Adaptive Compensation Strategy For The Tracking/Rejection of Signals with Time-Varying Frequency in Digital Repetitive Control Systems

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Abstract

Digital repetitive control is a technique which allows to track periodic references and/or reject periodic disturbances. Repetitive controllers are usually designed assuming a fixed frequency for the signals to be tracked/rejected, its main drawback being a dramatic performance decay when this frequency varies. A usual approach to overcome the problem consists of an adaptive change of the sampling time according to the reference/disturbance period variation. However, this sampling period adaptation implies parametric changes affecting the closed-loop system behavior, that may compromise the system stability. This article presents a design strategy which allows to compensate for the parametric changes caused by sampling period adjustment. Stability of the digital repetitive controller working under time-varying sampling period is analyzed. Theoretical developments are illustrated with experimental results.

1. Introduction

Repetitive control [24], [13] is an Internal Model Principle-based control technique [9] that allows both the tracking and rejection of periodic signals. Its use has reported successful results in different control areas, such as CD and disk arm actuators [3], robotics [23], electro-hydraulics [15], electronic rectifiers [27], pulse-width modulated inverters [26, 25] and shunt active power filters [5].

Repetitive controllers are usually designed assuming a fixed frequency for the signals to be tracked/rejected. This entails a selection of a fixed sampling period and a further structural embedding of these data in the control algorithm. However, it is also a well known fact that even slight changes in the frequency of the tracked/rejected signals result in a dramatic decay of the controller performance. An example of such situation may be found in the frequency variations experimented by shunt active filters connected to the electric distribution network [6].

Several approaches have been introduced in order to overcome this problem. The proposals may be grouped in two main frameworks, namely, the one that deals with the preservation of the sampling time and the one that changes it adaptively. For the first approach there are also two branches: improving robustness by using large memory elements [21] or introducing a fictitious sampler operating at a variable sampling rate and later using a fixed frequency internal model [2]. Both ideas work well for small frequency variations at the cost of increasing the computational burden. An alternative technique is to adapt the controller sampling rate according to the reference/disturbance period [16, 17, 11]. This allows to preserve the steady-state performance while maintaining a low computational cost but, on the other hand, it implies structural changes in the system behavior which may destabilize the closed-loop system. Regarding this issue, an LMI gridding-based stability analysis for digital repetitive control systems with time-varying sampling is carried out in [18].

This paper presents a design strategy for a digital repetitive controller working under time-varying sampling period which allows to compensate for the parametric changes originated by the adaptation of the sampling rate. This is achieved with the introduction of a compensator that annihilates the effect of the time-varying sampling on the closed-loop system and forces its behavior to coincide with that of the closed-loop system under an a priori selected nominal sampling period. The theoretical results

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are experimentally validated in a mechatronic laboratory plant [7].

The structure of the paper is as follows. Section 2 contains a brief description of a digital repetitive controller and a study of stability issues in case of constant sampling period. Section 3 introduces the compensation scheme and analyzes the stability of the overall closed-loop system under aperiodical sampling. Experimental results are collected in Section 4, while conclusions and further research lines are presented in Section 5.

2. Digital repetitive control under constant frequency

Repetitive controllers are composed by two main elements: the internal model, $G_r(z)$, and the stabilizing controller, $G_x(z)$. The internal model is the one in charge of guaranteeing null or small error in steady state, while the stabilizing controller assures closed-loop stability. Several types of internal models are used depending on the concrete periodical signal to deal with [7, 14, 10, 8]. In this work the generic internal model will be used, i.e.

$$G_r(z) = \frac{H(z)}{z^N - H(z)},$$

where $N = \frac{T_p}{T_s} \in \mathbb{N}$, T_p being the period of the signal to be tracked/rejected and T_s being the sampling period. $H(z)$ plays the role of a low-pass filter in charge of introducing robustness in the high frequency range [4]. Although the internal model and the stabilizing controller can be arranged in different manners, most repetitive controllers are usually implemented in a "plug-in" fashion, as depicted in Figure 1: the repetitive compensator is used to augment an existing nominal controller, $G_c(z)$. This nominal compensator is designed to stabilize the plant, $G_p(s)$, and provides disturbance attenuation across a broad frequency spectrum.

Assume that either T_p and T_s are constant, which makes N also constant, and let $P(z)$ stand for the discrete-time plant model. Sufficient stability criteria are given in the next Proposition:

Proposition 1 *[7, 14] The closed-loop system of Figure 1 is stable if the following conditions are fulfilled:*

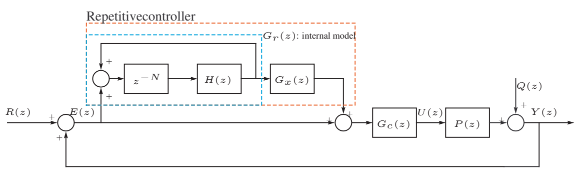

Figure 1. Discrete-time block-diagram of the proposed repetitive transfer function.

1. *The closed-loop system without the repetitive controller $G_o(z)$ is stable, where*

$$G_o(z) = \frac{G_c(z)P(z)}{1 + G_c(z)P(z)}.$$

2. $\| H(z) \|_\infty < 1$.

3. $\| 1 - G_o(z)G_x(z) \|_\infty < 1$, *where $G_x(z)$ is a design filter to be chosen.*

Remark 1 *These conditions hold for a proper design of $G_c(z)$, $H(z)$ and $G_x(z)$. Namely [7, 14]:*

- *It is advisable to design the controller $G_c(z)$ with a high enough robustness margin.*
- *$H(z)$ is designed to have gain close to 1 in the desired bandwidth and attenuate the gain out of it.*
- *A trivial structure which is often used for $G_x(z)$ in case that $G_0(z)$ is minimum-phase is [22]:*

$$G_x(z) = k_r \left[G_o(z)\right]^{-1}.$$

Otherwise, alternative techniques should be applied in order to avoid closed-RHS plane zero-pole cancellations [22]. Moreover, there is no problem with the improperness of $G_x(z)$ because the internal model provides the repetitive controller with a high positive relative degree. Finally, as argued in [12], k_r must be designed looking for a trade-off between robustness and transient response.

3. Adaptive compensation of the time-varying frequency effect

The repetitive controller introduced in the previous section contains the ratio $N = \frac{T_p}{T_s}$, which is embedded in the controller implementation. This setting renders a well-known good performance if the reference/disturbance periodic signal has a known constant period. However, as it has been previously mentioned in Section 1, the controller performance decays dramatically when a variation of T_p appears. This article propounds to deal with this problem by means of an adaptive compensation scheme based on three issues:

1. The repetitive controller is designed and implemented to provide closed-loop stability for an a-priori selected nominal sampling time $T_s = T_s^N$ to the nominal, Linear Time Invariant (LTI) plant

$$G_p(z, T_s^N) \triangleq \frac{Num(z, T_s^N)}{Den(z, T_s^N)}, \qquad (1)$$

in accordance with Proposition 1 and Remark 1. Hence, G_r, G_x and G_c are kept invariant.

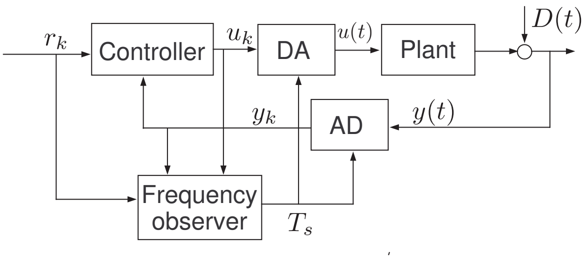

Figure 2. Accommodation of the sampling period T_s to possible variations of T_p.

2. Aiming the maintenance of a constant value N for the ratio $T_p(t)/T_s(t)$, which preserves the reference/disturbance signal reconstruction quality, the controller sampling time T_s is accommodated to the time variation of the reference/disturbance period $T_p(t)$, i.e. $T_s(t) = T_p(t)/N$. Therefore, the discrete-time representation of the plant $G_p(s)$ is that of a Linear Time Varying (LTV) system[1]

$$G_p(z, T_s) = \frac{Num(z, T_s)}{Den(z, T_s)}. \qquad (2)$$

See Figure 2 for details.

3. The structural changes caused by the sampling time variation of T_s are annihilated through an additional compensator

$$C(z, T_s) = G_p(z, T_s^N) G_p^{-1}(z, T_s) \qquad (3)$$

that premultiplies the LTV plant $G_p(z, T_s)$. Thence, under the assumption of internal stability for the compensator-plant system, its behavior is that of the nominal LTI system $G_p(z, T_s^N)$. Figure 3 depicts the overall system, while Figure 4 details the compensator-plant subsystem.

Remark 2 *The compensation strategy yields*

$$P(z) \triangleq C(z, T_s) G_p(z, T_s) = G_p(z, T_s^N), \qquad (4)$$

i.e. a time-invariant block representing the nominal plant for which the repetitive controller provides of closed-loop stability. Furthermore, when T_s remains constant at the nominal sampling time T_s^N, the closed-loop system does not experiment further changes and behaves as the nominal system (i.e. $C(z, T_s^N) = 1$).

[1]The use of the z-transform notation in a LTV framework is not formally correct. In this paper this notation is preserved in order to achieve a compact and simple notation (z^{-1} should be read as a one sample time delay, but the sampling interval may change from sample to sample).

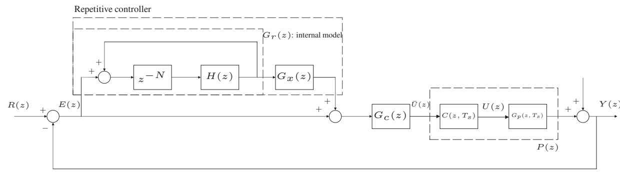

Figure 3. Discrete-time block-diagram of the closed-loop system with the proposed repetitive controller structure.

It is also worth mentioning that the compensator has not causality problems. Indeed, using (1) and (2) in (3) allows to write

$$C(z, T_s) = \frac{Num(z, T_s^N)}{Den(z, T_s^N)} \cdot \frac{Den(z, T_s)}{Num(z, T_s)}. \qquad (5)$$

Therefore, the possible improperness of the second factor is removed by the properness of the first factor.

One last consequence is that the system, $P(z)$ response at the sample instants $\{t_0, t_1, \ldots, t_k, \ldots\}$ for a fixed sampling period are the same in spite of the sampling rate value. This corresponds to a time scaling effect as a result of forcing the system to stay invariant in the discrete-time space.

The above exposed discussion allows to prove the following result:

Proposition 2 *Let $G_p(s)$ be a continuous LTI plant, and let $G_p(z, T_s)$ be its discrete-time transfer function corresponding to the sampling time T_s. Let also $T_s = T_s^N$ be a nominal sampling time, and let $G_p(z, T_s^N)$ defined in (1) be its associated discrete-time transfer function. Assume that the compensator $C(z, T_s)$ of Figure 3 is defined as in (3), and that the subsystem $P(z) = C(z, T_s) G_p(z, T_s)$ detailed in Figure 4 is internally stable for all $T_s \in \mathcal{T} \subset \mathbb{R}^+$. Finally, assume that the repetitive controller elements $G_r(z)$, $G_x(z)$, $G_c(z)$ are designed according to Proposition 1 and Remark 1 in order to provide closed loop stability to the LTI system, $P(z)$. Therefore, the closed-loop system depicted in Figure 3 is stable for all $T_s \in \mathcal{T}$.*

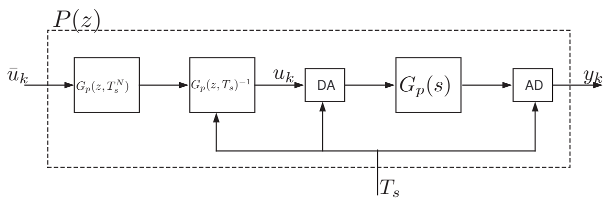

Figure 4. Detail of the compensator-plant system.

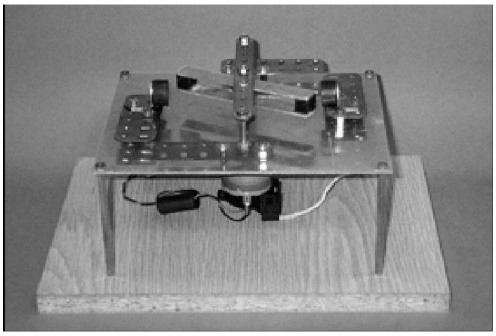

Figure 5. Picture of the main part of the plant: DC motor, optical encoder, magnetic system (load), and supporting structure.

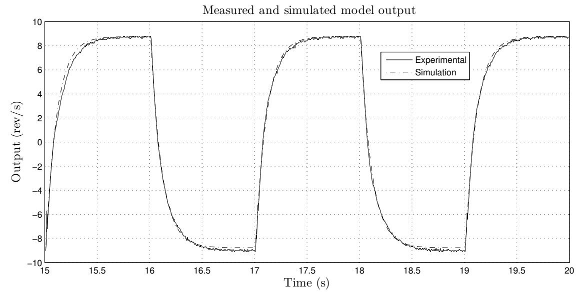

Figure 7. Open-loop time response of the plant without the fixed magnets.

Remark 3 *(i) The internal stability of the compensator-plant subsystem $P(z)$ can be checked using an LMI gridding approach [1, 20].*
(ii) In most practical applications the plant $G_p(z)$ admits a relative degree 1, first order model description, which ensures internal stability for $P(z)$ whenever $G_p(z)$ is stable.

4. Experimental setup and results

4.1. Plant description

Systems with rotary elements are usually affected by periodic disturbances due the movement of these parts (e.g. electrical machines, CD players...). This kind of system is supposed to be moving, in some cases, at a fixed angular speed. Under these working conditions any friction, unbalance or asymmetry appearing on the system generates a periodic disturbance that affects its dynamical behavior. Figure 5 shows the picture of device designed to reproduce this working conditions [7]. This device is composed of a bar holding a permanent magnet in each end, with each magnet magnetically oriented in the opposite way, and attached to a DC motor and two fixed permanent magnets (see a sketch in Figure 6). The rotation of the DC motor causes a pulsating load torque (Γ_p) that depends on the mechanical angle θ of the motor axis. When the motor axis angular speed ω is constant ($\ddot{\theta} = \dot{\omega} = 0$), the pulsating torque is a periodic signal with a fundamental period directly related to the axis speed: $T_p = \omega^{-1}$, with ω expressed in $\mathrm{rev/s}$. The control goal for this plant is maintaining the motor axis angular speed constant at a desired value.

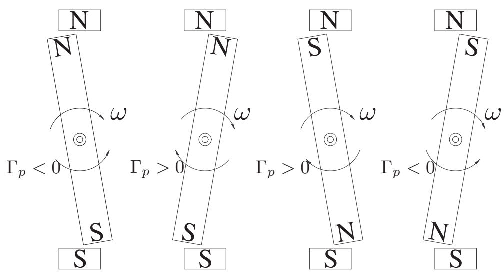

Figure 6. Mechanical load: fixed and moving permanent magnets sketch (ω and Γ_p stand for the angular speed and the disturbance torque, respectively).

Figure 7 shows the open-loop time response of the plant without the fixed magnets. From the observation of this time response and after a parametric identification procedure, the following plant model can be derived:

$$G_p(s) = \frac{8.762}{0.106s + 1} \frac{\mathrm{rev/s}}{\mathrm{V}}. \tag{6}$$

This is a first order parametrization with characteristic time response $\tau = 0.106\ s$ and stationary state gain $K = 8.762\, rev/(V \cdot s)$

Figure 8 shows the open-loop time response of the plant containing the fixed magnets. It is important to note that the speed describes an almost periodical signal. This type of disturbances may not be rejected by a regular controller, so a repetitive controller is designed in next subsection.

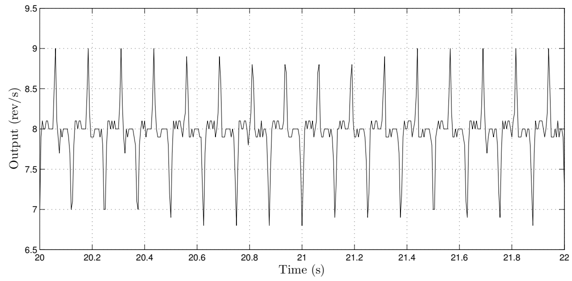

Figure 8. Open-loop time response of the plant with the fixed magnets.

4.2. Control design

The controller is constructed from model (6), for a speed of $\omega = 8$ rev/s and obtaining 25 samples per period, i.e. $N = 25$. These conditions imply a nominal sampling period of $T_s^N = T_p N^{-1} = (\omega N)^{-1} = 5$ ms. Under these assumptions the nominal discrete time plant is:

$$G_p(z, T_s^N) = \frac{0.4012}{z - 0.9542}$$

According to Remark 1, the following design issues have been taken into account:

- $G_c(z) = 0.25$ provides a very robust inner loop.
- The first order linear-phase FIR filter

$$H(z) = 0.02z + 0.96 + 0.02z^{-1}$$

  provides good performance in the present case.
- The fact that $G_p(z)$ is minimum-phase allows $G_x(z) = k_r G_0^{-1}(z)$, with $k_r = 0.3$.

According to Section 3, the compensator $C(z, T_s)$ is designed as follows. On the one hand, the discrete-time model of the plant at the nominal sampling period is taken as the time-invariant component of $C(z, T_s)$ (see (1)):

$$G_p(z, T_s^N) = \frac{Num(z, T_s^N)}{Den(z, T_s^N)} = \frac{0.4012}{z - 0.9542}. \quad (7)$$

On the other hand, the plant (see (2))

$$G_p(z, T_s) = \frac{Num(z, T_s)}{Den(z, T_s)} = \frac{K(1 - e^{-T_s/\tau})}{z - e^{-T_s/\tau}} \quad (8)$$

is first order, stable and minimum phase; hence, its inversion is possible and one can define:

$$\frac{Den(z, T_s)}{Num(z, T_s)} = \frac{z - e^{-T_s/\tau}}{K(1 - e^{-T_s/\tau})}. \quad (9)$$

Therefore, (7), (9) and (5) yield

$$C(z, T_s) = \frac{0.4012\left(z - e^{-T_s/\tau}\right)}{K(1 - e^{-T_s/\tau})\,(z - 0.9542)}, \quad (10)$$

which is a time-varying model that depends on the sampling period T_s and this, in turn, depends on the disturbance period variation T_p.

It is important to emphasize that, in this case, the function $P(z)$ introduced in (4) is composed of the series connection of:

1) The system $C(z, T_s)$ described in (10), which admits a LTV state-space representation $(A_c(k), B_c(k), C_c(k), D_c(k))$ with $A_c(k) = 0.9542$, $B(k) = 1$,

$$C_c(k) = \frac{0.4012e^{-T_k/\tau} + 0.3828}{K(1 - e^{-T_k/\tau})},$$

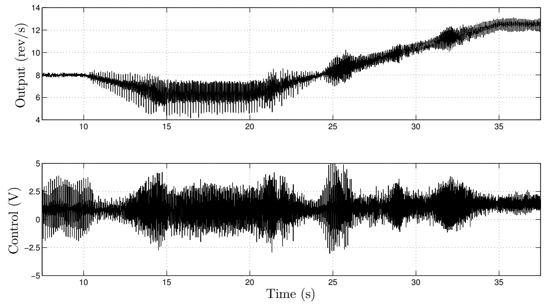

Figure 9. Closed-loop system behavior using a repetitive controller and with sampling period fixed at the nominal value ($T_s^N = 5$ms).

$$D_c(k) = \frac{0.4012}{K(1 - e^{-T_k/\tau})},$$

the sampling period being $T_k = T_s(k) = t_{k+1} - t_k$. Its internal stability is guaranteed by the fact that the state equation is time-invariant, A_c being a 1×1 real matrix with modulus less than 1, which yields uniform exponential stability [19]. Moreover, as $B_c(k)$ is constant and $C_c(k)$, $D_c(k)$ are bounded for all T_k belonging to any compact interval $\mathcal{T} \subset \mathbb{R}^+$, uniform BIBO stability is also ensured [19].

2) The system $G_p(z, T_s)$ described in (8), which is internally and uniformly BIBO stable because it corresponds to the sampled version of the LTI, continuous-time stable plant, $G_p(s)$.

As the series connection of internally and BIBO stable systems is also internally and BIBO stable, in the case under study $P(z)$ fulfills the hypothesis of Proposition 2, so the overall closed-loop system will be stable.

In order to derive the control action applied to the plant, i.e. the signal $\bar{U}(z)$ (see Figure 2), it has to be taken into account that the compensator makes the system time-invariant. Therefore, $\bar{u}_k$ is the invariant control law obtained from the nominal repetitive control, that is:

$$\begin{aligned} \bar{u}_k \;=\; & 0.25e_k + 0.015e_{k-23} + 0.70e_{k-24} + \\ & -0.84e_{k-25} - 0.018e_{k-26} + 0.02\bar{u}_{k-24} + \\ & +0.96\bar{u}_{k-25} + 0.02\bar{u}_{k-26}, \end{aligned}$$

with $e_k = r_k - y_k$, r_k, y_k being, respectively, the system output and the reference velocities.

The derivation of u_k according to Figure 4 yields:

$$u_k = \frac{0.4012}{K(T_s)}\bar{u}_k - 0.4012\frac{1 - K(T_s)}{K(T_s)}\bar{u}_{k-1} + 0.9542u_{k-1}$$

with $K(T_s) = 1 - e^{-T_s/\tau}$. It can be easily seen by straightforward calculation than when the sampling interval remains constant at nominal the sampling time, i.e. $T_s = T_s^N$, then $u_k = \bar{u}_k$.

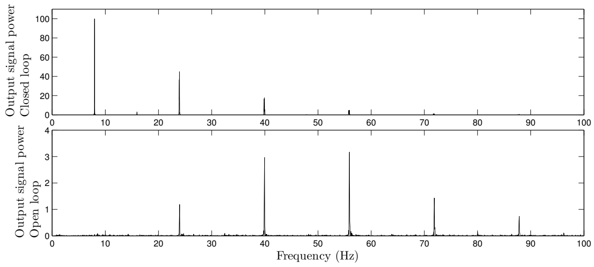

Figure 10. Frequency spectra in open and closed-loop working at the nominal speed (8 rev/s).

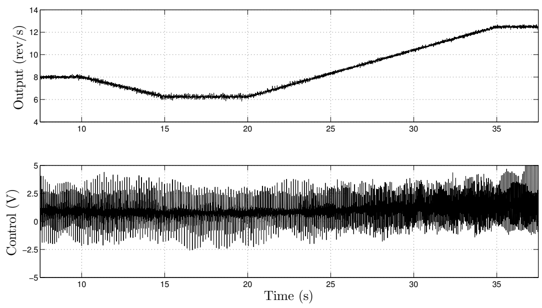

Figure 11. Closed-loop system behavior using a repetitive controller with adaptive sampling rate.

4.3. Experimental results

Figure 9 contains the experimental results of the repetitive controller designed in Section 4.2. During the time interval $[0, 10]$ s, the reference is maintained constant at speed value of $\omega = 8$ rev/s: it is important to realize that, in comparison to the uncompensated speed profile of Figure 8, now disturbances are almost rejected. Figure 10 contains both the uncompensated and compensated speed spectrum when working at the nominal speed; note that most relevant harmonics are eliminated or highly attenuated due to the repetitive control action. At $t = 10$ s a ramp reference change, from $\omega = 8$ rev/s to $\omega = 6.25$ rev/s, is introduced in the system, then the speed is kept constant for 5s and finally at $t = 20$ s the speed is gradually augmented at constant acceleration until it reaches the value $\omega = 12.5$ rev/s at $t = 35$s: Figure 9 reveals that after time 10s the system can no longer reject the disturbances. In addition, the action generated by the control law is also portrayed in Figure 9: it can be seen that the controller generates the necessary action to compensate disturbances when working at the nominal speed, i.e. up to $t = 10$ s, while when it works at the new speeds, i.e. for $t > 10$ s, the control action is reduced or is not suitable so disturbances cannot be properly compensated.

Figure 11 shows the experimental results using an adaptive compensation of the sampling rate, which is accommodated to the desired reference. One may observe that, the controller is capable of preserving the system performance and references are tracked with small error. At the bottom of Figure 11 the control action generated by the control law is shown. Now the control signal is the appropriate one in all cases, so the controller is capable of rejecting the disturbances.

5. Conclusions

In this work, the sampling period of the digital repetitive controller is adapted in order to maintain the rejection performance of the controller when reference/disturbance periodic signals with varying period are present. The design strategy presented here allows to compensate for the parametric changes caused by this sampling period adjustment. This is accomplished by the inclusion of an additional compensator which makes the system invariant in discrete time domain.

With this approach the repetitive controller design defined under constant sampling time remains valid also in case of variable sampling time; therefore, the tracking/rejecting performance can be preserved by adapting the sampling rate of the system in concordance with the period variation of the signal to be tracked/rejected.

Stability of the digital repetitive controller working under time-varying sampling period is analyzed. Since nominal design is made invariant it remains valid and since it is thought to fulfill the stability conditions these always hold despite of sampling time changes. Theoretical developments are illustrated with experimental results.

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