Expert Systems With Applications 216 (2023) 119471

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

A hierarchical integration method under social constraints to maximize satisfaction in multiple criteria group decision making systems

David Boix-Cots [a,1], Francesc Pardo-Bosch [b,c,2], Pablo Pujadas [b,c,*,3]

[a] Department of Civil and Environmental Engineering, Universitat Politècnica de Catalunya BarcelonaTech (UPC), C/ Jordi Girona 1-3, Barcelona 08034, Spain
[b] Department of Project and Construction Engineering, Universitat Politècnica de Catalunya BarcelonaTech (UPC), Av. Diagonal 647, Barcelona 08028, Spain
[c] Group of Construction Research and Innovation (GRIC), C/ Colom, 11, Ed. TR5, Terrassa, Barcelona 08222, Spain

ARTICLE INFO

Keywords:
Decision making
Multi-source decision system
Multiple criteria group decision-making
MCGDM
Weight of expert
Consensus

ABSTRACT

Aggregating multiple opinions or assessments in a decision has always been a challenging field topic for researchers. Over the last decade, different approaches, mainly based on weighting data sources or decision-makers (DMs), have been proposed to resolve this issue, although social choice theory, focused on frameworks to combine individual opinions, is generally overlooked. To resolve this situation, a novel methodology is developed in this paper based on social choice theory and statistical mathematics. This method innovates by dividing the assessment into components which provides a multiple assessment analysis, assigning weights to each source regarding their position compared to the group for each considered criteria. This multiple-weighting process maximises individual and group satisfaction. Furthermore, the method makes it possible to manage previously assigned influence. An example is given to illustrate the proposed methodology. Additionally, sensitivity analysis is performed and comparisons with other methods are made. Finally, conclusions are presented.

1. Introduction

Modern decision science is becoming an essential concept across all fields. Multiple criteria decision-making (MCDM) methods have been extensively used to assess, evaluate, or prioritise a set of alternatives concerning a finite set of criteria or attributes in all fields. Healthcare (Bharsakade et al., 2021), public administration (Pardo-Bosch & Aguado, 2016; Pujadas et al., 2017), robotics (Abd et al., 2014) or environmental and engineering sciences (Boix-Cots et al., 2022; Casanovas-Rubio et al., 2019; Pujadas et al., 2019) are just a few examples where MCDM methods are applied.

However, the constant development of science and increasingly specific expert knowledge have made problems more complex. Nowadays, a single decision-maker (DM) may not be able to consider all relevant decision aspects (Yue, 2013). Consequently, multiple criteria group decision-making (MCGDM) methods are widely used in real-world decisions (Yue et al., 2009). Nevertheless, reconciling the opinions of different DMs to reach a consensus is still a challenge (Kerr & Tindale, 2004; Roigé et al., 2020).

In this regard, a wide range of MCGDM methods has been proposed to address this challenge, covering multiple perspectives (Koksalmis & Kabak, 2019). Some examine the negotiation among DMs, proposing discussion and debate processes. Others focus on assigning weights to decision-makers, either by analysing who is giving the opinion (DM-based methods) and factors such as their skills, experience or education or by analysing the position of their evaluations relative to the group (Data-based methods). Finally, some studies propose methods of direct aggregation assessments or rankings.

Nevertheless, even with such a variety of approaches and methods, it is striking how three fundamental issues have been overlooked:

(1) First, while these methods have been presented as specialised approaches to obtain the best group solution, it is noticeable how they have omitted the social choice theory scheme (Davis, 1973). This theory, originally based on the studies of the election theory (Black, 1958) and the choice according to individual values

* Corresponding author at: Department of Project and Construction Engineering, Universitat Politècnica de Catalunya BarcelonaTech (UPC), Av. Diagonal 647, Barcelona 08028, Spain.

E-mail address: Pablo.pujadas@upc.edu (P. Pujadas).

[1] https://orcid.org/0000-0002-8462-8887.
[2] https://orcid.org/0000-0001-9532-8508.
[3] https://orcid.org/0000-0001-5634-7431.

https://doi.org/10.1016/j.eswa.2022.119471
Received 7 September 2022; Received in revised form 2 December 2022; Accepted 22 December 2022
Available online 26 December 2022

(Arrow, 1963), aims on aggregating multiple individual opinions and preferences to reach an acceptable collective decision for social welfare. To ensure a fair, ethical, and objective consensus considering all participants, this theory states a set of Axioms, or social rules, which have to be considered in the aggregation procedure. Even though the importance that this theory has nowadays, being used in most democratic decisions, most MCGDM methods apply mathematical approximations without considering the contributions that this science has made.

(2) Current methods only consider a scenario in which all participants have an equivalent and equative influence on the decision. However, in the real world, DMs may have different influences on a decision because of the impossibility of having a homogeneous expert group in terms of experiences, attitudes, knowledge or power positions (Koksalmis & Kabak, 2019). Such circumstance is particularly striking in examples such as governments trying to introduce citizens' opinions into their decisions (Georgiou, 2022).

(3) The Data-based weighing methods, based on assigning weights to DMs regarding their assessments and opinions, consider the multi-criteria DM's evaluation as an indivisible dataset. Even though, some experts have stated that a component separation would significantly improve the group consensus (Kerr & Tindale, 2004).

Considering the above-mentioned shortcomings, this paper proposes a new approach to conceal different stakeholders' opinions, overcoming the limitations of current methods. With this aim, a novel methodology is presented to determine DM weights considering social choice theory constraints, giving researchers and practitioners a tool that enhances and guarantees the decision equity and fairness for all participants while considering the influence factor in non-homogeneous DMs and which improves the group response by analysing the multi-criteria data set component by component. The paper is organised as follows: the next section presents the literature review. Section 3 explains the novel method. Section 4 illustrates our proposed method with an example. Sections 5 and 6 compare the proposed method with other approaches and show a sensitivity analysis. The final section gives a general conclusion.

2. Literature review

As mentioned in the previous section, there are several proposed methodologies to handle an MCGDM problem. Generally, they can be labelled as subjective or objective techniques depending on how much an individual DM can affect the aggregated decision (Liu & Li, 2015), as shown in Table 1. The subjective methodologies are divided into two main subgroups: negotiation methods and DM-based methods, and the objective methodologies, in turn, are divided into three main subgroups: Data-based methods, Assessment aggregation operators and Ranking aggregation methods, depending, in both cases, on the approach that they use to aggregate the different group opinions or assessments.

Regarding the subjective methodologies, the negotiation methods reach a consensus based on discussions and debates among DMs. In this subgroup are classified examples such as the DELPHI method and its extensions (Greatorex & Dexter, 2000; Okoli & Pawlowski, 2004), which propose an interactive discussion between DMs that can become iterative after the results are known, the Devil's advocacy (Schwenk & Cosier, 1993), where a DM takes the role to point out all the flaws and risks of the proposed solution, and the Ward method (Schielke et al., 2009), in which DMs have periods of group discussion and periods of individual thinking.

Rather, the DM-based methods assign a weight to each DM based on interpersonal evaluations or using their individual characteristics as a benchmark. Within this subgroup can be included the following approaches: manager interpersonal evaluations, group interpersonal evaluation and individual characteristics. The manager interpersonal evaluations consist of either direct DMs valuations given by a manager or superior (Hafezalkotob & Hafezalkotob, 2017; Tabatabaei et al., 2019). The interpersonal evaluations consider the direct DMs valuations given by group members using Markov chains (Bodily, 1979), an analytic hierarchy process (van den Honert, 2001), or by confidence surveys (Chen et al., 2018; Wu et al., 2015). These direct evaluations show the evaluators' opinions regarding other DMs. Finally, among the individual characteristics, Slevin et al. (1998) propose a form to state the self-confidence of each DM in their individual assessments. Similar is the idea of Elbarkouky and Fayek (2011), who send each DM a form to fill in that considers their years and diversity of work experience, position and time in the company, plus enthusiasm and willingness to participate. DMs are weighted by comparing the group forms. Experience and knowledge (Borissova, 2018; Chunhua et al., 2020) and formation and expertise (Ivlev et al., 2015) comparisons have also been used as weighting characteristics. Finally, there are some discrimination and consistency capability comparisons among DMs (Cheng et al., 2018; Herowati et al., 2014).

Even though subjective techniques are easier to apply, some scientists and experts considered dishonesty or unfairness, among other issues, could take place in the decision, leading to a biased consensus. Therefore, techniques which ignore DMs and only look at their

Table 1
MCGDM literature approaches

Label	Subgroup	Type	References
Subjective	Negotiation methods	DELPHI	Greatorex and Dexter (2000); Okoli and Pawlowski (2004)
		Devil's advocacy	Schwenk and Cosier (1993)
		Ward method	Schielke et al. (2009)
	DM-based methods	Manager interpersonal evaluation	Hafezalkotob and Hafezalkotob (2017); Tabatabaei et al. (2019)
		Group interpersonal evaluation	Bodily (1979); van den Honert (2001); Chen et al. (2018); Wu et al. (2015)
		Individual characteristics	Slevin et al. (1998); Elbarkouky and Fayek (2011); Borissova (2018); Chunhua et al. (2020), Ivlev et al. (2015); Cheng et al. (2018); Herowati et al. (2014)
Objective	Data-based methods	Group distance	Theil (1963); Xu and Zhou (2017); Zeng et al. (2016); Thong et al. (2020)
		Distance to a centroid	Chen et al. (2021); Lin et al. (2018); Yue (2012); Yue (2011)
		Optimisation algorithms	Xu and Wu (2013); Lin and Wang (2018); Li et al. (2019); Ma et al. (2020); Meng et al. (2016); Wan et al. (2016); Dong and Cooper (2016); Ji et al. (2021)
		Evaluation quality	Cabrerizo et al. (2010); Wu et al. (2018); Toloie-Eshlaghy and Farokhi (2011); Ye (2014); RazaviToosi and Samani (2019); Wu et al. (2020)
	Assessment aggregation operators	Direct aggregation	Akram et al. (2019); Garg and Kaur (2020); Zindani et al. (2020)
	Ranking aggregation methods	Probability distribution techniques	Thurstone (1927a, 1927b)
		Heuristic techniques	Aslam and Montague (2001); Deconde et al. (2006); Dwork et al. (2001)

evaluation have been proposed as possible solutions to these problems (Liu & Li, 2015). These are known as objective techniques, and the last three subgroups can be labelled as such.

The first objective subgroup encompasses the Data-based methods, which use DM evaluations to determine their weight based on four characteristics: Their group distance, their distance to a centroid, an optimisation algorithm or the evaluation quality. The group distance methods analyse the distance between DM assessments. A short distance can be considered to have a negative effect, reflecting inefficacy (Theil, 1963) or not helping to discern between options (Xu & Zhou, 2017). On the other hand, it can be considered positive as a short distance implies compatibility (Zeng et al., 2016) or correlation (Thong et al., 2020) among DMs. The centroid distance methods use the distance to a central point, considered the agreement, to determine DM weights in a directly relative way. The total average has been the most widely used central point (Chen et al., 2021; Lin et al., 2018), also applied in the projection method (Yue, 2012) to determine distance among DMs and average vectorial projections. However, the same author has also modified the well-known TOPSIS (Yue, 2011) to weigh DMs by their distance to the ideal and non-ideal solution. The optimisation algorithms methods generally generate dynamic weights to determine the minimum distance. This distance can be between DMs (Xu & Wu, 2013; Lin & Wang, 2018; Li et al., 2019; Ma et al., 2020), or among each DM and the group average (Meng et al., 2016; Wan et al., 2016). Some of these algorithms introduce a feedback phase to allow an assessment change depending on the results (Dong & Cooper, 2016; Ji et al., 2021). Finally, the evaluation quality methods use indicators to analyse the assessment worth. These indicators can be such as the assessment consistency (Cabrerizo et al., 2010; Wu et al., 2018), the iterations needed to obtain the convergence vector (Toloie-Eshlaghy & Farokhi, 2011) or the entropy degree, understood as the lack of information in each evaluation (Ye, 2014; RazaviToosi & Samani, 2019; Wu et al., 2020).

The Assessment aggregation operators, which is the second subgroup of objective approaches, encompasses the direct aggregation operators, such as the arithmetic or geometric means. These are mainly used to simplify MCGDM problems by converting them into MCDM. For example, some studies aggregate the DMs' preferences before applying TOPSIS, a Technique for Order of Preference by Similarity to the Ideal Solution (Akram et al., 2019; Garg & Kaur, 2020), or TODIM (Zindani et al., 2020), an acronym in Portuguese for Interactive Multi-criteria Decision Making.

Finally, the Ranking aggregation methods, based on combining the DM individual rankings once applied their assessments. This combination can be done using probability distribution techniques based on the Thurstone scale (Thurstone, 1927a) or binary comparisons (Thurstone, 1927b), or heuristic techniques to obtain a solution by simple and intuitive mathematical approximations, such as Borda methods (Aslam & Montague, 2001) or Markov chains (Deconde et al., 2006; Dwork et al., 2001).

Even though all the studies mentioned above have contributed to finding group consensus and explicitly aim to obtain the best group solution, as said in the introduction section, the social choice theory has been omitted in creating their proposed methodologies. Overlooking this theory could lead to a mathematically acceptable but socially unacceptable solution. In addition, these methods generally do not consider the existence of previous influence, which is an increasingly necessary feature. Lastly, the multi-criteria assessment data set is not considered component by component, which could lead to a consensus disagreement among DMs.

3. HIVES method

This section presents the proposed methodology to reach a consensus by weighting DMs. Firstly, the idea behind the development approach is explained plus why the method has been proposed. Then, the social constraints and new mathematical approaches and definitions encompassed by the methodology are laid out. Finally, the algorithm is given.

3.1. Developed approach

As presented in section 1, there is a lack of mathematical consensus approaches that include social choice theory characteristics. This is important as social choice theory has a long history of developing multiple participant aggregation approaches beyond mathematical approximations, including psychological or social welfare considerations to ensure fairness and equity in a decision. Furthermore, the literature review highlighted other possible improvements. For example, component division is proposed as some authors have presented a consensus group response improvement (Kerr & Tindale, 2004) with its use. Additionally, how the previous influence is addressed must be analysed as in real-world decisions, participants might not have same importance. With this in mind, we needed to develop a methodology that could attain 5 objectives.

First, consensus must be reached by maximizing group satisfaction. Second, social choice theory must be considered in the form of mathematical methodology constraints. Third, the method must be able to handle previously given weights or influence. Fourth, methodology must be applicable even with no alternative knowledge. Finally, analysis technique must divide the MCGDM into components to improve the satisfaction response.

The proposed methodology has been developed considering these objectives. It was named Hierarchical Integration of Values and Evaluations under Social constraints (henceforth HIVES). HIVES is a dynamic methodology which can use the initial participant influence and their assessments given in non-negative percentage values to simulate group behaviour, called hive behaviour (HB). HB is used to maximise group satisfaction in all components of the decision, as every component is specifically analysed. The representation of how HIVES works is shown in Fig. 1, inspired by beehive consensus.

Every DM assessment contains a weight vector regarding the analysed criterion set. This vector is affected by the DMs respective previous influence which is represented by different hive roles, such as a worker bee or a queen bee, and it means that HIVES can consider participants equally or unequally. Once assessments have been influenced by DM weights, every component of the set of criteria is allocated in a unique HIVES analysis. This analysis applies statistical mathematics with social theory constraints to find the most desirable group solution.

This solution is based on a game methodology where DMs obtain a score regarding their position compared to the group. This score is assigned using two new concepts: the social ideal consensus point (SICP) and the score bell. The DMs' score places their weights. Finally, the consensus is reached by a component aggregation.

3.2. Social choice theory constraints

The social choice theory has been used to analyse multiple ways of combining individual preferences and opinions to reach an acceptable collective decision. In this theoretical framework, numerous axioms, statements or premises taken to be true have been proposed to ensure fairness and equity. Ramanathan and Ganesh (1994) presented 4 of the most common Axioms and provided another one to consider. These five have been used to develop statistical mathematics HIVES social restraints and have been adapted to consider criteria preferences instead of alternatives due to the HIVES domain. The following are the social choice Axioms that are used:

- *Axiom 1:* Universal domain. The group preference aggregation method should define a group preference pattern for all individual preferences that are logically possible. In other words, it should not be impossible to provide the group preference for any set of individual preferences.

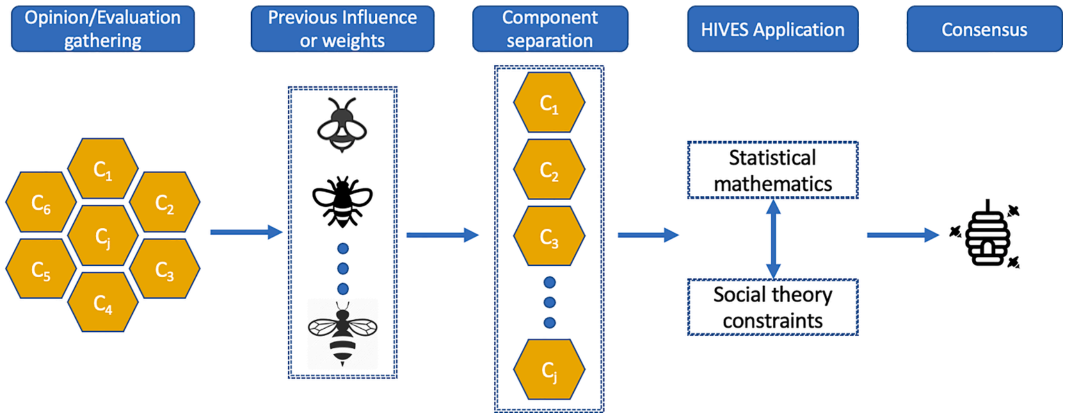

Fig. 1. Representation of how HIVES works.

- *Axiom 2:* Pareto optimality. Let A and B be two criteria. If all group members prefer A to B, then the group decision should favour A.
- *Axiom 3:* Independence of irrelevant criteria. If a criterion is eliminated from consideration, the new group ordering for the remaining criteria should be equivalent (i.e., same ordering) to the original group ordering for the same criteria.
- *Axiom 4:* Non-dictatorship. There is no individual whose preferences automatically become the group's preferences, independently of the preferences of other group members.
- *Axiom 5:* Recognition. Group preferences are reached only after considering all the members' preferences. This means that all DM or participants must be considered.

Regarding the HIVES method, two Axiom characteristics should be highlighted. First, Axiom 3 "Independence of irrelevant criteria" has been adapted to state criteria orderings instead of alternatives. As HIVES works with percentual criteria scores, the same ordering can be produced. It means that the new methodology must be able to accept the deletion of a criterion. Second, Axiom 4 "non-dictatorship" is directly related to previous DM weights. This means that, in an initial condition affected by equality, the method ensures this Axiom. If there are previous weights to be applied, that Axiom might be violated. For example, if one participant holds 60 % of the decision power before applying HIVES, it is obvious that this Axiom cannot be applied.

Besides these occasional highlights, HIVES ensures an adaptable methodology which improves the process's objectivity and neutrality. At the same time, social choice Axioms increase fairness and equity throughout the technique.

3.3. Mathematical concepts developed

As has been said, HIVES is based on an HB concept generated by DMs assessments. These group assessments are introduced to two new methodological concepts, the social ideal consensus point (SICP) and score bell, which create a game where all DM-introduced assessments obtain a score from SICP distance and the Axioms are used as mathematical restrictions. These scores are used to weight DMs in every component of the criterion set.

3.4. Social ideal consensus point (SICP)

Even though the data average has been widely used as an ideal consensus, some academic studies have questioned its usefulness (Crott et al., 1991; Davis et al., 1997). Moreover, its direct use may violate the presented Axioms. For example, an extremely biased DM might modify the result, violating Axiom 4. Therefore, in the present paper a novel ideal consensus, named as the social ideal consensus point (SICP), is developed.

The SICP is generated by the average of the criterion assessments encompassed between the first and third quartile data. These assessments have the highest probabilities of being accepted by the group due to its position and are placed in a range named influence zone (IZ). On the other hand, criterion assessments encompassed by the minimum and first quartile and third quartile and maximum are placed in a range named dispersion zone (DZ). These assessments are far from possible consensus, meaning these DMs must bend to reach a solution. As seen in Fig. 2, IZ and DZ representation creates a combination of boxplot and Japanese candle.

Therefore, the SICP is determined as follows. For convenience, in this paper, let $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$; $i \in M$ and $j \in N$. And let $C = \{c_1, c_2, c_3, \ldots, c_n\}$ be a discrete set of criteria and $D = \{d_1, d_2, d_3, \ldots, d_m\}$ a group of DMs. Suppose each $d_i \in D$ evaluates the importance of each criterion of the criteria set by a weight vector $W_i = \{w_{i1}, w_{i2}, w_{i3}, \ldots, w_{in}\}$, such that $0 \geq w_{ij} \geq 100$, $\sum_{j=1}^{n} w_{ij} = 100$ for $\forall$ j $\in$ N and $\forall$ i $\in$ M. Then, all member group evaluations constitute a decision matrix expressed by:

$$GE = (w_{ij})_{m\ x\ n} = \begin{array}{c} \\ d_1 \\ d_2 \\ \vdots \\ d_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_{m1} & w_{m2} & \cdots & w_{mn} \end{bmatrix} \end{array} \quad (1)$$

where w_{ij} expresses the weighting of DM d_i over the criterion c_j. For each criterion c_j there is a $SICP_j$ generated by the set of weights w_{ij} which are placed between the first and third quartile of c_j data. Therefore, let $F_j = \{j1, j2, \ldots, jf\}$; $jr \in F_j$ be the discrete subset of c_j in which its components meet $Q_1{}^j \leq w_{ij} \leq Q_3{}^j$, where Q_1^j and Q_2^j are the first and third quartiles, respectively. These considered weights form a set $W_{rj,j} = \{w_{j1,j}, w_{j2,j}, w_{j2,j}, \ldots, w_{jf,j}\}$ which are used to get the $SICP_j$, as follows:

$$SICP_j = \frac{1}{jf} \cdot \sum_{jr=j1}^{jf} w_{rj,j} \quad (2)$$

3.5. Score bell (SB)

Once the SICP has been presented, a scoring tool to judge each DM position over the group in every component is needed. The selected tool is the score bell (SB), a function similar to a probability density function for two reasons: (i) Its ability to link the assessments' influence status (IZ, DZ, SICP) shown in Fig. 2 and its actual relevance with the data set and (ii) the possibility of treating the function in parts, assigning fixed values that turn the considered Axioms into mathematical restrictions.

Hence, SB is formed by 4 functions depending on its respective application zone. The first encompasses from the minimum assessment to Q1 value. The second encompasses from Q1 value to the SICP. The

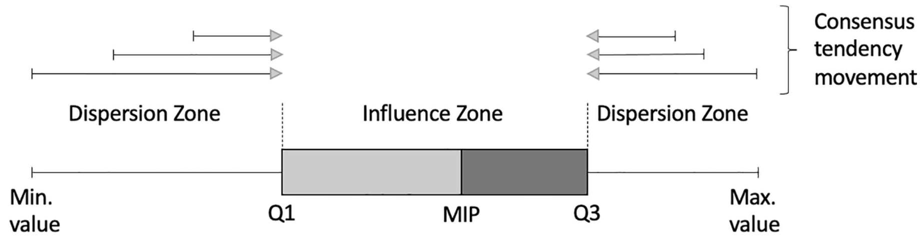

Fig. 2. Representation of influence zones.

third comprises from SICP to Q3 value. The fourth includes from Q3 value to maximum assessment. Every SB equation contains a relative position score. This means that a score value must be given to equation limits. Thus, for each criterion there are four SB equations:

$$SB_1 = e^{Ln(S_{Mean})\cdot\left(\frac{x_{rel}}{x_{Q1}-x_{\min}}\right)^f} \tag{3}$$

$$SB_2 = (S_{\max}+1) - e^{Ln(S_{\max}+1-S_{Mean})\cdot\left(1-\frac{x_{rel}}{x_{Max\ \inf.}-x_{Q1}}\right)^f} \tag{4}$$

$$SB_3 = (S_{\max}+1) - e^{Ln(S_{\max}+1-S_{Mean})\cdot\left(\frac{x_{rel}}{x_{Q3}-x_{Max\ \inf.}}\right)^f} \tag{5}$$

$$SB_4 = e^{Ln(S_{Mean})\cdot\left(1-\frac{x_{rel}}{x_{\max}-x_{Q3}}\right)^f} \tag{6}$$

where:

- S_{Mean}: Q1 and Q3 turning point scores.
- S_{Max}: SICP score.
- x_{rel}: Relative DM assessment for analysed criteria regarding SB domain as shown in Fig. 3.
- x_{min}: Minimum assessment value.
- x_{Q1}: Q1 assessment value.
- $x_{Max\ inf.}$: SICP value.
- x_{Q3}: Q3 assessment value.
- x_{max}: Maximum assessment value.
- f: SB shape form.

HIVES development highlighted some characteristics. Minimum score must be 1 due Axiom 5 restriction. Medium score of 50 is given to Q1 and Q3 values. A maximum score of 100 is given to SICP. A shape factor of 2 has been used to express the influence difference between zones. Thus, four SB equations are:

$$SB_1 = e^{Ln(50)\cdot\left(\frac{x_{rel}}{x_{Q1}-x_{\min}}\right)^2} \tag{7}$$

$$SB_2 = (101) - e^{Ln(51)\cdot\left(1-\frac{x_{rel}}{x_{Max\ \inf.}-x_{Q1}}\right)^2} \tag{8}$$

$$SB_3 = (101) - e^{Ln(51)\cdot\left(\frac{x_{rel}}{x_{Q3}-x_{Max\ \inf.}}\right)^2} \tag{9}$$

$$SB_4 = e^{Ln(50)\cdot\left(1-\frac{x_{rel}}{x_{\max}-x_{Q3}}\right)^2} \tag{10}$$

As expressed, these four equations shown in Fig. 4 are directly related to the influence treatment. DZ equations exponentially decrease the DM score once their opinion moves away from SICP. On the other side, IZ equations consider an exponentially flat rising score, given the SICP proximity. Once the SB is generated, each DM receives a score vector $S_i = \{S_{i1}, S_{i2}, S_{i3}, \dots, S_{in}\}$, for all $i \in M$, directly related to $W_i = \{w_{i1}, w_{i2}, w_{i3}, \dots, w_{in}\}$, which defines its score on each criterion. The DM weights will be defined by the score vectors.

3.6. Previous influence treatment

Once the social constraints have been presented, and the HIVES mathematical concepts explained, how HIVES manages previous influence will be shown. In the real world, equal influence is barely normal. Differing knowledge levels, work hierarchy or citizen representation percentages are all situations in which decision participants might have different weights. HIVES brings in the "vote-casting" concept to introduce different initial influences into the methodology.

As presented, HIVES uses DM assessments to generate IZ, DZ, SICP and SB. In other words, HIVES considers each assessment as a DM vote to generate the process. When a previous influence exists, HIVES translates it into a different number of votes cast, taking the DM with less influence as a basis. Thus, the number of votes cast by each d^i is:

$$v^i = \frac{Influence^i}{Influence_{\min}} \tag{11}$$

Then, Axiom 5 of representativity is ensured as the less weighted DM is represented. Slight differences between DM are dependent on the relative aspect of the weight, as in some cases they will be neglected. This system is widely used in democracy, where parliamentary positions are assigned by a minimum number of votes.

3.7. HIVES algorithm

Once the mathematical and social concepts used have been presented, the HIVES algorithm to determine DM consensus and weights is

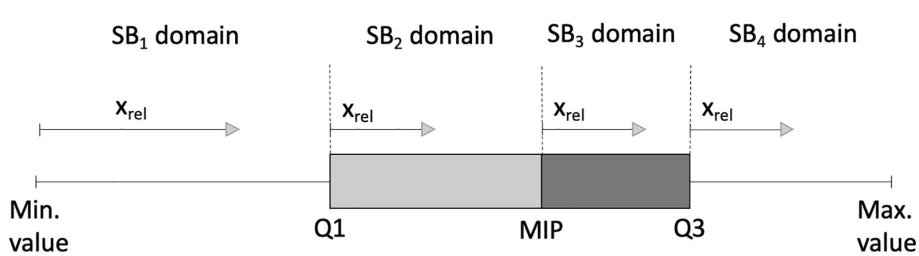

Fig. 3. x_{rel} position example regarding SB domain.

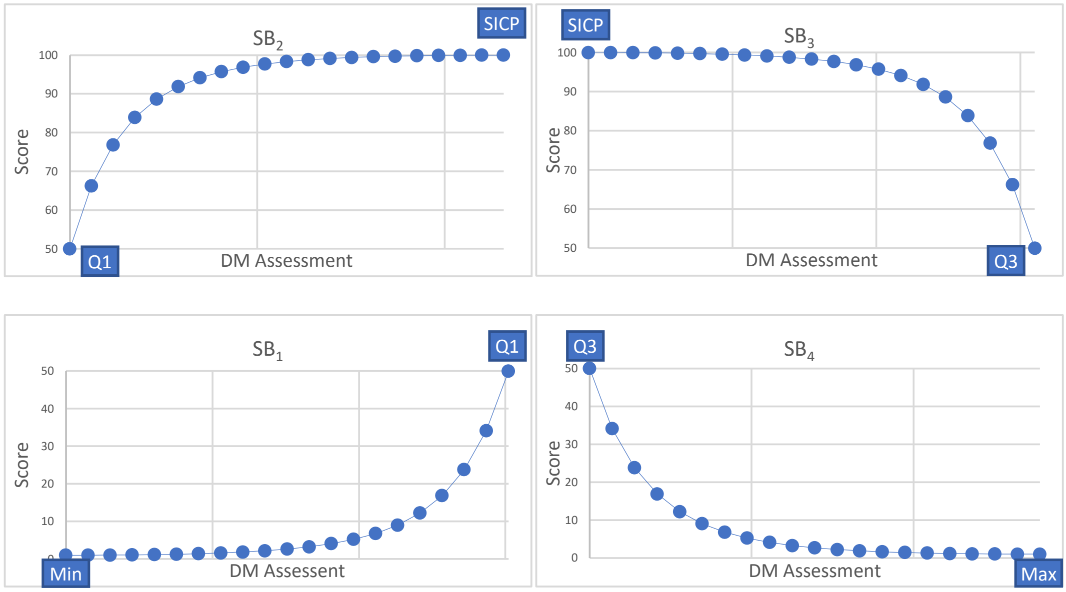

Fig. 4. Graphic representation of Score Bell.

shown in Fig. 5. Notice that previous MCDM preparation steps are included. These are DM selection, influence analysis and evaluation gathering. Even though these steps are common in every MCDM process, their inclusion is used to show the vote casting technique and component separation application.

As shown, representation of HIVES behaviour starts once the vote casting technique has been used to consider possible previous influence and component separation has been performed to divide the MCDM problem in c_j analysis. For each criterion, there is a set generated by decision maker' assessments regarding its importance. For each one of them, HIVES follows these steps:

- *Step 1*: SICP calculation. Minimum, Q1, Q3 and maximum assessments are used to generate IZ, and DZ. SICP is determined by Eq. (2).
- *Step 2*: DM assessment zone designation and x_{rel} derivation. As SB is generated by 4 functions with their respective domains, each DM assessment must be positioned in one of them as shown in Fig. 3 and its x_{rel} must be obtained.
- *Step 3*: Score bell application. With DM assessment x_{rel}, IZ, DZ and SICP knowledge, the score bell can be applied by equations (7) to (10). Each DM will obtain a score vector S_i directly related its W_i. These score vectors form a HIVES score matrix (HSM) as follows:

$$HSM = \left(S_{ij}\right)_{m\,x\,n} = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_m \end{array} & \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix} \end{array} \tag{12}$$

- *Step 4*: DM criteria weight. For every c_j each DM obtains a score percentage regarding other DMs. This percentage represents d_i weight over the analysed criteria and is obtained using Eq. (13). These weights constitute the percentage score matrix (14):

$$\lambda_{ij} = \frac{s_{ij}}{\sum_{i=1}^{m} s_{ij}} \cdot 100 \tag{13}$$

$$\lambda_{HSM} = \left(\lambda_{ij}\right)_{m\,x\,n} = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_m \end{array} & \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{m1} & \lambda_{m2} & \cdots & \lambda_{mn} \end{bmatrix} \end{array} \tag{14}$$

- *Step 5*: Criteria weights. Once DMs have obtained their respective decision weights, a weight sum method (WSM) is used to aggregate the assessments. Thus, criteria c_j weight γ_j is:

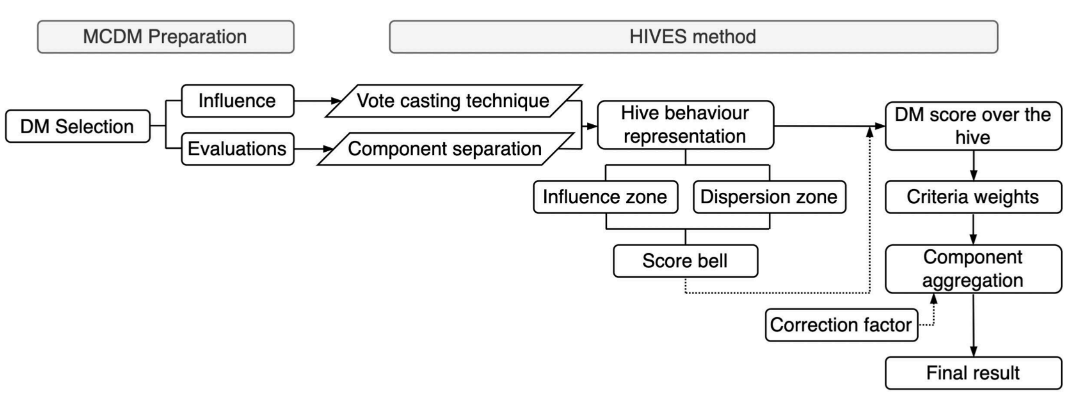

Fig. 5. HIVES process representation.

$$\gamma_j = \sum_{i=1}^{m} \lambda_{ij} \cdot w_{ij} \tag{15}$$

- *Step 6*: Component aggregation. The sum of the criteria may not be 100 as HIVES maximizes DM satisfaction over all criteria. This means that, in some cases, the correction factor shown in equation (16) is needed to transform the result into $\sum_{j=1}^{n}\gamma_j = 100$. This correlation factor does not affect the final decision as the relative aspect between criteria weights remains constant. Then, the final modified criteria weight γ'_j is (17):

$$\beta = \frac{100}{\sum_{j=1}^{n}\gamma_j} \tag{16}$$

$$\gamma_j' = \beta \cdot \gamma_j \tag{17}$$

4. Illustrative example

In the following, an instance adapted from Shih et al. (2007) is provided to illustrate the proposed approach.

A local chemical company is trying to recruit an online manager. The company's human resources department provides relevant selection tests as the benefit attributes to be evaluated. These objective tests include knowledge tests (language test, education test, professional test, and safety test), and skill tests (professional skills and computer skills). After these objective tests, the candidates (as alternatives marked as A1, A2, …, A15) obtain the scores shown in Table 2. Then, seven DMs (marked as d_1, d_2, …, d_7) give the elicited criteria weights shown in Table 3, which must be aggregated to rank the candidates set. Importance equity is supposed between DMs; thus, they have the same initial weight.

The HIVES method is applied using this information. First, component separation must be performed:

$$c_1 = \{23, 10, 7, 15, 38, 25, 30\} \tag{18}$$

$$c_2 = \{8, 9, 9, 10, 17, 25, 20\} \tag{19}$$

$$c_3 = \{14, 7, 14, 10, 8, 15, 10\} \tag{20}$$

$$c_4 = \{30, 45, 20, 35, 20, 10, 17\} \tag{21}$$

$$c_5 = \{15, 20, 25, 20, 2, 15, 13\} \tag{22}$$

$$c_6 = \{10, 9, 25, 10, 15, 10, 10\} \tag{23}$$

- *Step 1*: SICP calculation. For each criterion, IZ, DZ and equation (2) are applied to obtain Table 4 data.

Table 2
Decision matrix with the candidates set and their respective scores.

Candidates	c_1	c_2	c_3	c_4	c_5	c_6
A1	11	22	16	69	67	16
A2	92	72	10	31	45	36
A3	74	49	15	6	20	42
A4	77	87	72	94	35	98
A5	51	81	13	9	21	75
A6	90	68	12	4	57	39
A7	32	77	13	8	51	90
A8	87	15	37	6	4	31
A9	67	45	38	10	93	42
A10	55	79	14	33	41	61
A11	16	89	80	23	13	45
A12	28	96	11	74	19	17
A13	59	78	75	90	50	93
A14	83	30	68	94	39	13
A15	48	58	34	28	30	20

Table 3
Proposed criteria weights.

DMs	c_1	c_2	c_3	c_4	c_5	c_6
d_1	23	8	14	30	15	10
d_2	10	9	7	45	20	9
d_3	7	9	14	20	25	25
d_4	15	10	10	35	20	10
d_5	38	17	8	20	2	15
d_6	25	25	15	10	15	10
d_7	30	20	10	17	13	10

Table 4
Criteria assessments characteristics.

	c_1	c_2	c_3	c_4	c_5	c_6
Min.	7.00	8.00	7.00	10.00	2.00	9.00
Q1	12.50	9.00	9.00	18.50	14.00	10.00
SICP	21.00	11.25	12.00	23.33	17.50	10.00
Q3	27.50	18.50	14.00	32.50	20.00	12.50
Max.	38.00	25.00	15.00	45.00	25.00	25.00

- *Step 2*: DM assessment zone designation and x_{rel} derivation. With Table 4 data, each DM assessment can be placed in one SB zone as shown in Table 5. Table 6 contains the relative distance of each assessment placed in HB.
- *Step 3*: Score bell application. With DM assessment x_{rel}, IZ, DZ and SICP knowledge, the score bell is applied by Eqs. (7) to (10). Table 7 shows the DM score results over each criterion.
- *Step 4*: DM criteria weight. Given the Table 7 DM scores, equation (13) is applied. Thus, Table 8 DM weights are obtained for each criterion.
- *Step 5*: Criteria weights. Table 8 data is aggregated by equation (15) to obtain the criteria weight set shown in equation (24). A shown, the criteria sum is not 100 %. Therefore, equation (16) is applied to obtain the correlation factor, which is introduced to the criteria weight set to obtain the modified weight set shown in equation (25).

$$\gamma_j = \{21.25, 12.14, 11.34, 23.50, 16.40, 10.18\} \tag{24}$$

$$\gamma_j' = \{22.41, 12.80, 11.96, 24.78, 17.30, 10.74\} \tag{25}$$

These weight values can be directly applied in Table 2 to obtain Table 9 data. This table includes the criteria with their respective weights, the aggregated sum and the final ranking.

5. Comparisons with other methods

In this section, the HIVES method is compared to other methods. The methods to be compared here are ETOPSIS and Projection (Yue, 2011, 2012). These methods have been selected as they focus on assigning weights to DMs analysing their assessments, being part of the same HIVES subgroup, the objective Data-based methods. Furthermore, both are recently presented techniques that have had a huge scientific impact on the MCGDM field and use non-negative real numbers.

To begin the comparison, the theoretical part of each method is analysed in Table 10. This table contains a theoretical comparison with

Table 5
DM assessments assigned zones.

DMs	c_1	c_2	c_3	c_4	c_5	c_6
d_1	3	1	3	3	2	3
d_2	1	2	1	4	3	1
d_3	1	2	3	2	4	4
d_4	2	2	2	4	3	3
d_5	4	3	1	2	1	4
d_6	3	4	4	1	2	3
d_7	4	4	2	1	1	3

Table 6
DM assessments x_{rel} regarding their respective zones.

DMs	c_1	c_2	c_3	c_4	c_5	c_6
d_1	2.00	0.00	2.00	6.67	1.00	0.00
d_2	3.00	0.00	0.00	12.50	2.50	0.00
d_3	0.00	0.00	2.00	1.50	5.00	12.50
d_4	2.50	1.00	1.00	2.50	2.50	0.00
d_5	10.50	5.75	1.00	1.50	0.00	2.50
d_6	4.00	6.50	1.00	0.00	1.00	0.00
d_7	2.50	1.50	1.00	7.00	11.00	0.00

Table 7
DM scores once SB equations have been applied.

DMs	c_1	c_2	c_3	c_4	c_5	c_6
d_1	99.55	1.00	50.00	93.00	93.57	100.00
d_2	3.20	50.00	1.00	1.00	50.00	1.00
d_3	1.00	50.00	50.00	94.51	1.00	1.00
d_4	93.91	97.63	95.26	12.23	50.00	100.00
d_5	1.00	89.14	2.66	94.51	1.00	12.23
d_6	96.57	1.00	1.00	1.00	93.57	100.00
d_7	9.69	10.12	95.26	14.20	26.77	100.00

Table 8
DM weights for each studied criterion.

DMs	c_1	c_2	c_3	c_4	c_5	c_6
d_1	32.65 %	0.33 %	16.94 %	29.96 %	29.62 %	24.14 %
d_2	1.05 %	16.73 %	0.34 %	0.32 %	15.83 %	0.24 %
d_3	0.33 %	16.73 %	16.94 %	30.44 %	0.32 %	0.24 %
d_4	30.80 %	32.66 %	32.27 %	3.94 %	15.83 %	24.14 %
d_5	0.33 %	29.82 %	0.90 %	30.44 %	0.32 %	2.95 %
d_6	31.67 %	0.33 %	0.34 %	0.32 %	29.62 %	24.14 %
d_7	3.18 %	3.39 %	32.27 %	4.57 %	8.47 %	24.14 %

the main characteristics, making it possible to distinguish elements.

Although three methods share the same purpose, there are differences that must be raised. However, we must highlight that each method has its own advantages and disadvantages. No one method performs better in all situations. For example, ETOPSIS can be used not only to obtain maximum profit but to avoid maximum risk. There can be situations in which DMs might decide to forfeit profit to gain safety. The ETOPSIS and Projection method could also be easier to apply.

However, a practical comparison is also needed to compare the HIVES performance with other methods. With this aim, illustrative example data is used to obtain ETOPSIS and Projection criteria weight vectors shown in Table 11. This table also contains a resume of HIVES criteria assigned weights. As can be seen, even though the percentage values are similar, there are some noticeable differences. Criteria 1 and 5 obtain a higher value, as HIVES has been able to analyse the existence of discordant DMs, d_3 and d_5, and has raised its value. The opposite occurs in Criteria 2 and 6, as d_3, d_6, and d_7 gave discordant values and HIVES has decreased its value.

Nevertheless, to analyse each method's performance, a comparison through ranking correlation needs to be done. With this aim, Spearman's (Kendall & Smith, 1939) correlation coefficient to evaluate the statistical difference between rankings is selected, as it is one of the most widely used ranking comparison coefficients (Kannan et al., 2014). Its value ranges from −1 to 1 and is calculated as follows:

Table 10
Theoretical Comparison between ETOPSIS, Projection and HIVES method.

Characteristic	ETOPSIS	Projection	HIVES
Objective	Selection and ranking of a number of experts	Selection and ranking of a number of experts	Selection and ranking of a number of experts
Alternative knowledge needed	Yes	Yes	No
Number of DM	N≧2	N≧2	N≧3
Previous influence treatment	No	No	Yes
Component separation	No	No	Yes
Ideal solutions	(3) Average, negative, maximum	(1) Average	(1) Majority average
Key decision	Relative closeness	Projection	Punctuation over group behaviour
Core factors	The distance from each individual decision to ideal decision	Both distance and angle from each individual decision to ideal decision	Relativeness between individual punctuation and group punctuation
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives	Criteria weights decision and ranking of a number of alternatives
Goal(s)	Both maximum profit and minimum risk/ regret	Maximum profit	Maximum profit and group satisfaction

Table 11
Criteria weights for each method.

Method	c_1	c_2	c_3	c_4	c_5	c_6
HIVES	22.41 %	12.80 %	11.96 %	24.78 %	17.30 %	10.74 %
ETOPSIS	21.00 %	13.88 %	11.22 %	25.45 %	15.88 %	12.56 %
Projection	22.01 %	14.26 %	10.99 %	25.20 %	15.12 %	12.43 %

Table 9
Alternatives' weighted criteria, criteria aggregation and its ranking position.

Candidates	$c_1 \cdot \gamma_2$	$c_2 \cdot \gamma_2$	$c_3 \cdot \gamma_2$	$c_4 \cdot \gamma_2$	$c_5 \cdot \gamma_2$	$c_6 \cdot \gamma_2$	Σ	Raking
A1	246.56	281.58	191.43	1710.01	1159.17	171.81	3760.56	10
A2	2062.14	921.53	119.64	768.27	778.55	386.58	5036.70	4
A3	1658.68	627.15	179.47	148.70	346.02	451.01	3411.02	14
A4	1725.92	1113.51	861.43	2329.58	605.54	1052.35	7688.34	1
A5	1143.14	1036.72	155.54	223.04	363.32	805.37	3727.13	12
A6	2017.31	870.33	143.57	99.13	986.16	418.79	4535.30	7
A7	717.27	985.52	155.54	198.26	882.35	966.45	3905.38	9
A8	1950.07	191.98	442.68	148.70	69.20	332.89	3135.52	15
A9	1501.78	575.95	454.64	247.83	1609.00	451.01	4840.21	5
A10	1232.80	1011.12	167.50	817.83	709.34	655.03	4593.63	6
A11	358.63	1139.11	957.15	570.00	224.91	483.22	3733.03	11
A12	627.61	1228.70	131.61	1833.93	328.72	182.55	4333.11	8
A13	1322.46	998.32	897.33	2230.45	865.05	998.66	7312.27	2
A14	1860.41	383.97	813.57	2329.58	674.74	139.60	6201.87	3
A15	1075.90	742.34	406.79	693.92	519.03	214.77	3652.74	13

$$\rho = 1 - \frac{6S(d^2)}{(n^3 - n)} \tag{26}$$

where:

- $S(d^2)$ is the sum of differences between two ranks of each observation
- n are the considered alternatives

Therefore, HIVES, ETOPSIS and Projection final rankings must be compared with each DM's individual ranking. Applying Table 11 weights to Table 2 participants' scores, both ETOPSIS and Projection methods obtain the same final ranking R = {A_4, A_{13}, A_{14}, A_2, A_9, A_{10}, A_6, A_{12}, A_7, A_5, A_{11}, A_1, A_{15}, A_3, A_8}, but different from the HIVES ranking shown in Table 9. The Spearman correlation between these rankings and HIVES is shown in Table 12. This table gives a correlation value for each DM and the compared method, including the aggregated value.

As can be seen, this table highlights the correlation improvement that the HIVES method achieves. One of the main reasons for this good performance lies in the component separation. Even though some DMs have proposed "biased" assessments, HIVES has safeguarded their weightage in their "non-biased" criteria, leading to a more accurate and fair analysis. For example, d_3, d_5, d_6 and d_7, which received a lower weight in some criteria due to their discordance with the group, improved their correlation with the final ranking. However, d_1 and d_2 correlation drops have to be pointed out.

In resume, the application of the method has highlighted three points. (i) First, HIVES is easily computable even in complex situations. (ii) In fact, HIVES outperforms other methods is in these complex situations, with a larger number of criteria and DMs. However, for non-complex MCGDM problems the use of other methods would ensure an acceptable result. (iii) HIVES group profit maximisation has been proved. Using the component division, the method has been able to detect the majorities in each criterion and to generate a ranking with more DM correlation, improving the group profits.

6. Sensitivity analysis

Some technical HIVES aspects must be studied in detail to ensure that the proposed social axioms are met. In these cases, the method is typically proven in the most critical situations. In HIVES, critical situations are those generated by 3 DMs. This is the minimum number of participants required to obtain the statistic HIVES data. Furthermore, "biased" DM existence could lead to high alteration of results.

Once the number of participants has been selected, a double sensitivity analysis is proposed as shown in Fig. 6. The first focuses on HIVES behaviour with 3 DMs with extreme assessment combinations. It contains 6 different case studies with 3 possible assessment types. The first type is Neutral (N), in which all criteria have the same value. The second type is Extremist (E), in which just one criterion matters. The last type is Biased (B), in which one criterion is sacrificed to benefit another.

The second sensitivity analysis is focused on previous influence and vote casting technique. It contains 3 different case studies regarding minimal, subtle differences and huge influences.

Table 12
Spearman's correlation between each DM and the method ranking.

DMs	HIVES	ETOPSIS	Projection
d_1	0,414	0,471	0,471
d_2	−0,089	−0,032	−0,032
d_3	0,146	0,061	0,061
d_4	0,257	0,143	0,143
d_5	0,689	0,632	0,632
d_6	−0,161	−0,304	−0,304
d_7	0,571	0,514	0,514
Total	**1,829**	**1,486**	**1,486**

6.1. HIVES behaviour analysis

As presented in Fig. 6. HIVES behaviour analysis is carried out with 3 DMs, considering 6 situations. Moreover, these situations consider DMs + 1 criteria to highlight technical DM' weight distributions.

First, two situations are related to a unified point of view. 3 Neutral assessments consider criteria weight equity. 3 Homogeneous Extremist assessments consider the same criteria at the same time. Situations 3 to 5 are related to a consensus majority. In these cases, 2 out of 3 DMs share interests. The last situation presents total disagreement. All DMs use Extremist assessments although all of them focus on a different criterion.

As HIVES methodology has been clearly presented in the previous section, results are shown directly in Table 13. This table shows the DMs group, expressed as D = {d_1, d_2, d_3} and the proposed criteria set C = {c_1, c_2, c_3, c_4}. For each c_j, $j \in N$, each DM assigns a weight to each criterion obtaining the weight vector denoted as W_i = {w_1, w_2, w_3, w_4}, $i \in M$. The same table gives the HIVES DM weight allocation for each criterion, expressed as λ_i = {λ_1, λ_2, λ_3, λ_4}, $i \in M$, and the final aggregation is shown as the result.

6.2. Influence and vote casting analysis

The impact of considering the proposed HIVES previous influence method is studied by a vote casting analysis. This analysis allows to show how DM weights change as their assessments change while the initial influence is static, to shed light in the relation between the initial DMs influence and the DMs influence assigned by HIVES.

As presented in Fig. 6, the analysis considers a group of 3 DMs expressed as D = {d_1, d_2, d_3} in 3 different situations. Only one criterion, expressed as C = {c}, is considered to specifically analyse the weight flow between DMs. This weight flow is analysed by considering two DM assessment scenarios encompassed in each situation, as shown in Table 14. The study of the DMs' obtained weights in each scenario and its comparison, will allow to know how HIVES assigns these weights and how they change in the proposed situation. Furthermore, both samples are prepared to contain critical assessment points such as nullity, average and totality.

Once analysis characteristics have been presented, the vote casting technique is applied in all three situations. For each situation, an initial DM influence vector, expressed as α = {α_1, α_2, α_3} and $\sum_{i=1}^{3}\alpha_i = 100$, for each decision maker $i \in M$, is proposed.

The first situation positions a DM with minor influence on the decision. Public opinion concerning municipal projects or decisions could be one example of this kind of influence. The proposed α for this situation is α = {5, 40, 55} in which the first DM presents the minor influence. The second situation considers a subtle influence difference between DMs. A company role influence in a decision could represent this situation. In these cases, a CEO's double vote or influence is usual. The proposed α for this situation is α = {20, 40, 40}. The latter situation studies the impact of a dominant DM on the decision. As an example, it could be administrative or company decisions where secondary opinions might need to be considered. The proposed α for this situation is α = {10, 10, 80}.

The proposed α vectors are translated into votes by the vote casting equation (11). These are expressed as V = {v_1, v_2, v_3} corresponding to each DM's respective vote. With this equation, the obtained number of votes cast V^{S1} = {1, 8, 11}, V^{S2} = {1, 2, 2}, V^{S3} = {1, 1, 8} for each situation. As HIVES methodology has been clearly presented in the previous section, results are shown directly in Table 15. This table contains DM votes, DM assessments, the obtained DM weights, and the result.

6.3. Results discussion

Regarding the behaviour sensitivity analysis, HIVES has clearly

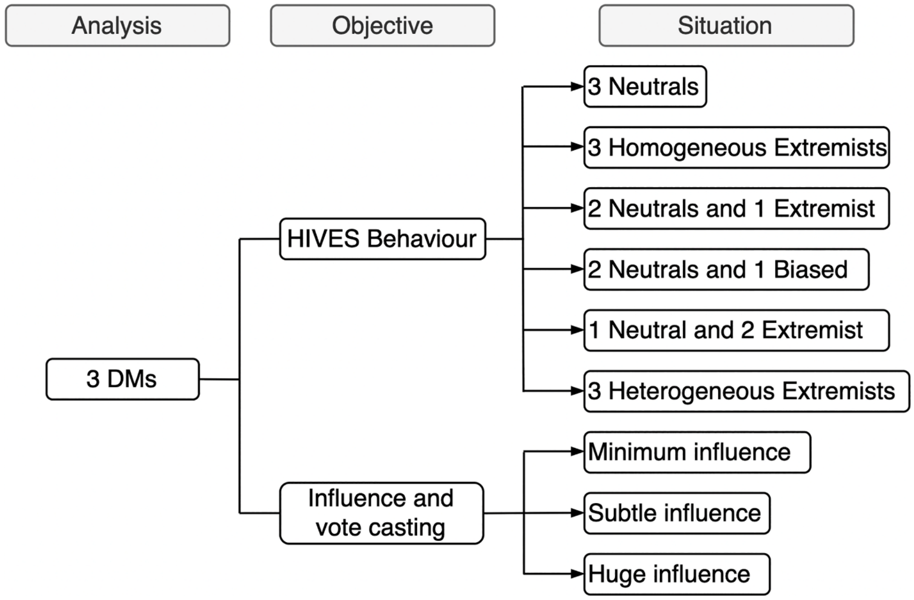

Fig. 6. Double sensitivity analysis representation.

Table 13
HIVES behaviour sensitivity analysis data results.

		c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4	
d_1	Situation 1	25.00 %	25.00 %	25.00 %	25.00 %	100.00 %	0.00 %	0.00 %	0.00 %	Situation 2
d_2		25.00 %	25.00 %	25.00 %	25.00 %	100.00 %	0.00 %	0.00 %	0.00 %	
d_3		25.00 %	25.00 %	25.00 %	25.00 %	100.00 %	0.00 %	0.00 %	0.00 %	
λ_1		33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	
λ_2		33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	
λ_3		33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	33.33 %	
Result		**25.00 %**	**25.00 %**	**25.00 %**	**25.00 %**	**100.00 %**	**0.00 %**	**0.00 %**	**0.00 %**	
d_1	Situation 3	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	Situation 4
d_2		25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	25.00 %	
d_3		100.00 %	0.00 %	0.00 %	0.00 %	25.00 %	25.00 %	0.00 %	50.00 %	
λ_1		49.75 %	49.75 %	49.75 %	49.75 %	33.33 %	33.33 %	49.75 %	49.75 %	
λ_2		49.75 %	49.75 %	49.75 %	49.75 %	33.33 %	33.33 %	49.75 %	49.75 %	
λ_3		0.50 %	0.50 %	0.50 %	0.50 %	33.33 %	33.33 %	0.50 %	0.50 %	
Result		**25.37 %**	**24.88 %**	**24.88 %**	**24.88 %**	**25.00 %**	**25.00 %**	**24.88 %**	**25.12 %**	
d_1	Situation 5	100.00 %	0.00 %	0.00 %	0.00 %	100.00 %	0.00 %	0.00 %	0.00 %	Situation 6
d_2		100.00 %	0.00 %	0.00 %	0.00 %	0.00 %	100.00 %	0.00 %	0.00 %	
d_3		25.00 %	25.00 %	25.00 %	25.00 %	0.00 %	0.00 %	100.00 %	0.00 %	
λ_1		49.75 %	49.75 %	49.75 %	49.75 %	0.50 %	49.75 %	49.75 %	33.33 %	
λ_2		49.75 %	49.75 %	49.75 %	49.75 %	49.75 %	0.50 %	49.75 %	33.33 %	
λ_3		0.50 %	0.50 %	0.50 %	0.50 %	49.75 %	49.75 %	0.50 %	33.33 %	
Result		**99.63 %**	**0.12 %**	**0.12 %**	**0.12 %**	**33.33 %**	**33.33 %**	**33.33 %**	**0.00 %**	

Table 14
DM assessment scenarios introduced in each situation.

	c^1	c^2
d_1	00.00 %	50.00 %
d_2	50.00 %	0.00 %
d_3	100.00 %	100.00 %

presented the expected reactions. In situations where DMs present a unified point of view, DM weights are equally distributed by HIVES. This parity is obtained because all DM assessments coincide with SICP. A similar condition is shown in situations 3, 4 and 5. HIVES can detect the group majority (2 DMs) and makes it possible to prioritize shared interests. Even then, the discordant DM consideration must be stated to meet Axiom 5. With a low decision weight, this DM can slightly modify the final decision. Furthermore, this slight modification is related to the strength of the discordant assessment. However, Axiom 4 of non-dictatorship is met as the other assessments have no preferences.

Table 15
HIVES previous influence sensitivity analysis data results.

	Situation 1		Situation 2		Situation 3	
	c^1	c^2	c^1	c^2	c^1	c^2
v_1	1		1		1	
v_2	8		2		1	
v_3	11		2		8	
d_1	0.00 %	50.00 %	0.00 %	50.00 %	0.00 %	50.00 %
d_2	50.00 %	0.00 %	50.00 %	0.00 %	50.00 %	0.00 %
d_3	100.00 %	100.00 %	100.00 %	100.00 %	100.00 %	100.00 %
λ_1	0.11 %	9.52 %	0.50 %	33.33 %	0.12 %	0,33 %
λ_2	42.06 %	38.10 %	49.75 %	33.33 %	0.33 %	0,12 %
λ_3	57.83 %	52.38 %	49.75 %	33.33 %	99.54 %	99,54 %
Result	**78.86 %**	**57.14 %**	**74.62 %**	**50.00 %**	**99.71 %**	**99.71 %**

When these DMs have a clear preference, as in situation 5, the neutral assessment does not modify the criteria prioritization. In this case, C_1 is maintained as the first criteria while c_2 to c_4 receive minimum weight. At the same time, situations 2, 3, 4 and 5 demonstrate compliance with Axiom 2. When the majority presents a criteria preference, the result is consistent with this preference. Obviously, neutral assessments do not count as a preference as every criterion is assessed equally.

On the other hand, situation 6 presents a complete disagreement between DMs. HIVES detect majority in the null assessments, assigning equity between weights. In this situation, as in situation 2, Axiom 3 is shown to be met completely. The independence of irrelevant criteria is pointed out as these (c_4 and c_2, c_3, c_4 in situations 6 and 2 respectively) can be deleted without affecting the process.

As a HIVES behaviour sensitivity analysis conclusion, good performance has been demonstrated. It was possible to compute all critical situations, which ensures that Axiom 1 is met. Furthermore, component division presents a breakthrough. It makes it possible to consider multiple analysis by assigning numerous weights to DMs regarding the group. These weights only depend on their given assessments, as they are compared with the group. When a bias or extreme assessment is given, HIVES can reduce its impact on the group while it is being considered. The quantity or consensus level of participants are factors related to this reduction.

The previous influence sensitivity analysis has been held by the vote casting technique proposed by HIVES. Over the 3 case studies, DM assessment allocation importance and the number of votes are highlighted. For example, situation 1 d_1 multiplies its decision weightage by more than 86 from sample 1 to sample 2. In this second sample, it even has 10 % of the decision weightage while it cast 5 % of votes (1 out 20), doubling its proposed initial influence. This large influence increase is generated by the d_1 assessment position, as the situation 1 case 2 SICP is placed at 57.50 %. As d_1 has proposed the SICP closest assessment, it obtains a huge influence rise. This SICP distance effect can also be seen in situation 2. With just one vote, in case 2, d_1 assumes the same weight has other DMs who double this DM initial influence.

Another HIVES vote casting technique characteristic is vote proportionality. In situations 1 and 2, decision-makers d_2 and d_3 obtain proportional weight regarding their vote numbers. In situation 1 case 1, each vote corresponds to 5.26 % weightage. 42.06 % and 57.83 % of weight is assigned to d_2 and d_3 with 8 and 11 votes respectively. In situation 2, case 2, d_2 and d_3 obtain a 49.75 % weight as every vote corresponds to a 24.88 % weightage.

This particularity can be explained by how the vote casting technique affects the inherent HIVES statistical mathematics. The first and third quartiles are being altered by N repeated assessments. An influence zone is generated between d_2 and d_3 assessments. If the remaining DM assessment is not placed inside this interval, HIVES considers SICP as the average of these values. In this case, every influence zone vote receives the same weight. On the other hand, if the remaining DM assessment is placed inside the influence zone, it affects SICP generation. This DM receives more weight as its assessment is closer to SICP than others, which generates Q1 and Q3.

Situation 3 examines how HIVES can be monopolized with enough initial influence. In both cases, this situation shows that Axiom 4 of non-dictatorship is not met. Nevertheless, this Axiom is pointless with this initial influence difference. In any condition when a DM possesses such influences, Axiom 4 must be ignored as it is violated before the aggregation process.

Moreover, HIVES vote casting technique has ensured Axiom 1 of universal domain and Axiom 5 of recognition. Any initial influence can be processed, and every DM is represented in the final decision. Also, the analysis has presented the technique's good performance. Different influence opinions can be successfully aggregated with a coherent result weightage. Furthermore, these initial weights are effectively and coherently affected by DM opinion position. In non-extreme situations such as situation 3, the decision weights are altered positively if the participant has a rational group opinion.

7. Conclusion

A specific method has been developed in this paper, based on social theories and statistical mathematics to aggregate DM opinions in a group decision environment.

HIVES has no data distribution limits, and it can handle an indefinite set of DMs and criteria if the minimum requirement of 3 DMs is met. Furthermore, no alternative knowledge is needed, allowing HIVES to be applied previously in the process. This could ensure interference avoidance generated by alternatives that had already been presented. The inclusion of social axioms and component division ensures fairness, equity, and neutrality for the participants. DMs receive specific attribute weights regarding their own proposed assessments and the group behaviour. When a DM proposes an assessment which facilities consensus, it obtains larger influence. This feature is improved considering HIVES previous ability to influence. DMs who support group consensus with commonly rational assessments can greatly improve their decision influence. In a non-extreme situation, this HIVES ability could even modify the DMs' weightage ranking.

Nevertheless, it should be made clear that the use of the proposed method is limited by the requirement that the given data is in crisp percentual numbers. In future research, the proposed method should be extended to support situations in other data forms, e.g., intervals, linguistic variables, or fuzzy numbers.

CRediT authorship contribution statement

David Boix-Cots: Conceptualization, Methodology, Formal analysis, Writing – original draft. **Francesc Pardo-Bosch:** Resources, Supervision, Validation, Writing – review & editing. **Pablo Pujadas:** Resources, Supervision, Validation, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The first author acknowledges support from the Spanish Ministry of Education, Culture and Sports [grant number FPU18/01471]. The second and third author wish to recognise their support from the Serra Hunter programme. Finally, this work was supported by the Catalan - agency AGAUR through its research group support program (2017SGR00227). This research is part of the R&D project IAQ4EDU, reference no. PID2020-117366RB-I00, funded by MCIN/AEI/10.13039/501100011033.

References

Abd, K., Abhary, K., & Marian, R. (2014). Application of a fuzzy multi-criteria decision-making approach for dynamic scheduling in robotic flexible assembly cells. *IEEE, 2015-January*, 379–382. 10.1109/IEEM.2014.7058664.

Akram, M., Dudek, W. A., & Ilyas, F. (2019). Group decision-making based on pythagorean fuzzy TOPSIS method. *International Journal of Intelligent Systems, 34*(7), 1455–1475. https://doi.org/10.1002/int.22103

Arrow, K. J. (1963). *Social Choice and individual values*. Yale Univ Press.

Aslam, J. A., & Montague, M. (2001). Models for metasearch. *SIGIR Forum (ACM Special Interest Group on Information Retrieval), 276–284*. https://doi.org/10.1145/383952.384007

Bharsakade, R. S., Acharya, P., Ganapathy, L., & Tiwari, M. K. (2021). A lean approach to healthcare management using multi criteria decision making. *OPSEARCH, 58*(3), 610–635. https://doi.org/10.1007/S12597-020-00490-5
Black, D. (1958). *Theory committees and elections*. Cambridge Univ Press.
Bodily, S. E. (1979). Note—A delegation process for combining individual utility functions. *Management Science, 25*(10), 1035–1041. https://doi.org/10.1287/mnsc.25.10.1035
Boix-Cots, D., Pardo-Bosch, F., Blanco, A., Aguado, A., & Pujadas, P. (2022). A systematic review on MIVES: A sustainability-oriented multi-criteria decision-making method. *Building and Environment, 223*, Article 109515. https://doi.org/10.1016/j.buildenv.2022.109515
Borissova, D. (2018). A group decision making model considering experts competency: An application in personnel selection. *Comptes Rendus de L'Academie Bulgare Des Sciences, 71*(11), 1520–1527. https://doi.org/10.7546/CRABS.2018.11.11
Cabrerizo, F. J., Heradio, R., Pérez, I. J., & Herrera-Viedma, E. (2010). A selection process based on additive consistency to deal with incomplete fuzzy linguistic information. *Journal of Universal Computer Science, 16*(1), 62–81.
Casanovas-Rubio, M. del M., Pujadas, P., Pardo-Bosch, F., Blanco, A., & Aguado, A. (2019). Sustainability assessment of trenches including the new eco-trench: A multi-criteria decision-making tool. *Journal of Cleaner Production*, *238*, 117957. 10.1016/j.jclepro.2019.117957.
Chen, X., Zhao, L., & Liang, H. (2018). A novel multi-attribute group decision-making method based on the MULTIMOORA with linguistic evaluations. *Soft Computing, 22*(16), 5347–5361. https://doi.org/10.1007/s00500-018-3030-3
Chen, Z. S., Zhang, X., Govindan, K., Wang, X. J., & Chin, K. S. (2021). Third-party reverse logistics provider selection: A computational semantic analysis-based multi-perspective multi-attribute decision-making approach. *Expert Systems with Applications, 166*, Article 114051. https://doi.org/10.1016/j.eswa.2020.114051
Cheng, D., Zhou, Z., Cheng, F., & Wang, J. (2018). Deriving heterogeneous experts weights from incomplete linguistic preference relations based on uninorm consistency. *Knowledge-Based Systems, 150*, 150–165. https://doi.org/10.1016/j.knosys.2018.03.013
Chunhua, F., Shi, H., & Guozhen, B. (2020). A group decision making method for sustainable design using intuitionistic fuzzy preference relations in the conceptual design stage. *Journal of Cleaner Production, 243*, Article 118640. https://doi.org/10.1016/j.jclepro.2019.118640
Crott, H. W., Szilvas, K., & Zuber, J. A. (1991). Group decision, choice shift, and polarization in consulting, political, and local political scenarios: An experimental investigation and theoretical analysis. *Organizational Behavior and Human Decision Processes, 49*(1), 22–41. https://doi.org/10.1016/0749-5978(91)90040-Z
Davis, J. (1973). Group decision and social interaction: A theory of social decision schemes. *Psychological Review, 80*(2), 97–125.
Davis, J. H., Hulbert, L., Au, W. T., Chen, X. P., & Zarnoth, P. (1997). Effects of group size and procedural influence on consensual judgments of quantity: The example of damage awards and mock civil juries. *Journal of Personality and Social Psychology, 73*(4), 703–718. https://doi.org/10.1037/0022-3514.73.4.703
Deconde, R. P., Health, P., Division, S., Clegg, N., Biology, H., Hutchinson, F., Knudsen, B., Hutchinson, F., Etzioni, R., Health, P., Division, S., Deconde, R. P., Hawley, S., Falcon, S., Clegg, N., & Knudsen, B. (2006). *Combining Results of Microarray Experiments : A Rank Aggregation Approach Combining Results of Microarray Experiments : A Rank Aggregation Approach*. *5*(1), 5–15. 10.2202/1544-6115.1204.
Dong, Q., & Cooper, O. (2016). A peer-to-peer dynamic adaptive consensus reaching model for the group AHP decision making. *European Journal of Operational Research, 250*(2), 521–530. https://doi.org/10.1016/j.ejor.2015.09.016
Dwork, C., Kumar, R., Naor, M., & Sivakumar, D. (2001). Rank aggregation methods for the web. *Proceedings of the 10th International Conference on World Wide Web, WWW 2001*, 613–622. 10.1145/371920.372165.
Elbarkouky, M. M. G., & Fayek, A. R. (2011). Fuzzy similarity consensus model for early alignment of construction project teams on the extent of their roles and responsibilities. *Journal of Construction Engineering and Management, 137*(6), 432–440. https://doi.org/10.1061/(asce)co.1943-7862.0000310
Garg, H., & Kaur, G. (2020). Extended TOPSIS method for multi-criteria group decision-making problems under cubic intuitionistic fuzzy environment. *Scientia Iranica*, *27*(1 E), 396–410. 10.24200/SCI.2018.5307.1194.
Georgiou, P. (2022). Nudging as a public policy tool: Exploring the relationship between nudge and the ability of citizens for rational thought and choice. *HAPSc Policy Briefs Series, 3*(1), 23–30. https://doi.org/10.12681/hapscpbs.30982
Greatorex, J., & Dexter, T. (2000). An accessible analytical approach for investigating what happens between the rounds of a Delphi study. *Journal of Advanced Nursing, 32*(4), 1016–1024. https://doi.org/10.1046/j.1365-2648.2000.t01-1-01569.x
Hafezalkotob, A., & Hafezalkotob, A. (2017). A novel approach for combination of individual and group decisions based on fuzzy best-worst method. *Applied Soft Computing Journal, 59*, 316–325. https://doi.org/10.1016/j.asoc.2017.05.036
Herowati, E., Ciptomulyono, U., & Parung, J. (2014). Expertise-based experts importance weights in adverse judgment. *ARPN Journal of Engineering and Applied Sciences, 9*(9), 1428–1435.
Ivlev, I., Kneppo, P., & Barták, M. (2015). Method for selecting expert groups and determining the importance of experts' judgments for the purpose of managerial decision-making tasks in health system. *E a M: Ekonomie a Management, 18*(2), 57–72. https://doi.org/10.15240/tul/001/2015-2-005
Ji, Y., Li, P., Wu, Z., & Qu, D. (2021). ReacHING CONSENSUS BASED ON THE OPINION DYNAMICS IN SOCIAL NETWOrks. *Arabian Journal for Science and Engineering, 46*(2), 1677–1690. https://doi.org/10.1007/s13369-020-04891-y
Kannan, D., de Sousa Jabbour, A. B. L., & Jabbour, C. J. C. (2014). Selecting green suppliers based on GSCM practices: Using Fuzzy TOPSIS applied to a Brazilian electronics company. *European Journal of Operational Research, 233*(2), 432–447. https://doi.org/10.1016/j.ejor.2013.07.023
Kendall, M. G., & Smith, B. B. (1939). The problem of "m" rankings. *The Annals of Mathematical Statistics, 10*(3), 275–287. https://doi.org/10.1214/aoms/1177732186
Kerr, N. L., & Tindale, R. S. (2004). Group performance and decision making. *Annual Review of Psychology, 55*, 623–655. https://doi.org/10.1146/annurev.psych.55.090902.142009
Koksalmis, E., & Kabak, Ö. (2019). Deriving decision makers' weights in group decision making: An overview of objective methods. *Information Fusion*, *49*(May 2018), 146–160. 10.1016/j.inffus.2018.11.009.
Li, Z., Zhang, Q., Du, X., Qian, X., & Lev, B. (2019). A hesitant fuzzy multi-criteria group decision making method for college applicants' learning potential evaluation. *Journal of Data, Information and Management, 1*(1–2), 65–75. https://doi.org/10.1007/s42488-019-00006-x
Lin, M., Zhan, Q., Xu, Z., & Chen, R. (2018). Group decision-making model with hesitant multiplicative preference relations based on regression method and feedback mechanism. *IEEE Access, 6*, 61130–61150. https://doi.org/10.1109/ACCESS.2018.2875400
Lin, Y., & Wang, Y. (2018). Group decision making with consistency of intuitionistic fuzzy preference relations under uncertainty. *IEEE/CAA Journal of Automatica Sinica, 5*(3), 741–748. https://doi.org/10.1109/JAS.2016.7510037
Liu, W., & Li, L. (2015). An approach to determining the integrated weights of decision makers based on interval number group decision matrices. *Knowledge-Based Systems, 90*, 92–98. https://doi.org/10.1016/j.knosys.2015.09.029
Ma, Z., Zhu, J., & Chen, Y. (2020). A probabilistic linguistic group decision-making method from a reliability perspective based on evidential reasoning. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 50*(7), 2421–2435. https://doi.org/10.1109/TSMC.2018.2815716
Meng, F., An, Q., & Chen, X. (2016). A consistency and consensus-based method to group decision making with interval linguistic preference relations. *Journal of the Operational Research Society, 67*(11), 1419–1437. https://doi.org/10.1057/jors.2016.28
Okoli, C., & Pawlowski, S. D. (2004). The Delphi method as a research tool: An example, design considerations and applications. *Information and Management, 42*(1), 15–29. https://doi.org/10.1016/j.im.2003.11.002
Pardo-Bosch, F., & Aguado, A. (2016). Sustainability as the key to prioritize investments in public infrastructures. *Environmental Impact Assessment Review, 60*, 40–51. https://doi.org/10.1016/j.eiar.2016.03.007
Pujadas, P., Cavalaro, S. H. P., & Aguado, A. (2019). Mives multicriteria assessment of urban-pavement conditions: Application to a case study in Barcelona. *Road Materials and Pavement Design, 20*(8), 1827–1843. https://doi.org/10.1080/14680629.2018.1474788
Pujadas, P., Pardo-Bosch, F., Aguado-Renter, A., & Aguado, A. (2017). MIVES multi-criteria approach for the evaluation, prioritization, and selection of public investment projects. A case study in the city of Barcelona. *Land Use Policy, 64*, 29–37. https://doi.org/10.1016/j.landusepol.2017.02.014
Ramanathan, R., & Ganesh, L. S. (1994). Group preference aggregation methods employed in AHP: An evaluation and an intrinsic process for deriving members' weightages. *European Journal of Operational Research, 79*(2), 249–265. https://doi.org/10.1016/0377-2217(94)90356-5
RazaviToosi, S. L., & Samani, J. M. V. (2019). A fuzzy group decision making framework based on ISM-FANP-FTOPSIS for evaluating watershed management strategies. *Water Resources Management, 33*(15), 5169–5190. https://doi.org/10.1007/s11269-019-02423-4
Roigé, N., Pujadas, P., Cardús, J., & Aguado, A. (2020). Water network renewal strategy: a case study of Aigües de Barcelona. *Proceedings of the Institution of Civil Engineers. Water Management*, *173*(2), 59–68. 10.1680/JWAMA.18.00100.
Schielke, H. J., Fishman, J. L., Osatuke, K., & Stiles, W. B. (2009). Creative consensus on interpretations of qualitative data: The ward method. *Psychotherapy Research, 19*(4–5), 558–565. https://doi.org/10.1080/10503300802621180
Schwenk R., C., & Cosier, R. (1993). *Effects of Consensus and Devil ' s Advocacy On Strategic Decision-Making*. 126–139.
Shih, H. S., Shyur, H. J., & Lee, E. S. (2007). An extension of TOPSIS for group decision making. *Mathematical and Computer Modelling, 45*(7–8), 801–813. https://doi.org/10.1016/j.mcm.2006.03.023
Slevin, D. P., Boone, L. W., Russo, E. M., & Allen, R. S. (1998). CONFIDE: A collective decision-making procedure using confidence estimates of individual judgements. *Group Decision and Negotiation, 7*(2), 179–194. https://doi.org/10.1023/A:1008650524782
Tabatabaei, M. H., Amiri, M., Khatami Firouzabadi, S. M. A., Ghahremanloo, M., Keshavarz-Ghorabaee, M., & Saparauskas, J. (2019). A new group decision-making model based on bwm and its application to managerial problems. *Transformations in Business and Economics, 18*(2), 197–214.
Theil, H. (1963). On the symmetry approach to the committee decision problem. *Management Science, 9*(3), 380–393. https://doi.org/10.1287/mnsc.9.3.380
Thong, N. T., Lan, L. T. H., Chou, S. Y., Son, L. H., Dong, D. D., & Ngan, T. T. (2020). An extended TOPSIS method with unknown weight information in dynamic neutrosophic environment. *Mathematics, 8*(3), 1–15. https://doi.org/10.3390/math8030401
Thurstone, L. L. (1927a). A law of comparative judgment. *Psychological Review, 34*(4), 273–286. https://doi.org/10.1037/h0070288
Thurstone, L. L. (1927b). The method of paired comparisons for social values. *Journal of Abnormal and Social Psychology, 21*(4), 384–400. https://doi.org/10.1037/h0065439
Toloie-Eshlaghy, A., & Farokhi, E. N. (2011). Measuring the importance and the weight of decision makers in the criteria weighting activities of group decision making process. *American Journal of Scientific Research, 24*(24), 6–12.

van den Honert, R. C. (2001). Decisional power in group decision making: A note on the allocation of group members' weights in the multiplicative AHP and SMART. *Group Decision and Negotiation, 10*(3), 275–286. https://doi.org/10.1023/A:1011201501379

Wan, S. ping, Xu, G. li, & Dong, J. Ying. (2016). A novel method for group decision making with interval-valued Atanassov intuitionistic fuzzy preference relations. In *Information Sciences* (Vol. 372). Elsevier Inc. 10.1016/j.ins.2016.08.019.

Wu, J., Chiclana, F., & Herrera-Viedma, E. (2015). Trust based consensus model for social network in an incomplete linguistic information context. *Applied Soft Computing Journal, 35*, 827–839. https://doi.org/10.1016/j.asoc.2015.02.023

Wu, P., Liu, J., Zhou, L., & Chen, H. (2020). Algorithm for improving additive consistency of linguistic preference relations with an integer optimization model. *Applied Soft Computing Journal, 86*, Article 105955. https://doi.org/10.1016/j.asoc.2019.105955

Wu, W., Kou, G., & Peng, Y. (2018). A consensus facilitation model based on experts' weights for investment strategy selection. *Journal of the Operational Research Society, 69*(9), 1435–1444. https://doi.org/10.1080/01605682.2017.1398203

Xu, J., & Wu, Z. (2013). A maximizing consensus approach for alternative selection based on uncertain linguistic preference relations. *Computers and Industrial Engineering, 64*(4), 999–1008. https://doi.org/10.1016/j.cie.2013.01.009

Xu, Z., & Zhou, W. (2017). Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. *Fuzzy Optimization and Decision Making, 16*(4), 481–503. https://doi.org/10.1007/s10700-016-9257-5

Ye, J. (2014). Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. *Journal of Intelligent and Fuzzy Systems, 27*(6), 2927–2935. https://doi.org/10.3233/IFS-141252

Yue, Z. (2011). A method for group decision-making based on determining weights of decision makers using TOPSIS. *Applied Mathematical Modelling, 35*(4), 1926–1936. https://doi.org/10.1016/j.apm.2010.11.001

Yue, Z. (2012). Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling, 36*(7), 2900–2910. https://doi.org/10.1016/j.apm.2011.09.068

Yue, Z. (2013). Group decision making with multi-attribute interval data. *Information Fusion, 14*(4), 551–561. https://doi.org/10.1016/j.inffus.2013.01.003

Yue, Z., Jia, Y., & Ye, G. (2009). An approach for multiple attribute group decision making based on intuitionistic fuzzy nformation. *International Journal of Uncertainty, Fuzziness and Knowlege-Based Systems, 17*(3), 317–332. https://doi.org/10.1142/S0218488509005899

Zeng, S., Palacios-Marqués, D., & Zhu, F. (2016). A new model for interactive group decision making with intuitionistic fuzzy preference relations. *Informatica (Netherlands)*, *27*(4), 911–928. 10.15388/Informatica.2016.117.

Zindani, D., Maity, S. R., & Bhowmik, S. (2020). Interval-valued intuitionistic fuzzy TODIM method based on Schweizer–Sklar power aggregation operators and their applications to group decision making. In *Soft Computing* (Vol. 24, Issue 18). Springer Berlin Heidelberg. 10.1007/s00500-020-04783-1.