

# A Cyclo–dissipativity Framework for Power Factor Compensation of Electrical Circuits

Eloísa García–Canseco<sup>†</sup>, Robert Griño<sup>‡</sup>, Romeo Ortega<sup>†</sup>, Miguel Salichs<sup>§</sup> and Alexander Stankovic<sup>#</sup>

<sup>†</sup>*Laboratoire des Signaux et Systèmes, SUPELEC, Plateau de Moulon, 91192, Gif–sur–Yvette, France.*

<sup>‡</sup>*Instituto de Organización y Control de Sistemas Industriales, Universitat Politècnica de Catalunya, Barcelona, Spain.*

<sup>§</sup>*Department of Electrical Engineering, Universitat Politècnica de Catalunya, Barcelona, Spain.*

<sup>#</sup>*Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115 USA.*

Emails: {garcia,ortega}@lss.supelec.fr, roberto.grino@upc.edu, salichs@ee.upc.es, astankov@ece.neu.edu

**Abstract**—The purpose of this paper is to bring to the attention of the control community some of the aspects of the practically important, and mathematically challenging, power factor compensation problem. Our main contribution is identifying the key role played by *cyclo–dissipativity* in the solution of the problem. Namely, we prove that a necessary condition for a (shunt) compensator to improve the power transfer is that the load satisfies a given cyclo–dissipativity property, which naturally leads to a formulation of the compensation problem as one of cyclo–dissipativity. Cyclo–dissipative systems exhibit a net absorption of (abstract) energy only along *closed paths*, while a dissipative system cannot create energy *for all* trajectories, henceforth, this concept generalizes the one of passivation.

## 1. Introduction to the Power Factor Compensation Problem

We consider the classical scenario of energy transfer from an  $q$ –phase ac generator to a load as depicted in Fig. 1. The voltage and current of the source are denoted by the column vectors  $\mathbf{v}_s(t), \mathbf{i}_s(t) \in \mathbb{R}^q$  and the load is described by a (possibly nonlinear and time varying)  $q$ –port system  $\Sigma$ . We make the following assumptions:

**Assumption A.1** All the signals in the system are periodic with fundamental period  $T$  and belong to the space

$$\mathcal{L}_2[0, T) := \{ \mathbf{x} : [0, T) \rightarrow \mathbb{R}^q \mid \|\mathbf{x}\|^2 := \frac{1}{T} \int_0^T |\mathbf{x}(\tau)|^2 d\tau < \infty \},$$

where  $\|\cdot\|$  is called the *rms value* of  $\mathbf{x}$  and  $|\cdot|$  is the Euclidean norm.

**Assumption A.2** The source is ideal, in the sense that  $\mathbf{v}_s(t)$  remains unchanged for all loads  $\Sigma$ .

Assumption A.1 captures the practically reasonable scenario that the system operates in a periodic, though not necessarily sinusoidal, steady state regime. This is the case of the vast majority of applications of interest for the problem at hand. Assumption A.2 is tantamount to saying that the source has no impedance and is justified by the fact that most ac apparatus operate at a given voltage, with the actual drained current being specified by the load.

The presence of distorted signals, in this case the current  $\mathbf{i}_s(t)$ , has the deleterious effect of reducing the power transmission efficiency. Let us discuss how this happens. The rated power of the source is the product of its maximum deliverable rms voltage and current. On the other hand, the *average (or active) power* delivered by the source is defined as

$$P := \langle \mathbf{v}_s, \mathbf{i}_s \rangle, \quad (1)$$

where  $\langle \mathbf{v}_s, \mathbf{i}_s \rangle := \frac{1}{T} \int_0^T \mathbf{v}_s^\top(t) \mathbf{i}_s(t) dt$  denotes the inner product in  $\mathcal{L}_2[0, T)$ . From (1) and the Cauchy–Schwarz inequality [1] we have

$$P \leq \|\mathbf{v}_s\| \|\mathbf{i}_s\| =: S,$$

where we have defined the *apparent power*  $S$ . From the inequality above we conclude that, under Assumption A.2,  $S$  is the highest average power delivered to the load among all loads that have the same rms current  $\|\mathbf{i}_s\|$ . The identity holds if and only if  $\mathbf{v}_s(t) = \mathbf{R} \mathbf{i}_s(t)$  for some unitary matrix  $\mathbf{R} \in \mathbb{R}^{q \times q}$ , that is  $\mathbf{R}^\top \mathbf{R} = \mathbf{I}_q$ . If this is not the case  $P < S$  and compensation schemes are introduced to reduce this mismatch. That is, to maximize the ratio  $\frac{P}{S}$ —that is called the *power factor* (PF) [2].

A typical compensation configuration is shown in Fig. 2 where, to preserve the rated voltage at the load terminals the compensator  $\Sigma_c$  is placed in shunt. Also, to avoid power dissipation,  $\Sigma_c$  is restricted to be lossless, that is,

$$\langle \mathbf{v}, \mathbf{i}_c \rangle = 0, \quad (2)$$

where  $\mathbf{i}_c(t)$  is the compensator current and we notice that  $\mathbf{v}_s(t) = \mathbf{v}(t)$ . Given these restrictions, and under the standing Assumption A.2, the problem of maximizing  $\frac{P}{S}$  admits an equivalent reformulation, which has a simple geometric interpretation and an explicit solution. First, referring to Fig. 2 we notice that the compensator losslessness condition (2) translates into

$$\langle \mathbf{v}, \mathbf{i}_s \rangle = \langle \mathbf{v}, \mathbf{i} \rangle. \quad (3)$$

Second, if  $\Sigma$  and  $\mathbf{v}(t)$  are fixed then  $\mathbf{i}(t)$ , and consequently  $P$ , are fixed. Hence:

*maximizing  $\frac{P}{S}$  with lossless compensators is equivalent to minimizing  $\|\mathbf{i}_s\|$  subject to the constraint (3).*

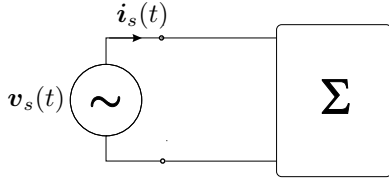


Figure 1: Circuit schematic of a polyphase ac system.

The optimization problems above are rather unprecise and considerations on the actuator are required for their clean formulation. Two types of compensator devices are available in practice, circuits with energy–storing components or regulated current sources—also called “with or without energy storage” in the circuits literature.

## 2. Lossless Shunt Compensation: A Cyclo–Dissipativity Framework

In this section we consider the use of lossless shunt elements for PF compensation and assume that  $\Sigma_c$  can be represented with its admittance operator  $\mathbf{Y}_c : \mathbf{v} \rightarrow \mathbf{i}_c$ . From the Projection Theorem [1] we have

**Fact 1** Fix  $\mathbf{v}(t), \mathbf{i}(t)$ . Among all currents  $\mathbf{i}_s(t)$  that satisfy the constraint (3), the one with minimal rms value  $\|\mathbf{i}_s\|$  is given by

$$\mathbf{i}_s^*(t) = \frac{\langle \mathbf{i}, \mathbf{v} \rangle}{\|\mathbf{v}\|^2} \mathbf{v}(t), \quad (4)$$

which is known in the literature as *Fryze’s current* [3].

Hence, from Fact 1 and  $\mathbf{i}_c(t) = \mathbf{i}_s(t) - \mathbf{i}(t)$  it is clear that any lossless operator  $\mathbf{Y}_c$  that solves

$$\frac{\langle \mathbf{i}, \mathbf{v} \rangle}{\|\mathbf{v}\|^2} \mathbf{v}(t) - \mathbf{i}(t) = (\mathbf{Y}_c \mathbf{v})(t),$$

for the given  $\mathbf{i}(t), \mathbf{v}(t)$ , is optimal. In spite of its apparent simplicity, it is not clear to these authors how to use this “data–interpolation” relationship in a practically meaningful way.

From the previous Section we have that the PF compensation problem is mathematically equivalent to the problem of minimization of  $\|\mathbf{i}_s\|$  subject to the constraint (3). From

$$\|\mathbf{i}_s\|^2 = \|\mathbf{i}\|^2 + \|\mathbf{i}_c\|^2 + 2 \langle \mathbf{i}_c, \mathbf{i} \rangle,$$

it is clear that a *necessary condition* to reduce the rms value of  $\mathbf{i}_s(t)$  is

$$\langle \mathbf{i}_c, \mathbf{i} \rangle < 0. \quad (5)$$

It turns out that the latter condition and the restriction of compensator losslessness can be nicely captured using the concept of *cyclo–dissipativity* and its associated *abstract energy*<sup>1</sup>. In our previous work [9–12] we used the more restrictive notion of passivity and the actual electric and magnetic energies that, unfortunately, impose extremely conservative conditions. Roughly speaking, the key advantages of cyclo–dissipativity are that it restricts the set of inputs of interest to those that generate periodic solutions—a feature that is intrinsic in PF compensation problems—it furthermore deals with “abstract” energies.

<sup>1</sup>Dissipativity theory has been extensively investigated by the control community in the last few years, see the books [4–6] or the recent survey papers [7, 8] for an extensive list of references.

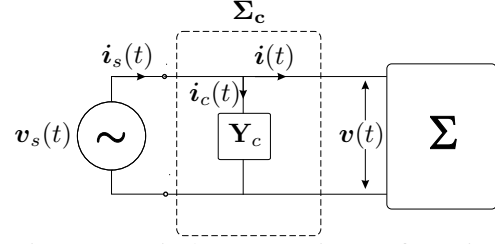


Figure 2: Typical compensation configuration.

**Definition 1** [13] Assume a dynamical system, with input  $\mathbf{u} \in \mathcal{L}_2[0, T]$  and output  $\mathbf{y} \in \mathcal{L}_2[0, T]$ , admits a state–space representation with state vector  $\mathbf{x} \in X$ . The system is *cyclo–dissipative* with respect to the supply rate  $w(\mathbf{u}, \mathbf{y})$ , where  $w : \mathcal{L}_2[0, T] \times \mathcal{L}_2[0, T] \rightarrow \mathbb{R}$ , if and only if

$$\int_0^T w(\mathbf{u}(t), \mathbf{y}(t)) dt \geq 0$$

for all  $\mathbf{u} : [0, T] \rightarrow \mathcal{L}_2[0, T]$  such that  $\mathbf{x}(T) = \mathbf{x}_0$  where  $\mathbf{x}(0) = \mathbf{x}_0$ . It is said to be *cyclo–lossless* if the inequality holds with identity.

In words, a system is cyclo–dissipative when it cannot create (abstract) energy over closed paths in the state–space. It might, however, produce energy along some initial portion of such a trajectory; if so, it would not be dissipative. On the other hand, every dissipative system is cyclo–dissipative. As an example, (possibly nonlinear) RLC circuits with input and output their port currents and voltages, respectively, are cyclo–dissipative with supply rate  $w(\mathbf{u}, \mathbf{y}) = \mathbf{u}^\top \mathbf{y}$  provided that all resistances are passive<sup>2</sup>. Notice that we do not assume the inductors and capacitors are passive—that is, that their stored energy is non–negative—if so, the circuit is in addition passive.

It has been shown in [13] that, similarly to dissipative systems, one can use storage functions and dissipation inequalities to characterize cyclo–dissipativity provided we eliminate the restriction that the functions—called virtual storage functions—be *non–negative* and they are only required to be bounded (from above and below).

**Theorem 1** [13] A system with state representation is *cyclo–dissipative* iff, for all  $\mathbf{x} \in X$  which are both *controllable* and *reachable*, there exists a virtual storage function  $\phi : X \rightarrow \mathbb{R}$ . That is, a function that satisfies

$$\phi(\mathbf{x}_0) + \int_0^T w(\mathbf{u}(t), \mathbf{y}(t)) dt \geq \phi(\mathbf{x}_1)$$

for all  $\mathbf{u} \in \mathcal{L}_2[0, T]$  such that  $\mathbf{x}(0) = \mathbf{x}_0$  and  $\mathbf{x}(T) = \mathbf{x}_1$ .

We are in position to formulate the PF compensation problem in terms of cyclo–dissipativity—a paradigm that we propose for future study.

**Definition 2 (Cyclo–dissipativity)** Consider the system of Fig. 2. Assume the load  $\Sigma$  is described by its admittance operator  $\mathbf{Y} : \mathbf{v} \rightarrow \mathbf{i}$  and it admits a state–space representation. Find a compensator with admittance  $\mathbf{Y}_c : \mathbf{v} \rightarrow \mathbf{i}_c$  and a state–space representation, such that

<sup>2</sup>This fact that can be easily proven using Tellegen’s Theorem [17].

- (i)  $\mathbf{Y}_c$  is cyclo-lossless with supply rate  $\mathbf{v}^\top \mathbf{i}_c$ .
- (ii) The overall system with input  $\mathbf{v}$  and output  $\text{col}(\mathbf{i}, \mathbf{i}_c)$  is cyclo-dissipative with supply rate  $-\mathbf{i}^\top \mathbf{i}_c$ .

A first step toward the solution of this general synthesis problem is to consider that the (lossless) *compensator* is given and we investigate under which conditions *on the load* the cyclo-dissipativity property is satisfied, that is, we want to characterize classes of loads for which it is possible to improve the PF with a given compensator.

### 3. Compensation with LTI Capacitors and Inductors and Load Cyclo-Dissipativity

Let us consider first capacitive compensation, for which we have,  $\mathbf{Y}_c = \mathbf{C}_c p$ , where  $\mathbf{C}_c = \{C_{ij}\} \in \mathbb{R}^{q \times q}$ ,  $C_{ij} \geq 0$  is the capacitance matrix and  $p = \frac{d}{dt}$ , then the necessary condition for PF compensation (5) becomes

$$\langle \mathbf{i}, \mathbf{C}_c \dot{\mathbf{v}} \rangle \leq 0 \quad \Leftrightarrow \quad \langle \hat{\mathbf{i}}, \mathbf{C}_c \mathbf{v} \rangle \geq 0, \quad (6)$$

where we have used the property  $\langle \dot{\mathbf{x}}, \mathbf{y} \rangle = -\langle \mathbf{x}, \dot{\mathbf{y}} \rangle$ , which holds for all periodic signals  $\mathbf{x}, \mathbf{y}$ . Let us assume that the load admits a state representation. If the voltage sources are in series with inductors the elements of  $\mathbf{i}$  qualify as state variables, that is,  $\mathbf{x} = \text{col}(\mathbf{i}, \chi)$ , with  $\chi \in \mathbb{R}^{n-q}$  denoting the remaining state variables. The dynamics of the load is then described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{v}) = \begin{bmatrix} \mathbf{f}_i(\mathbf{x}, \mathbf{v}) \\ \mathbf{f}_\chi(\mathbf{x}, \mathbf{v}) \end{bmatrix}, \quad (7)$$

where  $\mathbf{f}_i(\mathbf{x}, \mathbf{v}) \in \mathbb{R}^q$ . We can state the following:

**Fact 2** Consider the nonlinear polyphase load  $\Sigma$ , with port variables  $(\mathbf{v}, \mathbf{i})$ , and dynamics (7). If the PF can be improved (for all periodic, possibly non-sinusoidal,  $\mathbf{v}_s$ ) with shunt LTI capacitors then there exists a matrix  $\mathbf{C}_c = \{C_{ij}\} \in \mathbb{R}^{q \times q}$ ,  $C_{ij} \geq 0$  such that the system with output  $\mathbf{y} = \mathbf{C}_c^\top \mathbf{f}_i(\mathbf{x}, \mathbf{v})$ , is cyclo-dissipative with supply rate  $\mathbf{y}^\top \mathbf{v}$ . That is, the system is cyclo-passive.

The fact above indicates that cyclo-dissipative loads (in the sense defined above) constitute an extension, to the nonlinear non-sinusoidal polyphase case, of the so-called *inductive* loads. This characterization takes a particularly simple form for *single-phase* loads, i.e., (6) is equivalent to

$$\langle \hat{\mathbf{i}}, \mathbf{v} \rangle \geq 0, \quad (8)$$

establishing that: *If the PF can be improved (for all, possibly non-sinusoidal,  $\mathbf{v}_s$ ) with a shunt LTI capacitor then the single-phase load (possibly nonlinear) is cyclo-dissipative (with supply rate  $\hat{\mathbf{v}}\mathbf{i}$ ).*

For inductor compensators  $\mathbf{Y}_c = \mathbf{L}_c^{-1} \frac{1}{p}$ , with  $\mathbf{L}_c = \{L_{ij}\} \in \mathbb{R}^{q \times q}$ ,  $L_{ij} \geq 0$  the inductance matrix, and the inequality of interest is  $\langle \mathbf{i}, \mathbf{L}_c^{-1} \int \mathbf{v} \rangle \leq 0 \quad \Leftrightarrow \quad \langle \int \mathbf{i}, \mathbf{L}_c^{-1} \mathbf{v} \rangle \geq 0$ , that admits also a cyclo-dissipativity interpretation—provided the current sources have capacitors in parallel. As before, for  $q = 1$  the condition becomes  $\langle \int \mathbf{i}, \mathbf{v} \rangle \geq 0$ .

For brevity, we restrict in the sequel to capacitor compensation—a scenario which is very common since loads are typically assumed to be dominantly inductive—for *single-phase* loads, that is  $q = 1$ . Two questions arise immediately:

- Q1** How can we characterize cyclo-dissipative loads? (that is, loads for which (8) holds)
- Q2** If PF improvement is possible, what is the optimal value of the capacitance?

A solution to the second question is straightforward and well known in the circuits community. Indeed,  $\|i_s\|^2$  in this case takes the form

$$\|i_s\|^2 = \|i\|^2 - 2C_c \langle \hat{\mathbf{i}}, \mathbf{v} \rangle + C_c^2 \|\dot{\mathbf{v}}\|^2,$$

which is a quadratic equation in the unknown  $C_c$  and achieves its minimum at

$$C_c^* = \frac{\langle \hat{\mathbf{i}}, \mathbf{v} \rangle}{\|\dot{\mathbf{v}}\|^2}. \quad (9)$$

See also [3] for the polyphase case and some illustrative examples.

Similar optimization problems for other reactive circuit topologies have been studied in the circuits literature. See [14] for an extensive treatment of the topic. However, there seems to be many open problems, for instance in [15, 16], it is shown that for parallel *RL* circuits the optimal solution corresponds to a negative inductance and a switched series *LC* circuit is proposed as an alternative option. To the best of our knowledge, no systematic study of this kind of optimization problem—that would lead, among other things, to a better understanding of admissible topologies and sub-optimal solutions—has been carried out.

To explore question Q1, consider the following lemma

**Lemma 1** Assume a (possibly nonlinear) *RLC* circuit, then the cyclo-dissipativity of the circuit is independent of the average steady-state behavior of the resistors. Moreover, inner products for the resistive elements, either voltage or current-controlled, are zero. Henceforth,

$$\langle \hat{\mathbf{i}}, \mathbf{v} \rangle = \langle \mathbf{v}_L, \hat{\mathbf{i}}_L \rangle - \langle \dot{\mathbf{v}}_C, \mathbf{i}_C \rangle. \quad (10)$$

where the sub-indices *L* and *C* stand from the voltages and currents at inductive and capacitive elements, respectively.

Even though resistors voltages and currents do not explicitly appear in the average supply rate (10) it is clear that they play a role in the overall voltage and current distributions. To unveil the role of the resistors on the cyclo-dissipativity property we define the function

$$q(t) := \mathbf{v}_L^\top(t) \hat{\mathbf{i}}_L(t) - \dot{\mathbf{v}}_C^\top(t) \mathbf{i}_C(t),$$

in view of (10), we have  $\frac{1}{T} \int_0^T q(t) dt = \langle \hat{\mathbf{i}}, \mathbf{v} \rangle$ . With a slight modification to the construction proposed in [18] we can prove the following interesting result.

**Fact 3** Assume the inductors and capacitors are linear and passive and the circuit is topologically complete<sup>3</sup>. Then

$$\dot{q}(t) = -2 \begin{bmatrix} \hat{i}_L^\top & \hat{v}_C^\top \end{bmatrix} \begin{bmatrix} \nabla^2 G(\mathbf{i}_L) & \mathbf{\Gamma} \\ \mathbf{\Gamma}^\top & -\nabla^2 F(\mathbf{v}_C) \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_C \end{bmatrix},$$

where  $\mathbf{\Gamma} \in \mathbb{R}^{n_L \times n_C}$  is a constant matrix (with elements  $(1, 0, -1)$ ) determined by the interconnection between the inductors and capacitors.  $G(\mathbf{i}_L)$  and  $F(\mathbf{v}_C)$  are the content and the co-content of the current-controlled resistors and voltage-controlled resistors defined in [17].

Even though we have shown above that the resistors do not intervene in the inner product  $\langle \hat{i}, v \rangle$  we see that they contribute with sign-definite quadratic terms in the time evolution of  $q(t)$ <sup>4</sup>. As pointed out in [18] the content and co-content functions can be modified adding current and voltage sources, which suggests a procedure to regulate  $q(t)$  and henceforth modify the circuit cyclo-dissipativity. Current investigation is under way to further analyze the properties of this differential equation in some circuit configurations of interest for PF compensation.

**Fact 4** The cyclo-dissipativity condition  $\langle \hat{i}, v \rangle \geq 0$  has a clear energy interpretation for the case of linear  $L, C$ , but possibly nonlinear resistors elements. We do not pursue any further this topic here and only mention that for general nonlinear RLC circuits: *magnetic energy “much larger” than electrical energy  $\Rightarrow$  cyclo-dissipativity with supply rate  $\hat{i}^\top v$ , electrical energy “much larger” than magnetic energy  $\Rightarrow$  cyclo-dissipativity with supply rate  $\hat{i}^\top \hat{v}$*  [18].

#### 4. Concluding remarks

The main contribution of the paper is the proof that a certain cyclo-dissipativity property of the compensated load, namely (5), is *necessary for PF improvement*. This important observation suggests an analysis and compensator design framework based on cyclo-dissipativity, which is a natural alternative candidate to replace (standard) dissipativity for applications where we are interested in inducing periodic orbits, instead of stabilizing equilibria.

Although the framework applies for general polyphase—possibly unbalanced—circuits, for the sake of clarity, we have presented in some detail the problem of PF compensation with LTI capacitors or inductors of single phase loads only. It is our belief that the full power of the proposed approach will become evident for polyphase unbalanced loads with (possibly nonlinear) general lossless compensators, where the existing solutions are far from satisfactory [3]. It is not clear at this point whether

<sup>3</sup>basically, this means that all current-controlled and voltage-controlled resistors are in series with inductors and in parallel with capacitors, respectively.

<sup>4</sup>A similar quadratic form, but with a more complicated “matrix”, is obtained for the case of *nonlinear inductors and capacitors* [18].

this, unquestionably elegant, setting will yield practical solutions. The main obstacle been that in this application very little is known about the nature of the load—a piece of information that is essential for a successful design. Also, it would be highly desirable to formulate a clear parameter minimization problem, in particular, for other circuit topologies and poly-phase loads, where questions like compensator circuit complexity, existence of the optimal solution and sub-optimality could be addressed.

#### References

- [1] D. Luenberger, *Optimization by Vector Space Methods*, Wiley & Sons Inc, New York, 1969.
- [2] J.G. Kassakian, M. F. Scholecht, and G.C. Verghese, *Principles of Power Electronics*, Series in Electrical Engineering, Adison Wesley, 1991.
- [3] H. Lev-Ari and A. Stankovic, “Hilbert space techniques for modeling and compensation of reactive power in energy processing systems”, *IEEE Trans Circ Syst-I*, vol. 50, no. 4, pp.540–556, April 2003.
- [4] R. Ortega, A. Loria, P. J. Nicklasson and H. Sira-Ramirez. *Passivity-Based Control of Euler-Lagrange Systems*. Communications and Control Engineering. Berlin: Springer, September 1998.
- [5] R. Sepulchre, M. Jankovic and P. Kokotovic, *Constructive Nonlinear Control*, Springer-Verlag, London, 1997.
- [6] A.J. van der Schaft,  *$\mathcal{L}_2$ -Gain and Passivity Techniques in Nonlinear Control*, Springer-Verlag, London, 2000.
- [7] A. Astolfi, R. Ortega and R. Sepulchre, “Stabilization and disturbance attenuation of nonlinear systems using dissipativity theory”, *European Journal of Control*, vol. 8 no. 5, 2002.
- [8] R. Ortega and E. Garcia-Canseco, “Interconnection and damping assignment passivity-based control: A survey”, *European Journal of Control*, vol. 10, no. 5, pp. 432-450, December 2004.
- [9] E. Garcia-Canseco and R. Ortega, “A new passivity property of linear RLC Circuits with application to power shaping stabilization”, *American Control Conference (ACC04)*, June 30–July 2, 2004, Boston, MA, USA.
- [10] D. Jeltsema, R. Ortega and J. Scherpen, “On passivity and power-balance inequalities of nonlinear RLC circuits”, *IEEE Transactions on Circuits and Systems*, vol. 50, no. 9, pp. 1174–1179, September 2003.
- [11] R. Ortega, B.E. Shi, “A note on passivity of nonlinear RL and RC circuits”, *15th IFAC World Conf*, Barcelona, Spain, July 21–26, 2002.
- [12] E. Garcia-Canseco, D. Jeltsema, R. Ortega and J.M.A. Scherpen, “Characterizing inductive and capacitive nonlinear RLC circuits: A passivity test”, *Proc. IFAC Symposium on Systems, Structure and Control*, Oaxaca, Mexico, December 8-10, 2004.
- [13] D.J. Hill and P.J. Moylan, “Dissipative dynamical systems: basic input-output and state properties”, *Journal of the Franklin Institute*, vol. 309, no. 5, pp. 327–357, May 1980.
- [14] W. Shepherd and P. Zand, *Energy Flow and Power Factor in Nonsinusoidal Circuits*, Cambridge University Press, 1979.
- [15] C.H. Page, “Reactive power in nonsinusoidal situations”, *IEEE Trans. Instrum. Meas.*, vol. IM-29, pp. 420–423, December 1980.
- [16] L.S. Czarnecki, “A time domain approach to reactive current minimization in nonsinusoidal situations”, *IEEE Trans. Instrum. Meas.* vol. 39, no. 5, October 1990.
- [17] P. Penfield, R. Spence and S. Duinker, *Tellegen’s Theorem and Electrical Networks*, MIT Press, 1970.
- [18] D. Jeltsema, E. Garcia-Canseco, R. Ortega and J.M.A. Scherpen, “Towards a regulation procedure for instantaneous reactive power”, *Proc. 16th IFAC World Congress*, Prague, Czech Republic, July 4-8, 2005.