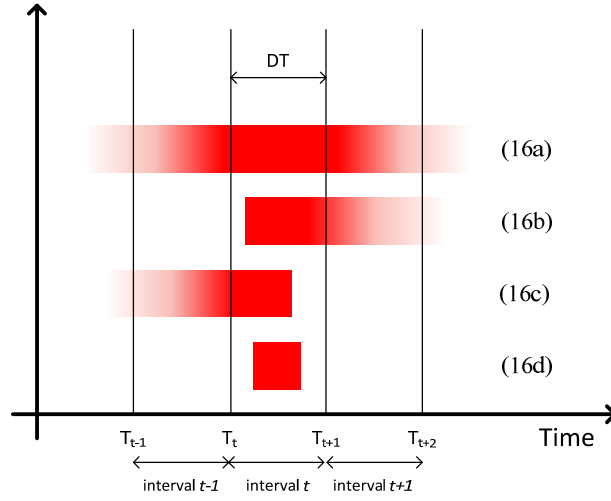


Figure 1. Time consumption within a time interval.

Obviously, the use of different time representations does not affect to the global energy balance, which in any case should include production, charge and discharge of the energy storage system, consumption and energy sales to the power grid, is enforced eq. (17):

$$\sum_{k \in K} SP_{k,t} + PT_t \cdot DT - Dem_t - \sum_{k \in K} Ld_{k,t} - \sum_{r \in R} Pg_{r,t} \cdot DT = 0 \quad \forall t \quad (17)$$

Furthermore, the economic aspects related to the energy management are introduced, considering energy production, energy storage and penalty costs in case of deviation from the initial target. The production cost is assumed to be proportional to the real amount of energy produced, and according to the different energy sources, eq. (18). In the same way, the storage cost is calculated as proportional to the real energy storage expected for each time period, according to eq. (19). Finally, the flexibility in the demand profile is tuned through the introduction of an additional term in the usual cost-based objective function penalizing the deviation from given initial consumption targets, eq. (20). The introduction of this penalty term and the flexibility to consider any arbitrarily availability profile in the energy generation systems previously described in eq. (1) and (2) are the basic elements to allow the simultaneous management of energy production and consumption. The value selected for the penalty coefficient of each requirement depends on the specific characteristics of this requirement: small values are associated to high flexibility, while high values will lead to very strict requirements. Hence, it is important to choose appropriate values for each one of the penalty coefficients to guarantee the adequate management of the energy demand.

$$CostPro = \sum_{t \in T} \sum_{i \in I} cpro_{i,t} \cdot P_{i,t} \cdot DT \quad (18)$$

$$CostSto = \sum_{t \in T} \sum_{k \in K} cstok_{k,t} \cdot SE_{k,t} \quad (19)$$

$$CostPen = \sum_{j \in J} \sum_{f \in F_j} cpen_{j,f} \cdot (Ts_{j,f} - Ts_{j,f}^{min}) \quad (20)$$

Also, eventual energy revenues from the power grid have been taken into account. No maximum limits on the energy that can be sold to the utility grid have been considered. Thus, incomes are computed as linear from the energy sold and its selling price to the power grid eq. (21), although this term may be modified according to applicable the market regulations.

$$Incomes = \sum_{t \in T} \sum_{r \in R} Price_{r,t} \cdot Pg_{r,t} \cdot DT \quad (21)$$



The profit of the microgrid, which corresponds to the objective function to be maximized, is calculated considering incomes and costs in eq. (22).

$$Profit = Incomes - (CostPro + CostSto + CostPen) \quad (22)$$

To sum up, two models are defined based on the presented formulation, namely:

- Discrete time model
  - ✓ Objective function: equation (22)
  - ✓ Subject to constraints (1-14) + constraint (15a) + constraints (17-21)
- Hybrid time model
  - ✓ Objective function: equation (22)
  - ✓ Subject to constraints (1-14) + constraints (15b-16d) + constraints (17-21)

In both cases, the already presented systems lead to a MILP problem. Both mathematical formulations can be extended by introducing typical features of the UC problem, such as ramping constraints and minimum up and down time constraints, as well as non-convex production costs and time-dependent startup costs, which would introduce non-linearities in the model. However, only linear constraints are included in the present formulation, since the objective of this work is to study the difference between the discrete and the hybrid time formulations, in terms of results and computational effort. Also, the model does not consider the presence of fixed costs associated to the investment and installation of energy production sources, since the design of the energy network related to the microgrid has not been taken into account and only short-term decisions are contemplated.

#### 4. Case study

Both proposed MILP formulations have been applied to a case study which includes a photovoltaic panel (*i1*) and a micro-wind turbine (*i2*) as renewable energy sources, as well as a bidirectional connection to the power grid (*i3*) to purchase and to sell energy. The possibility to purchase energy to the power grid ensures the feasibility of the optimization problem, disregarding weather conditions and abnormal demand requirements. Also, an energy storage system *k* has been considered.

Moreover, 30 different appliances or consumers *j* have been taken into account, with different energy requirements  $Cons_{j,f}$ , which are assumed to admit a certain degree of flexibility in their respective targets, in terms of accepting some delay from their expected schedule. Each consumption has associated a penalty cost, to be applied in case of that such deviations from their respective targets are introduced. Data related to minimum/maximum initial time, duration and penalty cost associated to each energy consumption can be found in Table A1 in Appendix A. A penalty cost has been assigned according to the characteristics of each energy consumption.

The results of the optimization procedure includes energy production, storage and consumption patterns. Decisions related to energy production are given every 15 minutes, according to the expected energy demand requirements and the anticipated energy availability resulting from the specific weather forecast. The considered time horizon extents to 24 hours, thus resulting in 96 time slots. According to the described discrete time model, decisions related to the consumption schedule are considered to be taken according to the indicated time grid (i.e., every 15 minutes), whereas in the hybrid time model, these decisions can be scheduled at any moment along the continuous time horizon.

Data related to energy production cost for each generator and energy storage cost are presented in Table 1. Also, the same table displays the minimum/maximum values for power supply and energy storage as well as other economic data. Variable production costs related to the energy production through solar panels and wind turbines are considered to be zero.

Table 1. Economic data and capacity constraints.

Unit	Description	Cost (m.u. / kWh)	$P_i^{min}$ (kW)	$SE_k^{min}$ (kWh)	$SE_k^{max}$ (kWh)
i1	Photovoltaic panel	0	0	-	-
i2	Wind turbine	0	0	-	-
i3	External power grid	0.153	0	-	-
k1	Energy storage system	$1 \cdot 10^{-4}$	-	13.44	16.80

## 5. Results and discussion

Different scenarios and solution approaches have been considered in order to compare, analyze and highlight the characteristics of the proposed models, including:

- (i) the absence of demand management opportunities (energy demands cannot be shifted),
- (ii) demand management using the discrete time model and
- (iii) demand management using the hybrid time model.

The same power availability (Figure 2) has been assumed and the energy storage level has been forced to be the same at the beginning and at the end of the scheduling horizon in order to ensure a fair comparison among the different obtained solutions.

The resulting MILP models have been implemented in GAMS 24.1 [25] and solved using CPLEX 12, in a Pentium Intel® Core™ i7 CPU 2600 @ 3.40 GHz, with 8.00 GB of installed memory (RAM).

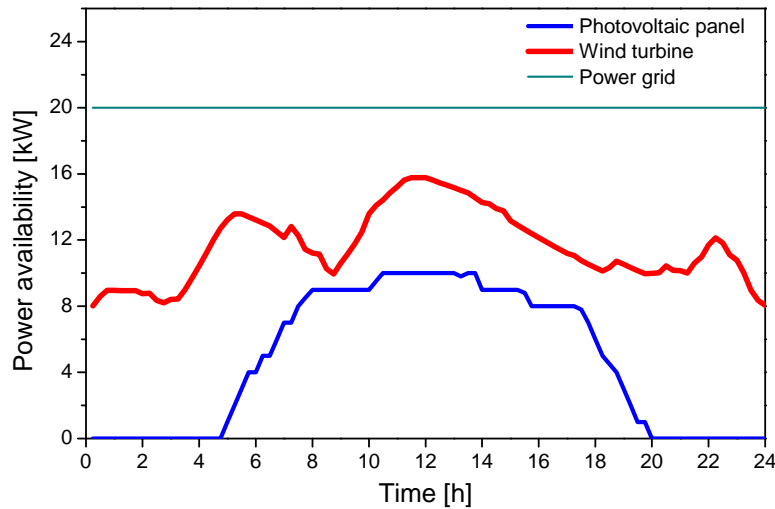


Figure 2. Power availability.

In the first situation (i), only production management is considered. The energy storage system is the only resource to be used in order to optimize the match between production and demand, since energy requirements do not match energy availability. The resulting energy storage level profile is represented in Fig. 4. The solution in this case shows that the generation (and storage) of energy from renewable sources is insufficient to timely satisfy the set of fixed energy consumptions, so the purchase of energy from the power grid is required (Fig. 3). Fig. 6a shows the optimum energy consumptions schedule obtained with this model.

The characteristics of the model proposed for the second situation (ii) allows to consider the possibility of delaying energy consumptions at the expense of a penalty cost for each delay. The optimum decisions in this case would lead to the independence from the power grid, so external energy purchases are not required to satisfy the energy demand (Fig. 3 and Table 3). In this case, some energy consumptions have been delayed one or more complete time intervals (15 minutes). The total accumulate delay is up to 19.35 hours (Table 2). These are delays are due to:

- The granularity of the model: The discrete time model forces all consumptions to start at the beginning of one time interval. Thus, all consumptions have been delayed to start in the boundary of the corresponding time interval. In this particular case study, this represents almost 40 % of the total accumulated delay.
- The decision making process itself which, in order to optimize the matching between production and demand, and the associated penalties, introduces additional changes which, at the end, results in about 60 % of the total accumulated time delay in this case study.

The third approach (iii) corresponds to the proposed hybrid time representation. Also with this model, the simultaneous management allows to satisfy the overall energy demand without acquiring energy from the grid, at the expense of

introducing consumption delays. Actually, the same energy production profile from the renewable (cheap) sources is obtained in all cases (Figure 3 and Table 3), since the microgrid should use all the available energy from renewable sources at their maximum capacity, and to sell the excess to the power grid (Figure 5). But in this case the value of the objective function is improved since the delays in the consumptions may be better adjusted (Figure 6c), and the penalty cost may be reduced accordingly. Now the total delay in the optimum case was 11.97 hours (Table 2). Obviously, and unlike the discrete model, only delays due to the decision making process are introduced.

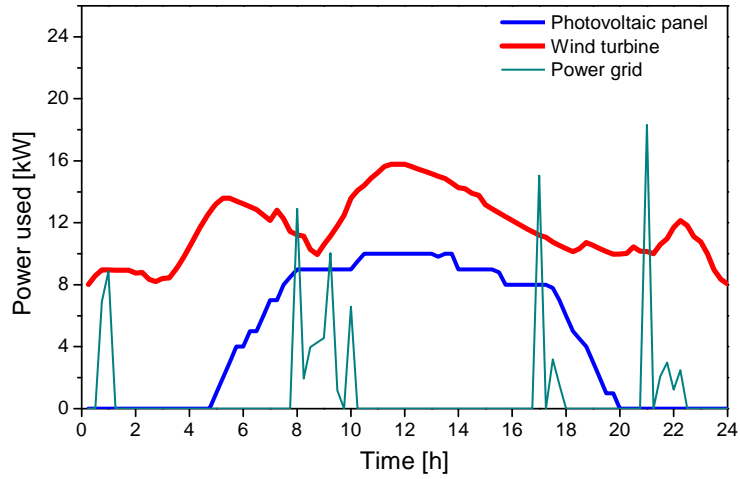


Figure 3. Power source in each time period (cases with demand side management do not require energy purchases from the power grid).

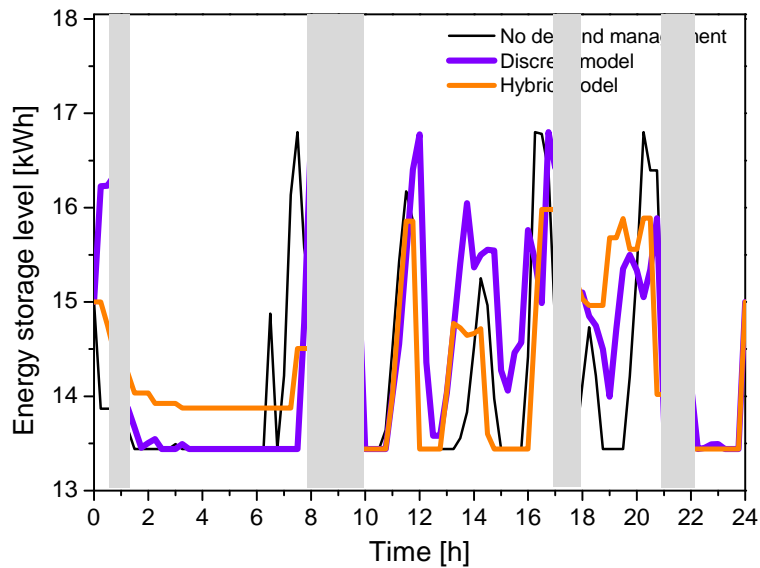


Figure 4. Energy storage level.

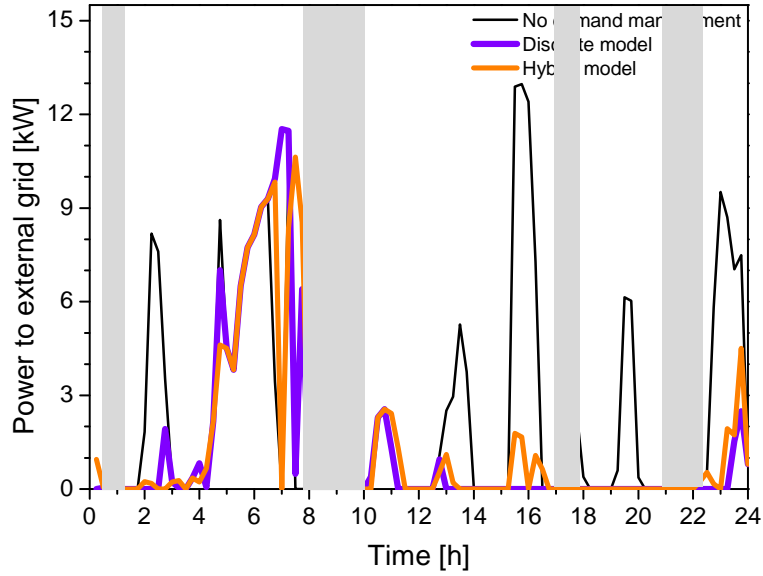


Figure 5. Energy sold to the power grid.

Table 2. Delays produced in *each* energy consumption.

Consumption <i>jf</i>	Delay using discrete time formulation with time interval of 15 minutes (h)		Delay using hybrid time formulation (h)
	Granularity	Decision	Decision
j5 f10	0.075	0.000	0.004
j5 f12	0.125	0.000	0.035
j6 f1	0.075	0.000	0.184
j7 f1	0.225	0.500	0.785
j7 f2	0.125	0.000	0.125
j7 f6	0.000	0.000	0.012
j7 f9	0.050	0.000	0.118
j7 f10	0.125	0.000	0.070
j11 f1	0.125	0.500	0.576
j14 f1	0.000	0.250	0.000
j15 f1	0.075	1.750	1.797
j16 f1	0.050	0.500	0.484
j19 f1	0.225	0.500	0.475
j19 f2	0.025	0.000	0.030
j20 f3	0.225	0.250	0.000
j21 f1	0.150	0.250	0.475
j22 f1	0.200	1.250	1.346
j22 f2	0.125	1.000	1.024
j23 f1	0.150	1.000	0.964
j24 f1	0.225	0.250	0.090
j24 f2	0.000	0.250	0.130
j25 f1	0.125	0.250	0.400
j27 f1	0.175	0.250	0.000
j28 f1	0.025	0.250	0.000
j29 f1	0.075	1.500	1.575
j29 f2	0.000	0.500	0.356
j30 f1	0.075	0.250	0.239
j31 f1	0.175	0.500	0.675
Other	4.575	0.000	0.000
Total delay (h)	7.600	11.750	11.969
	19.350		11.969

Table 3. Comparison of the obtained results through the different solution approaches.

Key performance indicator	No demand management	Discrete time model	Hybrid time model
Profit (m.u.)	2.63	3.08	3.51
Incomes (m.u.)	6.60	3.41	3.66
Production cost (m.u.)	3.97	0.00	0.00
Penalty cost (economic units)	0.00	0.33	0.15
Consumed energy (kWh)	359.0	359.0	359.0
Energy produced or purchased (kWh)	417.9	391.9	391.9
Energy from photovoltaic panels (kWh)	112.7	112.7	112.7
Energy from wind turbines (kWh)	279.2	279.2	279.2
Energy from power grid (kWh)	26.0	0.0	0.0
Energy from energy storage systems (kWh)	17.1	19.3	10.7
Energy sold to the power grid (kWh)	55.0	28.5	30.5
Energy loses and energy to load the energy storage system (kWh)	21.0	23.7	13.1

Figure 6 summarizes the main optimum decisions obtained using each one of the 3 presented solution approaches. In general, the introduction of delays is proposed in those energy requirements expected to start in the time slots in which the energy from less expensive sources is not enough (shaded areas), but the need to match decisions with a pre-established time grid (discrete time formulation) may introduce some unexpected behavior.

Some of these cases can be analyzed and compared in more detail in Figure 7: As an example, using the discrete time formulation a significant deviation from its initial target was proposed to consumer  $j27$  since the consumption cannot start until the beginning of the next time interval (due to the granularity of the model) and at this boundary of the time interval the optimal decision is to delay the consumption again because the lack of availability in the energy generation system. On the contrary, using the hybrid time model the possibility to satisfy this particular consumption according to its target is recognized so no delay was proposed at all.

Table 4 shows that, although the two time-representations involve the same number of discrete variables, the hybrid time representation model requires more equations and more continuous variables, as well as a substantial increase in the computational effort in order to determine the optimal solution. Moreover, in order to study how the length of the time interval affects the solution and the computational time, the same problem has been solved using different time interval sizes (Table 4). The solution using the discrete time formulation is improved when the length of the time interval decreases, since the model becomes more sensitive and delays due to the granularity of the model are reduced. But obviously, the computational effort increases when the duration of the time interval is reduced, since more binary variables are required in order to define the model. It is worthy to mention that the solution for the discrete time model using a time interval of 1 minute could not be obtained because the memory requirements exceeded that capacity of the computational system used for these tests. This was also the case of the hybrid model with time intervals of 5 and 3 minutes.

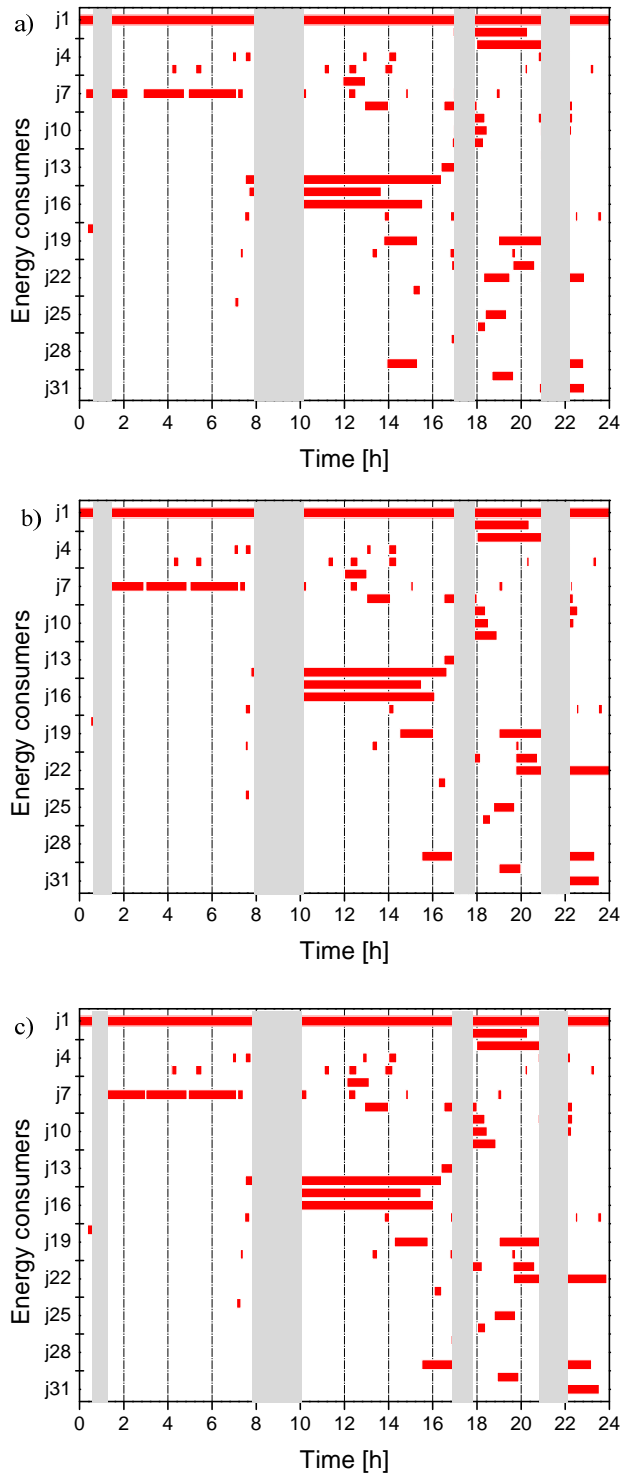


Figure 6. Active energy consumers in each time period for the a) no-demand side management and the demand side management using b) discrete model and c) hybrid model.

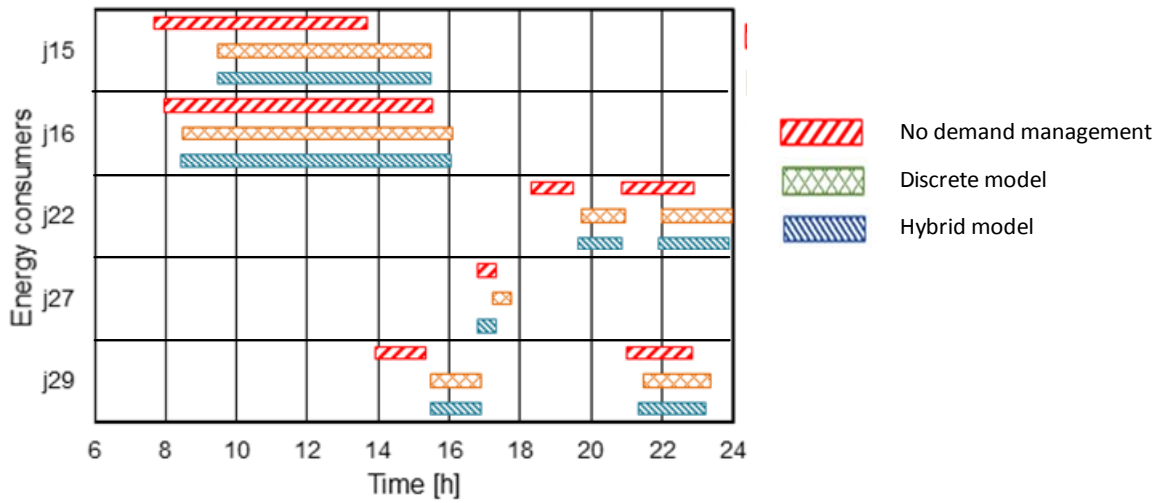


Figure 7. Comparison of the consumption schedule for some energy consumptions.

Table 4. Comparison of optimal solution and model statistics.

Key performance indicator	dt = 15 min		dt = 5 min	dt = 3 min
	Discrete time model	Hybrid time model	Discrete time model	Discrete time model
Profit (economic units)	3.08	3.51	3.42	3.50
Equations	93,606	312,978	582,726	1,477,350
Continuous variables	54,615	76,626	328,599	823,767
Discrete variables	33,468	33,468	210,554	535,034
Generation time (CPU, s)	4.1	6.1	71.5	302.5
Resource time (CPU, s)	0.8	242.7	10.5	31.8
Total computational time (CPU, s)	4.9	248.8	82.0	334.3
Memory required (Mb)	128	176	2,086	8,581
Relative gap (%)	0	< 1	0	0

GAMS 24.1 / CPLEX 12  
a Pentium Intel® Core™ i7 CPU 2600 @ 3.40 GHz

## 6. Conclusions

This work addresses the short-term scheduling problem of a smart grid in order to determine optimal decisions in terms of both energy production and consumption. This is achieved by maximizing an economic objective function including penalties for the delayed fulfilment of the energy requirements. In order to solve the inconveniences shown by traditional discrete and continuous time representations to manage the decision making process at both the energy production and at the energy requirement sides simultaneously, a new hybrid time formulation, combining elements from both discrete and continuous time representations, is presented, and the resulting discrete and hybrid MILP formulations have been presented and discussed.

The obtained results demonstrate that the proposed model is able to approach, with high level of detail, the real system for a short-term daily scheduling horizon using both time representations, and so its potential to exploit the advantages of the eventual flexibility in the energy demand side, which would allow enhancing the microgrid efficiency and autonomy. In addition, the hybrid time representation proposed improves the results due to a better adjust of the required starting time of consumptions, which improves the robustness of the model and the value of the optimal solution. This improvement is

achieved at the expense of an increase in the complexity of the model, which results in an affordable increase in the required computational effort.

The proposed approach has been used to solve a case study based on a single domestic microgrid. However, the results obtained show that it could be also used as the basis for solving future problems with higher complexity (i.e., industrial cases). Further work includes expanding the size of the microgrid or considering interconnected microgrids, as well as incorporating uncertainty related to weather conditions effects in energy suppliers and energy demand variations.

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### References

- [1] L. Wang and C. Singh, "Multicriteria design of hybrid power generation systems based on a modified particle swarm optimization algorithm," in *IEEE Trans. Energy Conversion*, vol. 24, pp. 163-172, 2009.
- [2] R. Krishnan, "Meters of Tomorrow," in *IEEE Power and Energy Magazine*, pp. 92-94, 2008.
- [3] T. Krishnamurti, D. Schwartz, A. Davis, B. Fischhoff, W.B. Bruin, L. Lave and J. Wang, "Preparing for smart grid technologies: A behavioral decision research approach to understanding consumer expectations about smart meters," in *Energy Policy*, vol. 41, pp. 790-797, 2012.
- [4] A. Ipakchi and F. Albuyeh, "Grid of the future," in *IEEE Power and Energy Magazine*, pp. 52-62, 2009.
- [5] C.A. Méndez, J. Cerdá, I.E. Grossmann, I. Harjunkoski, and M. Fahl, "State-of-the-art review of optimization methods for short-term scheduling of batch processes," in *Computers and Chemical Engineering*, vol. 30, pp. 13-946, 2006.
- [6] K.H. Lee, H.I. Park and I.B. Lee, "A Novel Nonuniform Discrete Time Formulation for Short-Term Scheduling of Batch and Continuous Processes," in *Industrial & Engineering Chemistry Research*, vol. 40, pp. 4902-4911, 2001.
- [7] I. Harjunkoski, C.T. Maravelias, P. Bongers, P.M. Castro, S. Engell, I.E. Grossmann, J. Hooker, C.A. Méndez, G. Sand and J. Wassick, "Scope for industrial applications of production scheduling models and solution methods," in *Computers and Chemical Engineering*, vol. 62, pp. 161-193, 2014.
- [8] C.A. Floudas and X. Lin, "Continuous-time versus discrete-time approaches for scheduling of chemical processes: a review," in *Computers and Chemical Engineering*, vol. 28, pp. 2109-2129, 2004.
- [9] C.T. Maravelias and I.E. Grossmann, I. E. "On the relation of continuous and discrete-time state-task network formulations," in *AIChE Journal*, vol. 52, pp. 843-849, 2006.
- [10] H. Stefansson, S. Sigmarsdottir, P. Jansson and N. Shah, "Discrete and continuous time representations and mathematical models for large production scheduling problems: A case study from the pharmaceutical industry," in *European Journal of Operational Research*, vol. 215, pp. 383-392, 2011.
- [11] S.M.S. Neiro, V.V. Murata and J.M. Pinto, "Hybrid time formulation for diesel blending and distribution scheduling," in *Industrial & Engineering Chemistry Research*, vol. 53, pp. 17124-17134, 2014.
- [12] M.A. Zamarripa, J.C. Vasquez, J.M., Guerrero and M. Graells, "Detailed operation scheduling and control for renewable energy powered microgrids," in *Computer Aided Chemical Engineering*, vol. 29, pp. 819-1823, 2011.
- [13] G.M. Kopanos, M.C. Georgiadis, and E.N. Pistikopoulos, "Energy production planning of a network of micro combined heat and power generators," in *Applied Energy*, vol. 102, pp. 1522-1534, 2013.
- [14] A. Bhardwaj, V.K. Kamboj, V.K., Shukla, B. Singh and P. Khurana, "Unit Commitment in Electrical Power System – A Literature Review," in *IEEE International Power Engineering and Optimization Conference*, pp. 275-280, 2012.
- [15] M. Carrión and J.M. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," in *IEEE Transactions on Power Systems*, vol. 21, pp. 1371-1378, 2006.
- [16] E. Zondervan, I.E. Grossmann and A.B. de Haan, "Energy optimization in the process industries: Unit Commitment at systems level," in *Computer Aided Chemical Engineering*, vol. 28, pp. 931-936, 2010.
- [17] M.L. Della Vedova and T. Facchinetti, "Real-time scheduling for industrial load management," in *IEEE International Energy Conference and Exhibition*, pp. 770-776, 2012.
- [18] T. Kato, K. Yuasa and T. Matsuyama, "Energy on demand: Efficient and versatile energy control system for home energy management," in *IEEE Smart Grid Communications*, pp. 392-397, 2011.
- [19] K. Nolde and M. Morari, "Electrical load tracking scheduling of a steel plant," in *Computers and Chemical Engineering*, vol. 34, pp. 1899-1903, 2010.



- [20] S. Mitra, I.E. Grossmann, J.M. Pinto and N. Arora, "Optimal production planning under time sensitive electricity prices for continuous power-intensive processes," in *Computers and Chemical Engineering*, vol. 38, pp. 171-184, 2012.
- [21] H. Hadera, I. Harjunkoski, I.E. Grossmann, G. Sandand S. Engell, "Steel Production Scheduling Optimization under Time-sensitive Electricity Costs," in *Computer Aided Chemical Engineering*, vol. 33, pp. 373-378, 2014.
- [22] A. Mohsenian-Rad and A. León-García, "Optimal residential load control with price prediction in real-time electricity pricing environments," in *IEEE Transactions on Smart Grid*, vol. 1, pp. 120-133, 2010.
- [23] J. Silvente, M. Graells, A. Espuña and P. Salas, "An optimization model for the management of energy supply and demand in smart grids," in *IEEE International Energy Conference and Exhibition*, pp. 424-429, 2012.
- [24] J. Silvente, A. Aguirre, G. Crexells, M.A. Zamarripa, C.A. Méndez, M. Graells and A. Espuña, "Hybrid time representation for the scheduling of energy supply and demand in smart grids," in *Computer Aided Chemical Engineering*, vol. 32, pp. 553-558, 2013.
- [25] R.E. Rosenthal, "GAMS – A User's Guide," in *GAMS Development Corporation*, Washington, 2012.

## APPENDIX A

Table A1. Input parameters.

Energy consumer $j$	Energy demand $f$	$Cons_{j,f}$ (kW)	$Ts_{j,f}^{min}$ (h)	$Dur_{j,f}$ (h)	$Tf_{j,f}^{max}$ (h)	Penalty cost (m.u./h)
j1	f1	2.557	0.000	0.250	0.250	40
j1	f2	1.012	0.250	0.250	0.500	40
j1	f3	0.957	0.500	0.250	0.750	40
j1	f4	0.916	0.750	0.250	1.000	40
j1	f5	0.955	1.000	0.250	1.250	40
j1	f6	1.693	1.250	0.250	1.500	40
j1	f7	1.705	1.500	0.250	1.750	40
j1	f8	0.520	1.750	0.250	2.000	40
j1	f9	0.605	2.000	0.250	2.250	40
j1	f10	0.728	2.250	0.250	2.500	40
j1	f11	0.683	2.500	0.250	2.750	40
j1	f12	0.174	2.750	0.250	3.000	40
j1	f13	0.157	3.000	0.250	3.250	40
j1	f14	1.187	3.250	0.250	3.500	40
j1	f15	1.341	3.500	0.250	3.750	40
j1	f16	1.606	3.750	0.250	4.000	40
j1	f17	1.143	4.000	0.250	4.250	40
j1	f18	1.843	4.250	0.250	4.500	40
j1	f19	1.684	4.500	0.250	4.750	40
j1	f20	1.719	4.750	0.250	5.000	40
j1	f21	1.776	5.000	0.250	5.250	40
j1	f22	1.686	5.250	0.250	5.500	40
j1	f23	1.672	5.500	0.250	5.750	40
j1	f24	1.070	5.750	0.250	6.000	40
j1	f25	1.008	6.000	0.250	6.250	40
j1	f26	0.565	6.250	0.250	6.500	40
j1	f27	0.565	6.500	0.250	6.750	40
j1	f28	0.945	6.750	0.250	7.000	40
j1	f29	0.335	7.000	0.250	7.250	40
j1	f30	0.492	7.250	0.250	7.500	40
j1	f31	0.560	7.500	0.250	7.750	40
j1	f32	0.233	7.750	0.250	8.000	40
j1	f33	0.288	8.000	0.250	8.250	40
j1	f34	0.260	8.250	0.250	8.500	40
j1	f35	0.230	8.500	0.250	8.750	40
j1	f36	0.348	8.750	0.250	9.000	40
j1	f37	0.386	9.000	0.250	9.250	40
j1	f38	0.163	9.250	0.250	9.500	40
j1	f39	0.481	9.500	0.250	9.750	40
j1	f40	0.168	9.750	0.250	10.000	40
j1	f41	0.600	10.000	0.250	10.250	40
j1	f42	1.129	10.250	0.250	10.500	40
j1	f43	1.311	10.500	0.250	10.750	40
j1	f44	0.400	10.750	0.250	11.000	40
j1	f45	0.200	11.000	0.250	11.250	40
j1	f46	0.655	11.250	0.250	11.500	40
j1	f47	0.694	11.500	0.250	11.750	40
j1	f48	0.494	11.750	0.250	12.000	40
j1	f49	0.681	12.000	0.250	12.250	40
j1	f50	0.436	12.250	0.250	12.500	40
j1	f51	0.723	12.500	0.250	12.750	40

j1	f52	1.195	12.750	0.250	13.000	40
j1	f53	0.212	13.000	0.250	13.250	40
j1	f54	0.295	13.250	0.250	13.500	40
j1	f55	0.342	13.500	0.250	13.750	40
j1	f56	0.209	13.750	0.250	14.000	40
j1	f57	0.421	14.000	0.250	14.250	40
j1	f58	0.179	14.250	0.250	14.500	40
j1	f59	0.296	14.500	0.250	14.750	40
j1	f60	0.236	14.750	0.250	15.000	40
j1	f61	0.174	15.000	0.250	15.250	40
j1	f62	0.155	15.250	0.250	15.500	40
j1	f63	0.423	15.500	0.250	15.750	40
j1	f64	0.734	15.750	0.250	16.000	40
j1	f65	0.075	16.000	0.250	16.250	40
j1	f66	0.680	16.250	0.250	16.500	40
j1	f67	0.573	16.500	0.250	16.750	40
j1	f68	0.171	16.750	0.250	17.000	40
j1	f69	0.465	17.000	0.250	17.250	40
j1	f70	0.461	17.250	0.250	17.500	40
j1	f71	0.479	17.500	0.250	17.750	40
j1	f72	0.010	17.750	0.250	18.000	40
j1	f73	0.457	18.000	0.250	18.250	40
j1	f74	0.077	18.250	0.250	18.500	40
j1	f75	0.224	18.500	0.250	18.750	40
j1	f76	0.632	18.750	0.250	19.000	40
j1	f77	0.441	19.000	0.250	19.250	40
j1	f78	0.028	19.250	0.250	19.500	40
j1	f79	0.088	19.500	0.250	19.750	40
j1	f80	0.743	19.750	0.250	20.000	40
j1	f81	0.931	20.000	0.250	20.250	40
j1	f82	0.792	20.250	0.250	20.500	40
j1	f83	0.906	20.500	0.250	20.750	40
j1	f84	1.299	20.750	0.250	21.000	40
j1	f85	0.918	21.000	0.250	21.250	40
j1	f86	0.894	21.250	0.250	21.500	40
j1	f87	0.572	21.500	0.250	21.750	40
j1	f88	0.193	21.750	0.250	22.000	40
j1	f89	0.725	22.000	0.250	22.250	40
j1	f90	0.187	22.250	0.250	22.500	40
j1	f91	0.443	22.500	0.250	22.750	40
j1	f92	0.259	22.750	0.250	23.000	40
j1	f93	0.689	23.000	0.250	23.250	40
j1	f94	0.631	23.250	0.250	23.500	40
j1	f95	0.862	23.500	0.250	23.750	40
j1	f96	0.312	23.750	0.250	24.000	40
j2	f1	1.500	16.675	3.375	20.250	0.4
j3	f1	1.500	17.725	3.550	21.500	0.4
j4	f1	0.350	6.675	0.200	7.000	0.04
j4	f2	0.350	7.250	0.275	7.750	0.04
j4	f3	0.350	12.575	0.213	13.000	0.04
j4	f4	0.350	13.750	0.375	14.250	0.04
j4	f5	0.350	20.525	0.198	21.000	0.04
j4	f6	0.350	21.750	0.250	22.000	0.04
j5	f1	2.000	0.525	0.175	1.000	0.04
j5	f2	2.000	3.925	0.250	4.250	0.04
j5	f3	2.000	5.000	0.300	5.500	0.04

j5	f4	2.000	7.625	0.300	8.250	0.04
j5	f5	2.000	8.725	0.350	9.250	0.04
j5	f6	2.000	10.825	0.250	11.250	0.04
j5	f7	2.000	11.950	0.375	12.500	0.04
j5	f8	2.000	13.575	0.375	14.250	0.04
j5	f9	2.000	17.000	0.175	17.250	0.04
j5	f10	2.000	19.925	0.125	20.250	0.04
j5	f11	2.000	20.750	0.375	21.250	0.04
j5	f12	2.000	22.875	0.175	23.250	0.04
j6	f1	2.640	11.675	1.025	13.000	0.04
j7	f1	8.000	0.025	1.925	2.750	0.04
j7	f2	8.000	2.625	1.875	5.000	0.04
j7	f3	8.000	4.675	2.200	7.250	0.04
j7	f4	8.000	6.900	0.275	7.500	0.04
j7	f5	8.000	8.875	0.275	9.500	0.04
j7	f6	8.000	9.750	0.300	10.250	0.04
j7	f7	8.000	11.925	0.350	12.500	0.04
j7	f8	8.000	14.525	0.125	15.000	0.04
j7	f9	8.000	16.700	0.313	17.250	0.04
j7	f10	8.000	18.625	0.188	19.000	0.04
j7	f11	8.000	20.675	0.313	21.250	0.04
j7	f12	8.000	21.800	0.088	22.750	0.04
j8	f1	0.400	12.650	1.100	14.250	0.02
j8	f2	0.400	16.250	1.525	18.250	0.02
j8	f3	0.400	20.700	1.375	22.750	0.02
j9	f1	0.240	16.725	1.400	18.500	0.02
j9	f2	0.240	20.525	1.575	23.000	0.02
j10	f1	0.240	16.700	1.525	18.750	0.02
j10	f2	0.240	20.650	1.400	22.750	0.02
j11	f1	12.000	16.625	1.425	19.250	0.02
j12	f1	2.000	20.625	1.125	22.250	0.02
j13	f1	8.000	16.125	1.375	18.000	0.02
j14	f1	7.000	7.250	8.900	16.750	0.02
j15	f1	7.000	7.425	6.000	15.750	0.02
j16	f1	7.000	7.700	7.600	16.250	0.02
j17	f1	2.000	7.225	0.250	7.750	0.02
j17	f2	2.000	7.750	0.300	8.250	0.02
j17	f3	2.000	8.225	0.600	9.250	0.02
j17	f4	2.000	13.550	0.250	14.500	0.02
j17	f5	2.000	16.550	0.438	17.500	0.02
j17	f6	2.000	20.700	0.250	21.000	0.02
j17	f7	2.000	21.000	0.175	21.750	0.02
j17	f8	2.000	22.200	0.125	22.750	0.02
j17	f9	2.000	23.225	0.188	23.500	0.02
j18	f1	7.500	0.100	0.750	1.500	0.02
j19	f1	1.500	13.525	1.550	16.250	0.02
j19	f2	1.500	18.725	1.975	21.000	0.02
j20	f1	0.240	7.025	0.150	7.750	0.004
j20	f2	0.240	13.000	0.250	13.500	0.004
j20	f3	0.240	16.525	0.250	17.250	0.004
j20	f4	0.240	19.325	0.175	19.750	0.004
j21	f1	0.350	16.600	0.925	18.000	0.004
j21	f2	0.350	19.375	1.000	20.500	0.004
j22	f1	5.000	18.050	1.200	20.750	0.004
j22	f2	5.000	20.625	2.000	23.750	0.004
j23	f1	10.000	14.850	0.350	16.750	0.004

j24	f1	20.000	6.775	0.200	7.500	0.004
j24	f2	20.000	7.750	0.250	10.000	0.004
j25	f1	4.000	18.125	0.975	19.750	0.004
j26	f1	0.192	17.775	0.375	18.750	0.004
j27	f1	0.440	16.575	0.313	17.500	0.004
j28	f1	0.660	16.725	0.300	17.500	0.004
j29	f1	4.000	13.675	1.400	17.000	0.004
j29	f2	4.000	20.750	1.850	23.500	0.004
j30	f1	0.200	18.425	1.000	19.750	0.004
j31	f1	1.500	20.575	2.050	23.750	0.004

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