Decentralized Robust Frequency Regulation of Multi-terminal HVDC-linked Grids

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Abstract—This paper proposes a new strategy for optimal grid frequency regulation (FR) in an interconnected power system where regional ac grids and an offshore wind farm are connected via a multi-terminal high voltage direct-current (MTDC) network. In the proposed strategy, decentralized $H_{\infty}$ controllers are developed to coordinate the operations of synchronous generators and MTDC converters, thus achieving optimal power sharing of interconnected ac grids and minimizing frequency deviations in each grid. To develop the controllers, robust optimization problems are formulated and solved using a dynamic model of the hybrid MTDC-linked grids with model parameter uncertainty and decentralized control inputs and outputs. The model orders of the controllers are then reduced using a balanced truncation algorithm to eliminate unobservable and uncontrollable state variables while preserving their dominant response characteristics. Sensitivity and eigenvalue analyses are conducted focusing on the effects of grid measurements, parameter uncertainty levels, and communication time delays. Comparative case studies are also carried out to verify that the proposed strategy improves the effectiveness, stability, and robustness of real-time FR in MTDC-linked grids under various conditions.

Index Terms—Grid frequency regulation, $H_{\infty}$ infinity, hybrid MTDC converters, offshore wind farm, optimal power sharing.

I. INTRODUCTION

SUSTAINABLE wind energy is critical to reducing fossil fuel emissions and mitigating global climate change [1]. In particular, the capacity of offshore wind farms (OWFs) is growing faster for many reasons, including a lack of suitable onshore sites and better wind conditions of offshore sites [2]. Meanwhile, high-voltage direct-current (HVDC) technology has been increasingly considered a promising solution for delivering offshore wind power to onshore power grids and sharing it between interconnected regional grids [3], [4].

In HVDC technology, line-commutated converter (LCC) and voltage source converter (VSC) have been widely used due to their distinct characteristics. An LCC is characterized by, for example, a high capacity for bulk power transmission, low power loss, and low installation cost [5]. However, it operates only as a rectifier or an inverter that can control either dc voltage or dc current [6]. By contrast, a VSC can change the direction of dc current and independently control active and reactive power, although it has lower power rating and higher power loss even when implemented in the form of modular multilevel converter [7]. In several projects, there is a recent interest in combining the advantages of both LCC and VSC, developing hybrid schemes.

An HVDC system can be further developed in the form of a multi-terminal dc (MTDC) system [9]–[14], for example, where LCCs, grid-side VSCs (GSVSCs), and OWF-side VSCs (WFVSCs) are connected via a dc network to connect regional ac grids with OWFs. This enhances the flexibility of inter-grid power delivery and sharing, thereby facilitating the integration of large-scale OWFs with regional load centers. However, due to the interconnection, each grid frequency is affected by power imbalances not only in the corresponding grid but also in other grids [8], implying that intermittent wind power causes frequency deviations in all interconnected grids. This motivates the development of a proper strategy for power balancing in MTDC-linked grids, essentially requiring the coordinated control of regional power generation and ac-to-dc power transmission.

Several studies have been conducted on frequency regulation (FR) in MTDC-linked grids. For example, in [9], VSCs were droop-controlled to mitigate grid frequency deviations and dc voltage variations resulting from wind power generation. In [10], communication-free coordination of VSC terminals was accomplished using a $V_{dC}$-f droop control scheme. In [11], adaptive droop control of a VSC-MTDC system was discussed, wherein the $V_{dC}$-I-f characteristics of the system were analyzed to determine the droop control gains. However, in [9]–[11], FR relied mainly on droop control or, equivalently, primary frequency control, rather than optimal secondary frequency control (SFC).

The optimal FR of MTDC-linked grids has been discussed in recent studies (e.g., [12]–[14]). In [12], a model predictive control (MPC) algorithm was adopted for the optimal power sharing in a VSC-MTDC system, wherein a discrete-time optimization problem was formulated considering grid operating costs. In [13] and [14], the MPC-based power sharing was also achieved to minimize the frequency deviations in the MTDC-linked grids with only VSCs and LCCs, respectively. However, in [12]–[14], hybrid MTDC systems were not considered. Moreover, in general, MPC is computationally intensive, because an optimization problem for MPC needs to be solved directly at every time step. MPC is also likely to cause oscillatory and even unstable operation of MTDC-linked grids due to communication time delays and model parameter uncertainty [15]. Therefore, the optimal FR strategies discussed in [12]–[14] need to be further analyzed under practical conditions of MTDC systems, for example, with regard to hybrid converters, communications links, and system parameter estimates.

When system parameters are uncertain, robust controllers can effectively reduce the effect of disturbance on system operation. For example, in [16], the robust dc voltage control of a VSC was achieved by establishing an $H_{\infty}$ mixed sensitivity function in a linear matrix inequality framework, wherein dc current injection was set as a disturbance input. In [17], a systematic method was provided to design the $H_{\infty}$ controller of a VSC for optimal and
robust control of active power and ac voltage. In [18] and [19], \(H_1\) and \(H_2\) controllers were discussed to optimally regulate the dc-link voltage and the converter internal energy of an HVDC system, respectively. However, in [16]–[19], robust control was applied to only a single converter; the coordination with synchronous generators and the power sharing between regional ac grids were not considered. In [19], a centralized optimization problem was formulated for centralized and decentralized \(H_\infty\) control of the dc-link voltage in a modular multilevel converter when applied to a VSC-HVDC system. The centralized controller slightly outperformed the decentralized controller. However, in practice, such a centralized controller is difficult to implement and apply for MTDC-linked grids due to computational and communication overloads.

This paper proposes a new decentralized strategy for optimal robust FR in regional ac grids that are interconnected via a hybrid MTDC system including LCCs and VSCs. For each ac grid, a decentralized \(H_\infty\) controller is developed for coordinated control of generated and transmitted power, optimizing inter-grid power sharing and thus minimizing frequency deviations. Specifically, for the proposed strategy, a dynamic model of hybrid MTDC-linked grids is implemented with decentralized control inputs and outputs. Given the dynamic model, a robust optimization problem for the optimal FR is formulated considering model parameter uncertainty. The solution to the problem leads to the optimal gain of the decentralized \(H_\infty\) controller. A balanced truncation algorithm is then applied to reduce the controller model order, thus facilitating its practical implementation and the reference signal generation. Sensitivity and eigenvalue analyses are conducted to verify grid frequency stability in the proposed strategy, focusing on the effects of system parameters, parameter uncertainty, and communication time delay. Comparative case studies are also carried out to verify that the proposed strategy improves the effectiveness and robustness in reducing frequency deviations under various grid conditions.

The main contributions of this paper are summarized below:

- To the best of our knowledge, this is the first study reporting the optimal robust FR of MTDC-linked grids using decentralized \(H_\infty\) controllers. Each \(H_\infty\) controller is designed with reduced requirement on inter-grid measurements and communications.
- Hybrid MTDC-linked grids are modeled with decentralized control inputs and outputs. For each grid, considering model parameter uncertainty, an optimization problem is then formulated to determine the corresponding optimal robust control gains.
- The model orders of the optimal decentralized \(H_\infty\) controllers are reduced using a balanced truncation algorithm, facilitating practical applications of the proposed FR strategy.

- The proposed strategy is comprehensively evaluated via comparison to conventional decentralized PI and \(H_\infty\) controller-based strategies with regard to effectiveness and robustness against time-varying power imbalance, communication failures and time delays, and different weighting function models.

II. HYBRID MTDC-LINKED GRIDS

Fig. 1 shows a schematic diagram of the proposed decentralized strategy for optimal robust FR in MTDC-linked grids, where regional ac grids and OWFs are interconnected with each other using hybrid MTDC converters (i.e., VSCs and LCCs). Each grid includes multiple synchronous generators (SGs), loads, and ac transmission lines, and each OWF contains several wind turbine generators (WTGs) using permanent magnet synchronous generators (PMSGs). Note that for brevity, Fig. 1 illustrates...
an SG and a WTG. Moreover, for each WTG, a back-to-back converter and its maximum-power-point-tracking controller are modeled using simple equations for power balance [35]. For each regional grid, a decentralized control structure is developed to establish the optimal control inputs for the local controllers of the SGs and the MTDC converter and adjust the generated and transmitted power in a coordinated manner, while receiving measurements from the corresponding grid and interconnected grids via communication systems with time delays [37]–[39]. Specifically, the decentralized controller $k$ receives the measurements of $\Delta P_{dk}, \Delta V_{dk}, \Delta f_k, \Delta V_{mg_k}, \Delta P_k$, and $\Delta V_{dc_k}$ from grid $k$ and the measurements of $\Delta f_j$ and $\Delta V_{dc_j}$ from interconnected grids $j \neq k$, as shown in Fig. 1 and discussed in Appendix.

A. Modeling of Hybrid MTDC-Linked Grids

Without loss of generality, this paper considers three regional ac grids and one PMSG-type OWF, all of which are linked to a dc network via two GSCVs, one LCC, and one WVFSC. Specifically, Grids 1 and 2 are interfaced using GSCVs and Grid 3 is connected with LCC at the points of common coupling (PCCs): i.e., $n = 2$ and $m = 3$ in Fig. 1. The GSCVs are equipped with local $P-V_{dc_k}$ droop controllers, so that power transmitted from Grids 1 and 2 are instantaneously adjusted according to variations in the corresponding dc terminal voltages. On the other hand, the LCC operates as a rectifier with a local controller that transmits constant or time-varying power from the regional grid to the dc network. Note that the modeling and control procedures, discussed in Sections II and III, can readily be applied to a general case with more than three regional grids.

A dynamic model of the hybrid MTDC-linked grids is implemented by integrating the individual models of the ac grids, OWF, and dc network, as discussed in Appendix, as:

$$\dot{X} = A_C \cdot X + B_r \cdot r + B_w \cdot w_d + E_c \cdot Z_{dc_k},$$

$$Y = C \cdot X,$$  

where $A_c = diag(A_1, A_2, A_3, A_{owf}, A_{dc}),$  

$$B_r = \begin{bmatrix} B_{r1} & 0 & 0 & 0 \\ B_{r2} & B_{r3} & 0 & 0 \\ 0 & 0 & B_{n1} & 0 \\ 0 & 0 & 0 & B_{n2} \end{bmatrix},$$  

$$B_w = \begin{bmatrix} B_{w1} & B_{w2} & 0 & 0 \\ 0 & 0 & B_{w3} & B_{w4} \end{bmatrix},$$  

$$C = \begin{bmatrix} C_1 \cdot O & O & 0 & 0 \\ C_2 \cdot O & 0 & O & 0 \\ C_3 \cdot O & 0 & O & 0 \end{bmatrix},$$  

$$E_c = diag(E_{1}, E_{2}, E_{1}, E_{owf}, E_{dc}),$$  

$$X = [X_1^T, X_2^T, X_3^T, X_{owf}^T, X_{dc}^T]^T,$$  

$$Y = [Y_1^T, Y_2^T, Y_3^T, Y_{owf}^T]^T,$$  

$$r = [r_1^T, r_2^T, r_3^T]^T,$$  

$$w_d = [\Delta P_{L1}, \Delta P_{L2}, \Delta P_{L3}, \Delta V_{dc}]^T,$$  

$$Z_{dc} = [I_{d1}, I_{d2}, I_{d3}, I_{d4}, V_{dc}]^T.$$  

The detailed procedure for (1)–(8) is provided in Appendix. In (6), $X_k$ includes the frequency $f_k$ and dc terminal voltage $V_{dc_k}$ for each grid $k$. Moreover, $X_{dc}$ includes the dc currents $I_{dc_k}$ [see (8)] that flow through the dc transmission lines, as:

$$I_{dc_k} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \cdot X_{dc} = F_{dc} \cdot X_{dc}.$$  

Similarly, $V_{dc}$ also can be extracted from $X_{dc}$ and $X_{owf}$ as $V_{dc} = F_{dc} \cdot [X_{dc}^T, X_{owf}^T]^T$. Then, $Z_{dc}$ can be represented using $X$ as:

$$Z_{dc} = \begin{bmatrix} F_{dc} \\ F_{dc} \end{bmatrix} \cdot X = F_{dc} \cdot X.$$  

Given (10), (1) can be expressed in a standard form as:

$$\dot{X} = (A_C + E_C \cdot F_{dc}) \cdot X + B_r \cdot r + B_w \cdot w_d,$$  

$$Y = C \cdot X.$$  

with coefficient matrices augmented as:

$$A_k = \begin{bmatrix} A & 0 \\ A_k & 0 \end{bmatrix},$$  

$$B_r = \begin{bmatrix} B_r \\ B_{kw} \end{bmatrix},$$  

$$B_w = \begin{bmatrix} B_{w1} & B_{w2} & 0 & 0 \\ 0 & 0 & B_{w3} & B_{w4} \end{bmatrix},$$  

$$C_k = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ C_2 & 0 & 0 & 0 \\ C_3 & 0 & 0 & 0 \end{bmatrix},$$  

$$E_{kc} = diag(E_{1}, E_{2}, E_{1}, E_{owf}, E_{dc}),$$  

$$X = [X_1^T, X_2^T, X_3^T, X_{owf}^T, X_{dc}^T]^T,$$  

$$Y = [Y_1^T, Y_2^T, Y_3^T, Y_{owf}^T]^T,$$  

$$r = [r_1^T, r_2^T, r_3^T]^T,$$  

$$w_d = [\Delta P_{L1}, \Delta P_{L2}, \Delta P_{L3}, \Delta V_{dc}]^T,$$  

$$Z_{dc} = [I_{d1}, I_{d2}, I_{d3}, I_{d4}, V_{dc}]^T.$$  

B. Decentralized Control Inputs and Outputs

The proposed decentralized controllers are developed in the form of output feedback controllers, as shown in Fig. 2, so that the optimal FR in MTDC-linked grids can readily be achieved in practice using the measurement of $Y_k$, rather than the estimation of $X_k$. In (13), $Y_k$ is decentralized to $Y_{tk}$ for the design of the proposed controller in grid $k$, discussed in Section III, as:

$$Y = [Y_{tk}^T, Y_{jk}^T]^T = C_{tk} \cdot X_k,$$  

$$[C_{tk}, C_{tk}, C_{tk}, C_{tk}] \cdot X_k, \ \forall \ k , \ j \neq k.$$  

In (16), $Y_{tk}$ and $Y_{jk}$ are the measurements delivered from grids $k$ and $j$, respectively, as shown in Fig. 2. For grid $k$, $C_{tk}$ and $C_{tk}$ are obtained by decentralizing the upper and lower parts, respectively, of $C_k$ in (14): i.e., $[C_k] = [C_{tk}, \ldots, C_{tk}, \ldots, C_{tk}]^T$ and $[O] = [C_{tk}, \ldots, C_{tk}, \ldots, C_{tk}]^T$. Moreover, in (16), $Y_{jk}$ is a sub-vector of $Y_{ek}$ and $C_{jk}$ is the corresponding sub-matrix of $C_{ek}$, because the design and activation of controller $k$ do not require all of the measurement data in $Y_{ek}$, as discussed in Section III-C.

Similarly, the controller outputs or, equivalently, the reference signals $r$ in (11) can be decentralized to $r_k$; i.e., $B_{r_e} \cdot r = \Sigma B_{r_e} \cdot r_k$. The reference signals $r_j$ generated by controllers $j \neq k$ are...
regarded as disturbances for controller \( k \). This implies that (12) can be represented from the perspective of controller \( k \), as:

\[
\begin{align*}
\dot{X}_k &= A_{mk} \cdot X_k + B_{mk} \cdot r_k + [B_{mk} \cdot B_{mk}] \cdot [w_d \ r_f] \ , \\
&= A_{mk} \cdot X_k + B_{mk} \cdot r_k + B_{mk} \cdot w_{dc} \ , \ \forall \ k \neq k.
\end{align*}
\]

III. OPTIMAL ROBUST FREQUENCY REGULATION

A. Design of an Optimal Decentralized \( H_\infty \) Controller

Fig. 2 shows a closed-loop model of MTDC-linked grid \( k \) with decentralized controller \( k \). A general time-domain model of the decentralized controller \( k \) can be established as:

\[
X_{dk} = A_{dk} \cdot X_{Ik} + B_{dk} \cdot Y_{IK},
\]

and, consequently, its frequency-domain model becomes:

\[
r_k = [C_{h_k} \cdot (sI - A_{hk})^{-1} \cdot B_{hk} + D_{hk}] \cdot Y_{IK}.
\]

Specifically, for the optimal robust FR in grid \( k \), the controller \( k \) minimizes the target performance outputs \( Z_{dk} \) and inputs \( Z_{dk} \) against disturbance \( d_k \). In the proposed strategy, \( Z_{dk} \) includes not only the weighted values of \( \Delta d_k \) and \( \Delta f_b \) \( dt \) as but also the weighted values of \( \Delta V_{dc} \) and \( \Delta f_b \) \( dt \) to prevent excessive variations in dc terminal voltages during the optimal FR. Similarly, \( Z_{dc} \) contains the weighted values of \( \Delta P_{ref}^{dc} \), \( \Delta P_{ref}^{dc} \), and \( \Delta V_{dc}^{ref} \) when grid \( k \) is linked via a VSC. For the case of an LCC, it consists of \( \Delta P_{ref}^{dc} \) and \( \Delta P_{ref}^{dc} \). Furthermore, \( W_c \), \( W_a \), and \( W_d \) are the weighting functions for \( Z_{dk} \) and \( d_k \), respectively, as:

\[
Z_k = \left[ \begin{array}{c} Z_{dk} \\ Z_{dc} \end{array} \right] = \left[ \begin{array}{c} \begin{bmatrix} -C_{ek} & O \\ O & W_c \end{bmatrix} \cdot X_k \\ r_k \end{bmatrix} = \left[ \begin{array}{c} [C_1 \ D_{12}] \cdot X_k \\ r_k \end{array} \right],
\]

and

\[
w_{dc} = W_c \cdot d_k,
\]

where \( C_{ek} \) is a sparse matrix with elements of ones to extract \( \Delta d_k \), \( \Delta V_{dc} \) \( dt \), \( \Delta f_b \) \( dt \), and \( \Delta V_{dc} \) \( dt \) from \( X_k \). Note that in (20), \( C_1 \) and \( D_{12} \) are additionally defined for brief notation. Moreover, in Fig. 2, \( \Delta_l(s) \) reflects the uncertainty in the model parameters of grid \( k \) [20], [21]; i.e., \( G_l(s)(1+\Delta_l(s)) \), rather than \( G_l(s) \). In time domain, \( \Delta_l(s) \) affects the accuracy of \( A_{ek}, B_{ek}, B_{ak}, \) and \( C_{ek} \) [see (16) and (17)]. This implies that the proposed decentralized controllers are developed considering uncertainty in the model parameter estimates of SGs, converters, and ac and dc transmission lines.

Given the weighting functions and parameter uncertainty, the model of the MTDC-linked grid with the decentralized control inputs and outputs [i.e., (16) and (17)] can be completed as:

\[
\begin{align*}
X_k &= A_k \cdot X_k + B_k \cdot d_k + B_{r2} \cdot r_k \ , \\
Z_k &= C_k \cdot X_k + D_{r2} \cdot r_k \ , \\
Y_{rk} &= C_k \cdot X_k,
\end{align*}
\]

where the notations are simplified as \( A_1 = A_k, B_1 = B_{r2} \cdot W_{dc}, B_2 = B_{r2} \cdot B_{r2}, \) and \( C_2 = C_{rk} \). In frequency domain, (21) is expressed as:

\[
X_k = (sI - A_1)^{-1} \cdot [B_1 \ B_2] \cdot [d_k \ r_k]^T.
\]

Using (24), (22) and (23) are then equivalently represented in the frequency domain as:

\[
Z_k = \{C_1 \cdot (sI - A_1)^{-1} \cdot [B_1 \ B_2] + [O \ D_{12}]\} \cdot [d_k \ r_k]^T,
\]

and

\[
Y_{rk} = C_2 \cdot (sI - A_1)^{-1} \cdot [B_1 \ B_2] \cdot [d_k \ r_k]^T.
\]

In other words, for the optimal robust FR, decentralized controller \( k \) [see (18) and (19)] should be designed to generate \( r_k \) that minimizes \( Z_k \) for all unknown \( d_k \) in (25), given the measurements \( Y_{Ik} \) in (26).

In the proposed optimal FR strategy, the decentralized control gain \( K_k(s) \) for grid \( k \) is determined to minimize the \( H_\infty \)-norm of the transfer function \( T_{zd,k} \) from \( d_k \) to \( Z_k \) by solving the robust optimization problem:

\[
\min \left\| T_{zd,k} \right\|_{H_\infty} < \gamma,
\]

where

\[
\left\| T_{zd,k} \right\|_{H_\infty} = \sup_{\omega} \tilde{\sigma}(T_{zd,k}(j\omega)),
\]

\[
T_{zd,k}(s) = G_{11}(s) + G_{12}(s)K_k(s) \cdot (I - G_{22}(s)K_k(s))^{-1}G_{22}(s),
\]

\[
G_{11}(s) = C_k \cdot (sI - A_k)^{-1} \cdot B_1,
\]

\[
G_{12}(s) = C_k \cdot (sI - A_k)^{-1} \cdot B + D_{12},
\]

\[
G_{22}(s) = C_k \cdot (sI - A_k)^{-1} \cdot B_1,
\]

and

\[
G_{22}(s) = C_k \cdot (sI - A_k)^{-1} \cdot B.
\]

In (27) and (28), \( T_{zd,k} \) can be obtained as (29) by substituting \( T_{zd,k} \) into (24)–(26). Moreover, \( \gamma \) represents a maximum singular value of \( T_{zd,k} \) and \( \gamma \) is the performance level. In this study, a \( \gamma \)-iteration method is adopted to solve the optimization problem (27)–(29), wherein \( \gamma \) is repeatedly updated to a smaller value until convergence [22]. For each value of \( \gamma \), there exists a stable \( H_\infty \) controller that satisfies (27) if and only if the three conditions hold [23] as:

\[
X_c = -A_kA_k^{\dagger} + C_kC_k^{\dagger}(\gamma^{-2}B_kB_k^{\dagger} + B_kB_k^{\dagger}) \geq 0,
\]

\[
Y_c = -A_kA_k^{\dagger} + B_kB_k^{\dagger}(\gamma^{-2}C_kC_k^{\dagger} - C_kC_k^{\dagger}) \geq 0,
\]

\[
\rho(X_cY_c)^{\dagger} < \gamma^2.
\]

In (30c), \( \rho(X_cY_c)^{\dagger} \) is the largest eigenvalue of \( X_cY_c \). When \( \gamma \) converges, the optimal control gain \( K_k(s) \) is then used (30), as:

\[
K_k(s) = \left[ \begin{array}{c} A_k^{\dagger}Z_c \ L_c^{\dagger} \\ 0 \end{array} \right],
\]

where

\[
A_k = A_k + \gamma^{-2}B_kB_k^{\dagger}X_c + B_kF_c + Z_cL_cC_c.
\]

\[
F_c = -B_k^{\dagger}X_c, \ L_c = -Y_cC_k^{\dagger}, \text{ and } Z_c = (I - \gamma^{-2}Y_cX_c)^{-1}.
\]

From (19) and (31), the time-domain control gains \( A_{mk}, B_{mk}, B_{mk}, \) and \( D_{mk} \) in (18) also can be obtained, which is omitted here.

In each grid, the decentralized \( H_\infty \) controller operates as the optimal secondary frequency controller that establishes extra power references to restore the grid frequency to a nominal value [40]. The extra references do not disturb the references of the droop controllers, because the optimization problem (27)–(29) is formulated using the dynamic model of the MTDC-linked grids with the droop control. This also implies that the \( H_\infty \) control gain \( K_k(s) \) inherently reflects the physical operating conditions and limits of the SGs, MTDC converters, and ac and dc networks. It further enables \( K_k(s) \) to be deterministically obtained and updated for changes in the grid operating conditions, unlike conventional non-optimal strategies wherein control gains need to be tuned using a trial-and-error approach.
B. Model Order Reduction of Decentralized $H_c$ Controllers

In general, a dynamic model of MTDC-linked grids [i.e., (12)–(15)] is implemented in high dimensions including several uncontrollable and unobservable variables particularly when the numbers of SGs, VSCs, LCCs, and WTGs are large. For the case of Grids 1–3, discussed in Section II-A, each decentralized $H_c$ controller is designed with a model order $r$ of 87. The full-order decentralized $H_c$ controllers are difficult to implement in practice due to high computational burden and high sensitivity to measurement noises. In this paper, for the practical implementation, a balanced truncation algorithm [24] is applied to reduce the model orders of the decentralized $H_c$ controllers while still preserving their dominant response characteristics. Specifically, the optimal control gains $A_{hk}, B_{hk},$ and $C_{hk}$ of the full-order $H_c$ controller $k$ are first converted to $\tilde{A}_{hk} = T^{-1}A_{hk}T$, $\tilde{B}_{hk} = T^{-1}B_{hk}$, and $\tilde{C}_{hk} = C_{hk}T$ using the balancing transformation matrix $T$, as:

$$\tilde{A}_{hk} = \begin{bmatrix} A_{hk,11} & A_{hk,12} \\ A_{hk,21} & A_{hk,22} \end{bmatrix}, \quad \tilde{B}_{hk} = \begin{bmatrix} B_{hk,1} \\ B_{hk,2} \end{bmatrix}. \quad (32)$$

$$\tilde{C}_{hk} = \begin{bmatrix} C_{hk,1} & C_{hk,2} \end{bmatrix}. \quad (33)$$

In (32) and (33), the matrix decomposition can be adaptively achieved considering the cumulative energy and number of Hankel singular values (HSVs). The reduced-order $H_c$ controller $k$ can then be determined as:

$$\hat{X}_{hk} = A_{hk,11} \cdot \hat{X}_{hk} + B_{hk,1} \cdot Y_{T_k} + r_k = C_{hk,1} \cdot X_{hk} + D_{hk} \cdot Y_{T_k}. \quad (34)$$

Fig. 3(a) shows the HSVs of the full-order and reduced-order models of the decentralized $H_c$ controllers for the MTDC-linked grids. The HSVs were obtained for the step response to the load demand variation by 0.1 pu in each grid, given the test conditions discussed in Sections IV-A and IV-C. Fig. 3(b) shows the corresponding cumulative energy curves. The results of the HSV analysis indicate that the energy magnitudes of the three decentralized $H_c$ controllers are still higher than 99.9% of the total energy when their model orders are reduced to $r = 26, 29$, and 13, respectively. This implies that the reduced-order models of the $H_c$ controllers can successfully reflect the dynamic operating characteristics of the original, 87th-order models.

In other words, the reduction of the model orders to 26, 29, and 13, respectively, only marginally affects the FR performance, while effectively mitigating the computational burden. Note that $r$ can be set to smaller values, considering the trade-off between controller performance and implementation complexity.

C. Sensitivity and Eigenvalue Analyses

The closed-loop system, shown in Fig. 2, was analyzed with respect to the gain sensitivity and stability. Note that the analysis was conducted using reduced-order models of the three decentralized $H_c$ controllers, discussed in Section III-B. Specifically, Fig. 4 shows the optimal gains of the $H_c$ controllers for the GSVSC- and LCC-interfaced grids (i.e., $k = 1, 2, 3$) particularly with regard to the measurements from the corresponding grid and the interconnected grids: i.e., $B_{hk}$ and $D_{hk}$ for $Y_{T_k} = [Y_{T_k}^c Y_{T_k}^p]^T$ in Fig. 2 and (18). It represents the extent to which the measurements $Y_{T_k}$ affect the controller states $X_{hk}$ and the reference signals $r_k$ under disturbed grid conditions. It can be seen that the elements in $B_{hk}$ and $D_{hk}$ have values close to zero for all measurements from grid $j \neq k$, apart from the measurements of $\Delta f_j$ and $\Delta V_d j$. In other words, $X_{hk}$ and $r_k$ in each grid $k$ are mainly affected by the measurements of $\Delta f_j$ and $\Delta V_d j$ for grid $j \neq k$. This demonstrates that the decentralized $H_c$ controller for grid $k$ can be effectively designed only using the measurements of $\Delta f_j$ and $\Delta V_d j$ for grid $j \neq k$, thus mitigating the requirement for inter-grid measurement and communication systems and facilitating wide applications of the proposed strategy in practice.

In addition, Fig. 5 shows the eigenvalues of the closed-loop system with changes in system parameters, model uncertainty level, and communication time delay. In Fig. 5(a)–(c), the complex-conjugate eigenvalues move closer to the imaginary axis, implying increased overshoot and settling time, as the converter phase inductance $L_d$, the turbine time constant $T_{th}$, and the stator inductance $L_s$ increase from 0.03 mH to 1.50 mH, from 4 s to 10 s, and from 0.01 pu to 0.30 pu, respectively. However, all eigenvalues are still placed in the left-hand half plane (LHP), verifying that the proposed decentralized $H_c$ controllers ensure grid frequency stability for large variations in the electrical and electromechanical dynamics of the MTDC-linked grids.

Analogously, Fig. 5(d) shows the loci of the eigenvalues when the uncertainty level $\Delta(s)$ of the system model parameters increases from 0% to 50%. The dominant eigenvalues are still located in the LHP for a large value of $\Delta(s)$ (i.e., up to $\Delta(s) = 47\%$). This confirms that the proposed $H_c$ controllers can secure...
Variation in $L_f$ from 0.03 mH to 1.50 mH, (b) $T_d$ from 4 s to 10 s, (c) $L_s$ from 0.01 pu to 0.30 pu, (d) $\Delta(s)$ from 0% to 50%, and (e) $T_s$ from 0 ms to 60 ms.

the stable operation of MTDC-linked grids even when there is relatively large uncertainty in the system parameter estimates. Moreover, Fig 5(e) shows the eigenvalue analysis for an increase in $T_d$ from 0 ms to 60 ms; for brevity, the delay is assumed to be constant and identical for all decentralized $H_s$ controllers. Given $T_d$ (18) changes to:

$$X_m(t) = A_{m, \ell} \cdot X_m(t) + B_{m, \ell} \cdot Y_m(t - T_d)$$

$$r_t(t) = C_{m, \ell} \cdot X_m(t) + D_{m, \ell} \cdot Y_m(t - T_d).$$

Therefore, in the frequency domain, (19) is replaced by:

$$r_t(s) = K_1(s) \cdot e^{-T_d s} \cdot Y_T(s),$$

where $e^{-T_d s}$ can be approximated as:

$$e^{-T_d s} \approx \frac{T_d^2 s^2 - 6T_d s + 12}{T_d^2 s^2 + 6T_d s + 12}.$$ 

In Fig. 5(e), with an increase in $T_d$, the complex-conjugate eigenvalues move toward the imaginary axis. When $T_d$ increases to approximately 51ms, the eigenvalues cross the axis and are placed on the right-hand half plane (RHP). It was reported in [25] that the time delay in common fiber optic communications for application to MTDC systems ranges between 1 ms and 10 ms. Thus, the proposed $H_s$ controllers can successfully guarantee the stability and robustness of the closed-loop system in practice.

IV. CASE STUDIES AND RESULTS

A. Test System and Simulation Conditions

For comparative case studies, a dynamic model of the hybrid MTDC-linked grids was implemented in MATLAB/SIMULINK (or, simply, SIMULINK). Each ac grid consisted mainly of SGs, transmission lines, regional loads, and a GSVSC or LCC station for the interface with the MTDC system. In SIMULINK, the hybrid MTDC converter was modeled using average circuit

<table>
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<th>Device</th>
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<td>PI gains of GFVSC</td>
<td>$K_p, K_i$</td>
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</tr>
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![Fig. 6. Continuous variations in the (a) loads $\Delta P_{i1-3}$ and (b) wind speed $\Delta V_w$.](image)
models with inner-loop and outer-loop feedback controllers in a dq frame. The dc transmission line between the MTDC converters was then modeled using a pi-section model, rather than a T-section model, due to its higher modeling accuracy [41]. An PMSG-type OWF model was also implemented connected to the dc network via a WFVSC. Table 1 shows the model parameters of the SGs, MTDC converters, ac and dc networks, and PMSG-type OWF, as well as the corresponding control gains at the local level [26]–[28]. Fig. 6 shows the load demand variations \( \Delta P_{Lk} \) for all grids \( k \), reflecting the scaled-down RegD signals [29] over a period of 200 s. It also shows the intermittent wind speed for the OWF.

Given the test bed parameters and operating conditions, the proposed decentralized \( H_c \) controllers were developed with no loop shaping (i.e., unity weighting factors) [20] and analyzed in comparison with conventional decentralized controllers [19].

Table II shows the main features of the proposed (Case 1) and conventional strategies (Case 2 and 3). In Case 2, a decentralized PI controller was adopted to regulate the regional grid frequency deviations without the coordination between SGs and MTDC converters [30]. In Case 3, a centralized optimization problem was formulated to determine the decentralized control gains. Specifically, the centralized \( H_c \) control gains were first obtained from the solution to the optimization problem and, consequently, the decentralized controller for grid \( k \) was implemented by arbitrarily setting the off-diagonal elements of the centralized gains to zero, apart from the elements corresponding to \( Y_{E_k} \) and \( Y_{P_j} \) for \( j \neq k \), shown in (15). This leads the decentralized control for Case 3 to become non-optimal.

### B. Validating the Dynamic Model of MTDC-linked Grids

Fig. 7 shows comparisons between the responses of two models of the MTDC-linked grids to a step increase in \( \Delta P_{L1} = 0.1 \) pu and \( \Delta P_{L2} = 0.15 \) pu at \( t = 45 \) s: i.e., the small-signal model and the comprehensive SIMULINK model, discussed in Sections II and IV–A, respectively. The small-signal model was developed in a linear time-invariant form, whereas the SIMULINK model was implemented in a nonlinear time-varying form. Note that the comparisons were conducted only for MTDC-linked grids without the proposed controller. In Fig. 8, the dynamic responses of the two models were very similar to each other for each profile of the grid frequencies \( f_{1-3} \) and the average dc voltage \( V_{dc} \) for all MTDC terminals. This indicates good consistency between the two models even when relatively large variations in load demands occur in the MTDC-linked grids under normal conditions. It hence validates the accuracy of the results obtained by applying the proposed and conventional controllers to the small-signal dynamic model. Moreover, Fig. 7(a)–(c) shows that in each grid, the frequency fell off and then reached to a steady state value with a large overshoot and settling time. This confirms the motivation of this study on the coordinated FR of MTDC-linked grids. Note that \( f_3 \) was maintained at 60 Hz in the steady state, because the LCC operated with a constant power reference, unlike the GSVSCs with P-V\(_{dc}\) droop controllers.

### C. Comparisons for Stepwise Load Variations

Fig. 8 shows \( f_{1-3} \) and \( V_{dc} \) for the step responses of the MTDC-linked grids to load variations \( \Delta P_{Lk} \), which increased by 0.3 pu for all grids \( k \) at \( t = 10 \) s over a 10 s period. Table III lists the corresponding numerical results. In the proposed strategy (i.e., Case 1), the frequency deviations became smaller for all grids \( k \) than in the conventional strategies (i.e., Cases 2 and 3).

Specifically, Case 1 led to a decrease in the sum of the maximum frequency deviations (i.e., \( \Sigma|\Delta f|_{\text{max}} \)) by 75.1% and 11.1% compared to Cases 2 and 3, respectively. Moreover, in Case 1, \( f_{1-3} \) were restored back to the nominal values more rapidly, and had smaller overshoot responses than in Cases 2 and 3. Case 1 also resulted in smaller \( |\Delta V_{dc}|_{\text{max}} \) by 17.7% and 44.1% than Cases 2 and 3, respectively, as shown in Fig. 8(d). In other words, the proposed decentralized \( H_c \) controllers were more effective in reducing the frequency deviations in all the MTDC-linked grids while further mitigating the dc voltage variations. The improvement was made only using the measurements of \( \Delta f_j \) and \( \Delta V_{dc,j} \), rather than all the measurements in grid \( j \neq k \), thus lessening the requirement on inter-grid sensing and communication systems.

In addition, Fig. 9(a) shows the profiles of the total power generation in each grid for the stepwise load variations. Case 1 led to only moderate variations in the generated power. This is also the case for the power transmitted by the MTDC converters, as shown in Fig. 9(b). The comparison results demonstrate that the improved performance of Case 1 is attributable to the optimal, coordinated power sharing among the MTDC-linked grids through the dc transmission network in transient and steady states, rather than using any extra power source and via excessive variations in the generated and transmitted power.

### D. Comparisons for Continuous Load and Wind Variations

The proposed FR strategy was also tested with regard to continuous variations in the grid load demands \( \Delta P_{L1-3} \) and wind speed \( \Delta V_w \) over a time period of 200 s, as shown in Fig. 6. Fig. 10 shows \( f_{1-3} \) and \( V_{dc} \) for the responses of the MTDC-linked grids to \( \Delta P_{L1-3} \) and \( \Delta V_w \), and Table IV lists the numerical results as root-mean-square (rms) values. Each rms value was calculated as \( \langle \Sigma|\Delta f_j|_r^2/L \rangle^{1/2} \) and \( \langle \Sigma|\Delta V_{dc,j}|_r^2/L \rangle^{1/2} \), where \( L \) is the index of the measurement samples and \( L \) is the total number of samples. Case 1 led to significantly smaller variations in the frequencies and dc terminal voltages for all the grids than Cases 2 and 3. Specifically, the sum of \( \Delta f_{1-3,\text{rms}} \) in Case 1 was 92.5% and 62.5% smaller than those in Case 2 and 3, respectively. Moreover, in Case 1, \( \Delta V_{dc,\text{rms}} \) was reduced by 66.7% and 75.0%, compared to Cases 2 and 3. The case study results consistently verify that the
CASE 20

In continuous variations in $\Delta V_{dc}$, the proposed decentralized $H_\infty$ controllers more effectively improve the frequency recovery of the MTDC-linked grids while further suppressing the dc network voltage variations through optimal, coordinated sharing of the time-varying power outputs of the SGs and OWF.

**E. Effects of Communication Failures and Time Delays**

Additional case studies were conducted to evaluate the performance of the decentralized $H_\infty$ controllers in Cases 1 and 3, particularly, under different conditions of the communication systems for the MTDC-linked grids. Note that Case 2 requires no inter-grid measurements and communications, so the effects of different communication conditions were not analyzed here. Fig. 11 represents five different conditions of the inter-grid communication systems: i.e., no failure, single-link and double-link failures, and no active communication. When the communication between grids $k$ and $j$ fails, the decentralized $H_\infty$ controller for grid $k$ generates the optimal reference signals $r_k$ without receiving the $\Delta f_j$ and $\Delta V_{dc,j}$ from grid $j \neq k$. Fig. 12 shows $f_1$ and $V_{dc}$ for the step responses of the MTDC-linked grids to $\Delta P_{dc}$ = 0.1 pu for all $k$ at $t = 10$ s, and Table V lists the corresponding numerical results. In Fig. 12(a), Case 1 enabled the smaller deviation of $f_1$ (and also $f_2$ and $f_3$) and the faster recovery back to the nominal value with smaller overshoot responses for all the communication conditions than Case 3 in Fig. 12(b).

Fig. 12(a) shows that in Case 1, it took approximately 3.0 s for the frequency recovery, and the recovery time is relatively smaller than in the existing SFC schemes for the real-world SG-only grids [40], but similar to the case of the previous SFC schemes for the power-electronics-interfaced grids (e.g., [12] and [17]). The improved performance of Case 1 is mainly because the optimal gains for the decentralized $H_\infty$ controllers were directly obtained by solving the decentralized optimization problems (27)–(29). This stands in contrast to Case 3 where the non-optimal gains were acquired from the centralized optimization problem. In other words, the inter-grid communication
failure more affects the non-optimal controllers, thus leading to larger and longer overshoot responses of $f_1$ and $V_{dc}$. The case study results confirm that the proposed decentralized $H_c$ controllers are still more effective and robust in the optimal FR against the events of inter-grid communication failure.

In addition, Fig. 13 compares the step responses of $f_2$ and $f_3$ for variations in $\Delta P_{L_{1-3}}$ by $\pm 0.1$ pu over every 40 s between Cases 1 and 3, when increasing $T_d$ gradually from 20 ms to 50 ms. Table VI lists the corresponding numerical results. Note that for brevity, $T_d$ was assumed to be the same for all the decentralized $H_c$ controllers, as discussed in Section III-C. As $T_d$ increased, Case 3 led to larger $\Sigma \Delta f_L_{\max}$ than Case 1. Moreover, in Case 3, $T_d = 50$ ms led to unstable, large oscillations of $f_2$ and $f_3$; this was also the case for $f_1$. It was mainly because in Case 3, the centralized $H_c$ control gains were simply divided for the decentralized controllers by setting the off-diagonal elements to zero, which reduced the maximum allowable value of the communication delay time. In other words, the eigenvalues moved across the imaginary axis from the LHP to the RHP when $T_d$ increased to 50 ms. By contrast, Case 1 still ensured stable, smaller variations in $f_2$ and $f_3$ for all $T_d$ ranging up to 50 ms, although the damped oscillations occurred for large $T_d$. The comparison results prove that the proposed strategy still further enhances the performance and stability of the FR in the MTDC-linked grids where there are relatively large time delays in the communication systems.

### F. Effects of Dynamic Responses of SGs and VSCs

For Case 1, Fig. 14 shows $|\Delta f|_{\text{max}}$ and $(|\Delta f_{L}|_{\text{max}}+|\Delta f_{R}|_{\text{max}})/2$ due to stepwise load variation $\Delta P_{L_1}(t = 5^s) = 0.3$ pu when $T_f$ and $K_f$ range from 0.01 s to 0.10 s and from 0.1 s to 4.0 s, respectively. The SGs and VSCs respond slowly as $T_f$ increases and $K_f$ decreases, respectively. In general, as $T_f$ increases, $|\Delta f|_{\text{max}}$ increases for all $k$, verifying that the test bed and the $H_c$ controllers successfully reflect the dynamic responses and practical operating conditions of SGs. In addition, the dynamic responses of the VSCs affect the FR, because they transfer the power outputs of the SGs throughout the MTDC-linked grids. Moreover, Fig. 14 reveals that the fast dynamic responses of the VSCs mitigate the frequency deviation in the corresponding grid...
(i.e., Grid 1), but increase the deviations in other grids (i.e., Grids 2 and 3). In other words, the VSCs conflict with each other. The case studies verify the need for coordinated control of both SGs and VSCs, wherein the optimal amounts of power to be generated and transferred are simultaneously determined, with consideration of the different dynamic responses and operating conditions of SGs and VSCs. Note that for all \( T_g \) and \( K_p \), Case 1 still led to significantly smaller \(|\Delta f_{\text{rms}}|\) than Case 2 wherein no coordination among the SGs and VSCs was achieved.

G. Performance under Various Conditions

The continuous response tests in Section IV-D were repeated to analyze the FR performance of the proposed strategy under various conditions. In the proposed and conventional \( H_c \) control strategies, the only hyper-parameters are the weighting functions \( W_p, W_r, \) and \( W_d \) for the system inputs \( r_k \), outputs \( y_{k,M} \), and disturbances \( d_k \), respectively (see Fig. 2). Fig. 15(a) shows comparisons between Cases 1 and 3 when all the weighting functions were modeled as LPFs with a cutoff frequency \( \omega_c \) ranging from \( 10^5 \) rad/s to \( 10^7 \) rad/s, rather than the unity function. Similarly, Fig. 15(b) shows the comparison results when BPFs were adopted with a bandwidth \( B \) from \( 10^5 \) rad/s to \( 10^7 \) rad/s. For all different weighting functions, Case 1 led to considerably smaller \( \Sigma |\Delta f_{\text{rms}}| \) and \( \Delta V_{\text{dc, rms}} \) than Case 3. This verified the improved effectiveness and robustness of the proposed FR strategy when applied to practical grids with different local controllers of the SGs and MTDC converters and different characteristics of the input measurement data and disturbances.

In the test bed (see Fig. 1), Cases 1–3 were implemented using the measurements of both the generator rotor speeds \( \omega_g \) and the grid frequencies \( f \) at the PCCs of the MTDC converters. This aimed to further improve the FR performance for all Cases 1–3, because \( \omega_g \) and \( f \) differ from each other in transient state due to the distant locations between the SGs and MTDC converters and the large inertia moments of the SGs [42] and [43]. Table VII shows that even when \( \omega_g \) was not measured, Case 1 still led to smaller \( \Sigma |\Delta f_{\text{rms}}| \) and \( \Delta V_{\text{dc, rms}} \) than Cases 2 and 3, confirming the robustness and applicability of the proposed FR strategy.

For further comparisons of Cases 1–3, another test bed was implemented using a CIGRE benchmark system (i.e., CIGRE B4 DC Grid Test System) with modifications [44] and [45]. Briefly, the test bed consists of three regional ac grids with three GSVCSCs and one PMSG-type OWF with a WVFSC: i.e., \( n = 3 \) and \( m = 0 \). The length of each dc transmission line is 200 km and the GSVSC for Grid 1 is not directly connected with the WVFSC. Given the test bed, Table VIII shows the test results of Cases 1–3 for the continuous variations in \( \Delta P_{L1-3} \) and \( \Delta V_r \). In Case 1,

\[ \Delta f_{\text{rms}} \text{ and } \Delta V_{\text{dc, rms}} \text{ were still smaller than in Cases 2 and 3. This consistently verifies the effectiveness and applicability of the proposed } H_c \text{ controllers in real-world MTDC systems.} \]

The performance of the proposed optimal strategy was also analyzed in comparison with the conventional non-optimal strategy using a consensus algorithm (i.e., Case 4) [9]. Table IX shows the test results of Cases 1 and 4 for the continuous variations in \( \Delta P_{L1-3} \) and \( \Delta V_r \). Case 1 led to the significant reductions of \( \Sigma |\Delta f_{\text{rms}}| \) and \( \Delta V_{\text{dc, rms}} \) by 91.2% and 88.7%, respectively, compared to Case 4. The improved performance of

![Fig. 16. Computation time required to solve the optimization problems of Cases 1 and 3 for an increase in the number of the SGs in each MTDC-linked grid from 1 to 10.](attachment:fig16.png)

![Fig. 15. Comparison between the proposed and conventional } H_c \text{ controllers (i.e., Cases 1 and 3) for different weighting functions: (a) LPF and (b) BPF.](attachment:fig15.png)

![Fig. 14. (a) \(|\Delta f_{\text{max}}| \) and (b) \((|\Delta f_{\text{max}}|+|\Delta f_{\text{rms}}|)/2 \) for \( \Delta P_{L1}(t=5 \text{ s}) = 0.3 \text{ pu in Case 1.} \)](attachment:fig14.png)
Case 1 is attributable to the fact that the dynamic response capabilities of the SGs and MTDC converters could be fully exploited and optimally coordinated for fast and accurate compensation for the power imbalances in all grids k. By contrast, the control gains in Case 4 (and also in Case 2) need to be tuned using a trial-and-error approach and hence there is a high possibility that the fast operating capabilities of the MTDC converters are not effectively exploited.

In addition, the computational efficiency of the proposed strategy was evaluated while gradually increasing the number of the SGs in each MTDC-linked grid. Specifically, Fig. 16 shows the computation time required to solve each decentralized optimization problem [i.e., (27)–(29) for k = 1, 2, and 3] for Case 1. It also shows the computation time to solve the centralized optimization problem [19], as discussed in Section IV-A. In Case 1, the optimal $H_c$ control gains were determined within shorter computation time than in Case 3. The maximum computation time for Case 1 was estimated as 22.99 s when 10 SGs were included in each grid (i.e., 30 SGs in total), whereas it was 42.17 s for Case 3. Moreover, Case 1 became more time efficient for a higher number of the SGs, compared to Case 3. The case study results verified the computational efficiency and hence scalability of the proposed decentralized $H_c$ controllers. Note that in Cases 1 and 3, the $H_c$ controllers were designed offline and then executed online for the real-time FR, as in Case 2. The computational efficiency of Case 2 was not discussed here, because the PI control gains were determined using a trial-and-error approach, rather than the optimization problem solving.

V. CONCLUSIONS

This paper proposes a new strategy for optimal FR in MTDC-linked grids via coordinated, decentralized control of SGs and hybrid converters. A dynamic model of hybrid MTDC-linked grids was implemented considering the decentralized control inputs and outputs. Given the dynamic model, a robust optimization problem was formulated to develop a decentralized $H_c$ controller for each grid, considering the inter-grid communication systems and the model parameter uncertainty. A balanced truncation algorithm was applied to reduce the model orders of the centralized $H_c$ controllers, while still preserving their dominant response characteristics. Sensitivity and eigenvalue analyses were then conducted, focusing on the effects of inter-grid measurement, system parameter, uncertainty level, and communication time delay. The results of comparative case studies verified that the proposed strategy was more effective in reducing the frequency deviations in all MTDC-linked grids than conventional strategies, while maintaining smaller variations in dc network voltages. Moreover, compared to the conventional $H_c$ control strategy, the proposed strategy also improved the effectiveness, stability, and robustness in the optimal FR of hybrid MTDC-linked grids under various conditions.

APPENDIX

A. Modeling of AC Grids with Hybrid Converter Interfaces

A regional ac grid includes SGs with reheating turbine. For each SG, a synchronous machine is modelled using the 6th-order differential equations, discussed in [31]. A governor and exciter are modelled using the 1st-order transfer functions, based on [32]. The dynamic model of an SG can then be established as:

$$\dot{X}_g = A_g \cdot X_g + B_g \cdot r_g + E_g \cdot I_{bus}, \quad (A1)$$

where $X_g = [\Delta \theta_g, \Delta \omega_g, \Delta \delta_g, \Delta \Delta x, \Delta \Delta y]$, $r_g = [\Delta r, \Delta \omega_r]$,

$$\Delta x_{PV1}, \Delta x_{PV2}, \Delta x_{PV3}, \Delta V_{gd}$$.  

Furthermore, in each grid, a phase-locked loop (PLL) is used to measure the grid frequency at the PCC of an MTDC converter and calculate the dq values of the PCC voltages and currents [33]. Considering the PLL operation, an ac network is modeled as:

$$X_{ac} = A_{ac} \cdot X_{ac} + E_{ac} \cdot V_{ac} + B_{ac} \cdot I_{ac} + E_{ac} \cdot \mathbf{I}_{dc}, \quad (A3)$$

where $V_{ac} = [\Delta E_{d}, \Delta E_{q}]$, $L_{ac} = [\Delta L^d, \Delta L^q]$, and $w_{ac} = \Delta P_L$.

In an MTDC system, a VSC includes inner and outer controller loops with a $P-V_{dc}$ droop function. For an LCC, dc terminal voltage or current is controlled to transmit constant or time-varying power from an ac grid to a dc network. The dynamic models of a VSC and an LCC are given [33], [34] as:

$$\dot{X}_{vsc} = A_{vsc} \cdot X_{vsc} + B_{vsc} \cdot r_{vsc} + E_{bus} \cdot V_{bus} + E_{dc} \cdot \mathbf{I}_{dc}, \quad (A4)$$

$$\dot{X}_{lcc} = A_{lcc} \cdot X_{lcc} + B_{lcc} \cdot r_{lcc} + E_{bus} \cdot V_{bus} + E_{dc} \cdot \mathbf{I}_{dc}, \quad (A5)$$

where $X_{vsc} = [\Delta V_{dc}^d, \Delta V_{dc}^q, \Delta m_{iq}^c, \Delta m_{dq}^c]$, $X_{lcc} = [\Delta V_{dc}^d, \Delta V_{dc}^q, \Delta m_{iq}^c, \Delta m_{dq}^c]$, $r_{vsc} = [\Delta \omega_r, \Delta \omega_r]$, and $r_{lcc} = [\Delta \omega_r, \Delta \omega_r]$. By combining (A1)–(A6), the dynamic model of ac grid k is represented as:

$$\dot{X}_k = A_k \cdot X_k + B_k \cdot U_k + [E_k, E_k] \cdot [I_k, V_k]^T + E_k \cdot \mathbf{I}_{dc}, \quad (A7)$$

where $I_k = [I_{bus}^T, I_{dc}^T]$, and $V_k = [V_{bus}^T, V_k^T]$. Moreover, $[I_k, V_k]^T$ in (A7) can be equivalently expressed as:

$$[I_k, V_k]^T = [F_k \cdot X_k, E_k \cdot X_k]^T$, \quad (A9)$$

where $F_k$ and $E_k$ are sparse matrices to extract $I_k$ and $V_k$ from $X_k$. Substitution of (A9) into (A7) gives:

$$\dot{X}_k = A_k \cdot X_k + B_k \cdot r_k + B_k \cdot w_k + E_k \cdot \mathbf{I}_{dc}, \quad (A11)$$

where $X_k = [X_k^T, X_k^T]^T$, \quad $F_k = [\Delta \omega_r, \Delta \omega_r]$, \quad $w_k = \Delta P_{L}, \quad \forall k$, \quad (A15)
paper focuses on optimal FR. In the proposed and conventional $H_\infty$ control strategies, $\Delta \nu_{\text{avg}}$ still need to be measured for the feedback into the $H_\infty$ controllers, so that the device- and grid-level controllers can regulate $dq$-axis currents of the MTDC converters for the optimal power sharing with the SGs.

B. Modeling of a PMSG-type OWF and a DC Network

A PMSG-type OWF is modeled based on [35] and [36], as:

$$X_{\text{off}} = A_{\text{off}} X_{\text{off}} + B_{\text{off}} w_{\text{off}} + E_{\text{off}} F_{\text{dc}}, \quad (B1)$$

where $X_{\text{off}} = [\Delta I_d, \Delta I_q, \Delta I_{d1}, \Delta I_{q1}]$, and $w_{\text{off}} = \Delta V_{\text{dc}}$, (B2) and

$$\Delta I_{dc,i} = (-R_{dc,i} \Delta I_{dc,i} + (\Delta V_{dc,i} - \Delta V_{dc,j})) / L_{dc,i}, \quad (B3)$$

where $\Delta I_{dc,i}$ and $\Delta \nu_{dc,i}$ are the incremental dc current from bus $i$ to $j$ and the incremental dc voltage at bus $i$, respectively. Moreover, $R_{dc,i}$ and $L_{dc,i}$ are the dc resistance and inductance. According to the configuration, the dynamic model of a dc network is given as:

$$X_{dc} = A_{dc} X_{dc} + E_{dc} V_{dc}, \quad (B4)$$

where $X_{dc} = [\Delta I_{dc1}, \Delta I_{dc2}, \Delta I_{dc3}, \Delta I_{dc4}]$, and $V_{dc} = [\Delta V_{dc1}, \Delta V_{dc2}, \Delta V_{dc3}, \Delta V_{dc4}]$.  

REFERENCES


