Adaptive backstepping control of a class of hysteretic systems

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ABSTRACT

A backstepping-based adaptive control is designed for a class of one degree of freedom hysteretic system. The true hysteretic behavior does not need to be known for the controller design. A polynomial description is assumed with uncertain coefficients and an uncertain residual function. These uncertainties are bounded and lump the discrepancies between the adopted description and the real hysteretic behavior. The adaptive controller is able to handle these uncertainties and make the closed loop globally uniformly ultimately bounded when the system is subject to an unknown excitation from which a bound is known. The efficiency of the approach is tested by numerical simulations on a hysteretic system under a seismic excitation. This system is mathematically described by the differential Bouc-Wen model, which is widely used in structural dynamics.

Keywords: Hysteretic systems, structural control, base isolators, adaptive control, backstepping

1. INTRODUCTION

Hysteresis is one of the classes of nonlinearities that have attracted a lot of interest in recent years within the context of analysis and control of dynamic systems. Hysteresis is encountered in a wide variety of processes in which the input-output dynamic relations between variables involve memory effects. Examples are found in biology, optics, electronics, ferroelectricity, magnetism, mechanics, structures, among other areas. This paper is primarily concerned with hysteresis in mechanical and structural systems. In these systems, hysteresis appears as a natural mechanism of materials to supply restoring forces against movements and dissipate energy.[1,2] This mechanism has been exploited in recent years in the design of hysteretic damping devices and vibration isolation schemes as those encountered in seismic base isolated buildings.[3–5] Another recent source of interest for hysteresis in mechanical and structural systems comes from the new "smart" materials and devices used for vibration control. Materials such as shape memory alloys[6] and electro/magnetorheological fluids[7,8] have been proposed for this purpose, which exhibit complicated hysteretic behaviors.

While there is an extensive literature about physical characterization and mathematical modelling of hysteretic systems in different areas, only a few references are found reporting feedback controllers in the general literature on control systems.[9–12] In structural systems, feedback controllers in the presence of hysteretic components have been primarily encountered when dealing with smart actuators and base isolation schemes. A passivity-based controller has been recently proposed for a class of shape-memory alloys actuators.[13] In the case of base isolated buildings, feedback control problem arises when the hysteretic isolators are coupled with active controllers acting on the base within a hybrid scheme. One way of addressing this problem has been to consider a linear model of the structure and the isolator, which is augmented with a nonlinear term describing the hysteretic restoring forces. Stochastic optimal control[14] and covariance control[15] have been proposed based on a stochastic linearization of the hysteresis nonlinearity. Another works have presented hybrid schemes in which stabilizing nonlinear controllers have been derived considering that the hysteresis nonlinearity can be treated as an unknown uncertain function under linear bounds.[16,17]

In a related vein, this paper considers a one degree of freedom system described by a model with a linear part and a nonlinear hysteretic restoring force. This force has not to be known for the controller design, but it is

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assumed to be modelled by a polynomial function plus a residual function. The residual and the coefficients of the polynomial function do not need to be known. It is assumed that they are bounded by known constants. To cope with the design of a controller under this class of uncertain nonlinear hysteretic component, a backstepping-based adaptive control is derived. Backstepping refers to a recent approach for design of stabilizing controllers for nonlinear systems.[18] In this paper, a switching σ-modification[19] and a new term that incorporates part of the information on the uncertainty is used with the objective of driving the output to a neighborhood of zero, thus reducing the vibrations induced by external excitations. The resulting closed loop is globally uniformly ultimately bounded[20] and the output can be made asymptotically arbitrarily small by choosing appropriate design parameters.

The efficiency of the controller is numerically tested on a system whose unknown hysteresis is mathematically represented by the Bouc-Wen model.[21] This model lies within the group of differential hysteretic models and has been widely used in structural dynamics, particularly to describe rubber bearing base isolation schemes.[4]

2. PROBLEM STATEMENT

We are interested in controlling the one degree of freedom system:

$$m\ddot{x} + c\dot{x} + \Phi(x,t) = f(t) + f_c(t), \tag{1}$$

where x is the displacement, m and c are the mass and the viscous damping coefficient, respectively, $f(t)$ represents the external exciting force and $f_c(t)$ is an active control force to be designed. The function $\Phi(x,t)$ describes a non-linear hysteretic uncertain restoring force. We assume that this function has the following structure:

$$\Phi(x,t) = \Psi(x) + R(x,t); \qquad \Psi(x) := \phi_0 + \phi_1 \frac{x}{a} + \phi_2 \left(\frac{x}{a}\right)^2 + \ldots + \phi_n \left(\frac{x}{a}\right)^n, \tag{2}$$

where $\Psi(x)$ is a n degree polynomial and $R(x,t)$ is a residual function. In the polynomial, a is a known constant with dimension of a displacement. It is introduced so that all the coefficients ϕ_i have the same dimension (of a force). This representation of the hysteretic force lies within the class of so-called nonparametric models.[22–24] In general, these models attempt to approximate unknown hysteretic behaviors by functional expansions with appropriate coefficients, which are usually chosen through identification experiments. In (2), a simple polynomial plus a residual function is adopted as a description of the hysteresis. Its purpose is not to give a precise model of the hysteresis, but to be used as a work model for the controller design. In fact, the coefficients ϕ_i and the function $R(x,t)$ do not need to be known exactly for the controller design. The following assumptions complete the description of system (1)–(2):

Assumption 1. The mass m is known. The damping coefficient c is unknown but lies within an interval $[0, c_{\max}]$, where $c_{\max}$ is known.

Assumption 2. The constant vector $\theta_\phi = (\phi_0, \phi_1, \phi_2, \ldots, \phi_n)^T$ is unknown but lies within a known sphere. That is $\|\theta_\phi\| \leq M_\phi$ for a known positive constant M_ϕ.

Assumption 3. The function $R(x,t)$ is unknown but bounded in the form $|R(x,t)| \leq m\bar{R}$ for all x and $t \geq 0$, where $\bar{R}$ is a known positive constant.

Assumption 4. The exciting force $f(t)$ is unknown but bounded in the form $|f(t)| \leq mF$ for all $t \geq 0$, where F is a known positive constant.

Our objective is to design a backstepping based adaptive feedback control law for the system (1)–(2) such that all the signals of the closed loop are bounded and such that the displacement x can be made as small as desired by an appropriate choice of the design parameters.

3. CONTROLLER DESIGN

First, by using the state variables $(x_1, x_2) := (x, \dot{x})$, the system (1)–(2) is transformed into the form

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\dfrac{cv}{m}\dfrac{x_2}{v} - \dfrac{\Phi(x_1,t)}{m} + \dfrac{f(t)}{m} + \dfrac{f_c(t)}{m} = \theta^T \varphi(x_1, x_2) - \dfrac{R(x_1,t)}{m} + \dfrac{f(t)}{m} + u(t), \end{cases} \tag{3}$$

where

$$\theta = \frac{1}{m}(cv, \phi_0, \phi_1, \phi_2, \ldots, \phi_n)^T \qquad \text{and} \qquad \varphi(x_1, x_2) = -\left(\frac{x_2}{v}, 1, \frac{x_1}{a}, \frac{x_1^2}{a^2}, \ldots, \frac{x_1^n}{a^n}\right)^T, \tag{4}$$

and v is a known constant which has the dimension of a velocity. It is introduced to have dimensionless variables in φ and coefficients of the same dimension in θ. We denote $u(t) = f_c(t)/m$ as the control signal.

From Assumptions 1 and 2, it follows that

$$\|\theta\| \leq \frac{1}{m}\sqrt{(c_{\max} v)^2 + M_\phi^2} := M_\theta. \tag{5}$$

Let us introduce the following variables:

$$\begin{aligned} z_1 &= x_1 - y_r, \\ z_2 &= x_2 - \alpha_1, \\ \alpha_1 &= -c_1 z_1, \end{aligned} \tag{6}$$

where y_r is the target, which is constant.

The following adaptive control law is proposed:

$$\left\{ \begin{array}{rcl} u(t) &=& -\varphi(x_1, x_2)^T \hat{\theta} - z_1 - c_2 z_2 - c_1 x_2 - \mathrm{sg}(z_2)\,\mathrm{cf}(|r z_2|) r \\ \dot{\hat{\theta}} &=& \Gamma \varphi(x_1, x_2, t) z_2 - \Gamma \sigma_\theta(\|\hat{\theta}\|)\hat{\theta} \\ \hat{\theta}(0) &=& \hat{\theta}_0. \end{array} \right. \tag{7}$$

In these expressions, c_1 and c_2 are positive design parameters and $r = \bar{R} + F$ is a bound on the uncertainty in the model (3). It is obtained by adding the bound on the residual term $R(x_1, t)$ and the bound on the excitation force $f(t)$ (see Assumptions 3 and 4). Γ is a positive definite design matrix and

$$\begin{aligned} \mathrm{cf}(y) &= \left\{ \begin{array}{ll} 0 & y < \varepsilon_1, \\ \frac{1}{\varepsilon_1} y - 1 & y \in [\varepsilon_1, 2\varepsilon_1], \\ 1 & y > 2\varepsilon_1, \end{array} \right. \\ \mathrm{sg}(y) &= \left\{ \begin{array}{ll} -1 & y < -\frac{1}{1+r}\varepsilon_2, \\ \frac{1+r}{\varepsilon_2} y & y \in \left[-\frac{1}{1+r}\varepsilon_2, \frac{1}{1+r}\varepsilon_2\right], \\ 1 & y > \frac{1}{1+r}\varepsilon_2, \end{array} \right. \\ \sigma_\theta\left(\|\hat{\theta}\|\right) &= \left\{ \begin{array}{ll} 0 & \|\hat{\theta}\| \leq M_\theta, \\ \frac{\bar{\sigma}_\theta}{M_\theta}\|\hat{\theta}\| - \bar{\sigma}_\theta & \|\hat{\theta}\| \in [M_\theta, 2M_\theta] \\ \bar{\sigma}_\theta & \|\hat{\theta}\| \geq 2M_\theta. \end{array} \right. \end{aligned} \tag{8}$$

where ε_1, ε_2 and $\bar{\sigma}_\theta$ are positive design parameters and M_θ is the parameter bound defined in (5).

The following result (proved in a more extended work[25]), assures that system (3) can be stabilized using the control scheme presented above, and that its performance can be improved by an accurate selection of the design parameters:

Theorem 1. The orbits of the closed loop composed of the system (3) and the controller defined in (7)–(8) are globally uniformly ultimately bounded. Furthermore, the asymptotic performance is given by:

$$\lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{t_0+T} z_1^2(t)\,dt \leq \frac{4\varepsilon_1 + 2\varepsilon_2}{c_1} \qquad \text{for any} \quad t_0 \geq 0. \tag{9}$$

This means that by decreasing the design parameters ε_1 and ε_2 we can obtain an asymptotic performance as good as desired. The same result may be obtained by increasing the gain c_1. Simulations show that the price to be paid is an increase of the control amplitude. That is, a trade-off between good performance and reasonable control amplitude should be made.

4. APPLICATION: CONTROL OF A HYSTERETIC OSCILLATOR

The purpose of this section is to illustrate the design and the application of the backstepping control law presented in the previous section to a one degree of freedom hysteretic system like the one in (1), which is excited by an external seismic force $f(t) = -ma(t)$, where $a(t)$ is the ground acceleration. Since a real system is not available for a true implementation, a mathematical model is adopted as the "true" system. Specifically, we use the well-known Bouc–Wen model,[4, 21, 26] which represents the nonlinear restoring force ϕ in the form

$$\begin{aligned} \Phi(x,t) &= \alpha k x(t) + (1-\alpha)kDz(t), \\ \dot{z} &= D^{-1}\left[A\dot{x} - \beta|\dot{x}|\,|z|^{n-1}z - \gamma\dot{x}|z|^n\right]. \end{aligned} \tag{10}$$

This model represents the restoring force $\phi(x,t)$ by the superposition of an elastic component αkx and a hysteretic component $(1-\alpha)kDz$, in which $D > 0$ is the yield constant displacement and $\alpha \in [0,1]$ is the post to pre-yielding stiffness ratio. The hysteretic part involves a nondimensional auxiliary variable z which is the solution of the nonlinear first order differential equation in (10). In this equation, A, β and γ are nondimensional parameters which control the shape and the size of the hysteresis loop, while n is an integer that governs the smoothness of the transition from elastic to plastic response. The Bouc-Wen model belongs to the class of differential hysteretic models and has been widely used in structural dynamics, particularly to describe rubber bearing base isolation schemes.[4]

In order to design the backstepping controller, we need first to find a representation of the "true" system in the polynomial form assumed in (2). This is described in the next subsection. Numerical simulations assessing the efficiency of the control law will be further presented.

4.1. Polynomial model

Let us consider the system (1) with the hysteretic function Φ characterized by the Bouc-Wen model (10). The purpose now is to perform an off-line identification to get an approximation of the function Φ in the form (2). To do this, we perform an open-loop numerical experiment in which a slowly varying periodic signal $f(t)$ with amplitude mF is applied to the open-loop system:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = m^{-1}\left[-cx_2 - \alpha k x_1 - (1-\alpha)kDz + f(t)\right], \\ \dot{z} = D^{-1}\left[Ax_2 - \beta|x_2|\,|z|^{n-1}z - \gamma x_2|z|^n\right]. \end{cases} \tag{11}$$

For a number of time instants t_i $(i = 1, \ldots, N)$, the values of the displacement and the restoring function are obtained:

$$x_1(t_i), \qquad \Phi(x_1(t_i), t_i) = \alpha k x_1(t_i) + (1-\alpha)kDz_i(t). \tag{12}$$

Then, a n-order least-square regression polynomial

$$\Psi(x_1) = \phi_0 + \phi_1 \frac{x_1}{a} + \phi_2 \left(\frac{x_1}{a}\right)^2 + \ldots + \phi_n \left(\frac{x_1}{a}\right)^n \tag{13}$$

is sought.

Let e_i denote the errors between the fitted values an the "true" values:

$$e_i = \left|\Phi(x_1(t_i), t_i) - \Psi(x_1(t_i))\right|. \tag{14}$$

If the residual error $R(x_1,t) = \Phi(x_1(t),t) - \Psi(x_1(t))$ of the least-square approximation is a normally distributed random variable with mean $\mu_R = 0$ and variance σ_R^2, then a good estimation for σ_R^2 is given by

$$\sigma_R^2 = \frac{1}{N-(n+1)} \sum_{i=1}^{N} e_i^2, \tag{15}$$

(see[27] - Chapter 13 - for more details). In such a case, $3\sigma_R$ would be a good estimation for a bound $\bar{R}$ of $|R(x_1,t)|$. Since, in general, this will not be the case, we adopt the following estimation of $\bar{R}$:

$$\bar{R} := \max(e_{\max}, 3\sigma_R), \tag{16}$$

where $e_{\max} = \max_{i=1,\ldots,N} e_i$.

4.2. Simulation results

We have considered the system (1) and the Bouc-Wen model (10) with the following parameter values: mass $m = 156 \times 10^3$ [Kg], stiffness $k = 6 \times 10^6$ [N/m], damping $c = 2 \times 10^4$ [Ns/m], $\alpha = 0.6$, $D = 0.6$ [m], $A = 1$, $\beta = 0.5$, $\gamma = 0.5$ and $n = 3$.

The purpose is to check the control efficiency against the action of an earthquake whose acceleration is roughly bounded by 1.2 [m/s^2]. Thus, for the polynomial identification described in Section 4.1, we perform an open-loop simulation introducing a slow-varying excitation given by $f(t) = 1.5m\cos(0.2t)$, which is a signal with a little more amplitude than the one expected to be controlled. We take $z(0) = 0$ and a simulation time of 100 seconds, with 8226 discrete time instants for the identification. The values of the constants a and v are determined from the open loop identification as the maximum output (displacement and velocity respectively) for the Thaft earthquake input. Thus we take $a = 0.03$ [m] and $v = 0.2$ [m/s]. The following third order least-squares regression polynomial is obtained:

$$\Psi(x_1) = \left(-0.02 + 1.14\frac{x_1}{a} - 0.001\left(\frac{x_1}{a}\right)^2 - 0.024\left(\frac{x_1}{a}\right)^3\right)m, \tag{17}$$

with a maximum error $e_{\max} = 0.4365m$ and a variance error $\sigma_R = 0.1446m$, so that $\bar{R} = 0.4365m$.

Figure 1 displays the results of the hysteresis identification, by comparing the hysteretic behaviour of the Bouc-Wen model ("true" system) and the identified polynomial.

For the control law implementation, we need first to set the values of the uncertainty bounds r and M_θ in (7) and (8). Assuming the excitation is bounded in the form $|f(t)| \le mF = 1.2m$, we have $r = \bar{R} + F = 1.6365m$. We consider $c^{(\text{nom})} = 2 \times 10^4$ and the vector $\theta_\phi^{(\text{nom})} = (-0.02,\, 1.14,\, -0.001,\, -0.024)^T m$ with the parameters identified in (17) as nominal model values. Compute $\|\theta_\phi^{(\text{nom})}\| = 1.14m$. Then, we assume that the real values c and θ_ϕ have the following bounds

$$0 \le c \le c_{\max} = 2c^{(\text{nom})} = 4 \times 10^4; \qquad \|\theta_\phi\| \le M_\phi = 2\|\theta_\phi^{(\text{nom})}\| = 2.28m.$$

With these values we obtain

$$M_\theta = \frac{1}{m}\sqrt{(c_{\max}v)^2 + M_\phi^2} = 2.28\,[m/s^2].$$

Finally, the following design parameters are judiciously chosen: $c_1 = 5$, $c_2 = 5$, $\varepsilon_1 = 0.01$, $\varepsilon_2 = 0.1$, $\bar{\sigma}_\theta = 1$ and $\Gamma = 5 \times 5$ identity matrix. For the parameter adaptive law in (7), the following initial parameter vector $\hat{\theta}_0$ has been chosen:

$$\hat{\theta}_0 = \frac{1}{m}\left(c_{\max}v, \theta_\phi^{(\text{nom})T}\right)^T.$$

Starting at time 0 with these values, the adaptive law in (7) updates on-line new values of these parameters.

The system is subjected to the Taft earthquake excitation, whose acceleration is plotted in Figure 2.

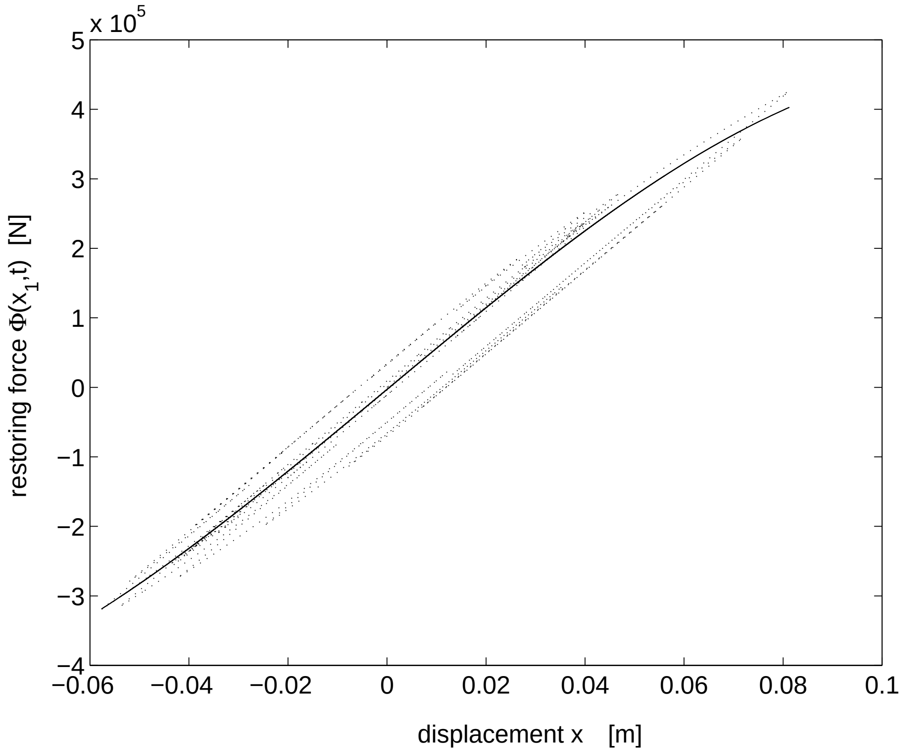

Figure 1. Hysteresis identification: hysteretic cycles (dashed line) and regression polynomial (solid line).

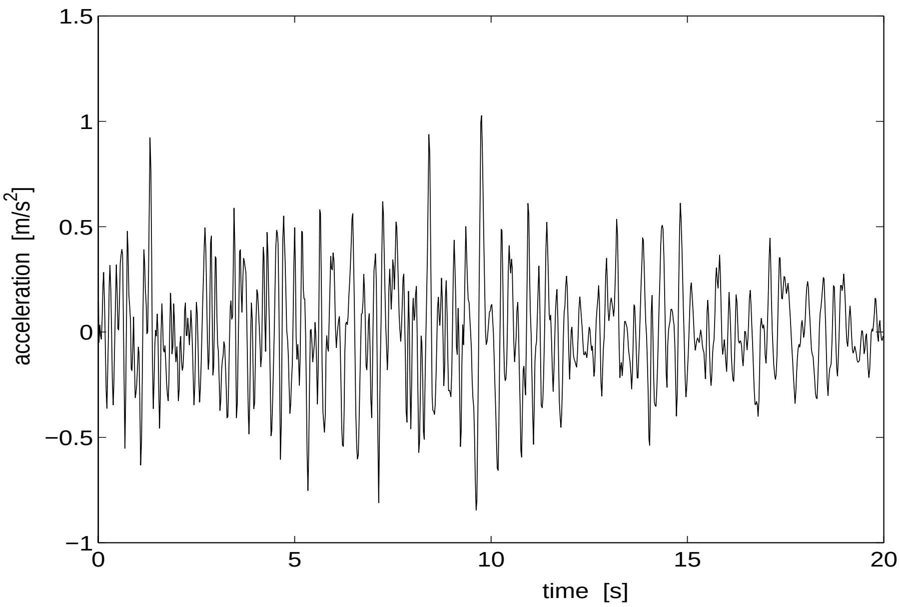

Figure 2. Taft earthquake acceleration signal.

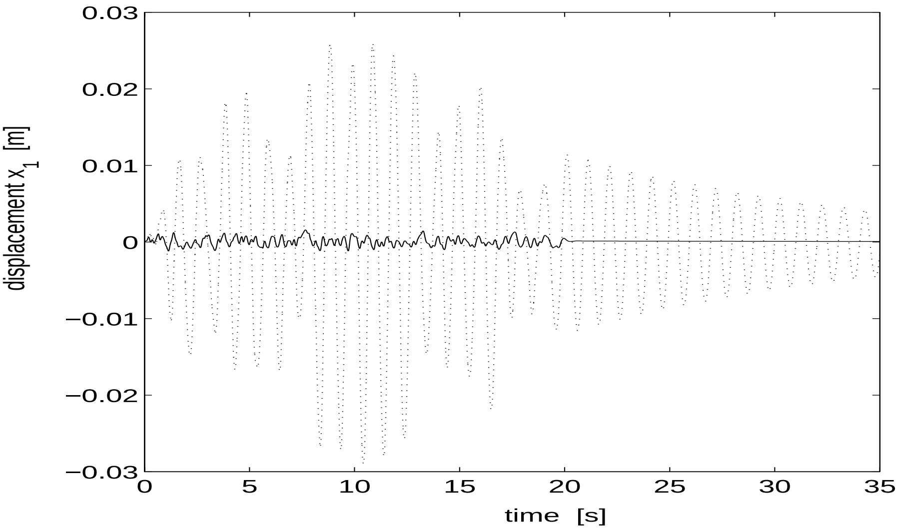

Figure 3. Displacement time history in open loop (dashed line) and closed loop (solid line).

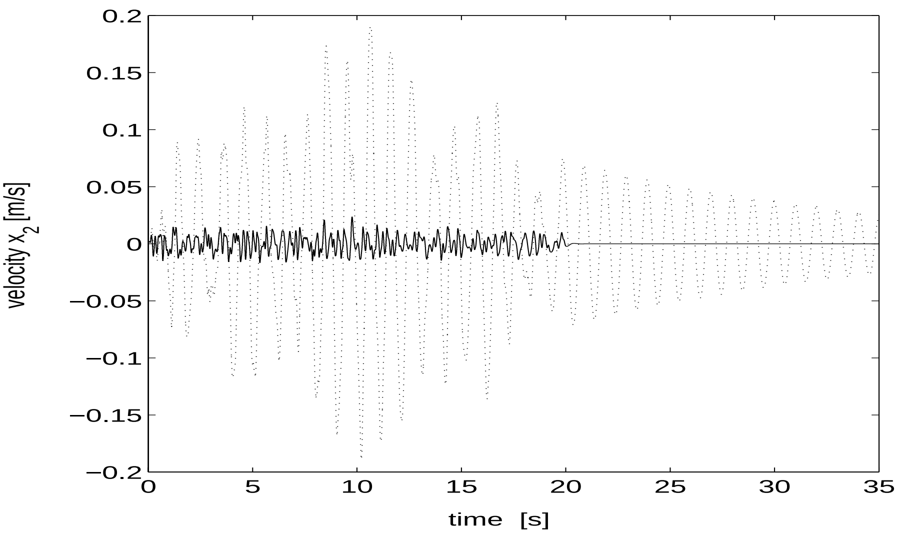

Figure 4. Velocity time history in open loop (dashed line) and closed loop (solid line).

Since the control objective is to mitigate the seismic displacement response of the system, the target for x_1 is set to $y_r = 0$.

The closed-loop behaviour is shown in Figures 3-6. Figures 3 and 4 show the time histories of the displacement and the velocity both in the case without control and with active control. A significant reduction can be observed due to the control. After $t = 20$ seconds, the excitation stops and the uncontrolled case corresponds to free vibration response. The open loop system exhibits a low damping damping behaviour. On the contrary, the control drives fast the response towards zero, thus introducing a significant damping effect into the system. These features are also observed in Figure 5, which depicts the solution of the open loop system (dashed line) and the closed loop system (solid line, in the center) in the phase space, including the auxiliary variable z that describes the hysteretic behaviour.

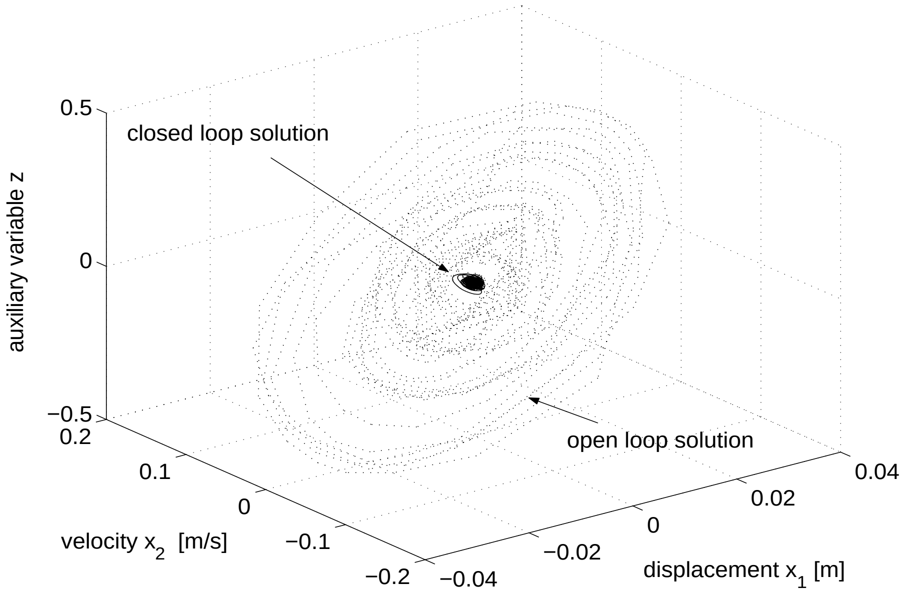

Figure 5. Phase portrait of the open and closed loop system.

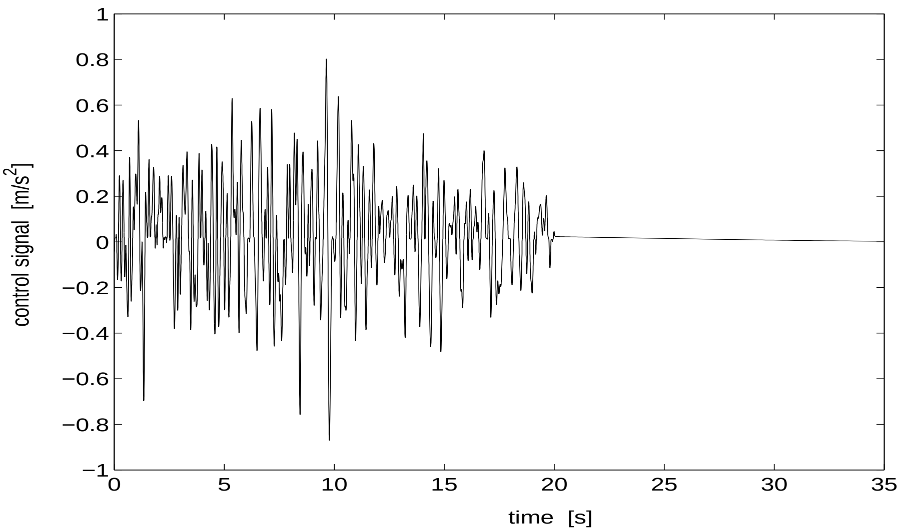

Figure 6. Control signal (acceleration).

Figure 6 shows the time history of the control signal. Physically, this signal is an acceleration. Its magnitude seems reasonable in comparison with the seismic excitation acceleration in Figure 2.

5. CONCLUSION

This paper has presented a backstepping-based adaptive controller for a class of one degree of freedom hysteretic system. For the control design, it is assumed that the hysteretic behaviour can be described by a polynomial function plus a residual function. This description lies within the family of non parametric models, which describe hysteretic elements by functional expansions. The coefficients of the polynomial and the residual function do not need to be known for the controller design. Indeed, an adaptive law is included in the control to estimate on-line these coefficients.

The behaviour of the closed loop is such that the response variables of the controlled system are globally uniformly ultimately bounded and they can be made arbitrarily small by an appropriate choice of design parameters. Since decreasing the ultimate bound of the controlled response requires increasing the control action, a trade-off between a desired response reduction and a reasonable control level has to be judiciously considered.

In this paper, the proposed control scheme has been tested on a hysteretic system described mathematically by the Bouc-Wen model and subjected to an earthquake excitation. Since this model is based on a differential equation, a regression based identification procedure has been proposed to obtain "experimentally" an approximation of the theoretical hysteretic behaviour within the polynominal framework of the control scheme. The discrepancies between the identified hysteresis and the theoretical one are lumped in the uncertain coefficients of the polynomial and the residual function. The numerical results show that the combination of this uncertain description of the hysteresis and the backstepping adaptive control law is satisfactory in that the response induced by the seismic action is significantly reduced. These results are encouraging towards the applicability of the control scheme proposed in this paper. In effect, both the Bouc-Wen model and non parametric approximations are widely used in structural dynamics, particularly in base isolation schemes where hysteretic behaviours pose challenging problems for designing active control systems.

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