A qualitative approach for aggregating people's perceptions

Francisco J. Ruiz\(^1\), Núria Agell\(^2\) and Mónica Sánchez\(^1\)

\(^1\)BarcelonaTech. Barcelona Spain
{Francisco.javier.ruiz, Monica.sanchez}@upc.edu
\(^2\)ESADE-Ramon Llull University. Barcelona Spain
Nuria.agell@esade.edu

Abstract

The concept of perceptual map is introduced in this paper to capture the semantics of linguistic assessments of an individual in a qualitative reasoning scenario, concretely in hesitant fuzzy linguistic term sets. In addition, the projected perceptual map is considered to provide a space to aggregate different perceptual maps. Qualitative distances and measures of centrality and agreement or consensus are revised based on this projected perceptual map.

1 Introduction

Qualitative Reasoning (QR) is an area of research within Artificial Intelligence (AI) that automates reasoning about continuous aspects of the physical world by discretizing these continuous variables. Absolute order of magnitude qualitative models consider intervals (in some cases with the same length and in other cases following some symmetry) and assign a linguistic label to each interval, for instance, negative large or positive small.

Qualitative models based on discretization of continuous spaces are present in many environments. For instance, in group decision making, where experts or decision makers (DMs) use linguistic terms to express their judgements with the aim to reach a joint decision. Also, in customer reviews and star ratings, where customer summarizes their opinion of a product giving a rating of 1 to 5 stars or 1 to 3 stars. These customer reviews help the company to understand overall customer satisfaction and allow new consumers to know the product (most of consumers always read online reviews before purchasing a product).

In star rating, the meaning of each score is not the same for all reviewers. There are very generous reviewers and others more reluctant to give a good grade. On the other hand, in group decision making, not all DMs might feel comfortable using the same linguistic terms set when expressing their judgements and the same linguistic term may have a different meaning depending on the DM who use it, that is, each DM may have his or her own manner of expressing the judgment using words.

In this paper we present a new concept: the perceptual map. It gives an extended meaning to each basic label of the linguistic term set. The concept is framed in the context of the qualitative model called hesitant fuzzy linguistic term sets (HFLTSs) that has recently attracted significant attention from researchers. A state of the art and list of applications of HFLTS can be found in [Liao et al, 2018]. Additionally, recent publications have also operated with this hesitant fuzzy linguistic approach to solve complex decision problems [Chen et al. 2021].

Each DM in group decision making or each customer in customer reviews has his or her own perceptual map. To find out the perceptual map of an individual it is necessary to extract it from previous judgements of previous reviews. In this paper, we will not focus on how to obtain that perceptual map from each individual but how to use it to obtain fusion information of groups of individuals, each managing their own perceptual map. For that, we introduce the projected perceptual map that enables us to merge all this information to obtain the measures of central tendency and degree of consensus.

In the next section, all the new tools and structures needed for the method are developed. The third section introduces the perceptual map. In addition, distances and measures of centrality and agreement or consensus are revised based on the concept of a perceptual map. Section 4 generalizes the concepts introduced in the previous section using the projected perceptual map. Finally, the conclusion summarizes the major points and suggests directions for future work.

2 Hesitant fuzzy linguistic term sets

This section provides the preliminary theoretical framework on the specific qualitative model used in this paper to model expert assessments, i.e. hesitant fuzzy linguistic term sets (HFLTS). Also, a brief review on distances and consensus measures previously developed for this model is provided.

The concept of HFLTS was introduced by Rodriguez et al. in [Rodriguez et al., 2012]. A state-of-the-art survey on HFLTSs
and its applications in decision-aiding can be found in [Liao et al. 2018].

**Definition 1.** [Rodriguez et al. 2012] Let $S$ be a totally ordered set of linguistic terms (or basic labels), $S = \{s_1, ..., s_n\}$, with $s_1 < \cdots < s_n$. A hesitant fuzzy linguistic term set (HFLTS) over $S$ is a subset of consecutive linguistic terms of $S$, i.e., $\{x \in S | s_i \leq x \leq s_j\}$, for some $i, j \in \{1, ..., n\}$ with $i \leq j$. We call $[s_i, s_j]$ to this HFLTS, or $[s_i, s_j]$ if $i = j$.

The set of all non-empty HFLTSs over $S$ is denoted by $\mathcal{H}_S$. The set $\mathcal{H}_S \cup \{\emptyset\}$, jointly with the two-binary operation intersection ($\cap$) and connected union ($\cup$), form a lattice [Montes-Rodriguez et al., 2017].

**Definition 2.** $H_1, H_2 \in \mathcal{H}_S$, the distance between $H_1$ and $H_2$ is defined as:

$$d(H_1, H_2) = 2 \cdot \text{card}(H_1 \cup H_2) - \text{card}(H_1) - \text{card}(H_2)$$

(1)

The distance between HFLTS allows to obtain the concept of centroid or central measure from a group of individuals that evaluates the same issue:

**Definition 3.** Given a group of $k$ individuals, $G = \{d_1, ..., d_k\}$ with each providing an assessment $H_i \in \mathcal{H}_S$ over one alternative or product, then the centroid of the group, denoted as $H^C$, is defined as:

$$H^C = \arg \min_{H \in \mathcal{H}_S} \sum_{j=1}^{k} d(H, H_j)$$

(2)

The following proposition proves that the centroid can be calculated using the two medians of the two sets of indexes.

**Proposition.** If $H_i = [s_i^L, s_i^R] \in \mathcal{H}_S$ for all $i \in \{1, ..., k\}$ then the centroid is calculated as:

$$H^C = \{[s_L, s_R] \in \mathcal{H}_S \mid L \in \mathbb{M}(s_1^L, ..., s_k^L), R \in \mathbb{M}(s_1^R, ..., s_k^R)\}$$

(3)

where $\mathbb{M}$ is the set that contains just the median if $k$ is odd, or two central values and any integer number between them if $k$ is even. The proof of this proposition can be found in [Porro et al., 2021].

The sum $\sum_{j=1}^{k} d(H^C, H_j)$ can be considered a measure of disagreement among the DMs. The bounds of this sum enable us to define the degree of agreement or consensus of $G$ as:

$$\delta(G) = 1 - \frac{\sum_{j=1}^{k} d(H^C, H_j)}{\zeta} \in [0,1]$$

(4)

where $\zeta = [k/2] \cdot (2 \cdot \sum \mu_i - \mu_1 - \mu_n)$.

### 3 The perceptual map on $\mathcal{H}_S$

The previous definition of distance does not distinguish between different basic labels, and only considers the number of them. To overcome this situation, the concept of perceptual map is defined and developed in this paper. This concept enables us to give an extended meaning to each basic label and to each HFLTS.

**Definition 4.** Let $S$ be a totally ordered finite set $S = \{s_1, ..., s_n\}$, then a basic perceptual map is a pair $(S, \mu)$, where $\mu$ is a measure over $S$, that is, $\mu : S \rightarrow R^+ = (0, +\infty)$. If $s_i \in S$, we call $\mu(s_i)$ the width of the basic label $s_i$.

This measure over $S$ can easily be extended to $\mathcal{H}_S$ using that the width of the HFLTS $[s_i, s_j]$ is $\mu([s_i, s_j]) = \sum_{k=1}^{j} \mu_k$. We use the expression perceptual map for the pair $(\mathcal{H}_S, \mu)$ that we also denote as $\mathcal{H}(\mathcal{S}, \mu)$.

Considering the perceptual map $\mathcal{H}(\mathcal{S}, \mu)$, the distance is redefined as follows:

**Definition 5.** $H_1, H_2 \in \mathcal{H}(\mathcal{S}, \mu)$, the distance between $H_1$ and $H_2$ is:

$$d(H_1, H_2) = 2 \cdot \text{width}(H_1 \cup H_2) - \text{width}(H_1) - \text{width}(H_2)$$

(5)

Using the distance defined in (5) the calculus of centroid and consensus is obtained in exactly the same way as indicated in the expressions (3) and (4).

### 4. The projected perceptual map

Given that a basic linguistic perceptual map $(S, \mu)$ can be considered as a partition of some real interval of length $L = \sum \mu_i$, we consider $k$ different basic perceptual maps $(S, \mu^j)$ such as $\sum \mu^j_l = L \forall j \in \{1, ..., k\}$. This last equality simply expresses a normalization that enables the comparison of different perceptual maps.

Given $k$ basic perceptual maps $(S, \mu^j)$, we can consider a basic projected perceptual map $(S^P, \pi)$ such as $N = \text{card}(S^P) \leq N + 1$ and where $\pi_i = \min \mu^j_l$ and $\pi_n = \min \mu^j_n$. This projected perceptual map forms a refinement of the partitions where all the HFLTSs of any individual have equivalence in this basic projected perceptual map. From the set $S^P$ and the measure $\pi$, the projected perceptual map $\mathcal{H}(S^P, \pi)$ is obtained as usual (see Figure 1).
If \( H_j \in \mathcal{H}_{(S_j, \mu_j)} \) is the assessment of individual \( j \) using the perceptual map \( \mathcal{H}_{(S_j, \mu_j)} \), this HFLTS has an equivalent \( H^*_j \) in the projected perceptual map \( \mathcal{H}_{(S, \mu)} \) as shown in Figure 1:

![Projected Perceptual Map Example](image)

**Fig. 1.** Example of projected perceptual map \( \pi \) from perceptual maps \( \mu^1, \mu^2 \) and \( \mu^3 \).

The centroid can now be considered using the projected perceptual map and its associated distance \( \mathcal{H}_{(S, \mu)} \) and using the projections of each \( H_j \) onto this projected perceptual map.

Similarly, the sum \( \sum_{j=1}^{k} d_{\mathcal{H}_{(S, \mu)}} (H_j, H^*_j) \) is upper bounded by \( \zeta = \lfloor k/2 \rfloor \cdot (2 \cdot \sum_{i=1}^{k} \pi_i - \pi_1 - \pi_k) \) and this enables defining the degree of consensus as:

\[
\delta(G) = 1 - \frac{\sum_{j=1}^{k} d_{\mathcal{H}_{(S, \mu)}} (H^*_j, H_j)}{\zeta}
\]  

(6)

To illustrate this calculus let us find the centroid and the degree of consensus in the example in Figure 1. Let us suppose the perceptual maps:

- \((S_1, \mu^1)\) where \( \mu_1^1 = 1, \mu_2^1 = 5, \mu_3^1 = 4 \).
- \((S_2, \mu^2)\) where \( \mu_1^2 = 2, \mu_2^2 = 3, \mu_3^2 = 3, \mu_3^2 = 2 \).
- \((S_3, \mu^3)\) where \( \mu_1^3 = 3, \mu_2^3 = 7 \).
- \((S, \pi)\) where \( \pi_1 = 1, \pi_2 = 1, \pi_3 = 1, \pi_4 = 2, \pi_5 = 1, \pi_6 = 2, \pi_7 = 2 \).

The assessment of the three individuals are: \( H_1 = [s_1, s_2] \), \( H_2 = [s_2] \), and \( H_3 = [s_1] \). Its projections are: that is \( H^*_1 = [s_1^*, s_2^*] \), \( H^*_2 = [s_2^*, s_3^*] \) and \( H^*_3 = [s_1^*, s_2^*] \). Therefore, the centroid is: \( H^C = \{[s_1^*, s_2^*] \in \mathcal{H}_{(S, \mu)} \mid L \in \mathcal{M}(s_1^*, s_3^*, s_2^*) \} \), \( R \in \mathcal{M}(s_3^*, s_4^*, s_2^*) \) = \( [s_1^*, s_4^*] \) and the degree of consensus is:

\[
\delta(G) = 1 - \frac{d_{\mathcal{H}_{(S, \mu)}} (H^C, H^*_1) + d_{\mathcal{H}_{(S, \mu)}} (H^C, H^*_2) + d_{\mathcal{H}_{(S, \mu)}} (H^C, H^*_3)}{\zeta} = 1 - \frac{\pi_3 + \pi_1 + \pi_2 + \pi_3}{(2 - 2 \pi_1 - \pi_7)} = \frac{13}{17} \approx 76.5\% .
\]

5. Conclusion and future work

DMs or product reviewers with different backgrounds or knowledge may feel more comfortable using their own linguistic term sets and their own semantics when expressing judgements. This paper presents a framework to aggregate people judgements which are made using different perceptual maps, that is, using their own linguistic labels with their own meanings. This framework is based on the projected perceptual map, a hyper-perceptual map where all HFLTSs from each of the perceptual maps from all individuals have equivalences. The concepts of distance, centroid, and degree of consensus can be interpreted directly in this projected perceptual map. An illustrative example is provided to demonstrate the overall approach.

Future research is oriented in various directions. Firstly, analyzing how to translate hesitancy among different perceptual maps without increasing granularity. Secondly, analyzing the evolution of the degree of consensus when new individuals join the decision group. Thirdly, designing a method to capture the perceptual map associated with each individual from previous assessments through machine learning techniques. Finally, analyzing how perceptual maps will revert positively in common-sense reasoning understanding and contribute to improving human-machine interaction.

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References


