On the Vaneless Space Vortex Structures in a Kaplan Turbine Model Operating at Speed No Load

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Abstract: The growing installation of intermittent renewable energy sources is forcing hydraulic turbines to work more frequently at speed no load when dangerous vaneless space vortex structures and stochastic flow phenomena can occur. An experimental campaign has been performed in a reduced-scale Kaplan turbine model at speed no load for different combinations of guide vane and runner blade angles under non-cavitation and cavitation conditions. Several simultaneous vaneless space vortex structures, all of them inducing torsional rotor vibrations, have been observed. Nonetheless, only one of them has been found to dominate over the rest depending on the blade and guide vane angles. Off-board pressures, torques and vibrations as well as on-board strains have been measured to characterize their nature, intensity, dynamic behavior and induced structural response. Their precession frequencies have been found to depend on the discharge factor, the number of vortices and their location inside the vaneless space. Under cavitation conditions, their dynamic behavior has not been significantly altered but the induced structural response has increased at the low-pressure side of the turbine.

Keywords: speed no load; vaneless space vortex structure; structural dynamic response; monitoring system; Kaplan turbine model

1. Introduction

The increase in the installation of new intermittent renewable energy sources, such as solar and wind power, must be complemented with other energy sources, such as hydropower, able to stabilize the grid, storage energy and supply peak load. Nevertheless, these conditions force turbines to be operated for longer periods of time in off-design and transient conditions [1–4]. For instance, a hydraulic turbine under a grid stabilization mode works 4% of the time at speed no load (SNL) and 24% at deep part load (DPL), while it only works 1% at SNL and 0% at DPL under a base load mode [5]. Deleterious hydrodynamic phenomena, such as vaneless space vortex (VSV) structures and stochastic flow phenomena, occur at SNL and DPL, which may cause high-pressure fluctuations, damage the runner blades [3] and impose more regular refurbishment requirements [6,7].

At SNL, turbines rotate at nominal speed without generating power while the discharge is not zero. The hydraulic energy is dissipated through the generation of large-scale unsteady vortex structures that break down into smaller vortices until the energy dissipation occurs at the smallest scales [8].

The flow at SNL is primarily stochastic and provokes high dynamic stresses on runner blades of Francis turbines [5,8–15]. Most investigations focused on determining the relative fatigue damage inflicted on runner blades when operated at SNL or DPL compared to that induced at best efficiency point (BEP). SNL and DPL operation have been concluded to generate most fatigue damage due to stochastic flow phenomena [5,8,10–15]. Other researchers have found that in addition to stochastic excitations, specific Francis turbines...
also present periodic excitations induced by coherent vortex structures [16,17]. Magnoli et al. [18] simulated several operating points of a Francis turbine from full load (FL) to DPL and identified up to three distinct runner channel vortices occurring simultaneously at low discharges. In the same study, the results of pressure measurements on the blades and on the band of a reduced-scale Francis turbine runner showed that the channel vortices caused significant induced pressure fluctuations. Yamamoto et al. [19] investigated the development and the flow characteristics associated with the formation of inter blade vortices in a Francis runner at DPL numerically. It was concluded that the appearance of a backflow region in the vicinity of the hub near the trailing edge was closely linked with the inter-blade vortex development. This finding was also proved experimentally by the same group of researchers in later studies [20].

In axial turbines, SNL operation also presents stochastic flow phenomena and coherent vortex structures, which are usually originated in the vaneless space and cause low-frequency pressure fluctuations and significant vibrations [21–33]. Yang et al. [21] investigated numerically the origin of strong pressure fluctuations and vibrations in an axial turbine operated at SNL. They observed vortices and a distorted flow in the runner and concluded that the origin of these instabilities was a combined action of the centrifugal force and the low flow rate. Yamamoto et al. [22] investigated numerically the unsteady flow characteristics and flow instabilities in the SNL condition of a bulb turbine. A rotating vortex array was found below the runner which was composed by two vortices self-rotating and precessing at frequencies lower than the runner rotating frequency. It was concluded that the vortex array was the major source of pressure and torque fluctuations at SNL. Soltani et al. [23,24] measured strains and pressure on a runner blade of a Kaplan turbine prototype at SNL and detected low-frequency pulsations. Additionally, torsional strains on the shaft were found to have 12-fold larger peak-to-peak values at SNL than those at BEP. Iovanel et al. [25] investigated numerically a Kaplan prototype at SNL and compared the results with experimental measurements. The mean quantities were predicted more accurately in the pressure side of the runner blade than in the suction side. However, the amplitude of the torque fluctuations was underestimated by a factor of 30. Four vortex structures below the blades, which joined in pair of two further downstream, were observed. Melot et al. [26] studied numerically the start-up and SNL operation of a bulb turbine and compared the results with experimental measurements. The largest discrepancies were found at SNL since most phenomena at this condition were a consequence of turbulence while the flow inertia effects dominated in the first stage of the start-up. Jonsson et al. [27] studied the flow-induced excitations coupled to the shaft vibrations on an old and a refurbished Kaplan turbine at SNL. In order to investigate the origin of the vibrations, experimental measurements were performed on both the model and prototype turbines and on a twin unit with the old runner design. They concluded that the origin of the vibrations was a VSV structure whose frequency coincided with a natural frequency of the shaft line and a resonance occurred. It was also observed that the frequency of the flow disturbances depended on the runner blade opening. Pupitel et al. [28] investigated experimentally and analytically the vortices occurring in the vaneless space of Kaplan turbines at SNL. Air injection was employed to visualize the VSV structures, which were formed by a particular number of vortices precessing at a particular frequency. They found that these VSV structures may cause strong vibrations. Houde et al. [29–31] studied numerically the VSV structures occurring at SNL on an axial turbine. The vortices attached to the inner head cover were formed in the radial transition between the main flow and a backflow region, which was extended from the inner head cover into the draft tube. Additionally, the topology of these vortices was observed to be independent of the runner. Skotak et al. [32] performed measurements in a Kaplan turbine and carried out numerical simulations to investigate the VSV structures at DPL and SNL. Similarly, it was also concluded that the VSV structures were originated at the interface between the backflow region and the main flow. Nennemann et al. [33] studied the VSV structures generated at DPL on a diagonal, a propeller and a Francis turbine through experiments and numerical simulations. Vortex
were confirmed to be generated due to mainly backflow. In the case of the propeller turbine, large VSV structures were found in the numerical simulations although vibrations on the turbine prototype were within acceptable levels.

As introduced previously, the VSV structures are hydraulic phenomena of periodic nature and, consequently, they generate periodic pressure pulsations and they exert periodic forces on the turbine. Their analysis in the frequency domain is then appropriate. In the current work, the time domain signals were transformed to the frequency domain through Discrete Fourier Transforms (DFT) solved by means of Fast Fourier Transform (FFT) algorithms. The FFT is an appropriate and efficient approach to solve DFT reducing the computation effort significantly. For instance, the computation effort to solve DFT of signals with more than 512 discrete samples is reduced to a 1% using the FFT evaluation relative to the direct evaluation [34].

Most literature related to VSV structures in Kaplan turbines focused on their characterization when generated in a particular SNL condition. However, in order to deepen the insight on these hydraulic structures, there is the need to also study their induced structural responses on different components of the turbine as well as their dynamic evolution with the SNL condition and with the presence of cavitation. In this sense, an experimental test campaign measuring strains, torques, pressure and acceleration on on-board and off-board components of a Kaplan turbine model was carried out when operated in a wide range of SNL conditions with and without cavitation.

2. Experimental Setup

2.1. Test Stand Description and Instrumentation

A 1:3.875 reduced-scale replica of the Porjus Kaplan U9 prototype was selected for this study. Figure 1 shows the shaft line below the generator, a hydraulic bearing supporting the bottom of the shaft, the penstock, the spiral casing and the entire draft tube. The runner has 6 blades and a reference diameter, $D$, of 400 mm. The distributor is composed of 18 unequally distributed stay vanes and 20 equally distributed guide vanes. The vaneless space presents a height, $B_0$, of 165.6 mm and a width, $V_0$, of 480 mm as shown in the bottom of Figure 1.

The experimental setup comprised a large number of sensors installed on the rotor (on-board) and on fixed parts of the test stand (off-board) to study the dynamic response of the turbine in off-design operating conditions as presented in detail by Roig et al. [35]. Among them, the following sensors shown in Figure 2 were selected for the present study:

- 1 accelerometer on the draft tube cone wall in radial direction ($A_{0y}$).
- 1 pressure transducer on the vaneless space in axial direction ($P_{VS_A}$).
- 1 torque sensor on a guide vane stem (TGV).
- 1 strain gauge on the pressure side of a runner blade (SB$_R$).
- 1 strain gauge on the shaft measuring the torque strain (SS$_T$).

2.2. Measurement Program

Several combinations of guide vane angles, $\alpha$, and runner blade angles, $\beta$, were selected as presented in Table 1. It must be noted that $\beta = -17^0$ and $\beta = 10^0$ correspond to closed and open runner positions, respectively. Hence, the selected angles covered all the possible opening range in SNL conditions and they corresponded to the same cam curve with a speed factor, $N_{11}$, of:

$$ N_{11} = \frac{nD}{H^{0.5}} = 127.1 $$

where $H$ is the head and $n$ is the rotational speed. As shown in Table 1, the turbine was tested at eight different discharges without cavitation. Moreover, three conditions were also tested with cavitation. A sufficient time was waited for each test to reach stable and steady conditions prior to taking continuous measurements of approximately 5 min. The
corresponding discharge factor, \( Q_{11} \), and Thoma number, \( \sigma \), have been calculated using Equations (2) and (3):

\[
Q_{11} = \frac{Q}{D^2 H^{0.5}} \tag{2}
\]

\[
\sigma = \frac{NPSH}{H} \tag{3}
\]

\[
NPSH = \frac{P_2 - P_{vap}}{g \rho_2} + \frac{V_2^2}{2g} - (z_r - z_2) \tag{4}
\]

where \( Q, NPSH, P_{vap}, g \) and \( z_r \) are the discharge, the net positive suction head, the vapor pressure, the gravity and the height of the runner, respectively, and \( P_2, V_2, \rho_2 \) and \( z_2 \) are the reference pressure, flow velocity, density of the water and height at Section 2, respectively, as indicated in Figure 1. All the signals were sampled at 5000 Hz and recorded simultaneously. The signals of the on-board sensors were extracted during the turbine rotation through a telemetry system.

Figure 1. Schematic of the test stand with Section A-A and heights \( Z_2 \) and \( Z_r \) indicated (top). Schematic of the vaneless space with dimensions \( (B_0, V_0 \text{ and } D) \) (bottom left). Top view of section A-A (bottom right).
2.2. Measurement Program

Several combinations of guide vane angles, $\alpha$, and runner blade angles, $\beta$, were selected as presented in Table 1. It must be noted that $\beta = -17^\circ$ and $\beta = 10^\circ$ correspond to closed and open runner positions, respectively. Hence, the selected angles covered all the possible opening range in SNL conditions and they corresponded to the same cam curve with a speed factor, $N_{\text{rev}}$, of:

$$N_{\text{rev}} = \frac{nD}{H_0H} = 127.1 \ (1)$$

where $H$ is the head and $n$ is the rotational speed. As shown in Table 1, the turbine was tested at eight different discharges without cavitation. Moreover, three conditions were also tested with cavitation. A sufficient time was waited for each test to reach stable and steady conditions prior to taking continuous measurements of approximately 5 min. The corresponding discharge factor, $Q_{\text{rev}}$, and Thoma number, $\sigma$, have been calculated using Equations (2) and (3):

$$Q_{\text{rev}} = \frac{Q}{D^2H_0H} \ (2)$$

$$\sigma = \frac{N_{\text{rev}}} {PSH} H \ (3)$$

$$N_{\text{rev}} = \frac{P_{\text{rev}} - P_{\text{out}}}{g\rho_{\text{ref}}} + \frac{V_{\text{ref}}^2}{2g} - (z_{\text{ref}} - z_{\text{rev}}) \ (4)$$

where $Q$, $N_{\text{rev}}$, $P_{\text{out}}$, $g$ and $z_{\text{ref}}$ are the discharge, the net positive suction head, the vapor pressure, the gravity and the height of the runner, respectively, and $P_{\text{ref}}$, $V_{\text{ref}}$, $\rho_{\text{ref}}$ and $z_{\text{ref}}$ are the reference pressure, flow velocity, density of the water and height at Section 2, respectively, as indicated in Figure 1.

Compressed air or vacuum were applied on the water free surface in the downstream tank to change the NPSH. Then, a manometer and a mass flow meter were used to measure $P_2$ and $V_2$, respectively.

2.3. Methodology

The recorded time signals were converted to the frequency domain by means of the Fast Fourier Transform (FFT) with a Hanning window. This has permitted to identify the different frequencies associated with the VSV structures and to quantify their root mean
square (RMS) amplitude levels. Each signal was divided in five segments of 60 s each to average results and calculate standard deviations. The FFT was directly applied to all signals except for those of the accelerometers which were firstly integrated to obtain the vibration velocity.

Since the VSV structures are periodic phenomena dependent on the shaft rotating frequency, all the spectra were plotted as a function of the reduced frequency, \( f^* = f / f_n \), where \( f_n = 14.5 \) Hz is the turbine rotating frequency.

3. Results

3.1. Identification of the VSV Structures

A VSV structure is a hydraulic phenomenon originated in the vaneless space which is formed by a certain number of vortices, \( z_D \), ideally distributed equidistant to each other. The entire structure precesses at a particular frequency, \( f_0 \), resulting in all the vortices following a circular orbit around the rotor. Based on the work of Pålpetel et al. [28], the frequency of a particular VSV structure, \( f_S \), detected directly with an off-board sensor depends on the number of vortices of the structure, \( z_D \), and its precession frequency, \( f_0 \):

\[
f_S = z_D \cdot f_0
\]

The frequency detected with an on-board sensor, \( f_R \), depends on \( z_D, f_0 \) and \( f_n \), as shown in Equation (6), if the turbine rotates in the same direction as the precession of the VSV structure:

\[
f_R = z_D \cdot (f_n - f_0)
\]

But if the turbine rotates in opposite direction to the precession of the VSV structure, the frequency detected with an on-board sensor, \( f_R \), would depend on \( z_D, f_0 \) and \( f_n \) as follows:

\[
f_R = z_D \cdot (f_n + f_0)
\]

Combining Equations (5) and (6) or Equations (5) and (7), \( z_D \) can be isolated to obtain Equation (8) or Equation (9), respectively. Hence, \( z_D \) can be determined only knowing \( f_S, f_R \) and \( f_n \).

\[
f_R + f_S = z_D \cdot f_n
\]

\[
f_R - f_S = z_D \cdot f_n
\]

Figure 3 presents the spectra of the guide vane torque (TGV), the pressure (PVS\(_A\)), the blade strain (SB\(_R\)) and the shaft torque strain (SS\(_T\)) for \( \alpha \) equal to 3.98°. The TGV and PVS\(_A\) off-board spectra show two frequency peaks, each one corresponding to a particular VSV structure. Similarly, the SB\(_R\) and SS\(_T\) on-board spectra also show two frequency peaks, corresponding to the same particular structures when detected from a rotating frame of reference. The frequency peaks measured off-board, \( f_S \), and on-board, \( f_R \), can then be combined according to Equation (8) to calculate the number of vortices \( z_D \) forming the VSV structure. For instance, considering the first frequency peaks at \( f^*_S = 1.25 \) and \( f^*_R = 0.77 \), \( z_D \) equates to 2. Then, considering the second frequency peaks at \( f^*_S = 1.71 \) and at \( f^*_R = 1.28 \), \( z_D \) equates to 3. These results indicate that the first frequency peak is associated with a VSV structure formed by two vortices and the second peak with a VSV structure formed by three vortices which precess in the same direction as the turbine rotation. Throughout this article, the VSV structures with two vortices and three vortices will be named as 2V and 3V, respectively.
will be referred to as 4V. For the frequency peaks highlighted in purple, inducing low amplitudes can also be distinguished in the spectra shown in Figure 3. Parallel trends when plotted as a function of Q precession frequencies, \( f_p \), of the first frequency peaks at \( z_2 = 2 \) and \( z_2 = 3 \), respectively.

In addition to these dominant 2V and 3V structures, two additional frequency peaks with significantly lower amplitudes can also be distinguished in the spectra shown in Figure 4. For the frequency peaks highlighted in orange, \( z_D \) equates to 4, and the structure will be referred to as 4V. For the frequency peaks highlighted in purple, inducing low strains on the blades, \( z_D \) equates 6, and the structure will be referred to as 6V GV. Similarly, for the frequency peaks highlighted in green, inducing low torques on the guide vanes, again \( z_D \) equates 6, and the structure will be referred to as 6V R. Surprisingly, these two structures present the same number of vortices. Nevertheless, the 6V GV presents higher amplitudes than the 6V R in the TGV spectra, suggesting that it occurs closer to the guide vanes. Meanwhile, 6V R presents higher amplitude than the 6V GV in the SB R spectra, confirming that it occurs closer to the runner blades.

No integer values for \( z_D \) could be found when combining the frequencies observed, according to Equation (9), at \( f^*_S \) on the TGV spectra and at \( f^*_R \) on the SB R spectra, which confirms that there are not VSV structures precessing in the opposite direction to the turbine rotation.

### 3.2. Precessing Frequencies of the VSV Structures

Once \( z_D \) is calculated with Equation (8), the precession frequency, \( f_0 \), of each structure can be derived from Equation (5) or Equation (6). As seen in Figure 5, the computed reduced precession frequencies, \( f^*_0 \), corresponding to 2V and 3V follow approximately linear and parallel trends when plotted as a function of \( Q_{11} \). The 3V presents lower precession frequencies than the 2V, suggesting that the higher the number of vortices per structure is, the lower \( f_0 \) is.
amplitudes than the 6VB in the TGV spectra, suggesting that it occurs closer to the guide vanes. Meanwhile, 6VB presents higher amplitude than the 6V GV in the SB R spectra, confirming that it occurs closer to the runner blades.

No integer values for $z$ could be found when combining the frequencies observed, according to Equation (9), at $f^*$ on the TGV spectra and at $f^*$ on the SB R spectra, which confirms that there are not VSV structures precessing in the opposite direction to the turbine rotation.

(a)

(b)

Figure 4. Spectra of (a) TGV and of (b) SB at all SNL conditions.
### 3.2. Precessing Frequencies of the VSV Structures

Once \( z_o \) is calculated with Equation (8), the precession frequency, \( f_\text{\(\omega\)} \), is calculated as a function of \( Q_{11} \) and \( z_o \), as depicted in Table 2. The fact that the location of the VSV structures influences the frequency evolution.

Reduced precessing frequencies corresponding to each VSV structure at all SNL conditions.

The 4V and 6V\(_{GV}\) follow the same trend as the 2V and 3V when plotted as a function of \( Q_{11} \) but presenting lower precession frequencies, which confirms that the structures with higher number of vortices precess slower. On the contrary, the 6V\(_B\), apparently closer to the runner blades than the guide vanes, follow a different trend to the 2V and 3V, suggesting that the location of the VSV structures influences the frequency evolution.

Based on these two distinct frequency evolutions, one followed by 2V, 3V, 4V and 6V\(_{GV}\) and another one followed by 6V\(_B\), it can be concluded that the precession frequencies of all the VSV structures are primarily governed by the SNL condition, and in turn by \( Q_{11} \), and \( z_D \). Similarly, other authors also found that the frequency of the VSV structures depended on the blade opening [27]. Moreover, per equal \( z_D \), the 6V\(_B\) apparently closer to the runner blades presents always higher precession frequencies than those of the 6V\(_{GV}\) apparently closer to the guide vanes.

### 3.3. Excitation Type of the VSV Structures

Based on Equation (10), the VSV structure induces an axial or a torsional vibration when \( N = 0 \), and a bending or whirling vibration when \( N = \pm 1 \) [35]. This result depends on \( z_D \), the integers \( j \) and \( k \) and the number of runner blades, \( z_R \).

\[
N = k \cdot z_R - j \cdot z_D \quad (N = 0, \pm 1) \tag{10}
\]

Values of \( N \) were calculated through different combinations of \( k \) and \( j \) for 2V, 3V, 4V, 6V\(_{GV}\) and 6V\(_B\). None of these combinations resulted in \( N = 1 \) but mainly in \( N < -1 \) or \( N > +1 \) with certain specific combinations resulting in \( N = 0 \), as depicted in Table 2. The fact that all the VSV structures induce a torsional rotor vibration and that their frequency evolutions are similar suggests that all of them may be related to each other, with the 2V and 3V being the main structures and the 4V, 6V\(_{GV}\) and 6V\(_B\) secondary structures.

### Table 2. \( k \) and \( j \) combinations for each VSV structure giving a \( N = 0 \).

<table>
<thead>
<tr>
<th></th>
<th>2V</th>
<th>3V</th>
<th>4V</th>
<th>6V(_{GV})</th>
<th>6V(_B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( j )</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### 3.4. Dynamic Behavior of the 2V and 3V Structures

The following sections are only focused on the 2V and 3V structures because, as seen in Figure 4, they present higher intensities than the rest.
Figure 6a,b present a zoom around 2V and 3V in the PVSA spectra as well as the amplitudes of their induced pressure fluctuations, respectively. At small guide vanes, angles from 2.6° to 4.97°, the 2V peaks appear at \( f_0^* \) from 1.09 to 1.32 and the 3V peaks from 1.53 to 1.79, which increase in amplitude and frequency linearly. The fact that the 3V presents higher and sharper frequency peaks than the 2V, which are lower and wider, suggests that 3V could be the dominant structure at low discharges. At 6.8°, the amplitude of 2V drops drastically and 3V keeps its dominance showing a sharp amplitude increase. At 7.35°, whereas no pressure fluctuations induced by 2V are detected, the 3V peak shows a \( f_0^* \) of 1.95 and the highest amplitude equal to 0.0185 bar. Suddenly, the dominance switches and only the pressure fluctuations induced by 2V are measured at 7.86° and 8.13° with \( f_0^* \) values of 1.46 and 1.47 and maximum amplitudes around 0.021 bar.

![Figure 6a](image1)
![Figure 6b](image2)

**Figure 6.** (a) Spectra and (b) amplitudes of the frequency peaks of 2V and 3V, on PVSA at all SNL conditions.

The present findings agree with the observations of other authors who reported sudden decreases and increases in amplitude of different VSV structures when changing the SNL condition [18,33]. The novelty of the present work is the coexistence of VSV structures with different dynamic behaviors, amplitudes and locations. The simultaneity and the various similarities observed in the frequency evolutions and induced rotor vibrations between different VSV structures supports the hypothesis that they are not independent.
as considered by previous studies but related to each other. Depending on the discharge factor as well as the guide vane and runner blade angles: (i) both 2V and 3V coexist, (ii) 2V dominates over 3V or (iii) 3V dominates over 2V.

3.5. Induced Responses of the 2V and 3V Structures

Figure 7 shows the amplitude of the 2V and 3V frequency peaks detected in the SBR, SST and TGV spectra. A first observation is that the amplitude evolutions resemble those observed on the pressure signals, which confirms that they are induced by the same phenomena detected from different locations. Additionally, it must be noted that although most sensors do not detect the 2V at 7.35° and the 3V at 7.86° and 8.13°, the SBR and TGV spectra show small peaks confirming the coexistence of both structures at all SNL conditions and reinforcing the hypothesis that they are not independent. Among all strain measurements, the blade ones (SBR) present higher values than the shaft torque ones (SST) with a maximum of 0.55 µm/m induced by the 2V at 8.13° and 0.9 µm/m by the 3V at 7.35°. Similarly, the TGV measurements also present a maximum of 0.16 Nm induced by 2V at 7.86° and 0.21 Nm by 3V at 7.35°. It must be noted that the maximum induced responses by 3V present higher values than 2V, unlike as observed in the pressures.

Equation (11) [28,36] permits to calculate the frequency of a VSV structure, \(f_{S,R}\), when measuring its induced vibration transmitted from the rotating rotor with an off-board sensor, using \(f_S\), \(k\), \(Z_R\), \(f_n\) and \(j\):

\[
f_{S,R} = k \cdot Z_R \cdot f_n - j \cdot f_S
\]

(11)

Table 3 presents the values of \(f_{S,R}\) for 2V and 3V calculated using Equation (11) when \(\alpha\) is 2.6°, 3.98°, 6.8° and 7.35°.
Table 3. Reduced frequencies calculated with Equation (11) that should be detected from an off-board sensor measuring the vibrations transmitted from the rotor for 2V and 3V.

<table>
<thead>
<tr>
<th>α [°]</th>
<th>2.6°</th>
<th>3.98°</th>
<th>6.8°</th>
<th>7.35°</th>
</tr>
</thead>
<tbody>
<tr>
<td>2V</td>
<td>2.72 Hz</td>
<td>2.27 Hz</td>
<td>1.78 Hz</td>
<td>1.66 Hz</td>
</tr>
<tr>
<td>3V</td>
<td>2.96 Hz</td>
<td>2.57 Hz</td>
<td>2.16 Hz</td>
<td>2.08 Hz</td>
</tr>
</tbody>
</table>

Figure 8a,b present the spectra of the vibration velocity measured on the draft tube cone (A0\text{y}) and the amplitudes of the peaks induced by 3V at 2.6°, 3.98°, 6.8° and 7.35°, respectively. It must be noticed that 2V is not detected by A0\text{y}, suggesting that 2V is not well transmitted to the turbine supporting structure. However, 3V is detected at the same frequencies listed in Table 3. The amplitude evolution of the 3V peak correlates with those previously found for the pressures, strains and torques, as presented in Figures 6b, 7a, 7b and 7c, respectively. In this case, the frequency peaks associated with 3V present lower amplitudes than other sources of excitation, such as the mechanical unbalance at $f^* = 1$ and their harmonics, as shown in Figure 8a. These results differ from the vibrations measured in a Kaplan turbine prototype by Půlpitel et al. [28] showing high peaks dominated by the VSV structures but agrees with the results found by Nennemann et al. [33] concluding that the vibrations derived from VSV structures have low intensity compared with other sources of excitation.

3.6. Cavitation Effects on 2V and 3V Structures

Figure 9 permits to compare the TGV, PVS$_A$, SBR, SST and A0\text{y} spectra when the turbine is operated under non-cavitation and cavitation conditions. A first observation is that the frequencies of the 2V and the 3V under non-cavitation conditions are approximately equal to those measured under cavitation conditions. For instance, the 2V peaks show exactly the same frequency for all SNL conditions and the 3V peaks only present a slight frequency decrease of less than 0.25% at 7.35°. The VSV structures originate at the vaneless space and thus above the runner blades, at the high-pressure side of the turbine. However, cavitation usually occurs at the low-pressure side, below the runner blades. The frequency of the VSV structures may not be altered under cavitation condition because the amount of cavitation generated at the vaneless space might be low.

![Figure 8a](image_url)  
![Figure 8b](image_url)  
![Figure 8c](image_url)  

Figure 7. Amplitudes of the frequency peaks of 2V and 3V on (a) SBR, (b) SST and (c) TGV, at all SNL conditions.
Figure 8. (a) Spectra and (b) induced amplitudes of draft tube cone vibration velocity (A0y) at α equal to 2.6°, 3.98°, 6.8° and 7.35°.

The amplitudes of the 2V peaks at 3.98° under cavitation conditions are approximately equal to those found under non-cavitation conditions. However, the 2V induced amplitudes under cavitation conditions are consistently lower than those under non-cavitation conditions at 8.13°. Similarly, the amplitudes of the 3V peaks at 3.98° and 7.35° under cavitation are also consistently lower than those under non-cavitation for most of the signals. Although there is a low degree of cavitation generated in the vaneless space, the intensity of the VSV structures decrease when the turbine is operated under cavitation conditions.

Exceptionally, A0y shows similar 3V amplitudes under cavitation and non-cavitation conditions at 3.98° and a 3V amplitude increase with cavitation at 7.35°. This amplitude increase cannot be explained considering a lineal transmission from the hydraulic excitation in the vaneless space to the turbine supporting structure. A0y is mounted on the walls of the draft tube cone and thus at the low-pressure side of the turbine. In this part, the amount of cavitation generated is high and thus it probably has a significant impact on the modal and dynamic response of the draft tube cone. This unexpected amplitude increase could happen due to a change in the natural frequency of the draft tube cone, resulting in a resonance with the 3V excitation, or of the modal properties of the draft tube cone, increasing the dynamic response at the 3V frequency.
Figure 9. Spectra of (a) TGV, (b) PVSA, (c) SBR, (d) SST and (e) A0y, under non-cavitation (black) and cavitation (blue) conditions for $\alpha$ equal to 3.98°, 7.35° and 8.13°.

4. Conclusions

Five simultaneous VSV structures with different number of vortices and inducing torsional rotor vibrations were found in a reduced-scale Kaplan turbine model operating at SNL. Each structure precessed at a particular frequency in the turbine rotation direction, which depended on the turbine discharge factor, its number of vortices and its location. More specifically, the precession frequencies decreased with an increasing number of vortices and increased for closer distances of the structures to the runner blades. Among all the VSV structures observed, those formed by two and three vortices presented the highest pressure fluctuations, strains and torques. The similarities observed in the frequency evolutions and in the induced rotor vibrations among the five different VSV structures suggest that there is some interrelation among them.
When the turbine was operated under cavitation conditions, the precession frequencies of the dominant VSV structures were not significantly altered. Their induced responses at different parts of the turbine test stand slightly decreased with the exception of the draft tube cone where they increased significantly.

Then, the runner and the whole rotor should be designed having their natural frequencies far away from the VSV excitation frequencies and their harmonics to avoid dangerous resonances at SNL. Additionally, the machine should operate in the SNL condition presenting the less intense VSV structures in order to minimize the wear and tear generated at this operating point.

As a future work, numerical simulations replicating the experimental tests should be performed for a better understanding of this particular flow field. The numerical results will help to understand the morphology of these structures and the mechanism driving the change in precession frequency and intensity observed experimentally.

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