



ENGIMATH ONLINE COURSE. EFFECTIVE FEEDBACK FROM UPC MATHEMATICS TEACHERS (CONCEPT)

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ABSTRACT

EngiMath is a 3 ECTS online course in engineering mathematics, in seven different languages, and it is the main and practical output of the ERASMUS+ project entitled “Mathematics online learning model in engineering education” in which the authors were participating. The course is integrated with Learning Management Systems such as Moodle and it is compatible with other platforms using Learning Tools Interoperability.

Once the project is finished, authors undertake, with the support of the Institute of Education Sciences at the Universitat Politècnica de Catalunya-BarcelonaTECH (UPC), an innovation project, **EngiMath@UPC**, with three practical objectives: a) to incorporate EngiMath into the teaching activity of the widest as possible range of students at UPC, b) to gather students and faculty feedback regarding the tracking of materials and their performance in the student training process, and c) to statistically

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analyze the data collected in order to validate and adjust the follow-up of the materials. In connection with the above mentioned objective b) a training course addressed to the math professors has been prepared at UPC.

The paper introduces, on the one hand, the EngiMath course as an online open educational resource for the academic community benefit, and on the other hand, analyzes the valuable feedback given by the training activity participants. Details on the online course implementation as well as main conclusions will be presented and discussed throughout the document.

1 THE ENGI MATH ERASMUS+ PROJECT

EngiMath, *Mathematics online learning model in engineering education*, is an ERASMUS+ innovation project granted in the 2018 call Cooperation for Innovation and the Exchange of Good Practices KA203 - Strategic Partnerships for higher education. The project (run from November 1st, 2018 to August 31st, 2021) aimed to create and develop shared medium for materials suitable for web-based assessment of mathematics for various types of engineering education curricula.

The EngiMath consortium was integrated by the following Higher Education institutions: Tallinna Tehnikakool (TTK UAS, Estonia, coordinator), Instituto Politecnico do Porto (IPP, Portugal), Universidad del País Vasco (UPV/EHU, Spain), Universitat Politècnica de Catalunya (UPC, Spain), Letterkenny Institute of Technology (LYIT, Ireland), Politechnika Koszalin (PK TUK, Poland) and Universitatea Tehnica Cluj-Napoca (UTC, Romania). Complete information on objectives, intellectual outputs, dissemination activities and contact can be found at <https://sites.google.com/tktk.ee/engimath>.

Contributions of the EngiMath project included a) a comprehensive report on pedagogical analysis for online assessment of mathematics for partner countries and needs of engineering mathematics, b) a 3 ECTS online course in engineering mathematics in seven different languages (English, Spanish, Portuguese, Catalan, Polish, Romanian and Estonian) and c) a mathematics online assessment model in order to engage the student voice more in the learning process [1-2].

2 THE ONLINE ENGI MATH COURSE

2.1 Introduction

The aim of this course is to promote the development of basic and structured knowledge and practical skills in the mathematical area of Linear Algebra, specifically in the sub-areas of Matrices and Matrix Calculus, Determinants and Linear Equations Systems related to Engineering. The direct target groups of the Project include students in engineering mathematics programs at higher educational institutions, academic staff teaching engineering mathematics in tertiary programs and research academics in the areas of technology-enhanced learning and online learning.

Stronger student engagement or improved student engagement are common instructional objectives expressed by Higher Educational Institutions. They aim, in particular, at reducing the early dropouts of Science, Technology, Engineering, and Mathematics (STEM) studies and to involve students in their own learning process. Finally, the EngiMath online course, implemented in Moodle and fully open, consists of a basic course on Linear Algebra composed by three topics: Matrices, Determinants and Linear Equations Systems. Firstly, an initial Introduction to Engineering Applications serves as a motivation. Secondly, each topic contains learning materials (26 Lessons), practice tests (2 Quizzes per Lesson) and at the end of each topic there is an Assessment Test. The final grade is the average of the three topic-specific assessments. At the end of the course the student is kindly requested to give his/her feedback for further development and improvement of the course. Fig. 1 allows to identify the components and the sequence of the course.

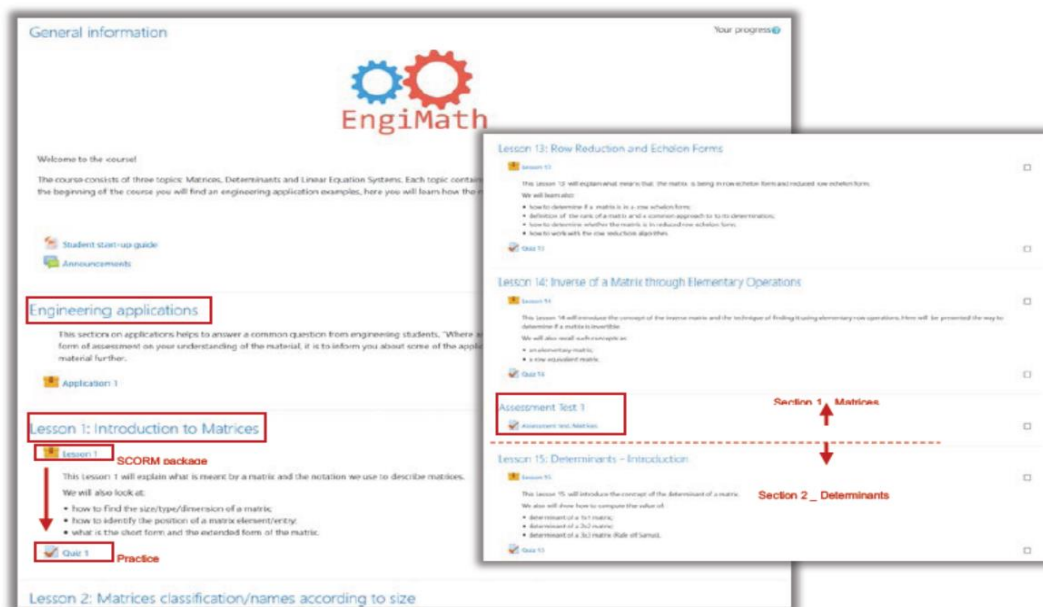


Fig. 1. Course sequence

2.2 Engineering Applications

In order to motivate and encourage students towards independent learning in mathematical topics at the beginning of the course there are some examples of Engineering Applications (in Civil, Electronic and Mechanical fields). On the one hand, they connect basic concepts of mathematics with relevant topics of Engineering. On the other hand, they propose and state contextualised problems which are solved stepwise using EngiMath tools and procedures.

2.3 Lessons

EngiMath course has adapted the PAR (Present, Apply, Review) model for online learning [3]. PAR structures the lesson by presenting new material, allowing the student to apply the learning followed by a review of the learning. This structure may be utilized several times within a lesson to maximize the learning potential.

Persistence and attrition factors were also taken into account when designing the

course [4]. The course adequately combines formative and summative aims in the e-assessment strategy [5].

The course is composed by 26 Lessons and is divided into three Blocks:

- Block I: Lessons 1–14. Matrices and Matrix Calculus
- Block II: Lessons 15–22. Determinants
- Block III: Lessons 23–26. Linear Equations Systems

All the theoretical concepts are presented in a very simple and comprehensive manner. Many examples are used to support the theoretical material. In total, there are more than 400 slides with over 3900 animations. Details in Fig. 2 and Fig. 3.

Lesson 1

Definition
The **Size** or **Type** or **Dimension** of a **Matrix**, or just the Matrix, is said to be **m-by-n** ($m \times n$) if it has **m** rows and **n** columns.

The size or type of a Matrix (or even Matrix Dimension), identifies directly its **number of rows** (the 1st to be stated) and **number of columns** (the 2nd)!

Examples...

Example 1 (2 rows and 3 columns): $\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \end{pmatrix}$

Example 2 (4 rows and 2 columns): $\begin{pmatrix} 1 & 0 \\ -1 & 2 \\ 4 & 3 \\ 7 & 10 \end{pmatrix}$

Example 3 (m rows and n columns): $\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$

Fig. 2. Slide of Lesson 1

Lesson 26

Example 2. Use Gauss elimination Method to solve the system:

$$\begin{cases} x_1 + 10x_3 = 5 \\ 3x_1 + x_2 - 4x_3 = -1 \\ 4x_1 + x_2 + 6x_3 = 1 \end{cases}$$

Just a curiosity
This system Graphic Solution shows that there is not a point of intersection for the three planes!
No Solution

Augmented Matrix: $\left[\begin{array}{ccc|c} 1 & 0 & 10 & 5 \\ 3 & 1 & -4 & -1 \\ 4 & 1 & 6 & 1 \end{array} \right]$

Row operations:
 $R_2 \leftarrow R_2 - 3R_1$
 $R_3 \leftarrow R_3 - 4R_1$

Equivalent system:
 $\begin{cases} x_1 + 10x_3 = 5 \\ x_2 - 32x_3 = -16 \\ 0 = -3 \end{cases}$

"IMPOSSIBLE" → **Inconsistent system: There is no solution**

We get the same result!... So... The "previous" rank analysis is "optional"!

Gauss Method (4 steps) 1. Augmented Matrix 2. Row Echelon 3. System 4. Backward

Fig. 3. Slide of Lesson 26

Furthermore, in each Lesson the student has the opportunity to check his/her skills using the "Try it" quizzes. These quizzes are composed of over 120 questions of different types: True or False, Multiple Choice, Matching, Correspondence, etc). Each question provides a step by step solution in order to help the student to arrive at the right answer. Fig. 4 and Fig. 5 show an example of a quiz and a quiz feedback.

Lesson 22

Given $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -3 & 1 \\ -1 & 0 & 0 \end{pmatrix}$ match all its following co-factors

$C_{12} = -$ $\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = -1$

$C_{21} = -$ $\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$C_{11} = -$ $\begin{vmatrix} 0 & -3 \\ -1 & 0 \end{vmatrix} = -3$

$C_{13} = -$ $\begin{vmatrix} -3 & 1 \\ 0 & 0 \end{vmatrix} = 0$

Fig. 4. "Try it" quiz

Question List

Question 1 of 5

Given $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -3 & 1 \\ -1 & 0 & 0 \end{pmatrix}$ match all its remaining co-f

Created with iSpring Suite evaluation version. Learn more

1. $C_{12} = -$ $\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = -1$

2. $C_{11} = -$ $\begin{vmatrix} 0 & -3 \\ -1 & 0 \end{vmatrix} = -3$

Partially Correct

You missed some match... 2

Please remember that:
 $C_{ij} = (-1)^{i+j} M_{ij} = \begin{cases} +M_{ij}, & i+j \text{ even} \\ -M_{ij}, & i+j \text{ odd} \end{cases}$

If there are still any doubts please see **LESSON 16**

CONTINUE >

Fig. 5. Example of quiz feedback

By using iSpring, the 26 powerpoint Lessons were transformed into e-courses, producing 26 SCORM packages that were uploaded into the Moodle platform.

2.4 Practices Materials

Regarding the practical materials, with the background experience applied to the frame of Erasmus+ project EngiMath, a huge question bank has been developed for the online engineering mathematics course.

Special attention was paid to the development of STACK (System for Teaching and Assessment using a Computer Algebra Kernel) questions and step-by-step teaching tasks that give students a certain logical sequence of mental actions that must be performed to solve the problem. Based on the objectives of the practical quizzes for each lesson, a testing model was compiled: a technological matrix containing competencies selected for practice and testing. For each competency, questions were created. Regarding the variability, in order to ensure the development of different practice experiences and tests, and also to allow randomness in the definition of the practical quizzes, each of these questions has, in average, 10 versions.

The question bank consists of more than 4500 questions, 2148 for practice in lessons and 2382 for assessment. There are close-ended type questions and open-ended type questions. Exhaustive feedback has been provided for each question (Fig. 6).

The screenshot displays the STACK quiz interface. On the left is a navigation menu with a tree structure: 'according to elements' restrictions (basic) > Lesson 4: Equal Matrices and Operations with Matrices - Matrix Addition > Lesson 4 > Quiz 4.1 > Quiz 4.2 > Lesson 5: Operations with Matrices - Multiplication by a scalar > Lesson 6: Operations with matrices - Matrix Multiplication.

Question 4: 'Solve for x: $\begin{pmatrix} 3 & -8 \\ 10 & 12 \end{pmatrix} - \begin{pmatrix} 4 & -x \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 12 \\ 15 & 11 \end{pmatrix}$ '. The student's answer is $x = 20$, which is marked as 'Correct' (1.00/1.00). A green checkmark is visible. The 'Worked solution' shows the equation $-8 - (-)x = 12 \Rightarrow x = 20$.

Question 5: 'Solve for a and b: $\begin{pmatrix} 2a & -1 \\ 3b & 5 \end{pmatrix} - \begin{pmatrix} 4 & 6 \\ 1 & 9 \end{pmatrix} = \begin{pmatrix} 10 & -7 \\ 8 & -4 \end{pmatrix}$ '. The student's answers are $a = 3$ and $b = 3$, which are marked as 'Partially correct' (0.50/1.00). A red 'X' is next to $a = 3$ and a green checkmark is next to $b = 3$. The 'Worked solution' shows $2a - 4 = 10 \Rightarrow 2a = 14 \Rightarrow a = 7$ and $3b - 1 = 8 \Rightarrow 3b = 9 \Rightarrow b = 3$.

Fig. 6. Quiz 4.2 and feedback

2.5 Assessment Tests

At the end of each Block student is invited to take a 90-minute Assessment Test in order to obtain the qualification of the corresponding topic. The grade of the Block is the maximum of those obtained in the three attempts allowed in the assessment test.

Assessment materials are a consequence and interconnected to the previous activities. As one important role of online technology is to facilitate teaching and promote learning and in the context of the EngiMath online course (or any other online assessment task) only the student/user can know if the learning requirements are being fulfilled. In this sense it is necessary to ensure a form of summative assessment (as well as already pointed out for the case of training in the previous section) that validates the work developed without too much entropy caused by external to the covered contents issues.



2.6 Course Feedback

Through a Google Form students are invited to give their feedback about different aspects of the course:

- general impression of the quality of theoretical, practice and assessment materials,
- the quality of graphics, the structure of the course and the presentation method of concepts,
- the level of difficulty of the presented materials and
- the interaction with the teacher.

Open questions on the three issues of the course they liked the most and the three issues that could be improved are also included.

3 TRAINING COURSE

3.1 Workshop Structure

According to the aims of the EngiMath@UPC project, in order to disseminate among the mathematics community at UPC, either the project or the online course, a training activity was organized, targeted to the math teachers (specially to Linear Algebra ones). The title of the activity was “EngiMath. A European project for UPC students” and it was planned as a 6-hour online workshop.

The schedule of the workshop was as follows:

- a) an initial 2-hour session for the introduction of the EngiMath project and the online course,
- b) an estimated 2-hour homework on working in detail (from a math teacher perspective) one or two lessons of the course, and
- c) a final 2-hour session for the joint discussion on the perceptions and comments (pros and cons) about the worked materials.

First session was mainly informative and descriptive of the project and the course as a main output. For the homework, participants had two days in order to deal with a preassigned lesson and provide the corresponding feedback. In the second session, the instructor described the results of the feedback and promoted an active and general discussion on the design and structure of the course, contents presentation, procedures, questions database and assessment strategy, in order to derive the main conclusions on validation and update of the materials. Last session also allowed to invite participants to be active part in the EngiMath@UPC project during the second semester of the 2021-22 academic year.

Teachers accessed the online course, available in Catalan, Spanish and English, via an External Tool implemented in the Moodle virtual campus allowing them to directly link the materials locate at the TTK University of Applied Sciences' server in Tallinn (Estonia).

3.2 Participants

The workshop proposal received a total of 27 registrations and 22 professors attended the sessions. Among the 22 attendees, 19 (86.4%) fruitfully contributed with their homework, feedback (similar to the one described in the subsection 2.6) and active discussion. Figures are based on the feedback provided by them.

Participants' profile: a) affiliation: 17 (89.5%) Dept. of Math, 2 (10.5%) others, b) online course language: 16 (84.2%) Catalan, 3 (15.8%) Spanish, c) age rang: 5 (26.3%) in [40, 49], 12 (63.2%) in [50, 60] and 2 (10.5%) above 60.

3.3 Results on Quality Issues

Fig. 7 and Fig. 8 display bar charts from the collected responses on quality for the EngiMath materials and resources. It can be seen, when grouping the two higher categories (4 and 5 in the Likert scale, colored green and purple, respectively) that the scores are really satisfactory in each item:

- Materials: Theoretical: 94.7%, Practice (84.2%) and Assessment (84.2%)
- Resources: Graphics: 78.9%, Structure (100%) and Presentation method (100%)

This findings state the good performance of the output and validates a high-quality standard of the EngiMath online course.

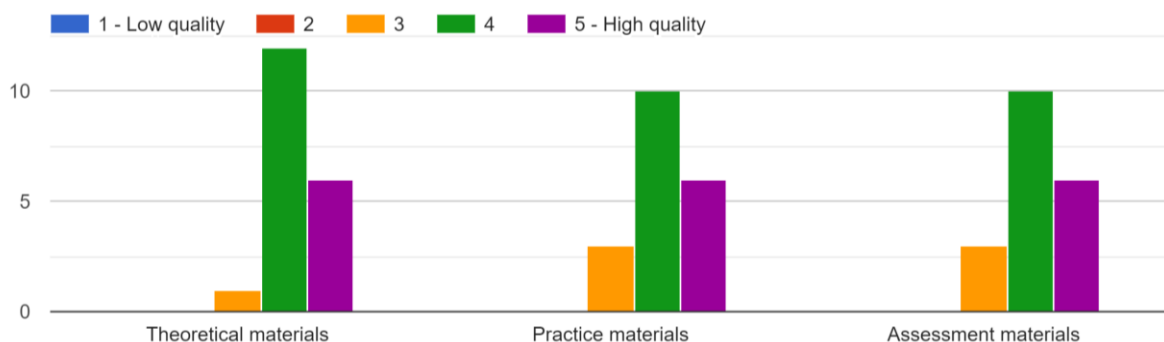


Fig. 7. Bar chart of the materials' quality feedback

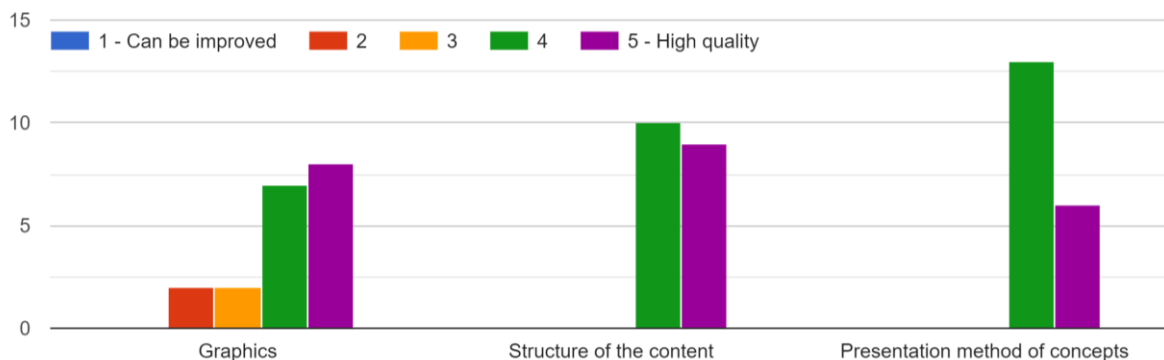


Fig. 8. Bar chart of the feedback about the designing resources' quality



4 FINAL CONCLUSIONS

Concerning the participants' positive experience, the answers focus on the following aspects (sorted from more to less frequency of responses)

- a) Clarity, conciseness, progression and summaries in explanations.
- b) Volume per lesson. Brief, which prevents giving up in the middle of the lesson.
- c) Illustrative examples. Helpful to understand the theory.
- d) Fully coverage of the subject and different levels of difficulty.
- e) Project and platform by themselves.

When proposing issues to be taking into account for improving the current version, proposals include (sorted from more to less frequency of responses)

- a) Correct errors in the answers or feedback in the tests. In particular in the assessment tests.
- b) Reduce overly calculative exercises (or too many boxes to fill) and add reasoning questions.
- c) Avoid repetitive exercises.
- d) Allow to follow the program in a non-sequential way.
- e) Incorporate some additional software like Mathematica, Matlab, Maple, SAGE, Geogebra, etc

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