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Capacity planning with working time accounts in services[ϒ]

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Abstract

This paper presents, formalises and classifies working time accounts (WTAs), a means of achieving flexible production capacity through the flexible organisation of working time. We propose linear programming models to plan working time, using WTA, at companies in the service industry. A computational experiment shows that the models can be solved for any planning problems of reasonable size.

Keywords: working time, working time accounts, capacity planning

1. Introduction

Adapting capacity to demand is very important in production systems of all types. Manufacturers can resort to inventories to fill gaps between demand and capacity, but service providers cannot. Therefore, most service providers must have management tools that allow flexible capacity.

Furthermore, in the service industry, capacity is often directly related to the number of workers present in a service centre. Hence, working time flexibility plays a major role in obtaining flexible capacity (Jack and Raturi, 2002).

Flexibility achieved by modifying the composition of the staff or using temporary workers is called external flexibility. Internal flexibility is obtained by only using existing staff (Keller and Seifert, 2004).

There are several types of internal working time flexibility models (Lehndorff, 1999 and Oke, 2003), which can be classified as follows:

a) *Overtime*. Overtime is work that exceeds the standard working time, for which workers are compensated by additional pay. If this payment is based on a standard rate, then the length of the working day will only be constrained by regulations or the workers' physical strength. Higher payments, bounds or both are usually applied.

b) *Working time accounted on average*. It is said that working time is accounted on average when a total amount is established for periods lasting longer than a day,

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such as a week, several weeks, several months, a year or more. In this case, the distribution of working hours within the period is flexible but the total number is not. Hours must be planned carefully in order to prevent a surplus or owed time when the end of the period approaches. If the period selected is one year, then this type of working type flexibility is called annualisation; it is also said that annualised hours are used. This is the most commonly case analysed in the literature (Azmat et al., 2004, Corominas et al., 2002, 2004a, 2004b, 2005a and Hung, 1999). Accounting time on average is useful for dealing with predictable seasonal variations or covering a broad operations calendar rather than individual work calendars. The use of annualised hours has been a particularly common practice in France due to the enactment of the 35-hour law (Bunel, 2004).

c) *Working time accounts (WTAs).* In this kind of working time flexibility each worker has an account of working hours in which are credited or debited the worked hours above or below, respectively, a standard value. Unlike working time accounted on average, working time accounts do not establish a point at which the balance between contractual hours and hours worked must be zero. This system provides real long-term flexibility. In some cases, working hours above or below the standard amount must be compensated for within a set time span. In this case, which is not discussed in this paper, it is possible to determine an expiry date for the hours in question.

Although staff scheduling has been extensively covered in the literature (Ernst et al., 2004a, 2004b), academia has given WTAs relatively little attention thus far.

This paper deals with this latter type of flexible working time. Section 2 discusses the concept of working time accounts and its variants. Section 3 formalises linear programming models for planning with WTAs. Section 4 presents the results of a computational experiment carried out using the models. Finally, Section 5 is devoted to conclusions and prospects for future research.

2. Working time accounts

A working time account is a personnel management tool in which the differences between an employee’s contractual hours and hours worked are recorded. Hours worked above or below a reference value are either credited or charged to the account, and the balance is then calculated, which of course may be a credit or debit balance.

Traditionally, salary and time to be worked were established by contract, including the possibility of overtime. Working time accounts appeared in Europe as early as 1967 and later spread to the United States (Owen, 1977). This concept was then called active flexibility—each employee decided his or her own timetable under certain conditions (Bosch and Lehndorff, 2001). In the United States, the percentage of workers having some flexibility in their timetables ultimately reached 28.8% (U.S. Bureau of the Census, 2001).

Active flexibility is always advantageous for the worker, but passive flexibility—in which the employer decides how to use working time—may be very inconvenient for the worker unless limits and conditions are respected (Bosch and Lehndorff, 2001).

Therefore, management and workers must negotiate in order to reach satisfactory agreements.

In Germany, collective agreements establishing flexibility clauses for managerial purposes arose from the 1984 agreement between the union IG Metall and the employers' association for the metalworking industry, Gesamtmetall. As a result, working hours may increase (or decrease) according to production needs, and workers are later compensated with less (or more) working time.

These schemes are becoming increasingly more prevalent in Europe (Lehndorff, 1999). Agreements on working time accounts in manufacturing firms have been widely published, probably due to the huge size of the companies involved. There are numerous cases in the service industry—for example, in banking (Croucher and Singe, 2004); in information technologies, including the cases of Debis and IBM's German operations described by Voss-Dahm (2001); in catering, including the clauses of the Liquor Trades Hotels contract for the Australian Capital Territory in 1998 (Australian Industrial Relations Comisión, 2006); in the civil service, as defined by the United States' amended Fair Labor Standards Act of 1938 (Employment Standards Administration, 2004); in the software industry (Haipeter, 2004); in airlines (Lufthansa, 2005); in airports (Fraport, 2005); and in the retail sector (Metro Group, 2003).

Working time accounts allow a great deal of flexibility, but they also give rise to complex planning problems, whose optimal solutions cannot be obtained unless mathematical models are used.

The manufacturing and service industries require quite different approaches in applying WTAs to planning. In manufacturing, it is common for all workers—or at least all workers in a given group—to work as a team and follow the same timetable. Some service companies apply the same conditions as manufacturing companies, but many others assign each worker a number of hours that may be different from that of her/his co-workers. A single timetable for all workers is, of course, easier to deal with, but manufacturing companies also deal with inventories (perishable or non-perishable) and the possibility of deferring demand, which complicates the problem.

The hours that the company owes the workers or that the workers owe the company may or may not have an expiry date. In cases in which there is an expiry date, when the date is reached, the company must pay the workers for any "expired" hours and, if the workers owe any hours to the company, this debt is forgiven. In either case, the hours are erased from the account. Obviously, expiry dates makes more difficult dealing with WTAs.

The solution to a WTA planning problem must comply with a variety of restrictions.

Firstly, there are restrictions on the number of working hours in each period of the planning horizon. In certain production systems, the decision may consist solely in eliminating a shift or establishing a shift on a day that is not normally a working day. In other systems, a worker may work any number of hours that falls within an interval that contains the average number of hours established by contract, or reference value. In this case, the hours worked in excess of the reference value can be dealt with in a variety of ways—they may either be registered in the WTA or considered as overtime. If the excess hours are considered as overtime, the worker is paid for them and the WTA is not affected. For example, with a reference value of 8 hours/day, we can establish that the length of the workday must be between 6 and 12 hours, that any hours in excess of 8 but less than 10 are credited to the WTA, and that any hours in excess of 10 and up to 12 are considered overtime. In other cases, the number of working hours during a period may need to be taken from a pre-established list.

Secondly, the WTA balance must be kept within the upper and lower bounds. For example, -100 and +75 hours, respectively: the worker can owe the company up to 100 hours and the company can owe the worker up to 75 hours.

There may or may not be a maximum limit on the hours a worker may work in excess of the normal workday in a given period (for example, one year or the planning horizon).

The overtime hours mentioned above may or may not be allowed. If they are allowed, an annual maximum may apply. There may also be “blocks” of overtime hours, with an upper bound for each block, such that the unit cost of an hour belonging to a block is greater than that of an hour belonging to the preceding one.

The aforementioned elements of a WTA system and the corresponding restrictions are common to many production systems or are extracted from documents describing implemented WTA. Moreover, we propose to consider two kinds of overtime, in order to provide the system with more flexibility. The first kind includes the working hours in a day that exceed a certain limit (10 hours in the above example). Henceforth, this category will simply be called “overtime”. The second kind (which will be called hereafter “overaccount”) includes hours worked within the range that may be entered in the WTA that cannot be totally or partially credited to the account because its balance is equal to (or too close to) the upper bound. The worker is paid for these hours and the WTA is not affected. For example, using the data from the previous paragraphs, let us assume that a worker's WTA balance is +74 and he or she works a 10-hour day, rather than the 8-hour reference workday. If the worker's balance were not so high, the two overtime hours would be credited to the WTA. In this case, however, only one of the hours can be credited (which brings the WTA balance to +75, the upper bound). The other hour must be considered overaccount. Logically, an upper bound should be placed on the number of hours of this type that can be used within the planning horizon.

As mentioned above, all flexible working time organisation systems must impose conditions that guarantee a result that is compatible with the health and personal life of the workers. With WTAs, the upper and lower bounds on the account balance and on the number of hours worked in a given day may be sufficient in many cases, but there may be additional conditions similar to those taken into account in the annualisation of the workday (Corominas et al., 2004b).

One significant characteristic to be taken into account in the planning process is the versatility of the workers, relative to their capacity to perform a variety of tasks. Let us assume that each staff member belongs to a single category (which does not imply any loss of generality, since there may be categories that contain a single individual) and that all members of a given category cost the same and are capable of performing a given set of tasks with a given degree of efficiency. If the sets of tasks of any pair of categories are disjoint, then each category may be planned independently of the others. Furthermore, subsets of categories may generally be established in such a way that each subset is planned independently of the others. We must distinguish between cases in which working time is planned for a single category and cases in which working time is planned for several categories at once. The latter case is only possible if the staff has a certain degree of versatility.

Lastly, the characteristics of the demand condition to a large extent the way to model WTA systems. The main distinction, of course, is between deterministic (and known) and stochastic demand. In deterministic cases, we assume perfect knowledge of demand, whose values are used to determine the capacity required over the course of the chosen planning horizon. Stochastic cases allow us to draw a distinction: (i) we assume that future demand may correspond to a finite number of scenarios with a known probability (Garg et al., 2002; Lusa et al., 2006); and (ii) we assume that the law of probability of demand is known for each period in the planning horizon (Pinker and Larson, 2003). Since inventories cannot be stored in the service industry, it would seem that capacity and demand should ideally coincide at all times. However, in general, the necessary capacity cannot be immediately determined based on a demand forecast, because even in cases of deterministic demand (considered overall) the random nature of the time between arrivals and the service time makes it necessary to define a desired service level. The calculation of the minimum capacity that makes the said service level possible is generally anything but trivial (Thompson, 1997, Atlason et al., 2004 and Corominas et al., 2005b).

WTA systems and their respective planning models may be classified according to the aforementioned considerations (Table 1).

Timtables	Same for all workers / by group / individual
Number of hours per period (day)	Zero or a fixed number / selected from a finite set of values / upper and lower bounded
Hours with extra pay	Overtime: allowed / not allowed If overtime is allowed: extra pay at a single rate / extra pay at two or more rates Overaccount: yes / no
Expiry of WTA hours	Hours worked above or below a reference value: expiry date / no expiry date
Number of categories	One / more than one
Demand	Deterministic / stochastic

Table 1. **Options for working time account systems and their planning models**

Unlike other flexible working time systems, WTA systems never have a natural planning horizon. Of course, a horizon is chosen for planning, but in general the system need not meet any special condition at the end of the horizon. Therefore, the future consequences on the state of the system at the end of the horizon must be evaluated. All traditional planning problems have this difficulty in common. In general, it is solved (or it is tried to be solved) by establishing a value for the final inventory of the product or products, sometimes without fully justifying said value. In WTA planning for service companies, intervals can be established for the final balance of each worker and the aggregated balance in accordance with the forecast of what might happen at the end of the horizon. In WTA planning for manufacturing industry, restrictions on the final value of the balances may be added to the traditional restrictions on the final value of the inventories.

Because the horizon is not pre-established, horizons of less than one year may be used (one-year duration is mandatory in cases of workday annualisations). This means that the periods can be short (one day, rather than one month or one week) without the model becoming too large.

3. WTA working time planning models in service companies

The case modelled in this paper involves a service centre (which therefore has no product inventory) with deterministic demand, multiple categories, individualised workdays with upper and lower bounds, overtime paid at a single rate, overaccount, and no expiry date for hours. Section 3.1 presents a preliminary model that includes the conditions described in Section 2. Given that some of the features for the solutions provided by this first model may not be wholly suitable, Section 3.2 presents a model with complementary characteristics that guarantee the desired properties.

3.1. Model for planning working hours with WTAs

Data:

W Set of workers $(i \in W)$.

K Set of categories of workers $(k \in K)$.

J Set of types of tasks or functions $(j \in J)$.

T Number of periods in the planning horizon $(t = 1, ..., T)$.

$\hat{W}_k$ Subset of workers belonging to category k $(\forall k \in K)$. $\bigcup_{k \in K} \hat{W}_k = W$; $\hat{W}_{k'} \bigcap \hat{W}_{k''} = \varnothing \quad \forall k', k'' \in K \mid k' \neq k''$

ρ_{kj} Efficiency associated with a worker from category k in tasks of type j $(\forall k \in K;\ \forall j \in J)$; $\rho_{kj} = 0$ indicates that workers of category k are unable to perform tasks of type j.

$\hat{K}_j$ Subset of categories whose workers can perform tasks of type j $(\forall j \in J)$: $\hat{K}_j = \{k \in K \mid \rho_{kj} > 0\} \quad \forall j \in J$

$\hat{J}_k$ Subset of types of tasks that can be performed by workers from category k $(\forall k \in K)$: $\hat{J}_k = \{j \in J \mid \rho_{kj} > 0\} \quad \forall k \in K$

κ_{jt} Desired capacity for tasks of type j in period t $(\forall j \in J; t = 1, ..., T)$.

s_{i0}	Balance of the WTA of worker i at the initial instant $(\forall i \in W)$, which may be negative, zero or positive.
h_i	Normal or reference number of hours per period for worker i $(i \in W)$.
h_i^-, h_i^+	Upper and lower bounds, respectively, for the number of ordinary working hours per period of worker i $(\forall i \in W)$. Working hours between h_i^- and h_i or between h_i and h_i^+ are taken into account in the WTA.
h_i^{++}	Upper bound for the number of working hours per period for worker i $(\forall i \in W)$. Working hours between h_i^+ and h_i^{++} are considered overtime (paid) and are not taken into account in the WTA. Hours worked in excess of the reference workday are also considered overtime when the WTA balance has reached the upper bound and all annual overaccount hours have been used.
S_i^-, S_i^+	Upper and lower bounds, respectively, of the WTA balance of the worker i $(\forall i \in W)$; notice that $S_i^- \leq 0$ $(\forall i \in W)$.
c_k	Cost of an hour of overtime for a worker from category k $(\forall k \in K)$.
$\hat{c}_k$	Cost of an hour of overaccount time for a worker from category k $(\forall k \in K)$. Overaccount hours are those worked in excess of the daily reference value when the WTA balance is equal to its upper bound S_i^+, for the worker i; we assume that the worker is paid for these hours and they are not credited to the WTA. We also assume that the management is able to consider hours already credited to the account as overaccount in order to adjust the balance to a given amount (for instance, at the end of the planning horizon). Logically, we also assume that $\hat{c}_k < c_k$.
u_i	Upper bound, for the duration of the planning horizon, on the number of overtime hours for the worker i $(\forall i \in W)$.
$\hat{u}_i$	Upper bound, for the duration of the planning horizon, on the number of overaccount hours for the worker i $(\forall i \in W)$.
γ_j	Cost of the lack of a unit of capacity for tasks of type j $(\forall j \in J)$.
α^-, α^+	Upper and lower bound on the sum of the WTA balances for all workers at the end of the planning horizon, T.

Variables:

$x_{it}^-, x_{it}^+ \geq 0$	Working hours, below and above, respectively, the reference value, h_i, of the worker i in the period t $(\forall i \in W; t = 1,...,T)$. The variables x_{it}^+ correspond to the regular hours, that is, those not considered overtime.
$e_{it} \geq 0$	Overtime hours of the worker i in the period t $(\forall i \in W; t = 1,...,T)$.
$\hat{e}_{it} \geq 0$	Overaccount hours of the worker i in the period t $(\forall i \in W; t = 1,...,T)$.

$y_{jkt} \geq 0$	Hours dedicated to tasks of type j by workers of the category k in the period t $\left(\forall j \in J; \forall k \in \hat{K}_j; t=1,\ldots,T\right)$.
s_{it}	WTA balance of the worker i in the period t $\left(\forall i \in W; t=1,\ldots,T\right)$; s_{it} may be negative (which indicates that the worker owes hours to the company), null or positive (which indicates that the company owes hours to the worker).
$\delta_{jt} \geq 0$	Capacity shortage for tasks of type j in the period t $\left(\forall j \in J; t=1,\ldots,T\right)$.

Model:

$$[MIN]\, z = \sum_{k \in K} c_k \sum_{\substack{i \in \hat{W}_k \\ t=1,\ldots,T}} e_{it} + \sum_{k \in K} \sum_{\substack{i \in \hat{W}_k \\ t=1,\ldots,T}} \left(\hat{c}_k - \frac{t}{100 \cdot T} \right) \cdot \hat{e}_{it} + \sum_{j \in J} \gamma_j \sum_{t=1,\ldots,T} \delta_{jt} \quad (1)$$

$$s_{it} = s_{i,t-1} + x_{it}^{+} - x_{it}^{-} - \hat{e}_{it} \qquad \forall i \in W;\ t=1,\ldots,T \quad (2)$$

$$\sum_{i \in \hat{W}_k} \left(h_i + x_{it}^{+} - x_{it}^{-} + e_{it} \right) = \sum_{j \in \hat{J}_k} y_{jkt} \qquad \forall k \in K;\ t=1,\ldots,T \quad (3)$$

$$\sum_{k \in \hat{K}_j} \rho_{kj} \cdot y_{jkt} + \delta_{jt} \geq \kappa_{jt} \qquad \forall j \in J;\ t=1,\ldots,T \quad (4)$$

$$\sum_{t=1,\ldots,T} e_{it} \leq u_i \qquad \forall i \in W \quad (5)$$

$$\sum_{t=1,\ldots,T} \hat{e}_{it} \leq \hat{u}_i \qquad \forall i \in W \quad (6)$$

$$\alpha^{-} \leq \sum_{i \in W} s_{iT} \leq \alpha^{+} \quad (7)$$

$$0 \leq x_{it}^{-} \leq h_i - h_i^{-} \qquad \forall i \in W;\ t=1,\ldots,T \quad (8)$$

$$0 \leq x_{it}^{+} \leq h_i^{+} - h_i \qquad \forall i \in W;\ t=1,\ldots,T \quad (9)$$

$$0 \leq e_{it} \leq h_i^{++} - h_i^{+} \qquad \forall i \in W;\ t=1,\ldots,T \quad (10)$$

$$S_i^{-} \leq s_{it} \leq S_i^{+} \qquad \forall i \in W;\ t=1,\ldots,T \quad (11)$$

$$0 \leq \hat{e}_{it} \leq x_{it}^{+} \qquad \forall i \in W;\ t=1,\ldots,T \quad (12)$$

$$y_{jkt} \geq 0 \qquad \forall j \in J;\ \forall k \in \hat{K}_j;\ t=1,\ldots,T \quad (13)$$

$$\delta_{jt} \geq 0 \qquad \forall j \in J;\ t=1,\ldots,T \quad (14)$$

(1) is the objective function that includes the cost of overtime plus the cost of overaccount time and the cost of the capacity shortage; (2) expresses, for each worker and for each period, the WTA balance; (3) is the balance between the hours available to a specific category of workers and those assigned to the different types of tasks; (4) expresses that the number of working hours assigned to a type of task added to the capacity shortage must not be less than the desired capacity; (5) and (6) impose the upper bounds, for the duration of the planning horizon, on the number of overtime hours and the number of overaccount hours for the worker i; (7) imposes the upper and lower bounds on the sum of the WTA balances for all workers at the end of the planning horizon; (8) to (12) impose the upper and lower bounds of the working hours below and above the reference value, the overtime hours, the WTA balance and the overaccount

hours, respectively; and (13) and (14) impose the non-negative character of the remaining variables.

In (1), a modification in the cost of overaccount hours (considered slightly more expensive the closer they fall to the start of the planning horizon) is included in order to prevent overaccount hours from being scheduled before they are strictly necessary, since without this modification two overaccount hours would be equivalent regardless of their allocation in the planning horizon.

If the planning horizon includes vacation periods for all or some of the workers, which we assume are established a priori, it is not necessary to define the variables x_{it}^{-}, x_{it}^{+}, e_{it}, $\hat{e}_{it}$ and s_{it} in the periods t in which the worker i will be not present in the workplace. This is not shown in the above model, so as not to complicate formalisation, but it was include in the model used to perform the computational experiment, described below.

3.2. Model that takes into account need and regularity in the use of WTA working hours

The model described above provides a minimum-cost solution. However, not all of the usually infinite number of minimum-cost solutions have the same properties. Some of these solutions may have characteristics that make them difficult to use, including:

1) *Use of the WTA when not strictly necessary*. Imagine a required capacity of 16 hours for two consecutive periods and two workers, P1 and P2, with a reference workday of 8 hours/period. Provided that the established annual limits are not exceeded, the following two solutions cover the required capacity at minimum cost: (8, 8) hours for P1 and (8, 8) hours for P2, or (7, 9) for P1 and (9, 7) hours and P2. Both solutions cover 16 hours in each period (8+8, 8+8) and (7+9, 9+7), respectively. However, the first solution does not use the WTA, whereas the second solution does use it even though it is not necessary.

2) *Irregular use of the WTA*. A solution is possible in which, at the end of the planning horizon, no WTA hours have been used for some workers and WTA hours have been used in all periods for other workers. This situation involves unequal treatment, since some workers enjoy stable, regular timetables and others have variable timetables.

3) *Irregular WTA balances*. Imagine two workers, P1 and P2, with WTA balances in a given period of -3 and +1 respectively. Provided that the established annual limits are not exceeded, if one overtime hour is needed, the following two minimum-cost solutions could be obtained: the hour is assigned to P1 (with final balances of -2 and +1) or to P2 (with final balances of -3 and +2). The first solution is clearly better than the second.

The following mathematical programming model overcomes these difficulties.

Let z^* be the minimum cost obtained in the initial model. We add a constraint to said model to guarantee that the cost of the new solution will not exceed z^*:

$$\sum_{k\in K} c_k \sum_{\substack{i\in \hat{W}_k \\ t=1,\ldots,T}} e_{it} + \sum_{k\in K} \sum_{\substack{i\in \hat{W}_k \\ t=1,\ldots,T}} \left(\hat{c}_k - \frac{t}{100\cdot T}\right)\cdot \hat{e}_{it} + \sum_{j\in J} \gamma_j \sum_{t=1,\ldots,T} \delta_{jt} \leq z^*$$

and obtain a correct and regular use of WTA hours using an appropriate objective function.

With the objective function $\sum_{t=1}^{T} \sum_{\forall i\in W} \varphi(s_{it})$, where φ is strictly convex and non-negative and $\varphi(0)=0$, the three possible situations are solved. Because the objective function is strictly convex, an optimal solution to the model will only use the WTAs if needed to minimise costs and balances, so that they are as low and as similar to each other as possible—that is, the various workers' WTAs will be used as equally as cost minimisation allows. Unfortunately, using strictly convex functions turns the linear program into a non-linear one, which makes the model significantly more difficult to solve. We therefore use, as an approximation to φ, a linear stepwise function with an increasing slope, $\hat{\varphi}$, which is convex but not strictly convex.

In order to introduce this objective function in the model, we replace the variable $s_{it} \in \left[S_{it}^-, S_{it}^+\right]$, not restricted in sign, with the difference of the two non-negative variables, each of which is expressed as the sum of the variables corresponding to the segments of the stepwise linear function:

$$s_{it} = s_{it}^+ - s_{it}^-;\ s_{it}^+ = \sum_{n=1}^{S_i^+} r_{itn}^+;\ s_{it}^- = \sum_{n=1}^{S_i^-} r_{itn}^-\ \ \left(\forall i \in W; t=1,\ldots,T\right);\ 0 \leq r_{itn}^+, r_{itn}^- \leq 1\ \ \forall i,t,n$$

And, adopting $\varphi(s_{it}) = s_{it}^2$, $\hat{\varphi}(s_{it}) = \sum_{n=1}^{S_i^+}\left[(2\cdot n-1)\cdot r_{itn}^+\right] + \sum_{n=1}^{S_i^-}\left[(2\cdot n-1)\cdot r_{itn}^-\right]$.

The number of variables increases considerably. If we must reduce the number of variables, due to the lack of computing power, then we must increase the size of the segments into which the interval $\left[S_{it}^-, S_{it}^+\right]$ is broken.

We have detected one occasionally occurring case that is not completely solved using, instead of φ, the function $\hat{\varphi}$. This case arises when the WTA is very scarcely used. Imagine four periods of time. Seventeen hours are required in the first and third periods and 15 hours are required in the second and fourth periods. Two workers (P1 and P2) are available, and the reference workday is 8 hours/period. Using the adopted function, the following two solutions are equivalent: (9, 7, 9, 7) hours for P1 and (8, 8, 8, 8) hours for P2, or (9, 7, 8, 8) hours for P1 and (8, 8, 9, 7) hours for P2. The first solution only uses the WTA for worker P1, and the second uses the WTA equally for both. We propose carrying out a simple final distribution process when this situation occurs. Nevertheless, it appears that this will not often be necessary in systems designed to use WTAs.

4. Computational experiment

A computational experiment was performed in order to evaluate the effectiveness of the presented models. Overall, the results show that optimal solutions can be obtained in managerially acceptable times for any reasonable size instance.

The basic data used for the experiment are as follows:

- $T = 250$ days (considering, for each worker, 220 working days and 30 holiday days).

- The number of holiday days for each worker is 30, distributed at random into two non-interrupted, non-overlapping periods of 10 and 20 days, respectively.

- Staff sizes ($|W|$) of 10, 50, 100, 150 and 250 workers.

- Three categories of workers $(|K| = 3)$: 50% of the workers belong to Category 1, 30% belong to Category 2 and 20% belong to Category 3.

- Three types of tasks $(|J| = 3)$.

- Three patterns of relative efficiency ρ_{kj}. Tables 2, 3 and 4 show the relative efficiency values for each pattern.

	Task 1	Task 2	Task 3
Category 1	1	0.9	0
Category 2	0	1	0.9
Category 3	0	0	1

Table 2. Relative efficiency values for pattern 1 of relative efficiency

	Task 1	Task 2	Task 3
Category 1	1	0.9	0.8
Category 2	0	1	0.9
Category 3	0	0	1

Table 3. Relative efficiency values for pattern 2 of relative efficiency

	Task 1	Task 2	Task 3
Category 1	1	0	0.7
Category 2	0.9	1	0
Category 3	0.8	0	1

Table 4. Relative efficiency values for pattern 3 of relative efficiency

- There are three different patterns of required capacity (in working hours) during the year. Required capacity type 1 is a non-seasonal capacity pattern (the required capacity is the same for every day in the planning horizon). Required capacity type 2 is a one-peak seasonal pattern. Required capacity type 3 is a two-peak seasonal pattern. In each case, the total requirement for the whole year is equal to the total number of available staff hours (we have assumed that the number of workers was fixed according to annual demand without noise). The whole required capacity is divided into three equal parts, corresponding to the three categories and a noise is added to each one of them. Figures 1 and 2 show the shape of the patterns, including noise, with one and two peaks respectively.

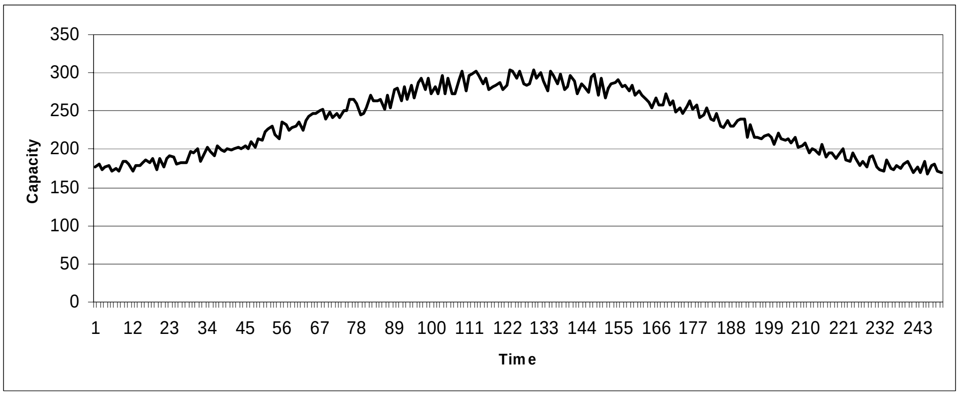

Figure 1. One-peak required capacity pattern

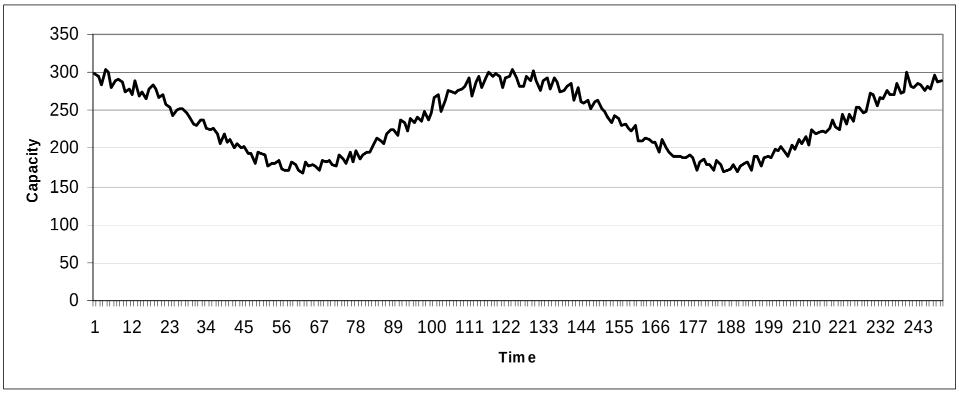

Figure 2. Two-peak required capacity pattern

- h_i, h_i^-, h_i^+ and h_i^{++}, equal to 8, 6, 10 and 11 hours/day $(\forall i \in W)$, respectively.
- s_{i0}, with a continuous uniform distribution between -25 and 25 $(\forall i \in W)$.
- S_i^- and S_i^+, equal to -200 and 200 $(\forall i \in W)$, respectively.
- u_i and $\hat{u}_i$, equal to 80 and 120 hours $(\forall i \in W)$, respectively.
- $\alpha^- = \alpha^+ = 0$.
- Costs: $c_k = 1 \ (\forall k \in K)$, $\hat{c}_k = 0.8 \cdot c_k = 0.8 \ (\forall k \in K)$ and $\gamma_j = 3 \cdot c_k = 3 \ (\forall j \in J)$.

For each combination of staff size, |*W*|, pattern of required capacity and pattern of relative efficiency, 100 instances were generated (with noise, holiday days and value of s_{i0} varying at random), giving 4,500 instances.

Firstly, the model described in 3.1 was applied to these 4,500 instances.

In spite of the large size of some of the models (Table 5 shows the number of variables and constraints for each staff size value $|W|$), they were solved to optimality using the ILOG CPLEX 9.0 optimiser and a Pentium IV PC at 3.4 GHz with 512 Mb of RAM.

$\|W\|$	No. of variables	No. of constraints
10	13,000 to 13,250	5,921
50	57,000 to 57,250	23,601
100	112,000 to 112,250	45,701
150	167,000 to 167,250	67,801
250	277,000 to 277,250	112,001

Table 5. Number of variables and constraints in the models

The 4,500 instances generated were solved optimally in computation times that would be suitable for an organisation that wished to implement a WTA system. Table 6 shows the minimum, average and maximum calculation times ($t_{\min}$, $\bar{t}$ and $t_{\max}$, respectively) for each staff size value $|W|$.

$\|W\|$	$t_{\min}$	$\bar{t}$	$t_{\max}$
10	0.41	0.59	1.13
50	4.30	7.29	12.57
100	14.64	24.40	37.98
150	28.81	48.64	78.28
250	71.74	112.25	158.14

Table 6. Minimum, average and maximum calculation times

An analysis of the results of the experiment reveals that the WTAs make it possible to satisfy the demand for various types of tasks at a minimum cost and to determine how and when WTA hours, overtime hours and overaccount hours are used.

As an example, the following are the results obtained for an instance with a staff of 50 workers, pattern 1 of relative efficiency and a two-peak required capacity pattern. In the planning horizon, a total of 87,859 hours are required. A total of 89,816 hours are scheduled, of which 1,787 are overaccount and 29 are overtime. A total of 8,482 hours above or below the reference workday are credited or charged to the WTAs. The capacity is equal to or greater than the demand in all tasks and all periods. Costs related to capacity shortage are therefore equal to zero.

Figure 3 shows the total desired capacity and the hours that the staff are present in each period of time in the planning horizon.

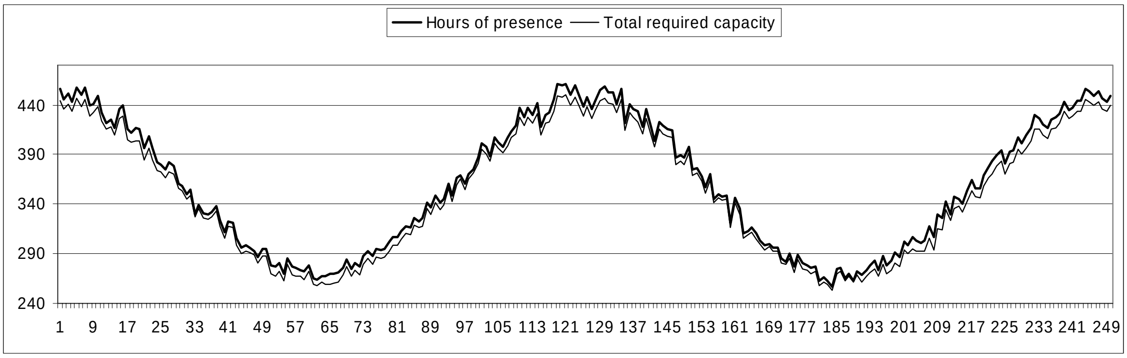

Figure 3. Total required capacity vs. total hours of presence

Figures 4 and 5 show the desired capacity for each type of task and each period of time, as well as the capacity obtained with this planning system.

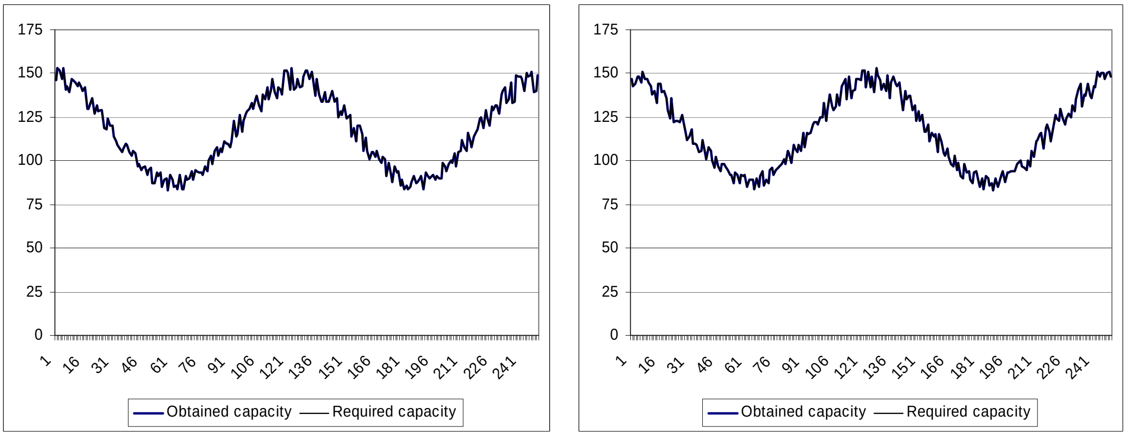

Figure 4. Required capacity for tasks 1 and 2 vs. capacity obtained

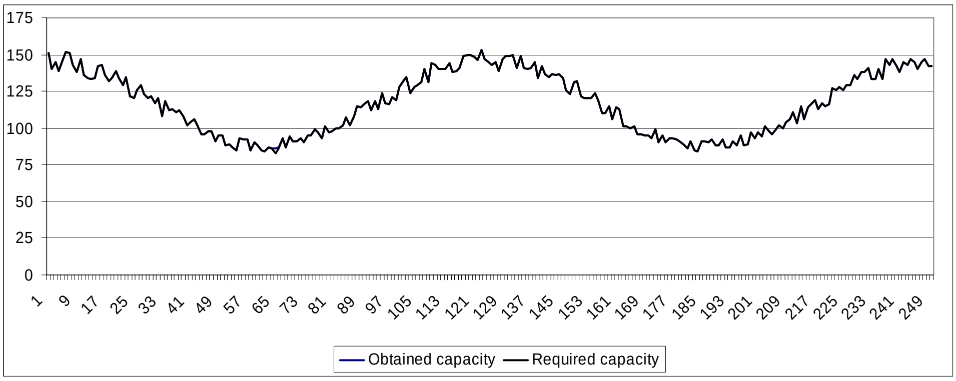

Figure 5. Desired capacity for task 3 vs. capacity obtained

Although the total number of hours of presence exceeded the required capacity, as shown in Figure 3, the available capacity for each task was equal to the required capacity for all tasks and all periods, except for periods 65 and 66, in which the required capacities for task 3 were 83 and 86 and the obtained capacities were 85.96 and 86.94, respectively. The apparent discrepancy that the number of hours available was greater than the required capacity, and that the available capacities almost coincided with the required ones was due to the fact that the workers had efficiency values equal to or less than one.

Figure 6 shows the sum of the 50 workers' WTAs for each period of time in the planning horizon. Starting from zero, the balance increased (that is, the company started to owe the workers hours since they had been working in excess of the reference workday) in response to an above-average initial required capacity. Later, the company took advantage of periods with lower required capacity to pay the workers back and obtain a negative balance of hours (the workers then owed hours to the company), which was subsequently used when the required capacity was greater.

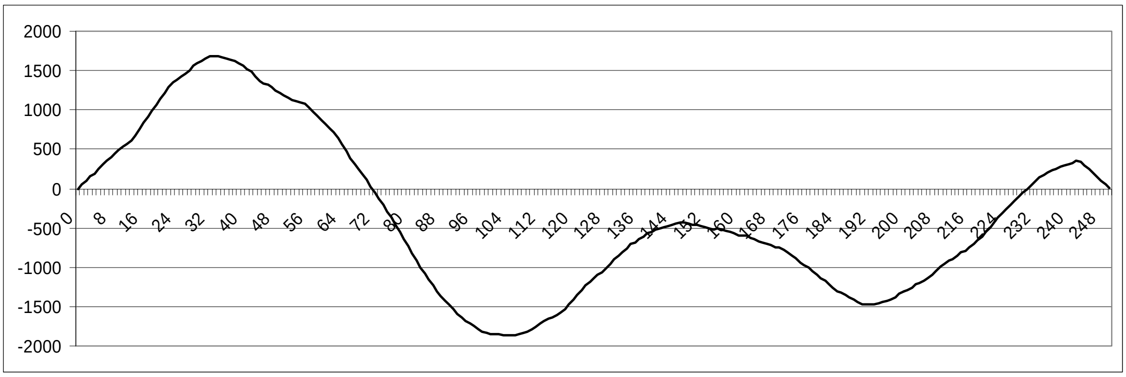

Figure 6. Sum of the WTA balances of all workers

However, a detailed analysis shows that, as predicted in Section 3.2, the obtained solutions include unsuitable values that are difficult to use, due to the irregular and unnecessary use of the WTAs and the wide range of individual balances obtained. In order to gain an overall perspective of this problem, below we discuss the results obtained for pattern 1 of relative efficiency, a two-peak required capacity pattern and $|W| = 10$. Figure 8 shows the WTA balances of the 10 workers over the course of the planning horizon and highlights the drawbacks mentioned above. Even when the need for workers was above average, several workers worked less than an ordinary workday. In a single period of time, some workers worked longer than the reference workday while others worked less than the reference workday (thereby cancelling out the effect). Finally, although the final balance for all workers was null overall, the balances of the individual workers were very irregular: nearly half of them had a balance of close to S_i^-, while the other half had a balance close to S_i^+.

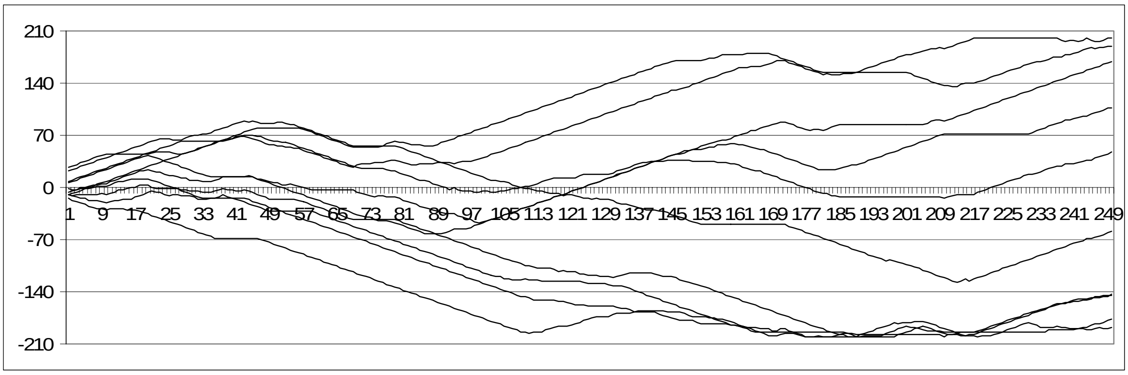

Figure 8. WTA balance for the 10 workers with the initial model

If the same instance is then solved using the model presented in Section 3.2, the drawbacks mentioned above are eliminated. Figure 9 shows the 10 workers' WTA balances if the instance is solved using the model proposed to regularise WTA use: the

WTAs of the various workers are used equally, and only when necessary. Figure 10 shows the overall balance of the 10 workers over the course of the planning horizon using the two models (WTA1 is the initial model and WTA2 is the model that considers regularity).

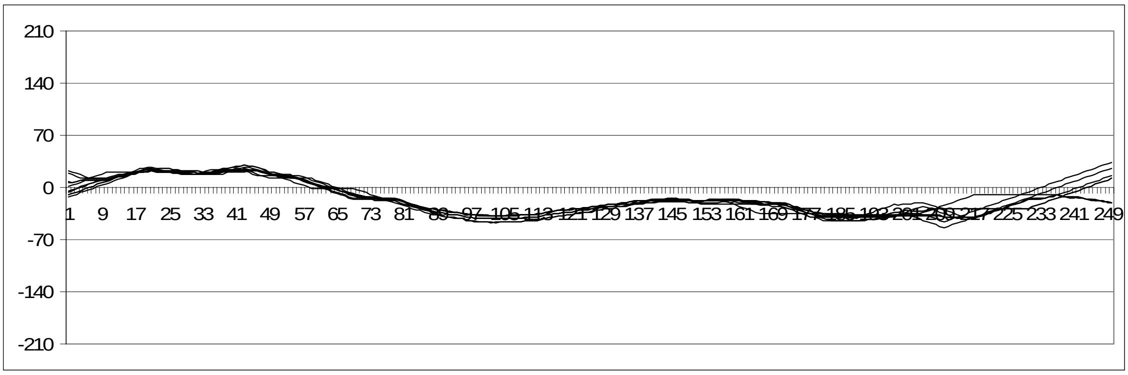

Figure 9. WTA balances of the 10 workers with the second model

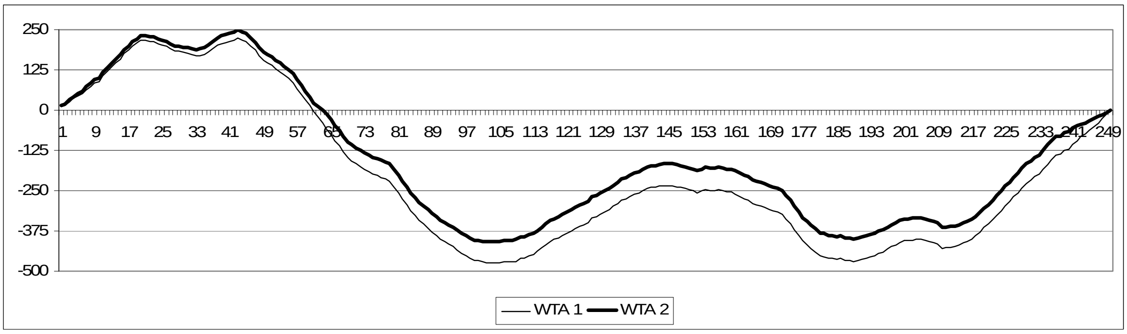

Figure 10. Overall WTA balance for the 10 workers

The cost obtained with the two models is obviously the same. The second model is larger: the 13,000 variables and 5,921 constraints in the first model increase to 1,014,500 variables and 10,222 constraints in the second (with the x-axis broken down into 1-unit segments). Moreover, the solving time increases from 0.58 to 18.91 seconds.

In the case of the 50 workers presented above, the 57,000 variables and 23,601 constraints increase to 1,314,500 variables and 45,102 constraints (with the x-axis broken down into 4-unit segments); the solving time increases from 6.80 to 272.28 seconds.

5. Conclusions and prospects for future research

This paper presents, formalises and classifies working time accounts as a working time flexibility tool, proposes linear programming models for planning by means of WTAs and presents a computational experiment, which shows that the proposed models are efficient to solve instances of realistic size.

Our prospects of future research consists in to model other WTA cases, in particular the case of service companies with expiry dates and industrial cases with inventories. We also plan to integrate working time planning with WTA in aggregate planning models that incorporate cash management and staff composition variations.

6. References

Atlason, J., Epelman, M. A., Henderson, S. G. (2004) "Call center staffing with simulation and cutting plane methods". *Annals of Operations Research*, 127, 333-358.

Australian Industrial Relations Comisión (2006). *Australian Industrial Registry Loose-Leaf Consolidation: Liquor Trades Hotels contract for the Australian Capital Territory in 1998 (modified in 2005).* Department of Employment and Workplace Relations, Camberra. Available at: www.wagenet.gov.au/WageNet.

Azmat, C., Hurlimann, T., Widmer, M. (2004) "Mixed integer programming to schedule a single-shift workforce under annualized hours". *Annals of Operations Research*, 128, 199-215.

Bosch, G., Lehndorff, S. (eds.), (2001): *New Forms of Employment and Working Time in the Service Economy (NESY), Final Report*. TSER Programme of the European Commission, Directorate General for Science, Research and Development. http://ec.europa.eu/research/social-sciences/pdf/finalreport/soe2ct983078-final-report.pdf

Bunel, M. (2004) "Modulation / annualisation dans le cadre des 35 heures : entreprises et salariés sous contrainte". *Travail et Emploi*, 98, 45-48.

Corominas, A., Lusa, A., Pastor, R. (2002) "Using MILP to plan annualised working hours". *Journal of the Operational Research Society*, 53, 10, 1101-1108.

Corominas, A., Lusa, A., Pastor, R. (2004a) "Planning annualised hours with a finite set of weekly working hours and joint holidays". *Annals of Operations Research*, 128, 217-233.

Corominas, A., Lusa, A., Pastor, R. (2004b) "Characteristics and classification of the annualised workings hours planning problems". *International Journal of Services Technology and Management*, 5, 5-6, 435-447.

Corominas, A., Lusa, A., Pastor, R. (2005a) "Annualised hours: a real flexibility tool". *OR Insight*, 18, 1, 10-14.

Corominas, A., Lusa, A., Muñoz, N. (2005b) "Cálculo de la capacidad necesaria para obtener un nivel de servicio predeterminado". IX Congreso de Ingeniería de Organización (CIO 2005), Gijón.

Croucher, R., Singe, I. (2004) "Co-determination and working time accounts in the German finance industry". *Industrial Relations Journal*, 35, 2, 153-168.

Employment Standards Administration (2004). *The Fair Labor Standards Act of 1938, as Amended*. Wage and Hour Division publication 1318. Employment Standards Administration, U.S. Department of Labor, Washington, DC.

Ernst, A. T., Jiang, H., Krishnamoorthy, M., Sier, D. (2004a) "Staff scheduling and rostering: A review of applications, methods and models". *European Journal of Operational Research*, 153, 1, 3-27.

Ernst, A. T., Jiang, H., Krishnamoorthy, M., Owens, B., Sier, D. (2004b) "An annotated bibliography of personnel scheduling and rostering". *Annals of Operations Research*, 127, 21-144.

Fraport (2005). *Annual report 2004*. Fraport, Frankfurt.

Garg, S., Vrat, P., Kanda, A. (2002) "Trade-offs between multiskilling and inventory in assembly line operations under demand variability". *International Journal of Operations and Production Management*, 22, 5, 565-583.

Haipeter, T. (2004) *Can norms survive market pressures?. The practical effectiveness of new forms of working time regulation.* Contribution to WES '04, Work, Employment and Society Conference, 1-3 September 2004, Manchester. Retrieved January 30, 2006, from Science Centre North Rhine-Westphalia, Institute for Work and Technology Web site: http://iat-info.iatge.de/english/publications.php

Hung, R. (1999) "Scheduling a workforce under annualized hours". *International Journal of Production Research*, 37, 1, 2419-2427.

Jack, E., Raturi, A. (2002) "Sources of volume flexibility and their impact on performance". *Journal of Operations Management*, 20, 5, 519-548.

Keller, B., Seifert, H. (2004) "Flexicurity- the German trajectory". *Transfer*, 10, 2, 226-247.

Lehndorff, S. (1999) *New working time systems, work organisation and re-distribution of work*. Institut Arbeit und Technik, Abteilung Arbeitsmarkt, Düsseldorf.

Lufthansa (2005). *Annual report 2004*. Lufthansa, Frankfurt.

Lusa, A., Corominas, A., Muñoz, N. (2006) *A scenario optimisation procedure to plan annualised working hours under demand uncertainty*. Working paper, IOC-DT-P-2006-11, http://hdl.handle.net/2117/317.

Metro Group (2003). *Annual report 2002*. Metro Group, Düsseldorf.

Oke, A. (2003) "Drivers of volume flexibility requirements in manufacturing plants". *International Journal of Operations and Production Management*, 23, 11, 1497-1513.

Owen, J. (1977) "Flexitime: some problems and solutions". *Industrial and Labor Relations Review* 30, 2, 152-160.

Pinker, E. J., Larson, R. C. (2003) "Optimizing the use of contingent labor when demand is uncertain". *European Journal of Operational Research*, 144, 1, 39.

Thompson, G. (1997) "Labor staffing and scheduling models for controlling service levels". *Naval Research Logistics*, 44, 8, 719-740.

U.S. Bureau of the Census (2001). *Current Population Survey. May supplement.* Bureau of Labor Statistics, U.S. Department of Labor, Washington, DC.

Voss-Dahm, D. (2001) *Long-term working time accounts: an innovative working-time-regulation for knowledge worker?.* Contribution to the 13th Annual Meeting on Socio-Economics, June 2001 in Amsterdam. Retrieved January 30, 2006, from Science Centre North Rhine-Westphalia, Institute for Work and Technology Web site: http://iat-info.iatge.de/english/publications.php