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History and Philosophy of Mathematical Notations and Symbolism

Organised by
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Antoni Malet, Barcelona

October 25th – October 31st, 2009

Abstract.
The conference aimed to discuss the nature of mathematical language with the focus set on the history of mathematical symbolism and symbolisation in a wide sense. Its contents were arranged along three main topics –symbolization from Mesopotamian, Egyptian, Greek, Latin, Indian and Chinese perspectives; algebraic analysis and symbolization in Europe, 1500-1750; the 19^{th} and 20^{th} century. The intention was to deal with the topic extensively, including all kinds of symbolism shaped in various traditions. Major problems addressed by the conference include but are not limited to why specific notations and symbolisms were introduced, how they were designed and how they were used. In addition, the lectures also dealt with the attitudes towards, and reflections on, mathematics and language that their introduction stirred. We hope that the workshop will contribute to shape future research in this domain.

Mathematics Subject Classification (2000): 01-02, 01-06.

Introduction by the Organisers

Contemporary mathematical writings are easily recognizable because of the distinctive feature that the use of symbolism constitutes. Generally speaking, we know that the connections between mathematics and language are pervasive, are important, and are elusive. Mathematics is the science where truth depends but on language, and yet we know that in ordinary language the relations between the words and the things they mean are so complex as to be impossible to formalize or pin them down. Questions of rhetoric as well as of language structure have been important in the many different languages used in mathematics throughout history. One of the biggest transformations in the history of mathematical thought came through the articulation of its proper symbolic language–a language made of graphic symbolism endowed with their proper rules of formal transformation, or syntax, and able to carry on or perform a great deal of logical deductions. We know now that this "artificial" (as it is sometimes called) symbolic language took form in Europe in a non too-short continuous process during almost three centuries, from the mid 15th through the mid 18th century. To be sure there have been many other important artificial languages, both within and without the realm of mathematics, both within and without the Greek-Latin Western traditions. The consolidation of the symbolic language of mathematics has had momentous implications methodologically, for the nature of mathematical objects, and for the scope and applicability of mathematics generally. It has also had a profound influence in philosophical discussions about the nature of logic and the nature of thought. However, in spite of its centrality, the formation of such an specific artificial language has proved largely impervious to historical investigation. More generally, the history of mathematical symbolism still presents many dark areas.

The Conference *History and Philosophy of Mathematical Notations and Symbolism* was organized by Karine Chemla (Paris), Eberhard Knobloch (Berlin) and Antoni Malet (Barcelona). It was held October 25th –October 31th, 2009 and consisted in 20 lectures. One of the organizers, Antoni Malet, gave the first talk, which consisted of a general introduction to the topic. The conference was concluded by a final discussion, which focused on six essential issues that had been addressed in the talks or were raised during the discussion after the talks.

In the conference, we discussed the nature of mathematical language with the focus set on the history of mathematical symbolism and symbolisation in a wide sense. There were three major themes in the lectures: early notations-symbolisms before the 16th century in the Eastern and Western tradition (Egyptian, Babylonian, Indian, Chinese and Medieval European); the 15th through the 18th century i.e. early modern mathematics; and finally the 19th and 20th century.

Major problems addressed by the conference included why symbolisms were introduced, how they were designed, and how they were used, and also the attitudes towards, and reflections on, mathematics and language that their introduction stirred.

25 scholars participated in this meeting. The workshop stimulated fruitful discussions between scholars of different research fields and will hopefully contribute to shape future research in the domain of history and philosophy of symbolism.

The organizers and participants thank the *Mathematisches Forschungsinstitut Oberwolfach* for providing an inspiring setting for the conference.

In the following, the abstracts are presented in the chronological and thematic order.

Workshop: History and Philosophy of Mathematical Notations and Symbolism

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Abstracts

Algebraic Symbolism and Wittgenstein's Concept of Pictorial Form

Ladislav Kvasz

In the paper the author first introduced some fundamental insights into the structure of language from Witgenstein's *Tractatus*, as:

– The boundaries of my language are the boundaries of my world.
– The subject does not belong to the world.
– A picture cannot depict its pictorial form, it displays it.

Then he tried to relate these insights to the analysis language of mathematics, and especially to algebraic symbolism. He proposes an approach to the study of the development of mathematical language as the evolution of the pictorial form. This approach consists in:

– the description of the emergence of the pictorial form
– the description of the different aspects of the pictorial form
– the description of the change of the pictorial form
– the creation of a possibly complete list of the different pictorial forms
– the identification of the various pictorial forms in the development of particular mathematical disciplines

Great deal of this program has been already realized (synthetic geometry, algebra). At the end of his presentation the author discussed the perspectives of the further progress of the project (towards logic, calculus, probability theory).

Does algebra need a (literal) formalism?

Marco Panza

Does algebra need a (literal) formalism? Generally, the origins of early modern algebra are intended to be connected with the introduction of a literal formalism (mainly thanks to the works of Viete, Harriot, Descartes, etc.). I have tried to question this claim. I have not denied that this formalism plays a crucial role and I have even wondered which is this role. But I have suggested that its introduction supervenes on a more essential phenomenon that has quite old roots: the admission of non positional geometrical inferences and the consequent practice of a specific form of analysis.

Ancient Egyptian notations of mathematical problems: rules and variations

Annette Imhausen

Egyptian mathematics, transmitted to us through collections of concrete problems and procedures for their solution, do no show the use of any mathematical symbols. Each problem is posed using concrete numerical values followed by a rhetoric procedure (using these values in a series of arithmetic operations) for its solution. However, this does not mean there are no formal rules in the notation of a mathematical problem. The limit of extant texts (half a dozen originating from a period of about 200 years) prohibits to make general or sophisticated claims concerning the formal structure of an Egyptian mathematical text. This talk attempted to use the available sources to point out regularities (e.g. the use of red & black ink, characteristic phrases and spatial arrangements) and variations in the available sources. These features may be used to connect Egyptian mathematics with other areas of Egyptian knowledge, e.g. medicine.

Mathematical representation in Babylonian astronomy

Mathieu Ossendrijver

Babylonian cuneiform tablets from the Late-Babylonian period (400 BC–100 AD) contain the first textual evidence of mathematical astronomy in the form of astronomical tables with computed times, positions, and other data for the Moon and the planets, and procedure texts with the corresponding instructions. In this paper I discussed aspects of the representation of numbers, quantities and arithmetical operations in these texts. Among other things I presented results concerning the treatment of additive and subtractive quantities based on a semantic analysis of the procedure texts. Substantial innovations with respect to Old-Babylonian mathematical texts were pointed out.

Working with notations: Extracting Square Roots in Sanskrit mathematical texts (Vth-Xth century)

Agathe Keller

Abstract

The use of the decimal place value notation in the operation to square roots was studied within an astronomical theoretical treatise, *Āryabhaṭīya* (499). Bhāskara's (628) and Sūryadeva's (XIIth century) [4] commentary of it contrasted with the same operation as described in a practical treatise, Śrīdhara's *Paṭigaṇita* (ca. Xth century) and its anonymous and undated commentary [3]. Symbolism and formalism, as these authors could have concieved it, may have less to do with signs used to designate a mathematical entity than with spatial configurations: different places in a table where what was important was the different operations/relations

that linked one place to another. Āryabhaṭa's mathematical puns highlighting square powers of ten (*varga*) will be substituted by commentators with a grid, reading the places of a decimal place value notation as an ordered list of positions on a line (a first, second, third, fourth position). Positions then will be considered as either even (*sama*) or odd (*viṣama*). Such a grid then, is not concerned with the mathematical meaning of the steps carried out. The algorithm is thus, on one level, described formally. Bhāskara's reading however has more to do with writing equivalent but less ambiguious statements than those of Āryabhaṭa. For Sūryadeva such a substitution allows the preliminary marking of places (*chin-*), although he does not detail how the algorithm is carried out. For Śrīdhara and his anonymous commentator, who go into the practical details of the algorithm, the operation develops into a tabular form, where operations are carried above and bellow, results "droping" on a line and "sliding like a snake". All authors recall the mathematical status of the quantity they are dealing with by renaming (*saṁjñā*) it. Such an expression, sometimes qualified as a synecdocque (*upalakṣaṇa*), is also used when commenting on technical definitions of mathematical terms and describing in algebra the use of syncopated notations.

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Does a symbolism require a permanent support of inscription? Reflections based on medieval sources

Karine Chemla

Some Chinese writings dating to the 13th century contain symbolic notations for equations and polynomials. The talk focused on notations for polynomials, algebraic equations and systems of linear equations.

These notations adhere to the specific practice of computation that developed in relation to the computing instrument to which all these writings refer. The instrument, which was used from at the latest the 3rd century BCE onwards, was a surface on which numbers were represented with counting rods. Accordingly, inscriptions on the surface were only temporary.

The talk argued that, long before we find notations in the writings *stricto sensu*, symbolic notations were designed on the surface and they emerged from the execution of algorithms in specific ways. However, these symbolic notations were not considered as such, mainly because they were written in a temporary fashion. Nevertheless, they are of the same nature as the notations encountered later in the writings. This is why I suggest that to develop fully a history of mathematical symbolism, we must gather all the extant evidence to restore how the surface for computing was used.

Moreover, one can analyze the processes of transfer of the notations elaborated on the surface onto paper, within the pages of the books, when the constitution of mathematical writings changed, allowing for non-discursive elements to be combined with discourse in the same texts.

It is unclear when these processes took place. However, in the 13th century, Chinese writings attest to the fact that the use of notations in the texts was by no means unified. This seems to indicate that the transfer of mathematical activity onto paper was not yet fully accomplished.

In India and in the Arabic world, similar instruments were used for computing. Writings in these two other areas attest to the use of a surface, on which numbers were inscribed in a temporary fashion, the material features of the notations allowing them to be moved or their value modified during a computation. Likewise, strict layouts for computations developed, sometimes quite similar to what can be found in China. Moreover, it seems that similar processes of transfer of the mathematical activity onto more permanent supports, like paper, also took place in the same way. The talk aimed at showing some Arabic documents in which symbolic notations similar to those found in Chinese texts began to be inserted.

Stumbling toward symbolism: Abbreviations and margin calculations in Italian abbacus manuscripts

Jens Høyrup

As discovered by Franz Woepcke and amply confirmed since then, Maghreb mathematicians made use of an algebraic notation from the twelfth century onward. It leaves no traces in early Latin writings on algebra (the translations of al-Khwārizmī, the *Liber mahamaleth*, the *Liber abbaci*), nor in the earliest abbacus algebra. However, from the mid-fourteenth century onward, standard abbreviations, schemes for arithmetical operations on polynomials and formal operations on fractions involving algebraic expressions begin to turn up which are likely to have been influenced by the Maghreb technique. As a rule, however, the abbreviations are used unsystematically, not as a standard notation, and the formal operations never go beyond the arithmetic of fractions. Three encyclopedic abbacus books from the 1460s, all written in Florence, all go slightly beyond what we find in fourtenth-century manuscripts, but indirect evidence suggests that most of the innovations we find in them go back to Antonio de Mazzinghi and thus to the late fourteenth century; they are repeated rather out of Humanist piety toward a famous predecessor than because symbolism and formal operations were seen as a promising new step in the development of algebra. Only toward the end of the fifteenth century do we see Pacioli and others try to present the various types of incipient formalism systematically, even they however without doing much more with it. German algebra until 1550 makes more systematic use of the Italian notations than the Italians had done themselves but does not innovate much itself; the same can be said of French algebraic authors like Peletier. The only French writer that surpasses what had been created in the Maghreb in certain respects is Chuquet, who however has no impact on this account.

It is suggested that the explanation for this failure to exploit what was almost at hand is that the mathematical practice within which algebra was applied until the mid-sixteenth century had no urgent use for the development of symbolism, and therefore did not enforce stabilization and integration of the various traces of incipient symbolism.

The Melancholy Mathematics of Albrecht Drer, Or the Dilemmas of Geometrical Representation in the Renaissance

J.B. Shank

Should the archive of the historian of mathematics include the famous masterpiece of European engraving, Albrecht Dürer's *Melencolia* I (1514)? This paper did not so much argue yes as explored the possibility so as to expose the dichotomies between art and geometry, and between visual representation and science as conceptual divisions during the European Renaissance. It further pushed this agenda so as to suggest that these seemingly natural divisions may be contingent and historical, and thus in need of reconsideration. I argued in particular that by reconstituting Albrecht Dürer's work in terms of the historical personae that sustained it, namely the dynamic between his self-conception as a Renaissance Neo-Platonic Humanist and an "artefice," or artisan-maker, we can see Dürer's work as a complex interplay between what we might isolate as art and geometry. I also argued that this interplay was typical of the Renaissance period in Europe, and the art-geometry entanglements in *Melencolia* I are therefore suggestive of the need to rethink the precise historical relationship between the two more generally.

Reading symbols in geometric diagrams: auxiliary constructions as means of proof

Bernardo Mota

Visualization is a part of geometric reasoning because the different elements of a geometric diagram play a unique ontological and epistemological role. One thing is the object that is given; another is the object that is constructed (or the result that is achieved); and another is the auxiliary construction that helps constructing an object or achieving a result. Along history, it was never unanimous what should be considered essential or accidental in a geometric diagram. In my presentation, I analyzed some opinions of philosophers and mathematicians on the function of each part of a diagram in geometric reasoning.

At first, I tried to present a common definition of what a construction is, based on the celebrated words of the philosopher Kant. He claims that geometrical reasoning cannot proceed "analytically according to concepts"; that is, purely logically; instead, it requires a further activity called "construction in pure intuition":

> Suppose a philosopher be given the concept of a triangle and he be left to find out, in his own way, what relation the sum of its angles bears to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, along with the concept of just as many angles. However long he meditates on these concepts, he will never produce anything new. He can analyze and clarify the concept of a straight line or of an angle or of the number three, but he can never arrive at any properties not already contained in these concepts. Now let the geometer take up this question. He at once begins by constructing a triangle. Since he knows that the sum of two right angles is exactly equal to the sum of all the adjacent angles which can be constructed from a single point on a straight line, he prolongs one side of the triangle and obtains two adjacent angles which together equal two right angles. He then divides the external angle by drawing a line parallel to the opposite side of the triangle, and observes that he has thus obtained an external adjacent angle which is equal to an internal angle-and so on (*Critique of the Pure Reason*, B743-745).

This vision of Euclidean geometrical construction is usual. There is construction of an object (in the case a triangle) and then there are auxiliary constructions which are added outside the object and which serve the purpose of proving some property about the object (extended side of the triangle and division of the external angle).

I then moved on to show that this understanding was not unanimous along history, by referring to opinions of a few philosophers and mathematicians. I began by mentioning some mathematical and epistemological objections to the external auxiliary constructions that were probably made before Euclid. One of the objections was that it may happen that there is no space (physical or geometrical) available in which to draw the external auxiliary construction, another was that the auxiliary construction is used to prove results, but we are not really entitled to assume that it actually provides a satisfactory explanation, given the fact that the mathematical result holds, even though no auxiliary construction is in fact drawn.

This made the late commentators of Euclid (Proclus, Heron) present several alternative proofs avoiding use of external auxiliary constructions. These scholars preferred to draw the auxiliary construction inside the figures that were the subject of demonstration. This allows us to distinguish between what they considered an auxiliary construction and the construction used to make geometrical objects. I presented some examples taken from Proclus' commentary on the first book of the *Elements* (propositions 5, 17 and 32), and I concluded that even if the production of one side of a triangle was avoided when possible, it was not really considered an auxiliary construction in some proofs; instead, it was taken as a construction of a second object (the external angle). On the other hand, when a property of the triangle itself was to be proved (for instance, that it has interior angles equal to two right angles), then Proclus made a construction inside the triangle, so that an external and accidental object would not disturb the perfectness of the proof. This means that the only true auxiliary construction in the alternative proofs to Euclid 1.17 and 32 is the line drawn parallel to one of the sides of the triangle, which represents the operation of division (of the external angle).

I then argued that other authors followed this path and considered that there are less auxiliary constructions in geometry than we would suspect because often what seems an auxiliary construction actually represents an autonomous object, which constitutes the subject of geometrical demonstration. In order to provide an example to support this claim, I presented the case of the Jesuit mathematician G. Biancani (17^{th} century). This author upholds that some of the constructions that we usually consider auxiliary (the drawing of circles to construct an equilateral triangle in the first problem of the *Elements*, or the construction of an external angle of the triangle in order to prove that it has the interior angles equal to two right angles in theorem 32 of the *Elements*), are the true subjects of demonstration (they correspond to geometrical objects). In the second place, where auxiliary constructions can in fact be found, they are really not external to the subject of demonstrations but internal (the drawing of a side parallel to one side of the triangle takes place inside the external angle). Finally, these auxiliary constructions represent in act operations that are potentially possible (namely, division), that is to say: the auxiliary constructions are used to perform operations on objects.

To sum up, along history, some authors presented a different view on what an auxiliary construction really is, because these constructions can be used to produce objects or to reproduce mental operations. There were always, sort to speak, different ways to read the constituent parts of geometrical diagrams.

Further reading on the role of geometrical construction in Ancient Geometry:

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Symbolism and diagrams in Descartes' and Pascal's Geometries

Sébastien Maronne

In my contribution based on [1], I studied the respective role of equations and construction in geometrical problem solving according to Descartes and Pascal.

For this purpose, I dealt with prototypical problems like Apollonius' problem of the three circles or Pappus' problem tackled by both mathematicians in their work or in their correspondence. I also paid attention to the correspondence between Pascal and Sluse through Brunetti's intermediacy of October-December 1657 and compared Sluse's conception of algebraic analysis in geometrical problem solving with Descartes' one. Indeed, these letters have not been thoroughly studied until now, whereas they supply an interesting discussion about what is to provide a solution to a geometrical problem.

I argued that Descartes' use of symbolism led to a transformation of the notion of problem in geometry. More precisely, I claimed that:

– The endorsement of a full geometrical interpretability of algebraic inferences in Cartesian geometrical problem solving raises difficulties. In particular, the constructions provided by the algebraic analysis can be very complex.

  That was recognized by Pascal and, later, Newton.

– An outcome of Cartesian geometry is a new conception of the role of equations in geometrical problem solving. It is based on the recognition of fundamental algebraic patterns or shapes, the clue being the method of indeterminate coefficients. This new conception is to be found for instance in Reyneau's treatise (among others) from the early 18th century onward.

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The role of symbolic language in Mengoli's quadratures

Mª Rosa Massa Esteve

This research on the development of symbolic language was framed within the context of more extensive research into the relations between the transformations of mathematics and natural philosophy that took place from the fifteenth century to the end of the seventeenth century.

The publication in 1591 of *In artem analyticen isagoge* by François Viète (1540–1603) constituted an important step forward in the development of a symbolic language. As his work came to prominence at the beginning of the 17^{th} century, other authors, like Pietro Mengoli (1626/7–1686), also began to consider the utility of algebraic procedures for solving all kind of problems [6].

Mengoli's *Geometriae Speciosae Elementa* (Bologna, 1659) is a 472-page text in pure mathematics with six *Elementa* whose title: "Elements of Specious Geometry" already indicates the singular use of symbolic language in this work and particularly in Geometry. He unintentionally created a new field, a "specious geometry" modelled on Viète's "specious algebra" since he worked with "specious" language, that is to say, symbols used to represent not just numbers but also values of any abstract magnitudes.

The arithmetic manipulation of algebraic expressions helped Mengoli to obtain new results and new procedures. For instance, in the *Elementum secundum* he invented a new way of writing and calculating finite summations of powers and products of powers. He did not give them values or wrote them using the sign + and suspension points (...), but rather created an innovative and useful symbolic construction that would allow him to calculate these summations (called *species* by him), which he considered as new algebraic expressions.

In order to compute the value of these summations Mengoli displayed them in a triangular table. Indeed, throughout the book he introduced triangular tables as useful algebraic tools for calculations. In the *Elementum primum*, the terms of the triangular tables are numbers and they are used to obtain the development of any binomial power. In the *Elementum secundum*, the terms are summations and are used to obtain their values. Finally, in the *Elementum sextum* and in the *Circolo*, the terms are geometric figures or forms and are used to obtain the quadratures of these geometric figures.

I affirmed that Mengoli's originality did not stem from the presentation of these tables but rather from his treatment of them. On the one hand, he used the combinatorial triangle and symbolic language to create other tables with algebraic expressions, clearly stating their laws of formation; on the other hand, he employed the relations between these expressions and the binomial coefficients to prove results like for instance the sum of the pth-powers of the first $t-1$ integers [3].

In fact, the formulae for sums of squares, cubes, and higher powers of integers were crucial to the development of integration in the 17^{th} century. Mengoli arrived at these results independently of Fermat [1] and Pascal [9], by using symbolic language to express the summations, a way that allowed him to achieve a certain level of generalisation. Like them, he found a rule in which the value of the sum of the pth powers is obtained. However, in addition to stating the rule, Mengoli also proved it and used it to perform these values expressing all calculations in symbolic language.

Mengoli's idea was that letters could represent not only given numbers or unknown quantities, but variables as well: that is, determinable [but] indeterminate quantities. The summations are indeterminate numbers, but they are determinate when we know the value of t [2]. By assigning different values to t, Mengoli explicitly introduced the concept of "variable", a notion that was quite new at the time. He applied his idea of variable to calculate the "quasi ratios" of these summations. The ratio between summations is also indeterminate, but is determinable by increasing the value of t. From this idea of quasi ratio, he constructed the theory of "quasi proportions" taking the Euclidean theory of proportions as a model, which enabled him to calculate the value of the limits of these summations. This theory constitutes an essential episode in the use of the infinite and would prove to be a very successful tool in the study of Mengoli's quadratures and logarithms [4].

Nevertheless, Mengoli's principal aim was the computation of the quadrature of the circle. Instead of just computing it, Mengoli created a new and fruitful algebraic method which involved the computation of countless quadratures [7]. He began in the *Elementum sextum* of the *Geometria* to compute quadratures between 0 and 1 of mixed-line geometric figures determined by $y = x^n(1-x)^{m-n}$, for natural numbers m and n.

Mengoli defined his own system of co-ordinates and described these geometric figures that he wanted to square as "extended by their ordinates" [5]. He denoted these geometric figures (which he referred to as forms) by means of an algebraic expression written, in Mengoli's notation, as $FO.a^n r^{m-n}.$, "$FO.$" denotes the form, a expresses the abscissa (x) and r the remainder $(1-x)$. He called this expression "Form of all products of n abscissae and $m-n$ remainders". I claimed that his approach here was deeply original. He used these new symbols, which he had associated with geometric figures, for algebraic calculations. Indeed, Mengoli had to ensure that each one of the algebraic expressions in the triangular table, which were new algebraic objects, could be associated with a definite geometric curve. He proved, for a given measure, how to construct the ordinate from the algebraic form corresponding to a curve using the composition of ratios. In this way, he established an isomorphic relation between algebraic objects and geometric figures that allowed him to deal with these geometric figures by means of their algebraic expressions.

He displayed these geometric figures in an infinite triangular table (*Tabula Formosa*), again inspired by the combinatorial triangle. He later explicitly identified these geometric figures with the values of their areas, which were also displayed in another triangular table (now called the harmonic triangle).

Later in the *Circolo* (1672), by interpolation, he computed quadratures between 0 and 1 of mixed-line geometric figures determined by $y = x^{\frac{n}{2}}(1-x)^{\frac{m-n}{2}}$, for natural numbers m and n. Note that in the special case $m = 2$ and $n = 1$, the geometric figure is the semicircle of diameter 1. First, he described these interpolated geometric figures and displayed again in an infinite interpolated triangular table (*Interpolata Tabula Formosa*). Then he obtained an infinite interpolated triangular table of values of their quadratures, which is nothing less than the interpolated harmonic triangle, and by homology he identified the values of both tables [8]. It is noteworthy that in the *Geometria*, there are only three drawings of the geometrical figures whereas in the *Circolo*, he did not include any drawing.

With the help of the properties of a combinatorial triangle, Mengoli was now able to fill the interpolated combinatorial triangle, except for an unknown number "a" which is closely related to the quadrature of the circle ($\frac{1}{2a} = \frac{\pi}{8}$). Mengoli obtained successive approximations of the number "a" in order to approximate the number π up to eleven decimal places [3].

I had to note that Mengoli used "specious" language both as a means of expression and as an analytic tool. Like Hérigone [7], at the beginning of the *Geometria* and on a separate page under the title *Explicationes quarundam notarum*, he explains the basic notation that he is going to use. Note that to represent the powers

Mengoli wrote the exponent in the right side to the letter. I wanted to emphasize that Mengoli's proofs were expressed with symbolic language more clearly in logical statements consisting of few lines, and moreover they were divided into parts such as "Hypothesis", "Praeparatio", and "Demonstratio".

Mengoli dealt with species, forms, triangular tables and quasi ratios using his specious language. I claimed that the triangular tables of quadratures that Mengoli constructed indefinitely as visual structures were true algebraic tools. Through these tables, he classified the geometric figures in three types and studied the properties of each group. When he proved a quadrature result, the proof was independent of the graphical representation of the geometric figure and could be used in all cases where the quasi ratio of the summation of powers was known. It is significant that he also used the symmetry of triangular tables and the regularity of their rows in order to generalise the proofs. Mengoli took it for granted that if a result was true for one row of the table, this result was also true for all rows and there was no need to prove it in the remaining rows.

However, I argued that the most innovative aspect of Mengoli's algebraic procedure was his use of letters to work directly with the algebraic expression of the geometric figure. On the one hand, he expressed a figure by an algebraic expression, in which the ordinate of the curve that determines the figure is related to the abscissa by means of a proportion, thus establishing the Euclidean theory of proportions as a link between algebra and geometry. On the other hand, he showed how algebraic expressions could be used to construct geometrically the ordinate at any given point. This allowed him to study geometric figures via their algebraic expressions.

Since I surmised that a possible reason for the emergence of symbolic notation in the 17^{th} century was the introduction of symbolic language in quadrature problems, I was able to conclude that Mengoli contributed to it because through his symbolic language in triangular tables and interpolated triangular tables he derived known and unknown values for the areas of a large class of geometric figures at once.

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From the "linea directionis" to the "vector"

Patricia Radelet de Grave

The pseudo-Aristotle and Hero of Alexandria realised that some mechanical entities as displacement and motion had to be compound using the parallelogram law and that the direction of the entity played a role. A first technical expression was introduced in 1546 by Tartaglia. He called *linea della direttione* the direction a weight would take if one would drop it. Roberval will generalize the use of the same expression to other forces.

I followed the evolution of the expression of that idea of direction, its terminology and how it is shown on the pictures until the first appearances of mathematical notations. The etymology of the word "vector" was also examined leading us from Kepler's second law to Hamilton's quaternions.

Leibniz's Philosophy of Mathematical Symbolism

David Rabouin

The aim of this paper was to shed some light on what Leibniz called "symbolic knowledge" (*cognitio symbolica*). He gave a precise definition of this type of knowledge in the *Meditationes de cognitione, veritate et ideis*, an article published in 1684 in the *Acta Eruditorum.* In his later writings, Leibniz always refers to this text, the ideas of which were already developed in 1676, just after his Parisian stay. According to Leibniz, symbolic knowledge is "blind" in the sense that it does not necessarily involve reference to an object or a state of facts. Hence the necessity of granting the possibility of the objects involved in symbolical procedures, be it in mathematics or in other types of knowledge. However, as I demonstrated, this situation did not imply that truth could not be accessed through purely symbolical means. This, in fact, was the very ground of Leibniz's original conception of the use of symbolism as a way of discovering new truths (*ars inveniendi*). I traced the origin and the meaning of the arguments supporting this view, particularly emphasising the role given to substitution as a key feature in Leibniz practice of symbolical procedures.

Leibniz's *ars characteristica*

Eberhard Knobloch

Leibniz's paramount interest in these three disciplines caused him to spend his entire life trying to perfect and organize the *ars characteristica*, that is the art of inventing suitable characters, signs; the *ars combinatoria*, that is the art of combining (the signs); the *ars inveniendi*, that is the art of inventing new theorems, new results, new methods. For that reason Cajori called him 'master-builder of mathematical notations'. These arts are strongly correlated with each other. There are two famous examples for the usefulness and success of Leibniz's invention of suitable signs in order to foster the mathematical development:

(1) the differential and integral calculus,
(2) determinant theory.

The lecture explained the role of signs in Leibniz's epistemology and focused on less well-known examples of Leibnizian inventions of mathematical symbolism related to infinite series, differential equations, number theoretical partitions and products of power sums, and elimination theory. To that end, Leibniz especially reintroduced numbers instead of letters thus consciously deviating from Viète's practice.

Mathematical Notations in the Japanese Tradition Wasan and the Acceptance of New Symbolisms

Tatsuhiko Kobayashi

The Japanese mathematics which developed during the Edo period (1603-1867) is called *wasan*. "Wa" means Japan or Japanese, and "san" means arithmetic or mathematics. *Wasan* has a root in ancient Chinese mathematics.

In the beginning of the seventeenth century two mathematical books were transmitted into Japan from China. One is *Saun fa tong zong* and was written by Cheng Dawei in 1592. The other is *Suan xue qi meng* and was written by Zhu Shijie in 1299. Of all others *Suan xue qi meng* played an important role in the development of pre-modern Japanese mathematics. Japanese mathematicians were able to acquire the knowledge of *Tian yuan yi* or an unknown x by studying this mathematical book which had not been studied in China. Takakazu Seki (?-1708) studied *Suan xue qi meng* in his young age and understood completely mathematical meaning of *Tian yuan yi* as an unknown. We may say that *Wasan* surpassed traditional Chinese mathematics in the latter half of the seventeenth century. This seems to have been caused by the introduction of a new effective mathematical symbolism. The method of this new algebra was originally called *Bôsho-hô*, and its procedure was named *Endan* or *Endan-jutsu*.

On the other hand, Western mathematical and astronomical books came to be imported into pre-modern Japan, because of the enforcement of the relaxation of the banned book policy in 1720 by the eighth Shôgun Yoshimune Tokugawa (1684-1751). The purpose that he carried out was to abolish an obsolete calendar, and to make newly a precise calendar. The Western scientific books which were imported into Japan were read by Japanese calendrical calculators and Dutch learning scholars and they made efforts to master up-to-date Western astronomy and to assimilate European mathematical terminology or notations. We can say that the acceptance of Western mathematical terminology and notation was led by calendrical calculators and Dutch learning scholars. In this sense they were front runners who opened up a new course to introduction of Western symbolisms in Japan. In this meeting, the author mainly discussed the following three points:

(1) Mathematics by Japanese and Classical Chinese
(2) Notation in Japanese traditional mathematics: *Wasan*
(3) The acceptance of Western symbolisms

The Acceptation of Western Mathematical Notation in China

Tian Miao

At the beginning of the 17^{th} century, European mathematics began to be transmitted to China. A survey on the translation and acceptation of European mathematical symbols can provide us a precious clue about the procession and characteristics of the early stage of modernization and internationalization mathematics in China. In this article, I presented my research in this topic in three aspects:

(1) the transmission of geometrical notations to China
(2) the transmission of notations of calculation and functions to China
(3) the transmission of hindu-arabic numerals to China

I focused on the way of translation of European mathematical notations, and the attitude of Chinese mathematicians toward these notations. In the conclusion, I stressed on two facts. First, Chinese mathematicians did not resist the transmission of new mathematical knowledge and the notations from the Europe. Secondly, there are really controversies concerning with the acceptation of European mathematical notations existed, and these usually happened when there was conflict between the tradition terms and the new introduced ones.

Speakable Symbols: Innovating Late Qing Mathematical Discourse

Andrea Bréard

The first Jesuits and sinologists have long defended (and some still do) the idea of Chinese being a symbolic, ideographic or universal language, i.e. a written language disconnected from speech. For mathematics, this would imply, that traditional algorithms can easily be translated into modern algebraic symbolism. But the introduction of symbolic algebra during the Qing dynasty (1644–1911) was not a straightforward process. I looked at the case of the transmission of algebra through translations and the discourses accompanying the introduction of Western' techniques against the background of Chinese Yuan dynasty algebraic techniques. Two prominent translators and mathematicians, Li Shanlan (1811–1882) and Hua Hengfang (1833–1902), in particular, maintained two different kinds of mathematical discourse in their commentatorial and translatory practices. Was this compartimentalization a purely conceptual or also a linguistic and political choice? Earlier results in this field have been published in [1] and [2].

I tried to make two statements in this paper, the first one was

– that the criteria in choosing a syncretistic symbolism for transcribing or translating formulas from Western mathematical writing were linguistic criteria: the retained formalism showed close links to natural language, where mathematical symbols were pronouncable and inserted into discursive text as part of a normal rhetoric and syntaxically correct phrase. The newly created symbols were thus not much different from a Chinese character word: non-phonetic and ideographic in nature they were nevertheless pronouncable, speakable signs. Strictly speaking in Saussurian or Peircean terms, one cannot speak of all of them as "symbols", since some are signs whose relation to its conceptual object referred to is not entirely arbitrary, but motivated by association with a character denoting a mathematical concept or the idea of an operation.
– My second statement was a historical one: Li Shanlan's attempt to synthesize Western and Chinese mathematics was successful for about half a century, and other translators followed his new conventions for writing algebraic formulae. This came to a sudden end during the educational and political reforms of the late Guangxu reign, when at the turn of the 20^{th}-century Chinese students were massively sent abroad, mainly to Japan, for their studies. They were the new generation of translators of scientific books, and became increasingly alienated with Chinas traditional past. The Education Departments were eager for textbooks that could serve in the newly established nationwide school system after 1904, replacing the traditional state examinations. Adopting a foreign symbolic system then clearly was a political choice and went well with the rejection of the old literary style.

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Symbols in Changes: The Transmission of the Calculus into Nineteenth-Century Japan

Lee Chia-Hua

This research project examined the process of introducing the calculus into nineteenth-century Japan, through the lens of changing symbols. The dramatic change of mathematical notations appearing in written and printed text materials has rarely been the main focus of research on the transmission of mathematical knowledge. However, this change sheds new light on the transmission of mathematics, to explore the facts and factors that caused symbols to change and more beyond changing symbols. The investigation into these changes is discussed, including symbols and terminologies, the content and form of text materials, the use of imported books, the academic backgrounds of involved scholars and institutions, the participation of intellectuals, and governmental policies.

The differential and integral calculus was introduced into Japan during the late Tokugawa and early Meiji periods in the mid-nineteenth century. It was particularly through the circulation of books that a triangular relationship between the West, China and Japan developed for the transmission process. During the nascent stage, calculus books written in Dutch, Chinese, and English were brought to Japan by Dutch traders and naval officers, Japanese samurais, and Western missionaries. A number of these imported books were partially or completely transcribed, re-edited, translated, rearranged, and compiled for producing Japanese textbooks. These reference books were particularly valuable for Japanese scholars who had not studied to abroad or been taught by Western teachers. Gradually, these works became the basis of and practical resources for newer editions of Japanese textbooks.

Among the imported reference books, *Dai Wei Ji Shi Ji*- an 1859 Chinese translation of the *Elements of analytical geometry and of the differential and integral calculus* by Elias Loomis (1811-1889)- was the first well-known and most used reference book when the calculus was introduced in Japan. This Chinese translation introduced to Japanese scholars not only the new subject of Western mathematics, but also the newlycoined terminologies in Chinese and the new notations transformed into Chinese symbolism. This investigation into the early text materials on the calculus found that, unlike the use of translated terminologies, the application of Chinese symbolism was eventually abandoned, after Japanese scholars were capable of directly consulting Western sources and soon encountered confusion in the use of Chinese or European notations. The process of changing from the exclusive use of Chinese notations, to the partial use of both Chinese and European notations, and to the eventual use of only European notations exemplified the changing attitudes of Japanese scholars towards the use of original Western sources for the creation of Japanese translations and textbooks on the calculus.

Although *Dai Wei Ji Shi Ji* was the most consulted reference at the time- based on the existing number of materials as transcriptions, re-editions, and translations of it, it is unclear to what extent Japanese scholars studied the calculus through this book. Using chapters on the differential calculus, for example, to compare the mistakes and mistranslations in the Chinese version with the corrections in related Japanese texts, there is not enough evidence to show that the concept of "limit," and the method of "rates" which Loomis used to explain the definition of the *differential* were clear to Japanese scholars. Later Japanese calculus texts that were translated, rearranged and compiled directly from other English and Dutch books also provide few clues to this question on the differential calculus. The possible reasons are that the explanation of these concepts had been changed in their source books and the few answers were given to the practice exercises by Japanese scholars in their texts.

It is worth noting among these later Japanese texts that the traditional Japanese mathematics was blended into the examples and exercise questions on the integral calculus by using European notations. One example is the first Japanese calculus textbook *Hissan Biseki Nyumon* (The Introduction of the Differential and the Integral Calculus), compiled from American and English textbooks by Fukuda Jiken (1849-1888), an army officer and scholar trained in both traditional Japanese mathematics and Western mathematics. The same application of European symbolism can also be found in the early issues of the *Tokyo Sugaku Kaisha Zasshi* (The Journal of the Tokyo Mathematical Society), when the calculus was introduced by traditional Japanese mathematicians. These examples led me to formulate the following hypotheses: 1) Scholars who had been trained in traditional Japanese mathematics might have found the integral calculus easier to learn and apply, and 2) the application of traditional Japanese mathematics to the examples and exercise questions on the integral calculus using Western notations might have been a way for scholars to keep promoting the traditional mathematics, and prevented it from being abandoned in the main trend of teaching Western mathematics at schools, which was the new education policy *Gakusei*, proclaimed by the Meiji government in 1872.

In the process of producing Japanese translations and textbooks on the calculus, intellectuals who were not sent abroad for education were the most active participants during the initial stage. Their academic backgrounds included training in Dutch and Chinese studies, and traditional Japanese and Western mathematics. By profession, they were officers of the Meiji government and mathematics instructors at governmental institutions, military schools, the Imperial University of Tokyo, and private schools. The majority of these early participants were unfamiliar with the calculus and Western languages, but they still enthusiastically pioneered the new subject of Western mathematics with the assistance of Chinese sources. They even challenged the practice of using different symbolisms, advocating instead using only Western notations and sources together with the rearrangement of the contents on the integral calculus. These changes reflect the great efforts and struggles of the early participants and the major response to the new educational system. Despite of their efforts, however, they could not prevent their eventual loss of influence in academic society as later participants who studied abroad took over.

As this study has shown, the transmission of the differential and integral calculus to Japan in the nineteenth century took place through the circulation of books in the early phase. The making of Japanese calculus texts initially consulted foreign sources in the following order: Chinese, English and finally Dutch. In later Japanese texts on this new subject, the Chinese sources provided the convenience of translated terminologies, but the Chinese symbolism became an obstacle to the use of European sources. Scholars who had an academic background in traditional Japanese mathematics found a way to blend the traditional mathematics with the Western symbols into the examples and exercise questions in their works on the integral calculus. These changes, beyond changing symbols, reflect the efforts of scholars who responded positively to new government policies. The changes also represent their struggles in facing the keen competition for influence in the new government and society from scholars who studied abroad and who were treated as important leaders, in the milieu of the emergence and promotion of Western mathematics.

Symbolic codings of topological objects: difficulties of an epistemic technique

Moritz Epple

(Mis-) interpreting Leibniz' remarks on the possibility of a new 'Analysis Situs', mathematicians of the 19th century hoped to be able to make an inroad into what C.F. Gauss termed 'Geometria situs' and his student and colleague J.B. Listing called "Topologie". Although both – and later mathematicians – found the field quite difficult, the initial hope was to be able to represent 'topological objects' (at the time: configurations of points, curves or surfaces in the plane or in 'space') by means of suitable symbolic techniques and to create a symbolism which could be manipulated in order to solve topological problems. The model for this hope clearly was the success of algebra and calculus in early modern geometry. The talk discussed examples of the early attempts at a 'symbolical algebra' of topological objects, including Gauss' discussion of a braid (first published in [1]), his problem of "Tractfiguren" and Listing's as well as P.G. Tait's attempts at representing knots and links by means of suitable "symbols". In all cases it turns out that the symbolic representation of a topological object is only possible after a prior stage of "representing" the same object by other means – such as drawings or verbal descriptions of suitable imaginations. Both these prior representations and their symbolic codings created many new problems (e.g. how do knot diagrams relate to spatial knots, etc.).

The talk concluded by a brief discussion of the early 20th century techniques of representing 3-manifolds as "polyhedra" in Poincaré's sense, as "Riemann spaces" (branched coverings of S^3) and by means of Heegaard diagrams. Even in these techniques one can recognize the same two levels of representation as in the 19th century examples.

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Notations, proof practices and the circulation of mathematical objects. The example of the group concept between 1830 and 1860.

Caroline Ehrhardt

In this talk, I examined a mathematical object, the group, in the works of three mathematicians who used it between 1830 and 1860. The first is Evariste Galois, the second one is Arthur Cayley and the third one is Richard Dedekind.

In my PhD, I studied how the way they dealt with groups was linked to the specific epistemological culture they were working in. In this talk, I took another point of view, focusing on *how* these mathematicians wrote the groups, on what it implied on what they were *doing* with it, and on what the group notion actually *meant* for them. Actually, this was an attempt to put to the test, in the field of mathematics, the conclusions of the anthropologist Jack Goody about lists and tables, as well as the ideas developed by the historian of books D. F. MacKenzie about the relations between the form of a text and the sense that readers give to it. In particular, mathematical notations could be seen as "intellectual technologies" in the sense defined by Goody. On the one hand, they depend on specific mathematical cultures and, as so, their diffusion is socially determined. But on the other hand, mathematical notations can change the very nature of the objects, because they change the way mathematicians can use them and think about them.

I hope that this historical example illustrated how different kind of mathematical notations, such as letters, lists or tables, allows mathematicians to practice different operations, to ask different questions or to solve different kind of problems, and, finally, to make mathematical proofs in very different ways. But I also hope to have emphasized the fact that the different notations that can be given to a mathematical object, as well as the meanings they are supposed to express, are linked to a particular time, person or place. What we call today "the group concept" is the result a historical process of readings and transmission of papers such as the ones of Galois, Cayley and Dedekind. And, in this process, symbolism should be seen as a way to convey a message about the mathematical object.

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