High-gain interval observer for continuous–discrete-time systems using an LMI design approach

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ABSTRACT

In this paper, a high-gain interval observer (HGIO) for a class of partially linear continuous-time systems with sampled measured outputs in the presence of bounded noise and additive disturbances is proposed. The design of the HGIO is formulated in the Linear Matrix Inequality (LMI) framework. The gain of the HGIO is designed to satisfy the cooperative property using a time-varying change of coordinates based on pole placement in separate LMI regions. Moreover, a procedure for designing the HGIO gain to minimise the effect of the noise and disturbance in the estimation is provided. The stability of the proposed HGIO is also guaranteed. The proposed approach is assessed in simulation using a numerical example.

KEYWORDS

High-gain observer; interval observer; continuous–discrete-time system; bounded uncertainty

1. Introduction

High-gain observer (HGO) is an important technique in non-linear control (Deza et al., 1992; Gauthier et al., 1992; Khalil, 2017). In Khalil and Praly (2014), a brief history of this technique is presented summarising the main ideas and results as well as some of their applications in control. HGOs were introduced in the context of linear feedback as a tool for the robust observer design. The use of HGOs in nonlinear feedback control started to appear in the late 1980s (Khalil & Praly, 2014). Since then intense research on this topic has been developed with the main results summarised in Khalil and Praly (2014).

As discussed in Khalil (2017), one of the most serious challenges in implementing HGOs is the effect of measurement noise and disturbances. One way to deal with this issue is to estimate these disturbances as proposed in Yao et al. (2019), where a disturbance observer is designed for singular Markovian jump systems. Another way is to work in a bounded-error context that assumes unknown but bounded disturbances as considered in this paper. Therefore, such an identified challenge will be faced in the context of an unknown but bounded description of noise and disturbances, such that instead of generating a nominal estimation, an interval that bounds the set of estimated states is proposed using an interval observer approach. Interval observers were introduced by Gouze et al. (2000) as a robust estimation approach. Since then it has been a very active area of research, as, e.g. in Mazenc and Bernard (2011), a time-varying exponentially stable interval observer can be constructed using the Jordan canonical form for a stable linear system with additive disturbances. In Mazenc and Dinh (2014), a new technique of construction of continuous–discrete interval observers for continuous-time systems with discrete measurements and disturbances in the measurements and the dynamics is introduced.

The principle of interval observers is to provide an interval for the state estimation bounding the effect of noise and disturbances that are assumed to be unknown but bounded with known bounds. There are two families of approaches for generating such an interval. The first family is based on an approximation of the set of states by means of some simple set (polytope, ellipsoid or zonotope) that is computed iteratively from the set obtained in the previous iteration

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(see, e.g. Alamo et al., 2005; Combastel, 2015). The second approach is to design two observers that provide, respectively, the upper and lower bounds of the interval that bounds the set of estimated states taking into account disturbance and measurement bounds. These observers are designed to satisfy the cooperativity condition that guarantees that just propagating the extreme values of uncertainty intervals is enough to produce the interval bounding the set of estimated states (Efimov & Raïssi, 2016). In Thabet et al. (2021), a high-gain interval observer (HGIO) is proposed for a class of partially linear systems affected by unknown but bounded additive disturbance terms and measurement noise. The proposed observer was designed based on a change of coordinates which ensures the cooperativity of the system and that the state matrix is Hurwitz stable and $\mathbb{C}$-diagonalisable under some assumptions. In this last work, there is no constructive method to design the observer gain. It was chosen arbitrarily such that the $\mathbb{C}$-diagonalisation and the Hurwitz character of the state matrix are satisfied. In addition, the measurements are considered known at each continuous-time instant t.

As discussed in Mazenc et al. (2015), in many real applications, measurements are collected in discrete time. This leads to continuous–discrete-time systems where the system dynamics evolves in continuous time while measurements are only available at discrete-time instants. In the literature, the problem of observers for continuous–discrete-time systems has already been investigated as a separate problem from the continuous-time and discrete-time cases because of the particular challenges it poses. Some extensions to continuous–discrete-time systems of some classical observers can be found for Kalman filters (Jazwinski, 2007), for HGOs (Deza et al., 1992) and for non-linear systems (Ahmed-Ali et al., 2009), among others. The case of interval observers for continuous–discrete-time systems has also been investigated in the literature (see, e.g. Goffaux et al., 2009; Mazenc & Dinh, 2014; Menini et al., 2021). In Mazenc and Dinh (2014), an interval observer approach for linear continuous–discrete-time systems including disturbances is proposed. This approach uses two copies of the studied system and a framer, accompanied by appropriate outputs which give component-wise, upper and lower bounds for the solutions of the studied system. Recently, in Menini et al. (2021), a high-gain practical observer with an interval arithmetic tool in the case of sampled data measurements is designed. In this work, two closed-form expressions are proposed in order to compute certified bounds based on the solution of the single input linear system. An extension of this method to the case of a nonlinear system in an observer normal form is also proposed by the authors. However, the class of systems treated in this paper, namely linear systems or nonlinear systems under the normal form, is quite restrictive. Moreover, neither the case of bounded noise and additive disturbances nor the continuous–discrete-time aspect is addressed.

The main contributions of this paper are the following:

- An HGIO for a class of partially linear continuous-time systems with sampled measured outputs in the presence of bounded noise and additive disturbances is proposed.
- The design of the HGIO is formulated in the Linear Matrix Inequality (LMI) framework.
- The gain of the HGIO is designed to satisfy the cooperative property using a time-varying change of coordinates based on pole placement in separate LMI regions.
- Moreover, a procedure for designing the HGIO gain to minimise the effect of the noise and disturbance in the estimation is provided.
- The stability of the proposed HGIO is also guaranteed.
- The proposed approach is assessed in simulation using a numerical example.

The structure of the paper is the following: Section 2 introduces some preliminary material. Section 3 presents the problem statement. Section 4 introduces the proposed HGIO. Section 5 illustrates the proposed approach with a numerical example. Finally, Section 6 draws the main conclusions and presents some future research paths.

2. Preliminaries

Before introducing the problem statement and the proposed HGIO for continuous–discrete-time systems, some preliminaries about complex intervals and $\mathbb{D}_{\mathbb{R}}$-regions are given. Complex intervals are usually defined by rectangles or disks in the complex plane (Boche, 1966; Petkovic & Petkovic, 1998). $\mathbb{D}_{\mathbb{R}}$-regions are convex subsets of the complex plane that include half-planes, conic sectors, disks, and vertical and horizontal strips, among others which can be symmetrical (Peaucelle et al., 2000) or not (Bosche et al., 2005) with respect to the real axis.

In the following, centred form concepts are introduced as well as the main notations and the technical propositions that will be further used to derive the main results of our work. The interested reader can refer to Combastel and Raka (2011) for more details about complex intervals and to Bosche (2003) for a more complete description of different types of $\mathbb{D}_\mathbb{R}$-regions.

The complex intervals used in this work rely on a partial order defined over $\mathbb{C}$ (the field of complex numbers) with three statements: $\forall (L, U) \in \mathbb{C} \times \mathbb{C}, L \star U \Leftrightarrow (L^R \star U^R) \wedge (L^I \star U^I)$, where $\star \in \{=, <, >\}$. $L^R \in \mathbb{R}$ and $L^I \in \mathbb{R}$ denote the real and the imaginary part of $L \in \mathbb{C}$ (idem for U), respectively. Similar statements also hold with $\star \in \{\leq, \geq\}$. In the following, this notation will be used to refer to the real and imaginary parts of scalar, vector or matrix complex arguments. A complex interval $[L, U]$ is defined as $[L, U] = [L^R, U^R] + \mathrm{i}[L^I, U^I]$ if $L \leq U$, where $[L^R, U^R] = [L, U]^R$ and $[L^I, U^I] = [L, U]^I$ are usual real intervals. A centred form can also be introduced using the operator $\pm$ which is defined as

$$\begin{array}{rcl} \pm : \mathbb{C} \times \mathbb{C}^+ & \to & \mathbb{IC} \\ (c, r) & \mapsto & c \pm r = [c - r, c + r], \end{array} \quad (1)$$

where $\mathbb{C}^+ = \{r \in \mathbb{C},\ r \geq 0\}$ is the set of *positive* complex numbers and $\mathbb{IC}$ is the set of scalar complex intervals. c (resp. r) denotes the centre (resp. the radius) of the complex interval $c \pm r$. Based on the partial order, which is previously introduced, it is clear that $r \geq 0 \Leftrightarrow c - r \leq c + r$. The restriction of $\pm$ to real numbers is defined by analogy to (1) with $\pm : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{IR}$. Hence, $(c \pm r)^R = c^R \pm r^R$ and $(c \pm r)^I = c^I \pm r^I$.

The sum of two centred complex intervals can be computed from the sum of the centres and the sum of the radius:

$$(c_1 \pm r_1) + (c_2 \pm r_2) = (c_1 + c_2) \pm (r_1 + r_2). \quad (2)$$

To compute $a(c \pm r)$, two operators namely *cabs* and *ctimes*, defined in Combastel and Raka (2011), are used in this paper. Their definitions for scalar values are as follows: *cabs*:$|a| = |a^R| + \mathrm{i}|a^I|$ and *ctimes*: $a \diamond b = |a||b| + 2|a^I||b^I|$, where i and $|.|$, respectively, referring to $\sqrt{-1}$ and the absolute value operator. In Combastel and Raka (2011), the element-by-element extension of these operators to vectors and matrices can be found. Then, the linear image of a complex interval matrix can be obtained by applying the following theorem that has been proved in Combastel and Raka (2011).

Theorem 2.1: *$\forall\ (M, C, R) \in \mathbb{C}^{n\times p} \times \mathbb{C}^{p\times q} \times \mathbb{C}^{p\times q}$, $M(C \pm R) = (MC) \pm (M \diamond R) \in \mathbb{IC}^{n\times q}$, where $M \diamond R = |M||R| + 2|M^I|R^I| \in (\mathbb{C}^+)^{p\times q}$*

Proposition 2.2: *Let z: $\mathbb{R}^+ \to \mathbb{C}^n$, $z^c : \mathbb{R}^+ \to \mathbb{C}^n$ and $z^r : \mathbb{R}^+ \to (\mathbb{C}^+)^n$ be three continuous functions (with respect to time). If $\forall\ t \in \mathbb{R}^+$, $z(t) \in z^c(t) \pm z^r(t)$ with $z^r(t) > 0$, then a continuous function $\sigma : \mathbb{R}^+ \to [-1, +1]^{2n}$ returning bounded real vector values exists and satisfies*

$$\forall\, t \in \mathbb{R}^+, \quad z(t) = z^c(t) + \triangle(z^r(t))\sigma(t), \quad (3)$$

where the operator $\triangle(.)$ is defined as

$$\forall\, \eta \in \mathbb{C}^n, \quad \triangle(\eta) = [\mathrm{diag}(\eta^R), \mathrm{i.diag}(\eta^I)] \in \mathbb{C}^{n\times 2n}. \quad (4)$$

Let A^T, A^* and A^H denote, respectively, the transpose of matrix A, the conjugate transpose and the Hermitian matrix defined as $A^H = A + A^*$. $\otimes$ represents the Kronecker product.

Let R, a $2d \times 2d$ Hermitian matrix, be defined as

$$R = \begin{bmatrix} R_{00} & R_{10} \\ R_{10}^* & R_{11} \end{bmatrix} \in \mathbb{C}^{2d\times 2d}, \quad R_{11} \in \mathbb{C}^{d\times d} \geq 0. \quad (5)$$

The subset of the complex plane defined according to

$$\mathbb{D}_\mathbb{R} = \{s \in \mathbb{C} : R_{00} + (R_{10}s)^H + R_{11}s^*s < 0\} \quad (6)$$

is called $\mathbb{D}_\mathbb{R}$-region of degree d.

Note that the interior of a disk with centre $c = c_1 + c_2 i$ and radius r is characterised by a matrix R equal to

$$R = \begin{bmatrix} c_1^2 + c_2^2 - r^2 & -c_1 + c_2 i \\ -c_1 - c_2 i & 1 \end{bmatrix}. \quad (7)$$

3. Problem statement

In this paper, a class of partially linear continuous-time systems with sampled measured outputs is considered in the presence of bounded noise and additive disturbances. The considered system is described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + \phi(t, u(t), x(t)) + d(t), \\ y(t_k) = Cx(t_k) + V(t_k), \end{cases} \tag{8}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector and $d(t) \in \mathbb{R}^n$ is the additive disturbance vector. Measured outputs $y(t_k) \in \mathbb{R}^s$ and noises $V(t_k) \in \mathbb{R}^s$ are available only at each sampling time t_k. Function ϕ is continuous w.r.t to the time.

Note that system (8) combines a continuous-time dynamic behaviour for the states and discrete-time measurements $y \in \mathbb{R}^s$ which are available only at the sampling times $t = t_k$. This type of system is known in the literature as continuous–discrete-time system as discussed in the introduction.

Assumption 3.1: *It is assumed that the additive disturbances $d \in \mathbb{R}^n$ are unknown but bounded with known elementwise bounds: $d \in [\underline{d}, \overline{d}] = d^c \pm d^r$ with $d^c \in \mathbb{R}^n$ and $d^r \in \mathbb{R}^n$ being, respectively, the centre and the radius of the interval.*

Assumption 3.2: *It is assumed that the measurements noise $V(t_k)$ is bounded and $|V(t)| \leq \overline{\delta}$ with $t \in [t_k,\ t_{k+1}]$, where $\overline{\delta} \in \mathbb{R}^s$. Therefore, we can assume that, for $\forall$ t, the noise is bounded with known elementwise bounds: $V(t) \in [-\overline{\delta}, \overline{\delta}] = \mathbf{0} \pm \overline{\delta}$, where $\mathbf{0} \in \mathbb{R}^s$ is a vector with all elements equal to zero.*

Assumption 3.3: *It is assumed that the initial system states are bounded: $x(0) \in x^c(0) \pm x^r(0) \subset \mathbb{R}^n$, where $x^c(0)$ and $x^r(0)$ are known.*

Assumption 3.4: *It is assumed that the input vector $u(t)$ is continuous w.r.t time t, known and bounded.*

In this paper, an HGIO is proposed for system (8) that considers the effect of unknown but bounded noise and additive disturbances providing upper/lower estimation bounds for states and outputs.

Let us first introduce the HGO structure proposed by Freidovich and Khalil (2008) for system (8) that will be used along the paper:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \phi(t, u(t), \hat{x}(t)) + L_1(y(t) - C\hat{x}(t)) + \hat{d}(t), \tag{9}$$

where $\hat{x}(t)$ and $\hat{d}(t)$ represent, respectively, the estimations of the state and the disturbance.

Following Ali et al. (2014), the observer gain $L_1 \in \mathbb{R}^{n\times s}$ is parameterised as follows:

$$L_1 = \theta \Delta_\theta^{-1} K, \tag{10}$$

where $\theta > 1$, $K \in \mathbb{R}^{n\times s}$ and the matrix Δ_θ is a diagonal matrix $n \times n$ defined by

$$\Delta_\theta = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{\theta} & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\theta^{n-1}} \end{bmatrix}.$$

Note that the product $\theta \Delta_\theta^{-1}$ should be understood as an elementwise product. Then, (9) can be reformulated as

$$\dot{\hat{x}}(t) = (A - \theta \Delta_\theta^{-1} KC)\hat{x}(t) + \phi(t, u(t), \hat{x}(t)) + \theta \Delta_\theta^{-1} Ky(t) + \hat{d}(t). \tag{11}$$

To implement such an observer, the measurements are needed at every time instant t. However, the measurements of system (8) are available only at each sampling time t_k. To overcome this problem, let us consider the following continuous–discrete time HGO:

$$\begin{cases} \dot{\hat{x}}_w(t) = A\hat{x}_w(t) + \phi(t, u(t), & \text{(12a)} \\ \hat{x}_w(t)) + \hat{d}(t) + L_1(w(t) - \hat{w}(t)), & \\ \hat{w}(t) = C\hat{x}_w(t) + \hat{V}(t), & \text{(12b)} \\ \dot{w}(t) = C[A\hat{x}_w(t) + \phi(t, u(t), & \\ \hat{x}_w(t)) + \hat{d}(t)], & \text{(12c)} \\ \text{with } t \in]t_k,\ t_{k+1}[, & \\ w(t_k) = y(t_k), & \text{(12d)} \end{cases}$$

where $\hat{x}_w(t)$, $w(t)$ and $\hat{w}(t)$ represent, respectively, the estimation of the state, the prediction of the output between two sampling times and the estimation of the noisy output based on the state estimation $\hat{x}_w(t)$. $\hat{d}(t)$ and $\hat{V}(t)$ are the estimation of the unknown disturbances and noise considering that the only knowledge we have is about the bounds introduced in Assumptions 3.1 and 3.2. L_1 is the gain of HGIO that will be designed in Section 4.1. For the implementation of this observer, two additional assumptions are considered:

Assumption 3.5: *It is assumed that the initial system states of (12a) are bounded: $\hat{x}_w(0) \in \hat{x}_w^c(0) \pm \hat{x}_w^r(0) \subset \mathbb{R}^n$, with $\hat{x}_w^r(0) \geq 0$, where $\hat{x}_w^c(0)$ and $\hat{x}_w^r(0)$ are known.*

Assumption 3.6: *It is assumed that the estimation of the disturbances $\hat{d}(t)$ and the estimation of the noise $\hat{V}(t)$ are unknown but bounded: $\hat{d}(t) \in d^c(t) \pm d^r(t)$ and $\hat{V}(t) \in \mathbf{0} \pm \bar{\delta}$, according to the bounds in Assumptions* 3.1 *and* 3.2.

Note that the observer in (12) will not be directly implemented using this structure. In this work, as a first step, we are interested in the design of an interval observer providing the lower and upper bounds of the output estimation for all t. Then, based on (12) and Assumptions 3.5 and 3.6, a continuous–discrete-time interval observer will be proposed in a new base introduced in Section 4.2.1 that will provide output bounds estimation.

Remark 3.1: The idea of designing an observer with a sampled measured output, where the output between two sampling periods is predicted, was originally proposed in Karafyllis and Kravaris (2009) using an inter-sample output predictor. In this paper, this idea is applied to design the so-called continuous–discrete HGO in the bounded-error context. In the following of this paper, a constructive method for designing the gain observer is given such that the effect of the noise, which is one of the main issues in the HGIO design, will be reduced.

Remark 3.2: Note that $w(t)$ is the estimation of the output without noise.

Remark 3.3: Continuous–discrete observer structure (12) is different from the one proposed in Mazenc and Dinh (2014). Indeed, observer (12) involves an output predictor term $w(t)$ which provides continuous-time estimation of the system outputs between two sampling periods. However, in Mazenc and Dinh (2014), the observer design is based upon a discrete model of the system using the zero-order hold approach. As a matter of fact, the update process at the sampling times t_k is performed only on the prediction term $w(t)$ for observer (12), while in Mazenc and Dinh (2014), this update process affects the whole system state.

In the next section, as a first step, an HGIO based on (12) will be proposed to provide the upper and the lower output bounds estimation at every time t. As a second step and based on (11), the obtained bounds will be used in the new HGIO to provide the lower and the upper bounds for state estimation values of system (8). This procedure will be further detailed later in the paper.

The design of the HGIO, which is a combination of HGO (9) and the interval observer approach, involves finding two observers that provide the lower and upper bounds of the state (resp. the output) estimation guaranteeing that the state (resp. the output) of system (8) satisfies $x(t) \in [\underline{x}(t), \overline{x}(t)]$ (resp. $y(t) \in [\underline{y}(t), \overline{y}(t)]$).

It should be noted that the design of such an observer requires some properties such as monotonicity (Gouze et al., 2000). Unfortunately, this property is hard to be satisfied in many cases. To overcome this difficulty, a time-varying change of coordinates, which is based upon the diagonalising of the observer state matrix ($A - \theta\Delta_\theta^{-1}KC = \nu^{-1}\text{diag}(\rho + \mathrm{i}\omega)\nu$) and depends on the parameter θ, has been proposed in Thabet et al. (2021). In this last work, where the measurements are considered known at each instant t, the parameter θ has been arbitrarily selected and the gain K has been arbitrarily chosen such that the matrix ($A - \theta\Delta_\theta^{-1}KC$) is $\mathbb{C}$-diagonalisable and Hurwitz stable, which is not always obvious to be satisfied. Furthermore, the selection of K becomes more and more difficult since the sufficient condition, given by Proposition 4.2 in Thabet et al. (2021) and which should be verified to ensure the stability of the radius dynamic of the proposed HGIO, depends on this gain K.

In this work, on the one hand, in order to overcome simultaneously the problem of the $\mathbb{C}$-diagonalisation and the Hurwitz character of the matrix ($A - \theta\Delta_\theta^{-1}KC$) when designing the HGIO, a constructive LMI design approach will be proposed to design the observer gain ($\theta\Delta_\theta^{-1}K$). On the other hand, in order to ensure the Metzler character (Smith, 1995) of the matrix ($A - \theta\Delta_\theta^{-1}KC$), a new time-varying change of coordinates, depending on the parameter θ, will be proposed. The choice of the parameter will not be arbitrary as in previous works, but it will be computed based on LMI and verifying some conditions in order to reduce the effects of the noise and the additive disturbances. It should be noted that the proposed methods to compute the parameter θ and the gain $\theta\Delta_\theta^{-1}K$ will guarantee the non-divergence of the bounds dynamics of the proposed observer. In this way, the sufficient condition, given in Thabet et al. (2021), of non-divergence will not be longer needed.

4. HGIO design

In the following, the design of the observer gain L_1 is firstly detailed. Secondly, a continuous–discrete-time HGIO structure for output bounds estimation is proposed and a new LMI procedure for computing the parameter θ is given. Finally, by using the resulting bounds of the output, an HGIO structure of state estimation of system (8) is given.

4.1. HGIO gain design

As discussed in the previous section, observer parametrisation (10) allows that the HGO state matrix in (11), $(A - \theta\Delta_\theta^{-1}KC)$, can be expressed as $(A - L_1C)$. In the following, the LMI approach to design L_1 such that the matrix $(A - L_1C)$ is Hurwitz and $\mathbb{C}$-diagonalisable is given. Note that the condition ensuring the stability of the proposed observers is also taken into account when designing the gain.

In a previous work (Thabet et al., 2021), this gain L_1 is chosen arbitrarily such that the matrix $(A - L_1C)$ is Hurwitz and $\mathbb{C}$ -diagonalisable. Therefore, there is no systematic design procedure for L_1. In this paper, an LMI design approach is proposed to design the observer gain L_1 which ensures simultaneously the Hurwitz character and the $\mathbb{C}$-diagonalisation of $(A - L_1C)$. In particular, all the eigenvalues of the matrix $(A - L_1C)$ will be located in pre-defined symmetric disks. Then, the task is to design the observer gain L_1 such that the eigenvalues of $(A - L_1C)$ are inside given $\mathbb{D}_\mathbb{R}$-regions ($\mathbb{D}_{\mathbb{R}j}$ with $j = 1, \ldots, n$). The main objective of designing L_1 using $\mathbb{D}_\mathbb{R}$-regions is to ensure simultaneously the $\mathbb{C}$-diagonalisation and the Hurwitz character of $(A - L_1C)$ such that

$$A - L_1C = \nu_1^{-1}\mathrm{diag}(\rho_1 + \mathrm{i}\omega_1)\nu_1, \tag{13}$$

where $\nu_1 \in \mathbb{C}^{n\times n}$, $\rho_1 = [\rho_{11} \cdots \rho_{1n}]^\mathrm{T} \in \mathbb{R}^n$, $\omega_1 = [\omega_{11} \cdots \omega_{1n}]^\mathrm{T} \in \mathbb{R}^n$, respectively, denote the eigenvector and the vector containing the real and the imaginary parts of the eigenvalues of $(A - L_1C)$. The function *diag* returns a diagonal matrix from its input vector.

In the following, the subsets of the eigenvalues $(A - L_1C)$ will be defined by particular convex regions called $\mathbb{D}_\mathbb{R}$-regions (Bosche, 2003), which are characterised by LMIs. Note that, in this paper, the considered class of $\mathbb{D}_\mathbb{R}$-regions is a class of open convex subsets of the complex plane defined by disks which can be symmetrical (Peaucelle et al., 2000) or not (Bosche et al., 2005) with respect to the real axis. Each region $\mathbb{D}_{\mathbb{R}j}$ ($j = 1, \ldots, n$), which in this work is a disk, with centre $c_{Rj} = c_{Rj_1} + c_{Rj_2}i$ and radius r_{Rj} is characterised by

$$R_j = \begin{bmatrix} R_{j_{00}} & R_{j_{10}} \\ R^*_{j_{10}} & R_{j_{11}} \end{bmatrix}, \tag{14}$$

where $R_{j_{00}} = c^2_{Rj_1} + c^2_{Rj_2} - r^2_{Rj}$, $R_{j_{10}} = -c_{Rj_1} + c_{Rj_2}i$, $R^*_{j_{10}} = -c_{Rj_1} - c_{Rj_2}i$ and $R_{j_{11}} = 1$. On the one hand, to ensure the Hurwitz character of $(A - L_1C)$, the real part c_{Rj_1} of each defined centre c_{Rj} should be chosen in the left half side of the complex plane such that $c_{Rj_1} + r_{Rj} < 0$. On the other hand, to guarantee the $\mathbb{C}$-diagonalisation of the matrix $(A - L_1C)$, at least one of the defined disks should have a centre with an imaginary part $c_{Rj_2} \neq 0$.

The previously mentioned objectives can be achieved using the following result:

Theorem 4.1: *There exists a gain $\hat{L}_{1_j}$ ($j = 1, \ldots, n$) that assigns the jth eigenvalues of the matrix $(A - L_1C)$ in a given region $\mathbb{D}_{\mathbb{R}j}$ of degree 1, if and only if there exist a positive scalar $\hat{X}_j$ and a matrix $\hat{S}_j \in \mathbb{R}^{s\times 1}$ such that the following LMI holds:*

$$\begin{bmatrix} R_{j_{00}} \otimes \hat{X}_j + (R_{j_{10}} \otimes (\hat{A}_j\hat{X}_j - \hat{C}_j\hat{S}_j))^H & Z_j^* \otimes (\hat{A}_j\hat{X}_j - \hat{C}_j\hat{S}_j)^* \\ Z_j \otimes (\hat{A}_j\hat{X}_j - \hat{C}_j\hat{S}_j) & -I_d \otimes \hat{X}_j \end{bmatrix} < 0 \tag{15}$$

with $\hat{A}_j = \Lambda_j$, $\mathrm{diag}([\Lambda_1 \cdots \Lambda_j \cdots \Lambda_n]^\mathrm{T}) = V^{-1}A^\mathrm{T}V$, $\hat{C}_j = \lambda_j V^{-1}C^\mathrm{T}$, λ_j is a row vector where the jth position element is equal to 1 and zero elsewhere (e.g. if $j = 1$, then $\lambda_1 = [1 \;\; 0_{1\times(n-1)}]$) and where Z_j is deduced from the Cholesky factorisation $R_{j_{11}} = Z_j^* Z_j$, then the gain is given by

$$\hat{L}_{1_j} = \hat{X}_j^{-1}\hat{S}_j. \tag{16}$$

Proof: The proof follows in a straightforward manner applying the duality principle between control and estimation to the result presented in Maamri et al. (2006) where the state feedback gain was designed. ■

It should be noted that Theorem 4.1 deals with a partial pole placement. In order to design L_1, we have to achieve a complete pole placement, using n different subsets of eigenvalues with each subset associated with a region $\mathbb{D}_{\mathbb{R}j}$ ($j = 1, \ldots, n$). Therefore, the previous procedure given in Theorem 4.1 should be repeated until all the eigenvalues of the matrix $(A - L_1C)$ have been placed using the following algorithm obtained by applying duality to the one proposed by Maamri et al. (2006) for the control design:

Remark 4.1: Note that Step 8 in Algorithm 1 is a preparation step to verify a sufficient condition given in previous works (Proposition 4.2 in Thabet et al., 2021) which will be used for ensuring the stability of the proposed observer.

Therefore, Algorithm 1 allows designing the gain L_1 for ensuring simultaneously the $\mathbb{C}$-diagonalisation and the Hurwitz stability. As previously mentioned, in order to design an HGIO, the matrix $(A - L_1C)$ is required to be not only $\mathbb{C}$-diagonalisable and Hurwitz stable, but also Metzler. In a recent work (Thabet et al., 2021), a time-varying change of coordinates ($T(t) = \mathrm{diag}(e^{(\frac{1}{\theta}+i\omega_1)t})v_1$), depending on the parameter Θ, where Θ is a column vector with all elements equal to θ, has been proposed to ensure the Metzler character of this matrix in a new base. However, in this reference, θ is chosen arbitrarily and the problem of minimising the effects of noise and disturbances was not addressed.

In this work, a new time-varying matrix for a change of coordinates (17) is proposed, not only to ensure the cooperativity of the transformed system, but also to minimise the effects of both noise and additive disturbances

$$T_1(t) = \mathrm{diag}(e^{-(\Theta+i\omega_1)t})v_1. \tag{17}$$

Parameter Θ will be tuned using an LMI approach presented in Section 4.2.2.

4.2. Design procedure of the continuous–discrete-time HGIO

In this section, a procedure for designing an HGIO for system (8) is presented. Note that the measurements of this system are available only at each sampling time t_k, as previously discussed. Then, to compute the upper and the lower state bounds of system (8), based on the dynamics given by (12), a continuous–discrete HGIO will be firstly proposed to estimate the upper and lower bounds of the output. Based on the obtained output

Algorithm 1

1. Initialise $j = 0$ and $A_0 = A^T$;
2. Actualise $j = j + 1$;
3. Compute Λ_{j-1} and V_{j-1} such that $A_{j-1}V_{j-1} = V_{j-1}\Lambda_{j-1}$ and rearrange Λ_{j-1} in the form

$$\Lambda_{j-1} = \begin{bmatrix} \Lambda^1_{j-1} & \cdots & 0 \\ \vdots & \Lambda^j_{j-1} & \vdots \\ 0 & \cdots & \Lambda^n_{j-1} \end{bmatrix},$$

where Λ^j_{j-1} is the jth eigenvalue to be shifted to $\mathbb{D}_{\mathbb{R}j}$ region. Then, calculate c_{j-1} as

$$c_{j-1} = \lambda_j V^{-1}_{j-1},$$

where λ_j is a row vector where the jth position element is equal to 1 and zero for the others.

4. Compute $\hat{A}_{j-1} = c_{j-1}A_{j-1}c^+_{j-1} = \Lambda^j_j$ and $\hat{C}_{j-1} = c_{j-1}C^T$
5. Find $\hat{X}_j > 0$ and $\hat{S}_j$ such that (15) holds, with $\hat{A}_j = \hat{A}_{j-1}$ and $\hat{C}_j = \hat{C}_{j-1}$, then calculate $\hat{L}_{1_j} = \hat{X}_j^{-1}\hat{S}_j$
6. Compute the observer gain L_{1j} at step j as

$$L_{1j} = \hat{L}_{1j}c_{j-1}$$

and the matrix A_j at step j as

$$A_j = A_{j-1} - L_{1j}^T C.$$

7. Compute Λ'_j and V'_j such that $A_jV'_j = V'_j\Lambda'_j$. Then, calculate $\tilde{V}_j$ as

$$\tilde{V}_j = \lambda_j V'^{-1}_j.$$

Note that $\tilde{V}_j$ represents the eigenvector which corresponds to the desired eigenvalue $\Lambda'_j(j,j)$.

8. Let $\tilde{\Lambda}_j = \mathrm{Real}(\Lambda'_j(j,j))$. If $\tilde{\Lambda}_j + ||\tilde{V}_jL_{1j}^TC\tilde{V}_j^{-1}|| < 0$ and if the matrix $[\tilde{\Lambda}_j + ||\tilde{V}_jL_{1j}^TC\tilde{V}_j^{-1}||\ \tilde{\Lambda}_j;\ \tilde{\Lambda}_j\ \tilde{\Lambda}_j + ||\tilde{V}_jL_{1j}^TC\tilde{V}_j^{-1}||]$ is stable, then go to step 9. Else, go to step 5.
9. If $j \neq n$ then go to step 2
10. Compute the observer gain that achieves the desired pole placement as

$$L_1 = \left(\sum_{j=1}^{n} L_{1_j}\right)^T.$$

bounds, the bounds of the states will be next computed. Note that the design of such an observer requires satisfying monotony conditions (Gouze et al., 2000). In the following, the new time-varying matrix for change of coordinates (17), which depends on the parameter θ, will be used for both output and state observers. The design of θ using an LMI approach will be given in Section 4.2.2, after the continuous–discrete HGIO structure, in order to show better how with the proposed procedure we can reduce the effects of both noise and additive disturbances and ensure the non-diverges of the observer bounds.

4.2.1. Continuous–discrete interval observer structure for output bounds estimation

Based on (12) and in order to design an HGIO, providing the lower and the upper bounds $[\underline{y}(t), \overline{y}(t)]$ for output values of the system, the matrix $(A - L_1C)$ is required to be Metzler, $\mathbb{C}$-diagonalisable and Hurwitz stable. The procedure given in Section 4.1 allows the design of L_1 such that the $\mathbb{C}$-diagonalisation and the Hurwitz stability of $(A - L_1C)$ are satisfied. Furthermore, the proposed time-varying matrix (17) will be used in the following to ensure the Metzler character of the state matrix in a new base.

Applying the time-varying change of coordinates (18) to the observer state equation (12)

$$z_w(t) = T_1(t)\hat{x}_w(t),\quad \text{with } T_1(t) = \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1 \tag{18}$$

and differentiating (18) with respect to time yields

$$\dot{z}_w(t) = \mathrm{diag}(-(\Theta + \mathrm{i}\omega_1))\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1\hat{x}_w(t) + \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1\dot{\hat{x}}_w(t). \tag{19}$$

Replacing $\dot{\hat{x}}_w(t)$ by its expression in (12), the matrix $(A - L_1C)$ by its decomposition (13) and using the fact that $\hat{x}_w(t) = T_1^{-1}(t)z_w(t)$, we derive the following transformed system:

$$\dot{z}_w(t) = \mathrm{diag}(\rho_1 - \Theta)z_w(t) + \Psi_w(t), \tag{20}$$

where $\Psi_w(t) = T_1(t)[\phi(t, u(t), T_1^{-1}(t)z_w(t)) + \hat{d}(t) + L_1CT_1^{-1}(t)z_w(t) - L_1\hat{V}(t)]$.

Remark 4.2: Note that the term $\Psi_w(t)$, in the new base z_w, depends on ϕ and the bounding of this term will be detailed in the proof of the next theorem using Proposition 2.2 and Theorem 3 in Combastel and Raka (2011).

Based on new dynamics (20), the upper and the lower bounds of $y(t)$ can then be computed based on the results given by the following theorem.

Theorem 4.2: *Considering HGO (12) and Assumptions 3.1, 3.2, 3.4, 3.5 and 3.6, as well as the following condition, hold: $z_w^c(0) = \nu_1\hat{x}_w^c(0)$, $z_w^r(0) = \nu_1 \diamond \hat{x}_w^r(0)$, $z_w^r(0) \geq 0$. Then,*

$$\begin{cases} \dot{z}_w^c(t) = \mathrm{diag}(\rho_1 - \Theta)z_w^c(t) + \Psi_w^c(t), \\ \dot{z}_w^r(t) = \mathrm{diag}(\rho_1 - \Theta)z_w^r(t) + \Psi_w^r(t) \end{cases} \tag{21}$$

is an HGIO for (12) where $z_w^c(t)$ and $z_w^r(t)$ represent, respectively, the state dynamics of both the centre and the radius in the new base ($z_w(t)$). The terms $\Psi_w^c(t)$ and $\Psi_w^r(t)$ are defined as follows:

$$\begin{cases} \Psi_w^c(t) = T_1(t)\phi^c(t, u(t)) + T_1(t)d^c(t), \\ \Psi_w^r(t) = |T_1(t)\hat{E}(t, z_w^r(t), z_w^c(t))|\mathbf{1}, \\ T_1(t) = \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1, \\ T_1^{-1}(t) = \nu_1^{-1}\mathrm{diag}(\mathrm{e}^{(\Theta+\mathrm{i}\omega_1)t}), \\ \hat{E}(t, z_w^r(t), z_w^c(t)) = [\phi^r(t, z_w^c(t), \Delta(z_w^r(t)))\ |d^r(t)\ | \\ L_1CT_1^{-1}(t)z_w^c(t)|, \\ \quad L_1CT_1^{-1}(t)\Delta(z_w^r(t))| - L_1\overline{\delta}]. \end{cases} \tag{22}$$

In addition, the given HGIO (21) satisfies the inclusion property in the new base

$$z_w^r(t) \geq 0 \wedge z_w(t) \in z_w^c(t) \pm z_w^r(t) \subset \mathbb{C}^n, \quad \forall\, t \in \mathbb{R}^+, \tag{23}$$

and also in the original base

$$\hat{x}_w^r(t) \geq 0 \wedge \hat{x}_w(t) \in \hat{x}_w^c(t) \pm \hat{x}_w^r(t) \subset \mathbb{C}^n, \quad \forall\, t \in \mathbb{R}^+, \tag{24}$$

$$\hat{x}_w^c(t) = T_1^{-1}(t)z_w^c(t), \quad \hat{x}_w^r(t) = T_1^{-1}(t) \diamond z_w^r(t). \tag{25}$$

Based on (25) and (12b), the bounds of the estimated output $\hat{w}(t)$ can be derived:

$$\hat{w}^c(t) = C\,\hat{x}_w^c(t), \quad \hat{w}^r(t) = |C|\,\hat{x}_w^r(t) + \overline{\delta}. \tag{26}$$

Finally, the bounds of the output y are obtained as follows:

$$[\underline{y}(t), \overline{y}(t)] = [C\,\hat{x}_w^c(t) - |C|\,\hat{x}_w^r(t) - \overline{\delta},\ C\,\hat{x}_w^c(t) + |C|\,\hat{x}_w^r(t) + \overline{\delta}], \quad \forall\, t \in]t_k, t_{k+1}[\tag{27}$$

$$[\underline{y}(t_k),\ \overline{y}(t_k)] = [C\,\hat{x}^c_w(t_k) - |C|\,\hat{x}^r_w(t_k) - \overline{\delta},\ C\,\hat{x}^c_w(t_k) \tag{28}$$

$$+\,|C|\,\hat{x}^r_w(t_k) + \overline{\delta}] \cap [Cx(t_k) - \overline{\delta},\ Cx(t_k) + \overline{\delta}]. \tag{29}$$

Proof: The proof of the theorem will be done in two steps. Firstly, the expressions of both $\Psi^c_w(t)$ and $\Psi^r_w(t)$ such that $\Psi_w(t) \in \Psi^c_w(t) \pm \Psi^r_w(t)$ are obtained using the time-varying change of coordinates (17). Secondly, based on the obtained expressions $(\Psi^c_w(t),\ \Psi^r_w(t))$ and Theorem 3 in Combastel and Raka (2011), the structure of the proposed observer as well as the inclusion property can be established. In what follows, we will derive the upper and lower bounds of the output estimation $(w(t))$, and then, the bounds of $y(t)$ can be directly deduced.

Based on (20) and replacing $\hat{d}(t)$ by its bounds ($\hat{d} \in d^c(t) \pm d^r(t) = d^c(t) + d^r(t)(\mathbf{0} \pm \mathbf{1})$), we have

$$\dot{z}_w(t) = \text{diag}(\rho_1 - \Theta)z_w(t) + \Psi_w(t) \tag{30}$$

with $\Psi_w(t) \in (T_1(t)[\phi^c(t, u(t)) + d^c(t)]) + (T_1(t)[\phi^r(t, z^c_w(t), \triangle(z^r_w(t))) + \hat{d}^r(t) + L_1CT_1^{-1}(t)z^c_w(t) + L_1CT_1^{-1}(t)\triangle(z^r_w(t)) - L_1\overline{\delta}])(\mathbf{0} \pm \mathbf{1})$.

From Theorem 2.1, $\Psi_w(t)$ can be expressed as $\Psi_w(t) \in \Psi^c_w(t) \pm \Psi^r_w(t)$, where

$$\Psi^c_w(t) = T_1(t)[\phi^c(t, u(t)) + d^c(t)]$$

and

$$\Psi^r_w(t) = |T_1(t)\hat{E}(t, z^r_w(t), z^c_w(t))|\mathbf{1}$$

with

$$\hat{E}(t, z^r_w(t), z^c_w(t)) = [\phi^r(t, z^c_w(t), \Delta(z^r_w(t))) \times |\, d^r(t)\, |\, L_1CT_1^{-1}(t)z^c_w(t)\, | \times L_1CT_1^{-1}(t)\Delta(z^r_w(t))\, |\, - L_1\overline{\delta}].$$

Under the assumptions considered in Theorem 4.2 and using Theorem 3 in Combastel and Raka (2011), one can see that HGIO (21) verifies the inclusion properties in the new base (23) and in the original base (24). Now, we just have to prove the expressions of $\hat{w}^c(t)$ and $\hat{w}^r(t)$. Since $\hat{w}(t) = C\hat{x}_w(t) + \hat{V}(t)$, where $\hat{x}_w(t) \in \hat{x}^c_w(t) \pm \hat{x}^r_w(t)$, then based on Theorem 2.1 and $\hat{V}(t) \in \mathbf{0} \pm \overline{\delta}$, we can show that

$$\hat{w}(t) = C\hat{x}_w(t) + \hat{V}(t) \in (C\hat{x}^c_w(t)) \pm (C \diamond \hat{x}^r_w(t) + \overline{\delta}). \tag{31}$$

Since C is a real matrix and $\hat{x}^r_w(t) \geq 0$, then Equation (31) can be rewritten as

$$\hat{w}(t) = C\hat{x}_w(t) + \hat{V}(t) \in \hat{w}^c(t) \pm \hat{w}^r(t)$$

with $\hat{w}^c(t) = C\hat{x}^c_w(t)$ and $\hat{w}^r(t) = |C|\hat{x}^r_w(t) + \overline{\delta}$. Based on the expressions of $\hat{w}^c(t)$ and $\hat{w}^r(t)$, and taking $y(t) = \hat{w}(t),\ \forall\ t \in]t_k,\ t_{k+1}[$, then the bounds of the output $\forall\ t \in]t_k,\ t_{k+1}[$ can be easily deduced as given in (27). For $t = t_k$, the bounds of the output result from the intersection between the estimated interval $[C\ \hat{x}^c_w(t_k) - |C|\hat{x}^r_w(t_k) - \overline{\delta},\ C\hat{x}^c_w(t_k) + |C|\hat{x}^r_w(t_k) + \overline{\delta}]$ and the computed bounds based on the real measurements $[Cx(t_k) - \overline{\delta},\ Cx(t_k) + \overline{\delta}]$. This ends the theorem proof. ■

Remark 4.3: As it can be seen in Equation (27), for all $t \in]t_k\ t_{k+1}[$, the bounds of the output $y(t)$ can be directly deduced based only on the estimated bounds of $\hat{w}(t)$. However, for all $t = t_k$ (i.e. at each sampling time), the only available information that we have of the output $y(t_k)$ is used at instant t_k. Indeed, as given in (28) at $t = t_k$, the bounds of the output are the results of the intersection of two intervals: the first one represents the estimated output bounds based on $\hat{w}^c$ and $\hat{w}^r$ and the second interval represents the output bounds using the 'real' measurements ($y(t_k) = Cx(t_k) + V(t_k)$) and taking into account the bounds of the noise ($V(t_k) \in [-\overline{\delta},\ \overline{\delta}]$). We shall notice that this update process at the instants t_k is quite different from the one adopted in Mazenc and Dinh (2014). This difference is a direct consequence of employing the continuous–discrete observer (12). Indeed in our approach, this update process is performed in one step in (28), whereas this update is performed in two steps in Mazenc and Dinh (2014) (see Equation (14) of this reference).

4.2.2. High-gain parameter design

In this section, based on the designed gain L_1, derived from Algorithm 1 and which is equal to $\theta\Delta_\theta^{-1}K$, an LMI approach presented in Theorem 4.4 is proposed to compute the parameter θ. Note that the proposed method of computing this parameter will not only reduce the effects of both noise and additive disturbances, but it will also guarantee the non-divergence of the radius dynamics of the proposed observer. In this way, the sufficient condition of non-divergence of the observer radius dynamics, given by Proposition 4.3 in Thabet et al. (2021), is no longer needed.

Proposition 4.3 (Thabet et al., 2021): *Let* $P = \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\| + |\alpha|$, *where* $\alpha \in \mathbb{R}_+^*$. *If* $(\rho - \frac{1}{\Theta}) < 0$ *(M is Hurwitz stable) and if the Metzler matrix* $[M + P, P; P, M + P]$ *is Hurwitz stable, then* $\forall\, t,\ 0 \leq z^r(t) \leq \bar{z}^r(t) < \infty$ *and* $\bar{z}^r(t)$ *follows a stable dynamics, so is* $z^r(t)$.

Note that z^r, P and M used in Proposition 4.3 are replaced, respectively, by z_w^r, $||\nu_1 L_1 C\nu_1^{-1}|| + \alpha I_n$ and $\mathrm{diag}(\rho_1 - \Theta)$ in this paper. As previously mentioned, this proposition is used in previous work to ensure the non-divergence of the radius dynamics. In this paper, an LMI approach will be proposed in Theorem 4.4 to ensure automatically this condition.

Theorem 4.4: *If there exists a symmetric positive definite matrix* $Q > 0,\ Q^T = Q$ *and a symmetric negative definite matrix* $D < 0,\ D^T = D$ *such that the following LMI is satisfied:*

$$M_1^{\mathrm{T}} Q + 2D + QM_1 < 0, \tag{32}$$

where $M_1 = [\mathrm{diag}(\rho_1) + P\ P;\ P\ \mathrm{diag}(\rho_1) + P]$ *with* $P = ||\nu_1 L_1 C\nu_1^{-1}||$, *then, the parameter* θ *is determined by*

$$\theta = \alpha - \lambda_{\min}(M_2), \tag{33}$$

where $M_2 = DQ^{-1}$, $\lambda_{\min}(M_2)$ *is the minimum eigenvalue of* M_2 *and* α *is a scalar which satisfies the following two conditions:*

$$\begin{cases} ||\mathrm{diag}(\mathrm{e}^{-(\Theta + i\omega_1)})\nu_1 d^r|| \leqslant \alpha_d \mathbf{1}, \\ ||\mathrm{diag}(\mathrm{e}^{-(\Theta + i\omega_1)})\nu_1 L_1 \bar{\delta}|| \leqslant \alpha_n \mathbf{1}', \end{cases} \tag{34}$$

where Θ *is a column vector with all elements equal to* $\theta = \alpha - \lambda_{\min}(M_2)$. α_d *and* α_n *are two defined scalars used to reduce the effects of the additive disturbances as well as the noise.* $\mathbf{1}$ *is a column vector with all elements equal to* 1 *and* $\mathbf{1}' \in \mathbb{R}^{n \times s}$ *is a matrix with all elements equal to* 1.

Proof: Based on Proposition 4.3, let us define matrix *Mat* as

$$Mat = \begin{bmatrix} \begin{matrix}\mathrm{diag}(\rho_1 - \Theta) \\ +||\nu_1 L_1 C\nu_1^{-1}|| \\ +\alpha I_n\end{matrix} & ||\nu_1 L_1 C\nu_1^{-1}|| + \alpha I_n \\ ||\nu_1 L_1 C\nu_1^{-1}|| + \alpha I_n & \begin{matrix}\mathrm{diag}(\rho_1 - \Theta) \\ +||\nu_1 L_1 C\nu_1^{-1}|| \\ +\alpha I_n\end{matrix} \end{bmatrix}, \tag{35}$$

where α is a positive scalar which will be computed later satisfying conditions (34). Note that the Hurwitz character of this Metzler matrix *Mat* will ensure the stability of the proposed observers, as it will be further discussed in Section 4.2.4. Let us rewrite matrix *Mat* (35) in a different manner as follows:

$$Mat = \begin{bmatrix} \mathrm{diag}(\rho_1) + P_1 & P_1 \\ P_1 & \mathrm{diag}(\rho_1) + P_1 \end{bmatrix} + \begin{bmatrix} \mathrm{diag}(-\Theta) + \alpha I_n & \alpha I_n \\ \alpha I_n & \mathrm{diag}(-\Theta) + \alpha I_n \end{bmatrix}, \tag{36}$$

where $P_1 = ||\nu_1 L_1 C\nu_1^{-1}||$. Until this step, ρ_1 and P_1 are known. Let us denote by M_2 the unknown part of the matrix *Mat*,

$$M_2 = \begin{bmatrix} \mathrm{diag}(-\Theta) + \alpha I_n & \alpha I_n \\ \alpha I_n & \mathrm{diag}(-\Theta) + \alpha I_n \end{bmatrix}. \tag{37}$$

Let us introduce M_1 as the known matrix $M_1 = [\mathrm{diag}(\rho_1) + P_1\ P_1;\ P_1\ \mathrm{diag}(\rho_1) + P_1]$. Until this step, L_1, ρ_1, ν_1 are known since the value of L_1 can be computed using Theorem 4.1 and Algorithm 1 and ρ_1, ν_1 can be deduced based on (13). Then, the only unknown parameters are θ and α. In the following, the expression of θ, which depends on α as given in (33), will be obtained.

It is clear that matrix *Mat* (36) is Hurwitz stable iff $\exists\, Q > 0$ and $D < 0$ with $Q^{\mathrm{T}} = Q$ and $D^{\mathrm{T}} = D$ such that

$$M_1^{\mathrm{T}} Q + 2D + QM_1 < 0, \tag{38}$$

where $D = M_2 Q$. Then, the unknown matrix M_2 can be deduced by solving LMI (38) as follows:

$$M_2 = DQ^{-1}. \tag{39}$$

Based on (37) and (39), the following inequality can be deduced:

$$-\theta + \alpha \geqslant \lambda_{\min}(M_2), \tag{40}$$

where $\lambda_{\min}(M_2)$ is the minimum eigenvalue of M_2. Then, from (40):

$$\theta \leqslant \alpha - \lambda_{\min}(M_2), \tag{41}$$

which allows deducing the expression of θ as follows: $\theta = \alpha - \lambda_{\min}(M_2)$.

Then, to compute θ, we have first to solve LMI (32) in order to compute $\lambda_{\min}(M_2)$ and then calculate α such that the two conditions (34) are verified. Conditions (34) can be rewritten element by element as

$$\begin{cases} ||\mathrm{e}^{-(\alpha-\lambda_{\min}(M_2)+\mathrm{i}\omega_{1j})}(\nu_1 d^r)_j|| \leqslant \alpha_d, \\ ||\mathrm{e}^{-(\alpha-\lambda_{\min}(M_2)+\mathrm{i}\omega_{1j})}(\nu_1 L_1 \overline{\delta})_j^i|| \leqslant \alpha_n, \end{cases} \tag{42}$$

where ω_{1j} is the jth element of the vector ω_1, $(\nu_1 d^r)_j$ is the jth element of the row vector $\nu_1 d^r$ and $(\nu_1 L_1 \overline{\delta})_j^i$ is element of the matrix $\nu_1 L_1 \overline{\delta}$ with jth row and ith column positions.

From (42), we can deduce the expression of α as

$$\alpha = \max(a_1, a_2) \tag{43}$$

with $a_1 = |\log((\nu_1 d^r)_j) - \log(\alpha_d) + \lambda_{\min}(M_2) - \mathrm{i}\omega_{1j}|$ and $a_2 = |\log((\nu_1 L_1 \overline{\delta})_j^i) - \log(\alpha_n) + \lambda_{\min}(M_2) - \mathrm{i}\omega_{1j}|$. This completes the proof of Theorem 4.4. ■

Remark 4.4: As previously mentioned, one of the main contributions of the given constructive method of computing θ is the guarantee of the radius dynamic (z_w^r) convergence. Furthermore, looking at the expressions of T_1 and $\hat{E}(t, z_w^r(t), z_w^c(t))$ in (22), we can easily deduce that Ψ_w^r depends on the products $\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1 d^r$ and $\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1 L_1 \overline{\delta}$ which are reduced due to parameter Θ. In fact, Θ satisfies the two conditions given in (34). Then, by reducing $||\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1 d^r||$ and $||\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1 L_1 \overline{\delta}||$, the effects of both noise and additive disturbances will be minimised as well as the radius, which depends on Ψ_w^r. Note that the choice of α_d and α_n depends implicitly on the following condition $\alpha = \max(a_1, a_2) > 1 + \lambda_{\min}(M2)$, since θ should be greater than 1 as known in the HGO context.

4.2.3. HGIO design for state bounds estimation

In the following, an HGIO structure is given to estimate the bounds of system states (8). As previously mentioned, the estimated output lower and upper bounds, given by Theorem 4.2, are used in the proposed structure of the HGIO in order to provide the two variables evaluating the lower and upper bounds for state values of system (8).

To guarantee the $\mathbb{C}$-diagonalisation and the Hurwitz and Metzler properties of the state matrix $(A - \theta\Delta_\theta^{-1}KC = A - L_1 C)$, the gain L_1 designed in Section 4.1 and the proposed change of coordinates (17) are used in the following.

Let us apply the time-varying change of coordinates (17) to the given HGO (11) where $y(t)$ is assumed to be unknown but with known bounds ($\overline{y}(t)$ and $\underline{y}(t)$ are obtained from Theorem 4.2):

$$z(t) = T_1(t)\hat{x}(t), \quad \text{with } T_1(t) = \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1. \tag{44}$$

Differentiating (44) with respect to time yields

$$\dot{z}(t) = \mathrm{diag}(-(\Theta + \mathrm{i}\omega_1))\mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1\hat{x}(t) + \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1\dot{\hat{x}}(t). \tag{45}$$

Replacing $\dot{\hat{x}}(t)$ by its expression in (11), the matrix $(A - L_1 C)$ by its decomposition (13) and using the fact that $\hat{x}(t) = T_1^{-1}(t)z(t)$, we can derive the following transformed system:

$$\dot{z}(t) = \mathrm{diag}(\rho_1 - \Theta)z(t) + \Psi(t), \tag{46}$$

where $\Psi(t) = T_1(t)[\phi(t, u(t), T_1^{-1}(t)z(t)) + \hat{d}(t) + L_1 y(t)]$.

Based on the new dynamics (46), the upper and lower bounds of $x(t)$ can then be computed based on the results given by the following theorem.

Theorem 4.5: *Given the dynamics of system* (8) *and considering that Assumptions* 3.1–3.4 *and* 3.6*, as well as the following condition , hold:*
$z^c(0) = \nu_1\hat{x}^c(0)$, $\quad z^r(0) = \nu_1 \diamond \hat{x}^r(0)$, $\quad \hat{x}^r(0) \geq 0$, $z^r(0) \geq 0$. *Then,*

$$\begin{cases} \dot{z}^c(t) = \mathrm{diag}(\rho_1 - \Theta)z^c(t) + \Psi^c(t), \\ \dot{z}^r(t) = \mathrm{diag}(\rho_1 - \Theta)z^r(t) + \Psi^r(t) \end{cases} \tag{47}$$

is an HGIO for system (8) *where* $(z^c(t))$ *and the* $(z^r(t))$ *represents, respectively, the state dynamics of both the centre and the radius in the new base* $(z(t))$*. The terms* $\Psi^c(t)$ *and* $\Psi^r(t)$ *are defined as follows:*

$$\begin{cases} \Psi^c(t) = T_1(t)\phi^c(t, u(t)) + T_1(t)d^c(t) \\ +T_1(t)L_1 y^c(t), \\ \Psi^r(t) = |T_1(t)\hat{E}(t, z^r(t), z^c(t))|\mathbf{1}, \\ T_1(t) = \mathrm{diag}(\mathrm{e}^{-(\Theta+\mathrm{i}\omega_1)t})\nu_1, \\ T_1^{-1}(t) = \nu_1^{-1}\mathrm{diag}(\mathrm{e}^{(\Theta+\mathrm{i}\omega_1)t}), \\ \hat{E}(t, z^r(t), z^c(t)) = [\phi^r(t, z^c(t), \\ \Delta(z^r(t))) \,|\, d^r(t) \,|\, L_1 y^r(t)]. \end{cases} \tag{48}$$

In addition, the given HGIO (47) *satisfies the inclusion property in the new base*

$$z^r(t) \geq 0 \wedge z(t) \in z^c(t) \pm z^r(t) \subset \mathbb{C}^n, \quad \forall\, t \in \mathbb{R}^+ \tag{49}$$

and also in the original base

$$\hat{x}^r(t) \geq 0 \wedge \hat{x}(t) \in \hat{x}^c(t) \pm \hat{x}^r(t) \subset \mathbb{C}^n, \quad \forall\, t \in \mathbb{R}^+, \tag{50}$$

$$\hat{x}^c(t) = T_1^{-1}(t) z^c(t), \quad \hat{x}^r(t) = T_1^{-1}(t) \diamond z^r(t). \tag{51}$$

Proof: The proof of Theorem 4.5 is similar to that of Theorem 4.2, where the only difference is on the expressions of $\Psi^c(t)$ and $\Psi^r(t)$, which is related to the use of the obtained bounds of the output $y(t)$.

Based on (46) and replacing $\hat{d}(t)$ (resp. $y(t)$) by its bounds ($\hat{d} \in d^c(t) \pm d^r(t) = d^c(t) + d^r(t)(\mathbf{0} \pm \mathbf{1})$) (resp. $y(t) \in y^c(t) \pm y^r(t) = y^c(t) + y^r(t)(\mathbf{0} \pm \mathbf{1})$), we have

$$\dot{z}(t) = \text{diag}(\rho_1 - \Theta) z(t) + \Psi(t) \tag{52}$$

with $\Psi(t) \in (T_1(t)[\phi^c(t, u(t)) + d^c(t) + L_1 y^c(t)]) + (T_1(t)[\phi^r(t, z^c(t), \triangle(z^r(t))) + \hat{d}^r(t) + L_1 y^r(t)])$ $(\mathbf{0} \pm \mathbf{1})$.

From Theorem 2.1, $\Psi(t)$ can be expressed as $\Psi(t) \in \Psi^c(t) \pm \Psi^r(t)$, where

$$\Psi^c(t) = T_1(t)[\phi^c(t, u(t)) + d^c(t) + L_1 y^c(t)]$$

and

$$\Psi^r(t) = |T_1(t)\hat{E}(t, z^r(t), z^c(t))|\mathbf{1}$$

with

$$\hat{E}(t, z^r(t), z^c(t)) = [\phi^r(t, z^c(t), \Delta(z^r(t))) \mid d^r(t) \mid L_1 y^r(t)].$$

Under the assumptions considered in Theorem 4.5 and using Theorem 3 in Combastel and Raka (2011), one can show that HGIO (47) verifies the inclusion properties in the new base (49) and in the original base (50). This ends the proof. ■

Figure 1 summarises the different main steps to follow to design the HGIO for partially linear system (8) with discrete-time measurements.

4.2.4. Stability analysis of the proposed HGIO

The stability of the proposed continuous–discrete HGIO for output bounds estimation, as well as for state bounds estimation, is derived from the stability of both centre and radius dynamics defined in (21) and (47), respectively.

Let us first prove the stability of the centre dynamics z^c_w (resp. z^c). Since the matrix $\text{diag}(\rho_1 - \Theta)$ is Hurwitz stable and both the input $u(t)$ and the disturbances $d(t)$ are assumed to be continuous and bounded, we can easily derive that $\Psi^c_w(t)$ (resp. $\Psi^c(t)$), which depends only on $u(t)$ and $d^c(t)$ that are bounded according to Assumptions 3.1–3.4, is bounded too. As a consequence, the stability of the centre dynamics of ($z^c_w(t)$) (resp. $z^c(t)$) is continuous, bounded and presents a stable dynamics.

On the other hand, usually, the stability of the radius dynamics ($z^r_w(t)$) (resp. $z^r(t)$) cannot be directly addressed in the same way as it was addressed for the centre dynamics $z^c_w(t)$ (resp. $z^r(t)$). Indeed, the term $\Psi^r_w(t)$ (resp. $\Psi^r(t)$) depends on $z^r_w(t)$, $z^c_w(t)$, $d^r(t)$ and $\overline{\delta}$ (resp. $z^r(t)$, $z^c(t)$, $d^r(t)$ and $y^r(t)$) which can be viewed as a bounded and continuous exogenous term in the second equation of (21) (resp. (47)). Meanwhile, $\Psi^r_w(t)$ (resp. $\Psi^r(t)$) still depends on the endogenous term $z^r_w(t)$ (resp. $z^r(t)$), and thus, in previous works (Thabet et al., 2021), a sufficient condition, which ensures the stability of the radius dynamics, was proposed.

In this paper, the stability of the radius dynamics ($z^r_w(t)$) (resp. $z^r(t)$) can be directly deduced because of the satisfaction of the condition given by step 8 in Algorithm 1. Indeed, the sufficient condition given in previous works (Proposition 4.2 in Thabet et al., 2021) is verified in this paper at each iteration of the observer gain design (step 8 in Algorithm 1 considering only the eigenvalues ρ_1 without θ), and by Theorem 4.4 which allows computing θ, based on α, and where the stability of matrix '*Mat*' (35) is still ensured in the new bases $z_w(t)$ and $z(t)$. Therefore, the sufficient condition is no longer needed and the stability of the proposed continuous–discrete HGIO for output bounds estimation, as well as the HGIO for state bounds estimation, is directly ensured.

5. Application example

5.1. Example description

To illustrate the efficiency of our proposed HGIO for the considered class of system (8), let us consider the same numerical example as in Thabet et al. (2021),

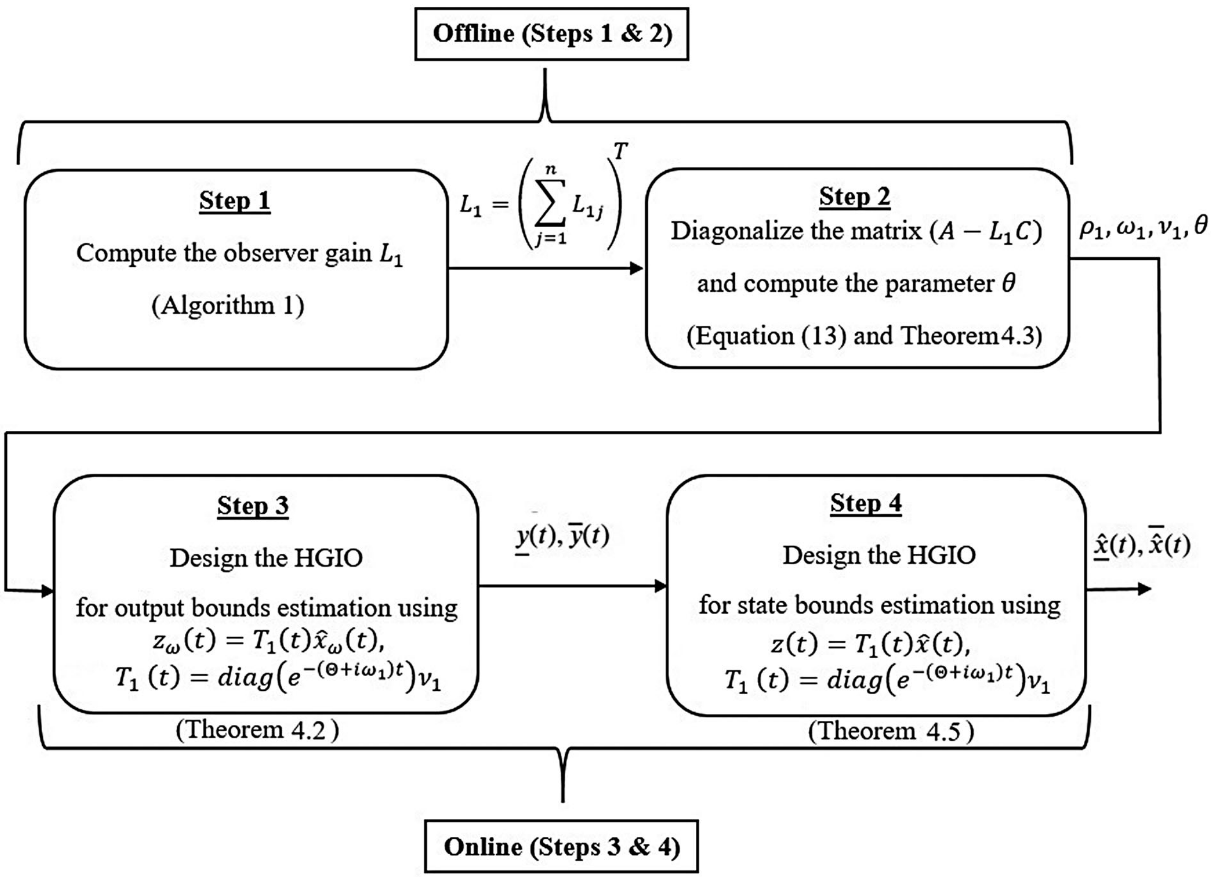


Figure 1. The main steps of the HGIO design.

where the output measurements y are considered in discrete time in this paper. Then, the system used for illustrative purposes is described by (8) with the following particular matrices and functions

$$A = \begin{bmatrix} -1 & -3 & -5 \\ 0 & 1 & -3 \\ 0 & 2 & -2 \end{bmatrix},$$

$$\phi(t, u(t), x(t) = \begin{bmatrix} 2u(t) + \exp(-|x_1|) \\ u(t) + \exp(-|x_2|) \\ -u(t) + \exp(-|x_3|) \end{bmatrix},$$

$$C = [1\ 0\ 0].$$

The input is given by

$$u(t) = 3\sin(t) - 2\sin(3t), \qquad (53)$$

while the output noise, known only at each instant t_k, and the additive disturbances are given, respectively, by

$$V(t_k) = 0.1\sin(0.1\pi t_k), \quad d(t) = \begin{bmatrix} 0.1\sin(t) \\ \sin(0.1\pi t) \\ \sin(2t) \end{bmatrix}. \qquad (54)$$

Both the additive disturbances and noise are bounded with the following bounds: $V^c = 0$, $V^r = \overline{\delta} = 0.1$, $d^c = [0\ 0\ 0]^{\mathrm{T}}$ and $d^r = [0.1\ 1\ 1]^{\mathrm{T}}$.

5.2. HGIO design

In this section, the constructive method presented in Section 4.1 is used to design the gain $L_1 = \theta\Delta_\theta^{-1}K$. As a first step, the different three regions (or disks) D_{R_j} $(j = 1, \ldots, 3)$ should be defined. Let the different centres and radius of these regions be, respectively, equal to: $c_{R1} = -0.9$, $c_{R2} = -0.6 + 1.8i$, $c_{R3} = -0.6 - 1.8i$, $r_{R1} = 0.5$, $r_{R2} = 0.2$, $r_{R3} = 0.2$. Based on these defined disks, using Algorithm 1 and solving LMI (15), the observer gain L_1 is obtained: $L_1 = 1e - 3\ [0.1111\ 0.125\ 0.1]^{\mathrm{T}}$. As a second step, once the gain L_1 is computed, the matrix $A - L_1C$ can

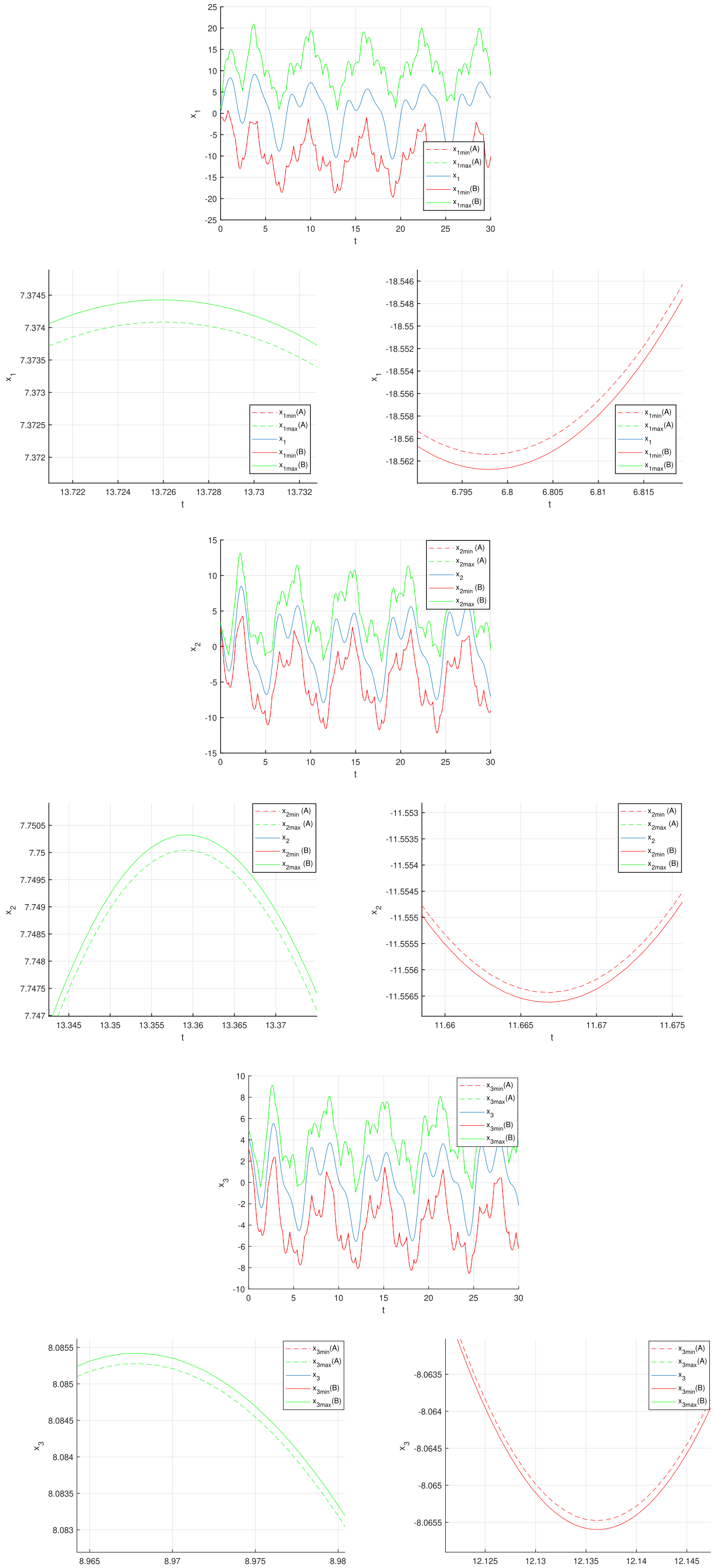


Figure 2. Simulation results for the HGIO applied to the continuous–discrete example. (a) The proposed method using the new change of coordinates (17) and (b) the method given in Thabet et al. (2021) with a continuous output case.

be diagonalised as in (13) where $\rho_1 = [-1.0003 \ -0.4999 \ -0.4999]^{\mathrm{T}}$, $\omega_1 = [0 \ 1.9365 \ -1.9365]^{\mathrm{T}}$, $\nu_1 = \begin{bmatrix} -0.9999 & -1.7500 & 0.2503 \\ -0.0001+0.0001i & 1.1154-0.6992i & -0.1596+1.6045i \\ -0.0001-0.0001i & 1.1154+0.6992i & -0.1596-1.6045i \end{bmatrix}$.

By applying Theorem 4.4 with $\alpha_d = 0.08$, $\alpha_n = 0.03$, $d^r = [0.1 \ 1 \ 1]^{\mathrm{T}}$ and $\overline{\delta} = 0.1$, the parameter α, which satisfies conditions (34), can be directly computed leading to $\alpha = 8.7586$. Then, θ parameter can be derived applying (33) leading to $\theta = 9.7586$.

As a third step, by applying Theorem 4.2 with $x_w^c(0) = [0 \ 3 \ 4]^{\mathrm{T}}$, $x_w^r(0) = [0.09 \ 0.2 \ 0.6]^{\mathrm{T}}$, the output upper and lower bounds can be calculated based on the change of coordinates (18), where $\Theta = [\theta \ \theta \ \theta]^{\mathrm{T}}$. The numerical simulation was conducted in the time range from $t = 0$ to $t = 30$ s with the Euler algorithm with a discretisation step $h = 0.001$ s and where the output estimation is updated at each instant $t_k = 0.005$ s.

Finally, based on the obtained bounds (27)–(29), using the change of coordinates (44) with $\Theta = [\theta \ \ \theta \ \ \theta]^{\mathrm{T}}$, where $\theta = 9.7586$, and by applying Theorem 4.5 with $x^c(0) = [0 \ 3 \ 4]^{\mathrm{T}}$, $x^r(0) = [0.09 \ 0.2 \ 0.6]^{\mathrm{T}}$, state bounds can be computed.

5.3. Simulation results

The numerical simulations of the proposed HGIO were conducted in the time range from $t = 0$ to $t = 30$ s with a discretisation step $h = 0.001$ s. The simulation results are presented in Figure 2.

For comparison, as in a previous work (Thabet et al., 2021), the parameter θ was chosen arbitrarily ($\theta = 1000$), as well as the gain K such that the matrix $(A - \theta\Delta_\theta^{-1}KC)$ is Hurwitz.

From the results presented in Figure 2, we can first conclude that the upper and lower state bounds provided by the proposed HGIO, with the new time-varying change of coordinates, are stable in spite of the presence of bounded additive disturbances and discrete noisy measurement. We can also see that the inclusion property is verified.

Comparing the existing method given in Thabet et al. (2021), the efficiency of our method is proved in spite of using the output bounds instead of the real output as was considered in Thabet et al. (2021). This is due to the constructive method for computing θ, such that the effects of both noise and additive disturbances are reduced, and to the new change of coordinates using $T_1(t) = \mathrm{diag}(\mathrm{e}^{-(\Theta + \mathrm{i}\omega_1)t})\nu_1$.

6. Conclusions

In this paper, an HGIO for a class of partially linear continuous-time systems with sampled measured outputs in the presence of bounded noise and additive disturbances has been proposed. The design of the HGIO has been formulated in the LMI framework. The gain of the HGIO has been designed to satisfy the cooperative property using a time-varying change of coordinates based on pole placement in separate LMI regions. Moreover, a procedure for designing the HGIO gain to minimise the effect of the noise and disturbance in the estimation has been provided. The stability of the proposed HGIO has also been proved. The proposed approach has been assessed in simulation using a numerical example.

As future research, the proposed method will be extended to other classes of continuous–discrete-time systems more generally than the ones presented in this paper. The integration of the proposed HGIO in control applications will also be explored.

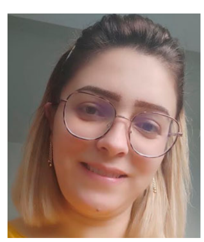
Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability

Data are available within the article.

Notes on contributors



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