Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit

Final Thesis developed by: 
Montero Vega, Mariana

Directed by: 
Badia Rodríguez, Hugo

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Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit

Author:
Mariana Montero Vega

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Supervisor:
Dr. Hugo Badia Rodríguez

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Abstract

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Demand Responsive Transit (DRT) is a type of transportation that can be either public or private and, in contrast to conventional services that aim to increase ridership, DRT adapts to the demand to provide maximum coverage in the service area.

Previous studies have shown that this technology is justified when service demand is low. However, when there is an increase in demand the total cost of the system increases, even over the total cost of the traditional fixed-line bus system. This occurs because the travel time of each passenger increases due to the increase in stops. This might be handled by growing the fleet of vehicles operating, but this has consequences for the operational cost of the system and the environment. In response to the limited resources that transport operators may have, a fleet-dependent model is derived from an existing headways-dependent model of DRT.

Through the analysis of economies and diseconomies of scale, this thesis explains which are the factors that make the costs of a DRT service increase, for a headway-dependent model and a fleet-dependent model. At the same time, it explains the methodology to find the critical demand where the system becomes more expensive and the optimal occupancy that a DRT vehicle should carry in order not to increase the travel time of passengers.

The externalities produced by several types of vehicles of various capacities were calculated. The ideal capacity from the previous analysis, as well as the social and environmental costs of operating various cars, were used to obtain the optimal vehicle fleet sizes for a DRT operation.

Finally, as a case study, three DRT lines in Sant Cugat that were previously evaluated in the MultiDEPART project where EIT Urban Mobility was part of, were analyzed. From this, the critical demand, optimal occupancy, and externalities generated with two different types of vehicles of different dimensions are calculated.

Keywords: Demand Responsive Transport, Fleet-Dependent Model, Diseconomies of Scale, Externalities,
Resumen

Comparación de Estrategias de Operación de Servicios de Buses: de Rutas Fijas a Buses a Demanda

El transporte adaptado a la demanda (DRT por sus siglas en inglés) es un sistema que responde a las necesidades del transporte público en zonas donde el objetivo es aumentar la cobertura del servicio más que el número de usuarios.

Estudios anteriores han demostrado que esta tecnología se justifica cuando la demanda del servicio es baja. Sin embargo, cuando hay un aumento de la demanda, el coste total del sistema aumenta, incluso por encima del coste total del sistema tradicional de bus. Esto ocurre porque el tiempo de viaje de cada pasajero aumenta debido al incremento de paradas. Esto podría solucionarse aumentando la flota de vehículos, pero esto tiene consecuencias medioambientales. En respuesta a los recursos limitados que pueden tener los operadores de transporte, se deriva un modelo dependiente de la flota a partir de un modelo existente dependiente de la frecuencia de operación.

A través del análisis de las economías y deseconomías de escala, esta tesis explica cuáles son los factores que hacen aumentar los costes, para un modelo dependiente de la frecuencia y para uno dependiente de la flota. Al mismo tiempo, se explica la metodología para encontrar la demanda crítica en la que el sistema se encarece y la ocupación óptima que debe llevar un vehículo de DRT para no aumentar el tiempo de viaje de los pasajeros.

Se calcularon las externalidades producidas por varios tipos de vehículos de distintas capacidades. La capacidad ideal del análisis anterior, así como los costes medioambientales de se utilizaron para obtener el tamaño óptimo de la flota de vehículos para una operación de DRT. Finalmente, como caso de estudio, se analizaron tres líneas de DRT en Sant Cugat, que fueron evaluadas previamente en el proyecto MultiDEPART del que formaba parte el EIT UM. A partir de esto, se calcula la demanda crítica, la ocupación óptima y las externalidades generadas con dos tipos de vehículos de diferentes dimensiones.

Palabras Clave: DRT, Modelo Dependiente de la Flota, Deseconomías de Escala, Externalidades
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1. Introduction

Fixed and traditional public transportation routes provide a shuttle service from one bus stop to another, that in optimal cases are close to the origin and destination of the entire trip of the users. The bus stop concept allows for a concentration of riders at a particular point which expedites the picking up of and dropping off of passengers. There are sources that indicate that this transportation concept was born in Great Britain in the middle of the seventeenth century and since then, this service has been multiplied and adapted in different cities around the world. Moreover, this service along with other means of public transport is the alternative offer to decongest and discourage the use of private transport. Many modern cities today are working to increase the demand for public transport in an attempt to minimize the use of private transport and all the externalities that go hand in hand with it. From this, the typical vicious cycle of public transport (bus service) is born. In short, this vicious cycle is based on the fact that when there is a drop in the demand for public transport, this will represent an increase in the number of people who travel by private vehicle. Consequently, congestion on the road increases, which affects the level of service of public transport because of the increase in travel time of users. As a result, this will attract even more people to use private transport and decreasing the demand for transit.

However, providing a quality public transport service to discourage people to use their cars is not always easy. Specially, in zones where there are comparatively low levels of population, such as rural areas, where there hasn't been always enough demand to justify such a public transport service. For the operator perceptive this could represent a service that is not cost effective. On the user's perspective, a fixed route would not necessarily meet the mobility needs because access times to the bus stops and final destinations would increase as the area of coverage of the service grows. In these cases, perhaps the best option is to adapt a DRT (Demand Responsive Transport/Transit) service. This consists of a bus service (with smaller capacities than regular services) which can be requested at any time of the day and which picks up and drops off the user at points that the users established before through a mobile app. The bus has the freedom to pick up more people along the way and change its route as many times as necessary to serve more users. This becomes a "pool" service and it can have a flat fare per service or a fare that depends on the mileage/time travel.
Nevertheless, in the DRT service the vicious cycle explained before is rather reversed. Instead of being a problem with a decrease in demand for the service, there is a problem with an increase in the number of people using the service. This is because as ridership increases, the service is forced to make more de-tours to pick up and drop off people. Each new rider represents an increase in travel time for other riders, decreasing the DRT's level of service. At the same time, by increasing travel time, the reliability of the service decreases, since many people depend on it to get to certain places at specific times or to transfer to other means of transportation. When this happens, the system will be forced to return to the old models of fix routes in order to serve all the demand. The fixed route would now represent as explained above, an increased in walking times to and from bus stops. As a result, people would no longer feel comfortable with the service and would switch back to private means of transport or taxis. Therefore, is safe to say that in the case of DRT services the average cost per trip only reduced when the demand density is low (Estrada et al., 2021) but are challenged when demand rises.

This thesis aims to understand how this vicious cycle works in an analytical way. By means of a theoretical model, the two main components of the price of the service will be analyzed, which are: the operator's cost and the user's cost. Also, taking into account that transport operators have limited resources, the amount of fleet (M) will be limited in order to understand how the variation of this component of the model can affect the travel time of users and therefore the total price of the service. This will be done by creating a new DRT fleet dependent model based on a headway dependent model presented in the study of Estrada et al. (2021).

The demand increases substantially because there is no readjustment of the fare, meaning a flat fare is still charged as those of a fixed route. Since an on-demand service represents a highly flexible door-to-door service, which can even be compared to a taxi service, the traditional fixed price for a regular service will not cover all the expenses of the service. Therefore, it is intended to understand better how to come to a balance point between costs and benefits of the system so that a DRT service is not losing its profitability and does not enter the vicious cycle of substantial increase in demand. However, the main hypothesis of this thesis is that the increase in the total cost per passenger of a DRT service is experienced by the user of the service rather than the DRT operator.

The economies and diseconomies of scale of the DRT service will be analyzed for both analytical models (fleet-dependent and headway-dependent). This will be done in order to find
the critical demand where the system starts to become more expensive with ridership growth and a comparison for DRT and conventional bus will give a greater perspective to the service capacity of both public transportation options. Nevertheless, even if the service is not economically profitable, the government may have the ability to subsidize DRT service at higher percentages than it does today, to ensure the service operation. Maintaining DRT service may be justified on two grounds, the first being that the overall benefit of the system is superior to that of a fixed route, or that the negative externalities of a fixed route outweigh those of an on-demand service. The following two paragraphs describe these two grounds:

The negative and positive externalities of the system besides the total cost of the operation are also something that needs to be highlighted. This is because for a DRT to be able to operate with high levels of demand, it is necessary to have higher levels of fleet operating at the same time. More vehicles in operation represent more pollutants per passenger, for which it is considered necessary to take into account the impact of these vehicles on the quality of life and air of the residents of the area. Parallel to this, more fleet in an on-demand responsive transport systems gives more accessibility for the community in general. Therefore, citizens have more freedom to go and do their daily activities at their convenience otherwise, if there were no DRT, they might do these activities in private vehicles or not at all, depending on how essential the activity is. This comparison is considered necessary to have a more holistic understanding of an on-demand and consequently, to justify the cases in which the negative externalities and the price of the service outweigh the positive externalities of the service and it is better to consider the closure of the service.

It is important to mention that this work is limited to a theoretical degree. This means that as mentioned above it is based on a specific analytical model for certain circumstances that may not always represent reality rigorously. Especially because DRT models operating today may have important variations such as: fixed pickup and drop off stops, circuit route model that only skips stops depending on demand or even fixed operating schedules. During the course of this work, explanations will be provided to understand under which circumstances the model is applicable and which are not.
2. State of the Art

2.1. On – Demand Responsive Transport Modeling and Background

In public transportation systems there is usually a tradeoff between having maximum ridership or maximum coverage. A maximum ridership system means that the routes go mainly through the areas where there is more demand, ensuring good connectivity in the areas where there is a greater concentration of users. Fixed bus lines usually provide maximum ridership services, where the stops have a logical order though the places where there is more concentration of people and in this way the operator ensures profitability of the service. On the other hand, a maximum coverage service means that the fleet operates in order to give more accessibility to the entire area of influence, regardless of whether there are high or low numbers of demand. In this way, the operation of the fleet is distributed throughout the region and it is guaranteed that service is also offered in remote towns or sectors. DRT (Demand Responsive Transport) services are designed to offer a service of this nature. Its aim is to be able to provide a service with a flexibility of stops and schedules that adapt to users and not the other way around and give the maximum coverage in the area of service.

One of the earliest definitions of a demand responsive transport was published in the Journal of Transport Economics and Policy (1986). Here it was established the demand-responsive paratransit, in US nomenclature, as a service which is non-schedule and may or maybe not have a variable route service in a van pool system. More recent definitions such as the one from Mageean and Nelson (2003), describe a DRT as a “intermediate form of transport, somewhere between bus and taxi which covers a wide range of transport services ranging from less formal community transport through to area-wide service networks” (p.255). Furthermore, it is discussed that this service has grown significantly in recent years thanks to transport telematics, which helps provide more efficiency and reliability for users.

A DRT service manage to solve many problems of traditional fixed route transportation service. For example, a DRT don't have to go through the inconvenience to visit specific stops or even zone where there are no users at the moment, increasing the travel time for the current passengers. Rather, this service has the flexibility to adjust its route to areas where there are
certainly passengers. This is especially practical in of peak hours where demand tends to drop and behave more arbitrarily. However, this can vary considerably depending on the street layouts and the type of demand in the area of study. It is for this reason that several researchers have been given the task of verifying in which situations a demand responsive transport solution can be more suitable than conventional routes and what are the real impacts of substituting one service for the other.

One of the pioneering on this topic was Daganzo (1984) in where he compared the cost versus efficiency of a regular public transport system against the door to door or “demand responsive transport” system. In this study he made several simplifications in order to came up with some equations that’s helps to estimate the operational cost of each system. A more recent study was the one from Kashani et al. (2016) in where it was found that by replacing a conventional public transport system with DRT in low demand area, users tent to increase significantly their mobility. In the same study is showed that DRT can compete with the conventional system even with a high frequency (7.5 minutes) in a grid network in terms of user performance and operator's cost. In the study of D. Edwards (2013) there is comparison between the effectiveness of fixed-route transit and a demand responsive transit in different streets and route system layouts. For example, comparing different station in Atlanta the study found that in the Chamblee Station Subnet the results show that that DRT could provide a less expensive alternative for handling trip requests for stations with relatively low demand at off-peak hours. Moreover, in other suburban street layouts such as Brookhaven Station and Springs station, the study showed similar improvements in the operational cost as in the Chamblee Station. The study of Diana et al. (2009) shows that regardless of demand level in areas where the structure is a grid, the kilometers traveled by the fix routes fleets are very similar to the ones traveled by the DRT services. Nevertheless, this was not the case for the ring radial scenarios with less demand density levels, where the DRT service always performs better in terms of distance traveled.

There are as well many studies regarding the routing problem that DRT are facing. Meaning, how is the optimal algorithm to solve the order of picking up and dropping of users of the system. For example, a study conducted by Columbia University and Uber (2017) explains its routing problem for their operations in New York City. They did a study assuming a constant speed of the ride, a one customer per ride operation and an average distance of one mile. Their goal was to create a routing schedule that minimizes the maximum lateness of a single vehicle
traveling on a tree-shaped network. A polynomial time algorithm was used with a depth-first routing constrains mention before. After finding the optimal number of drivers, the high demand areas across Manhattan and the customers/driver ratio, the routing algorithm used to minimize the route was the greedy algorithm, in where each point is added when it gives the lowest cost possible in each iteration. Regarding Uber-pool, Uber claims (2015) to use a A*search algorithm to find the most suitable route. This algorithm is a Dijkstra search that finds as long as certain conditions are met, the path of least cost between a source node and a target node with heuristics. This algorithm allows taking into consideration the traffics conditions without needing to do any precomputation but it only works well for short trip since it is a very slow tool of route calculation.

The study used as the basis for this thesis and perhaps one of the most complete analytical models for conventional systems, DRT and taxi services is that of Estrada et al, 2021. In this study, a continuous approximations model is used to optimize an objective function of the total costs for each means of transport mentioned. From this model, one of the most important conclusions and the main inspiration for this work is that as passenger demand increases for conventional service, the total cost per passenger is reduced; however, for DRT the total cost per passenger increases. The study therefore shows that DRT services are considered "profitable" for very small demands. Although the study makes the purchase for three different types of vehicles, there is no mention of the occupancy of the vehicles for different cases. Therefore, the curiosity arises to understand how the number of people transported affects the travel time and therefore the user's cost.

2.2. Externalities of On-Demand Services

One of the most common arguments used to justify a DRT line is that it provides more accessibility to isolated communities where connectivity is scarce or a fixed line is not justifiable. However, making the change from a fixed line to a DRT or implementing a DRT line from scratch can mean changes beyond people's means of transportation. For example, public transport fares may vary which generates a change in the net benefits from consumers and producers and therefore an increase (or decrease) in social welfare. In addition, hand in hand with any means of transport there are always negative externalities such as pollution, noise, accidents among others, and by making a change of the type of vehicle circulating in communities and the frequency of it, these agents will also vary. It is for this reason that authors
have already been challenged with understanding these two opposing consequences that the installation of a DRT might have on a community’s wellbeing.

One of the most robust studies on emissions in a DRT service is the one of (Diana, Quadrifoglio, & Ponello, 2007). The quantity of grams/km of CO2, CO, Nox, VOC, and PM released for a DRT service and a standard service were compared in this study. In the case of DRT, an eight-seat van was utilized as a vehicle, while buses were employed for the traditional service, in order to make the distinction between one service and the other. Twelve different scenarios were analyzed, and in 11 of these 12, a reduction in the number of pollutants was observed when switching to a DRT line. However, when the same vehicle is used for both services, this reduction is present in only 7 of the 12 scenarios.

A practical study that was carried out on different aspects related to replacing a fixed public transport line by a demand responsive transport system was that of Mariz et al. (2020) For this study, the Mokumflex service was taken as a case study, which was a one-year pilot (2017-2018) in a rural and low-density area in Amsterdam. In this study different indicators are taken to assess the impact of the change from fixed to DRT route, one of them being the amount of GHG (Green House Gases) and emissions. The examination of GHG and CO2 was made throughout all hours of operation and there is a considerable decrease throughout the day when implementing the DRT line. The transition to a more efficient (and cleaner) technology was an essential factor to consider, but it's worth noting that ridership fell from 78,1 to 15,9 passengers per day, resulting in a reduction in mileage from 1252,8 to only 136,6 kilometers per day. As a result, it is thought that, while technological advancements may have an influence on pollution reduction, the major factor influencing this decrease is the long mileage of traditional transportation.

In the study of Berrada & Poulhès (2020), the socio-economic assessment of replacing a fixed bus line with a DRT service is studied. Here, social welfare is considered as the sum of the operator profit, the user surplus, and the environmental surplus for both services (DRT and conventional). Here the equations of demand depended on a fixed elasticity for generalized costs of each service. The case study of this paper took place in Palaiseu, Paris and elasticity to generalized cost of -2.3 for the conventional and DRT service was defined. It was obtained that in order to maximize the operator's profitability, it would propose a taxi-type service. However, for the public authorities, fares should be regulated between 2 and 3 euros and fleets between
20 and 30 vehicles to maximize social welfare. In terms of greenhouse gases, there is a benefit in having fewer vehicles in operation and higher fares (which would decrease the ridership of the service). However, one of the main limitations of this study was that to derive the demand, the same and only one elasticity related to the cost of the service was used for both cases, which assumes that the ridership conditions are the same for a DRT as for a conventional service.

2.3. **Practical Examples of DRT Systems**

For more than fifty years different DRT services have been operated in different countries for different purposes. Some have been successfully in certain areas and extended to other cities and are highly profitable and scalable. On the contrary, there are many other DRT services that have not been operated successfully and have had to face the shame of being shut down and stopped operation. This subchapter will explain case studies where DRT services have and have not been successful. It will highlight the weaknesses and strengths of different services and reasons why some of them were forced to close.

One of the most important DRT cases among the Nordic countries was “Kutsuplus” in Helsinki, Finland. The initiative was born in Aalto University, and it started as a pilot project in 2012 with 10 buses and began public operation in the spring of 2013 with 15 on demand buses with the local tech start up, Alejo. Despite having a great mission to reduce the private car fleet in the city, the scheme ceased operation at the end of 2015. In the article of Kelly (2016) two important opinions are mentioned. The first one from: Teemu Shivola, the founder of Ajelo, who claimed that the operating costs were extremely high given that the driver contracts were fixed for three years and expensive compared to the driver contract model of Uber (Kutsuplus’ main competitor) and that the operation of the vehicles was expensive. Nevertheless, Shivola as well implied that the solution could have been becoming bigger (more fleet) as Uber and Lyft did at that time. The second from Kari Rissanen, a programmer director at the Helsinki Regional Transport Authority (HSL) who claims that rides were subsidized at almost 80% which was too much and that despite the technology there was not enough money to take it to scale. According to Hanlund et al. (2019), despite having vehicles in operation with a capacity of 9 seats, the average occupancy was 1.27 users. This is an important indicator of the weakness of the service operation since it is an exceedingly small number of passengers. Hanlund et al. (2019) also mentioned that the lack of identifiable users caused some limitations to the operation and for further advancements of DRT service algorithm should aim to improve the
demand prediction taking into consideration the different periods during the day and diverse types of users.

One of the European countries with a largest number of examples of applied DRT services is United Kingdom. The company ArrivaClick a DRT service operator under the big Arriva UK Bus company, has been characterized by operating in different cities: Liverpool (since 2018), Leicester (since, 2019) Watford and Ebbsfleet (since 2020). This service starts with a base fare of £1 and increasing depending on the distance of the journey and the day on which it operates. It also has different offers to users such as the possibility of discounts when buying with another person, when buying daily or weekly passes. It is a non-public-funded bus service. ArrivaClick is among the most successful DRTs in Europe with a total of 4 583 passengers per month with the highest passenger/revenue hours of 2.6, which is 60% higher than Kutsuplus, with a passenger/revenue hour of 1.5 (Pettersson, 2019). Their success could be attributed to the comfort, reliability, friendly drivers, and cash less payments of the system according to a survey made of users of the system (Highways, Transport and Environment Overview and Security Panel, 2017).

Nevertheless, not all examples in the UK have been positive. Dial-a-Bus in Milton Keynes is one of the earliest cases where DRT has not been successful. This company began operating in 1975 with a system of booking travel by telephone call from homes or workplaces. Enoch et al. (2006) claims that although it was a well-designed service, very innovative for the time and with good on-bus branding, it had many shortcomings. For example: the cost of operation was exceedingly high, the fares were incredibly low and did not reflect the quality of the DRT, there was no political support to make changes in the fare system, the dispersed design of Milton Keynes made the trips exceptionally long. Finally, there was not enough political commitment to keep it going. This service was closed after 2 years of operation and was replaced by a conventional fixed route service.

A successful example of a DRT system outside Europe is Keoride, which has been operating in trial mode since 2017 and recently became a permanent service in 2021. Keoride provides a specific service to the "Northen Beaches" of Sydney, Australia and bookings are done via the app and/or the website. The managing director, Mark Dunlop, assures that the vehicles chosen for the DRT service ensure safety, comfort, and accessibility for people with motor difficulties (Hino Australia, 2021). It is a subsidized, door-to-door service with physical and virtual stops.
Fares are dynamic, i.e., vary with distance and operate throughout the week for almost 16 hours a day. Patterson (2019) indicates that for the trial Keoride had 9,816 passengers per month which means 2.5 passengers per revenue hour of operation, which is an indicator of particularly good operation in comparison with many other DRT systems.

RideKC was an attempt to apply a DRT technology and service to connect downtown and suburban areas in Kansas, USA (KCATA, 2016). This company operated with 14-seat capacity minibuses and only physical stops at split times in the morning and afternoon. Booking was done only through a mobile phone application less than an hour before departure. The fare was a flat rate of 1.5 dollars, the same as the local bus fare. Their operation lasted only a one-year trial and according to Patterson (2019), has one of the lowest metrics of 0.06 passengers/revenue hour.

When analyzing the above cases, there is no single formula for DRT success; however, most of the successful cases have dynamic fares that reflect the total cost of the trip more accurately than a flat fare. However, dynamic fares also do not ensure the success of the service, as for example Kutsuplus, which despite having dynamic fares, operating costs were so high that they were not compensated by revenue and subsidy. This can also happen by operating for many hours, despite not having enough demand during the whole period, excessive flexibility can increase the operator's cost. According to Hanlund et al. (2019) for future DRT pilots it is essential to identify the user profile and the likelihood of acceptance of the new service otherwise, the users themself must make sure that the service is economically competitive with other means of transportation.
3. Analytic Model

This section will analyze two models are analyzed, one proposed by (Estrada, Salanova, Medina-Tapia, & Robusté, 2021) where an objective function is optimized depending on the cost of the user and the operator (total cost). The variables used for its modeling and the main limitations of it will be explained. In the second subchapter, the reasoning of the model of the previous chapter is used, but with a new approach. In this one, an original model is proposed in which the operator’s vehicle fleet becomes the independent variable. This is because, as will be explained below, the number of vans or buses is usually one of the main operating limitations. This second model is the one that for the future of this work will be called fleet-dependent model. The one proposed by Estrada will be mentioned in this document as the headway-dependent model.

3.1. Model Overview

This model assume that the service will be provided in a rectangular area of width “w” and length, “l”. The route length of l will serve an hourly demand of λd which is in km²-h. The demand is constant and homogeneous over the entire area served. The spatial representations of the service area for both cases (DRT and Conventional Line) are as follows:

![Figure 1. Conventional Bus Line Scheme](image1)

![Figure 2. Demand Responsive Transport Line Scheme](image2)
This model will analyze the cost of the operator and the cost of the user. The sum of both is called the total cost, and the purpose of the analysis was to make a comparison of the total costs of the conventional bus line and the demand responsive transport line when the same variables or very similar scenarios are used.

Both the agency cost $Z_a$ and the user cost $Z_u$ depend on several variables. Some of them are constants that will be defined depending on the characteristics of the service, vehicle, geography of the area and users. The list of variables to be used for both analytical models, the conventional bus model, and the demand responsive transport (DRT) model, is shown below:

**Table 1. Inputs for the Analytic Model**

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip length</td>
<td>$l$</td>
<td>km</td>
</tr>
<tr>
<td>Route length</td>
<td>$L$</td>
<td>km</td>
</tr>
<tr>
<td>Band width</td>
<td>$w$</td>
<td>km</td>
</tr>
<tr>
<td>Area</td>
<td>$R = w \times L$</td>
<td>km$^2$</td>
</tr>
<tr>
<td>Demand density per hour</td>
<td>$\lambda_d$</td>
<td>(pax/km$^2 \cdot$h)</td>
</tr>
<tr>
<td>Vehicle capacity</td>
<td>$C$</td>
<td>(pax)</td>
</tr>
<tr>
<td>Cruising speed</td>
<td>$v$</td>
<td>(km/h)</td>
</tr>
<tr>
<td>Acceleration rate</td>
<td>$a$</td>
<td>(m/s$^2$)</td>
</tr>
<tr>
<td>Boarding and alighting time per passenger</td>
<td>$\tau'$</td>
<td>(s/pax)</td>
</tr>
<tr>
<td>Walking speed</td>
<td>$v_w$</td>
<td>(km/h)</td>
</tr>
<tr>
<td>Headway variance</td>
<td>$s^2_H$</td>
<td>(h$^2$)</td>
</tr>
<tr>
<td>Unit distance cost</td>
<td>$c_d$</td>
<td>(€/veh-km)</td>
</tr>
<tr>
<td>Unit temporal cost</td>
<td>$c_t$</td>
<td>(€/veh-h)</td>
</tr>
<tr>
<td>Fare</td>
<td>$q$</td>
<td>(€/trip)</td>
</tr>
<tr>
<td>Value of time</td>
<td>$\beta_t$</td>
<td>(€/pax-h)</td>
</tr>
<tr>
<td>Street spacing</td>
<td>$D_s$</td>
<td>(km)</td>
</tr>
<tr>
<td>Spacing between consecutive stops</td>
<td>$s$</td>
<td>(km)</td>
</tr>
<tr>
<td>Headway</td>
<td>$H$</td>
<td>(h)</td>
</tr>
</tbody>
</table>
3.2. Analytic Model, DRT and Conventional Bus

For this model, the objective function is the following:

\[
\min_{H,S} Z = Z_a + Z_u
\]

subject to:
\[
s \geq s_{\text{min}}, \quad O \leq C, \quad H \geq H_{\text{min}}
\]

This means that the model aims to look for the lowest total cost (agency plus user) with a stop spacing bigger or equal to the minimum with certain capacity bigger than occupancy of the ride and a headway that needs to be bigger than or equal to a fixed one (minimal one).

3.2.1. Operator Cost

The cost of the operator consists of two main variables. The one that is directly proportional to the distance travelled by the operator and the one that is directly proportional to the operating time of the service:

\[
Z_a = c_d Q + c_t M
\]

The first factor in calculating the cost of the agency is the total distance traveled in one hour. In the case of the conventional bus, this distance is very simple since it is only one travel cycle (2L) divided by the headway at which it travels (H). As follows:

\[
Q = \frac{2L}{H} \text{ for conventional}
\]

In Figure 2, it is easy to notice that the longitudinal or L-axis distance of a DRT trip is the same as the total distance for a conventional bus in this way, the sum Lxi is L. However, the transverse distance varies for a DRT trip. The sum of Lyi is defined as the expected distance between two consecutive points i.e., w/3, multiplied by the number of boardings and alighting's operations in a roundtrip 2λ_dRH. Finally, the model takes into consideration an extra distance in cases where a point in the same street block has to be visited and is defined as follows:
\[ Q = \frac{2L}{H} + \frac{2\lambda_d LHw^2}{3} + \frac{L}{H} \sum_{i=2}^{\infty} (i - 1) * 2D_s \left( \frac{2D_s}{L} \right)^i \geq 1 \text{ for DRT} \] (4)

However, for the model explained in the next chapter the last factor will be omitted for simplicity. It is left to the reader to investigate further in cases where a DRT service is required in the same street block.

The fleet calculation for conventional and DRT service is the same and defined as the number of veh-km traveled in one hour divided by the operating speed of the service. Although the expression is the same, take into consideration that in the case of DRT and conventional bus the operating speed is different as will be explained later.: 

\[ M = \left[ \frac{Q}{v_c} \right]^+ \] (5)

3.2.2. User Cost

The cost of public transport users is basically based on the time spent on the journey. The longer the trip, the more expensive in terms of “the value of people’s time (\( \beta_t \))”. In the equation below the demand is multiplied by the €/hour of each passenger.

The first time taken into consideration is the access time (A), which is the time spent traveling to and from the bus stops. For the case of a DRT service, as it is considered door-to-door service, the access time is zero. The second time taken into consideration is the waiting time (W) which is the time a user has to wait until the bus arrives at the bus stop or pickup point. Finally, the in-vehicle time (IVTT) is all the time that elapses from the time the passenger boards the vehicle, makes the trip and gets off the vehicle.

\[ Z_u = \lambda_d Lw * z_u = \lambda Lw * [\beta_t * (A + W + IVTT)] \] (6)

To calculate the access time of a conventional bus it is assumed that a person travels a longitudinal distance of \( s/2 \) and a transverse distance of \( w/2 \). Both are divided by the walking speed to calculate the time it takes as shown below:
\[ A = \frac{s + w}{2 \cdot v_w} \text{ for conventional} \]

\[ A = 0 \text{ for DRT} \quad (7) \]

Regarding the waiting time, it is considered that passengers do not know the passing interval and simply wait the necessary time at the stop. Thus, this time will vary between zero and the passing interval (H) and on average it will be H/2 with the possibility of increasing due to the variability of the service:

\[ W = \frac{1}{2} \left( H + \frac{S^2_H}{H} \right) \quad (8) \]

In order to define the amount of time on the vehicle it is necessary to first define the number of stops for a conventional line. Two cases are considered, the first case represents a line that stops at all stops (total length between spacing) and the second case represents a situation in which there are fewer users than the number of stops. Therefore, there would be a maximum number of stops as the number of users in a whole cycle:

\[ S_e = \min \left\{ \left[ \frac{2L}{s} \right]^+, [2\lambda_d LwH]^+ \right\} \text{ for conventional} \quad (9) \]

Having defined the number of stops now, we can define the in-vehicle time for both cases. This is composed of three parts for both conventional and DRT cases. These are: the free flow time, the acceleration and deceleration time at each pick up or drop off point and finally the time it takes for all users to get on and off the bus. Respecting this same order in the following sequence we have:

\[ IVTT(l) = \frac{l}{v} + \frac{l \cdot S_e}{2L} \cdot \frac{v}{a} + \frac{l \lambda_d R H \cdot \tau'}{2L} \text{ for conventional} \quad (10) \]

\[ IVTT(l) = \frac{QHl}{2L} \left[ \frac{1}{v} + \frac{2\lambda_d R}{Q} \cdot \left( \frac{v}{a} + \frac{\tau''}{2} \right) \right] \text{ for DRT} \quad (11) \]
From the expressions of in vehicle travel time the operating speed can be derived. Just divide the route length (l) by IVTT and obtain the speed at which the bus operates. Below, in the following subchapters, the expression of the inverse of this speed for the DRT is presented for future calculations.

Finally, as seen in equation 1, once the model is run, it is necessary to verify that the occupancy of the vehicle is less than its capacity, therefore, it is necessary to calculate the occupancy in the following way where: \( \frac{l \lambda_d R}{2L} \left[ \text{pax} \right] \) is the passenger flow in any section which is multiplied by the headway to obtain O:

\[
O = \frac{l \lambda_d R}{2L} H
\]

(12)

3.3. Analytic Model of DRT and Conventional Line When Depending on the Fleet

This chapter will focus on a "new" analytical model for the DRT and the conventional line service. The novelty compared to the models explained in the previous chapter is that, now, the independent variable is the vehicle fleet (M), which was previously determined by the objective function. This approach stems from the idea of exploring how the model behaves when there is a vehicle constraint. Since the acquisition and operation of a new vehicle represents a very high increase in the operational cost of the system. As seen above, several failed DRT cases are due to the high operational costs of the system. In this chapter we will see how the objective equation changes and the demand dependent variables change as well. Ones again, for this model, the expression for agency cost and user cost are the followings:

\[
Z_a = c_d Q + c_t M
\]

\[
Z_u = \lambda_d Lw * z_u = \lambda Lw * [\beta_t * (A + W + IVTT)]
\]

In this way, the new objective function will be the following:

\[
\text{Min } Z = Z_a + Z_u
\]
\[ s.t \quad s \geq s_{min}, \quad O \leq C, \quad M_{max} \geq M \]

(13)

3.3.1. Conventional Line Fleet Dependent Model

In order to define the new Headway that is going to be fleet dependent, Equation 3 and Equation 5 are rewrite in the following way:

\[ M = \frac{2L}{Hv_c} \]

Now, operation speed can be derived from the in-vehicle time of Equation 10 since the IVTT is the inverse of the operating speed multiplied by the total length of the trip and substitute in the previous equation as follows

\[ M = \frac{2L}{H} \left( \frac{1}{v} + \frac{S_e v}{2La} + \frac{\lambda_d RH}{2L} \right) \tau' \]

As you can see, this expression depends on the number of effective stops (Se), this could vary as depending on the conditions of Equation 9. Consequently, there are two different expressions for the headway. The first one, when the number of boarding and alightings is lower than the number of predefined stops. And the second one, where more than one user is getting on and off the bus in each predefined stop.

When,

\[ S_e = 2\lambda_d LwH \]

Then:

\[ H = \frac{2L \left( Mv - \frac{2\lambda_d RV^2}{a} - \lambda_d RV \tau' \right)}{Mv - \frac{2\lambda_d RV^2}{a} - \lambda_d RV \tau'} \]

When,

\[ S_e = \frac{2L}{s} \]

Then:

\[ H = \frac{2L(as + v^2)}{asv(M - \lambda_d R \tau')} \]
From this point on, the same equations explain in the Section 3.2 can be used taking into account the two expressions of the headway explained before. In case of the waiting time, the second term of Equation 8. was considered too small to take into consideration and it was omitted from these equations for terms of simplicity. In this way the new expression for the total cost of the fleet dependent model are the followings:

Table 2. Expression for a Conventional Line in a Fleet Dependent Model

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\lambda_d LwH &lt; \frac{2L}{s}$</td>
<td>$H = \frac{2L}{(Mv - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau')}$</td>
</tr>
<tr>
<td>$S_e = \frac{2L}{s} &lt; 2\lambda_d LwH$</td>
<td>$H = \frac{2L(as + v^2)}{asv(M - \lambda_d R\tau')}$</td>
</tr>
<tr>
<td>$Q = (Mv - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau')$</td>
<td>$Q = \frac{asv(M - \lambda_d R\tau')}{(as + v^2)}$</td>
</tr>
<tr>
<td>$A = \frac{s + w}{2\nu_w}$</td>
<td></td>
</tr>
<tr>
<td>$W = \frac{1}{2}(Mv - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau')$</td>
<td>$W = \frac{1}{2}\left(\frac{2L(as + v^2)}{asv(M - \lambda_d R\tau')}\right)$</td>
</tr>
<tr>
<td>$IVTT(l) = \frac{l}{v} + \frac{2L \cdot \lambda_d Lw}{MV - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau'} \cdot \left(\frac{v}{a} + \frac{\tau'}{2}\right)$</td>
<td>$IVTT(l) = \frac{l}{v} + \frac{l + S_e \cdot v}{2L} + \frac{\lambda_d w}{2L} \cdot \frac{2L(as + v^2)}{asv(M - \lambda_d R\tau')} \cdot \tau'$</td>
</tr>
</tbody>
</table>

3.3.2. DRT Fleet Dependent Model

In a similar way than for a conventional line, to start the approach of the fleet dependent model for DRT, the first thing to do was to rewrite the headway (H) in terms of the fleet (M). For this purpose, Equation 5 and Equation 4 were used. It is important to mention again that for simplicity reasons, the last expression of Equation 4 will not be used. For future research it could be included if it is desired to know the impact of those trips that have pickup and/or drop off points in the same street block and a detour distance is included. For this study it was assumed that there would be no trips of this nature.
The expression of the operating speed for DRT system of the model of Estrada et al. (2021) is used as well. This can be derived from the in-vehicle time in Equation 11 since the IVTT is the inverse of the operating speed multiplied by the total length of the trip:

\[
\frac{1}{v_c} = \frac{1}{v} + \frac{2\lambda_d R}{Q} \ast \left( \frac{v}{a} + \tau' \right) \tag{14}
\]

Now, in order to calculate the fleet, Equation 5 is used by multiplying Equation 14 by \(Q\) in the following way:

\[
M = \frac{Q}{v} + 2\lambda_d R \ast \left( \frac{v}{a} + \tau' \right)
\]

And now, \(Q\) is substituted with the expression of Equation 4. again, the last expression will be omitted for the sake of simplicity in the following way:

\[
M = \frac{2L}{\frac{H}{v}} + \frac{2\lambda_d L w^2}{3v} + 2\lambda_d R \ast \left( \frac{v}{a} + \tau' \right)
\]

When solving the equation for \(H\) we would have the following expression which now depends on \(M\) as can be seen in Equation 15.

\[
H = \frac{2L}{v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right]} \tag{15}
\]

From this expression it is interesting to note that now, considering a fixed fleet, the headway is directly proportional to the demand. That is, if demand increases, the denominator of the equation decreases and therefore the headway increases. This can be explained in the following way: as demand increases, DRT buses must run longer routes to serve users, so the time between one service and another must be longer to "recover the bus" for the next trip.

Next, a similar operation is performed with the value of \(Q\). The expression calculated in the previous step is substituted into the expression for the number of kilometers traveled in one hour, \(Q\), of Equation 4 in the following way:
\[ Q = v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right] + \frac{2\lambda_d L w^2}{3} * v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right] \]

Simplifying the above equation, the following is obtained:

\[ Q = vM - \lambda_d R \left( \frac{2v^2 + v\tau' a}{a} \right) \]

(16)

For the user cost there are three main variables: access time, waiting time and in vehicle travel time. For the case of the DRT, as explained above, the access time is 0. Then we proceed with the next term, the waiting time, based on Equation 8 and the new expression of \( H \) of Equation 15 the waiting time can be expressed as follows:

\[ W = \frac{2L}{2v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right]} + \frac{S^2_h * v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right]}{2L} \]

(17)

Now, the in-vehicle travel time equation is rewritten taking into account Equation 11 and the new M-dependent approach.

\[ IVTT = \frac{QHL}{2L} \frac{1}{v} + \frac{\lambda_d RH}{L} \frac{v}{a} + \tau' \frac{QHL \lambda_d}{2L} \]

\[ IVTT = QH \left( \frac{1}{2Lv} + \frac{\tau' \lambda_d}{2L} \right) + \frac{\lambda_d RH}{L} \frac{v}{a} \]

\[ IVTT = \left[ vM - \lambda_d R \left( \frac{2v^2 + v\tau' a}{a} \right) \right] \left[ \frac{2L}{v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right]} + \frac{1}{2Lv} + \frac{\tau' \lambda_d}{2L} \right] + \frac{2\lambda_d R L}{2L} \frac{v}{a} \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right] + \tau' \lambda_d + \frac{1}{v} \]

\[ IVTT = \left( \frac{2\lambda_d R w}{3} \right) * v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right] \left( \frac{1}{2Lv} + \frac{\tau' \lambda_d}{2L} \right) + \frac{2\lambda_d R L}{2L} \frac{v}{a} \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) \right] + \tau' \lambda_d + \frac{1}{v} \]

(18)
4. Diseconomies of Scale

As explained before, one of the main objectives of this thesis is to understand the vicious cycle of the DRT transportation system, which is strongly linked to the number of users in the service. It is for this reason that this chapter makes a separation of all those terms in the total cost per passenger that depend on $\lambda_d$ and those that do not.

There will be certain factors that are inversely proportional, meaning that as demand increases the cost decreases. Then there will be other factors that are proportional, i.e. they grow with demand. Finally, there are those that are independent of demand, i.e., they are not affected by the number of users, the cost will be the same. In this way, first the factors of the traditional systems and then the DRT model will be analyzed.

4.1. Headway- Dependent Model

4.1.1. Conventional Bus Lines

For the conventional line, when convening the first expression of the Equation 2 and Equation 3 and diving it by $\lambda R$ (The total number of passengers per hour,) the result in the following:

$$\frac{Q \cdot c_d}{\lambda_d R} = \frac{2L \cdot c_d}{\lambda_d R}$$

Inverse to $\lambda_d$

Then, following Equation 5 the expression of Q for a conventional line is divided by the $v_c$ or the operational speed to obtained the fleet as follows:

$$M = \frac{2L}{H} \left( \frac{1}{v} + \frac{S_e}{2L} \cdot \frac{v}{a} + \frac{\lambda RH \cdot \tau'}{2L} \right)$$

Considering that the bus is not stopping in all the stops from Equation 9.

$$S_e = 2\lambda RH$$
And taking into consideration the second part of the Equation 2, and dividing again by the number of users, the following is obtained:

$$\frac{M \cdot c_t}{\lambda_d R} = \frac{2L \cdot c_t}{Hv\lambda_d R} + \frac{2 \cdot v \cdot c_t}{a} + \tau' \cdot c_t$$

Inverse to $\lambda_d$ Independent from $\lambda_d$

For the access time and following Equation 6 and Equation 7 for conventional bus line the following $\lambda_d$ independent expression is obtained:

$$\lambda_d R \cdot \beta_t (A) = \frac{s + w}{2 \cdot v_w} \cdot \beta_t$$

Independent from $\lambda_d$

For the waiting time and using the before mentioned Equation 6 and Equation 8, for conventional bus line the following $\lambda_d$ independent expression is obtained:

$$\lambda_d R \cdot \beta_t (W) = \beta_t \cdot \frac{1}{2} \left( H + \frac{S_H^2}{H} \right)$$

Independent from $\lambda_d$

Finally, the in-vehicle travel time can be expressed by taking into consideration Equation 6 and Equation 10 in where there is a part of the expression that is independent from the demand and another oner that is proportional as follows:

$$\lambda_d R \cdot \beta_t (IVTT) = \frac{l}{v} \beta_t + \frac{l \lambda_d RH}{L} \cdot \frac{v}{a} \beta_t + \frac{l \lambda_d RH \cdot \tau'}{2L} \beta_t$$

Independent from $\lambda_d$ Proportional to $\lambda_d$
Thus, the $Z_t$/pax equation can be rewritten in three parts, the first one is the one that is inverse to $\lambda_d$ the second one is independent from $\lambda_d$ and the third the one that is proportional to $\lambda_d$ as follows:

$$
\frac{Z}{pax} = \left[ \frac{2L \cdot cd}{H} + \frac{2L \cdot ct}{Hv} \right] \cdot \left( \frac{1}{\lambda_d R} \right) + \left[ A + W + \frac{l}{v} \right] \cdot \beta_t + \frac{2 \cdot v \cdot ct}{a} + \tau' \cdot ct + \left[ \frac{lH}{L} \cdot \frac{v}{a} \beta_t + \frac{lH \cdot \tau'}{2L} \beta_t \right] \cdot \lambda_d R
$$

Please note that, the diseconomies of scale are given only from the user cost. That is, it is the portion of the vehicle travel time price (IVTT) that increases as demand increases and which consequently generates the diseconomies of scale. From here we can see than even for the conventional line, the initial hypothesis stating that the users are the ones affected by the increase of demand, is proof. Nevertheless, in the result section the impact of this factor will be analyzed in greater depth.

On the other hand, access time and waiting time are independent of demand, so it is a constant cost regardless of whether there are many or few users. The factor that adds to the economy of scale is the cost of the operator, both for time and distance, since this is relatively constant with few or many passengers, if it is divided among more users the cost per passenger becomes lower.

4.1.2. Demand Responsive Transport Line

For the conventional line, when convening the first part of the Equation 2 and Equation 4 and diving by $\lambda R$ which represents the number of passengers to obtain the cost/pax will result in the following expression:

$$
\frac{Q \cdot c_d}{\lambda R} = \frac{2L}{H \lambda_d R} \cdot c_d + \frac{2w}{3} \cdot c_d
$$

Inverse to $\lambda_d$ Independent from $\lambda_d$
Then, considering the second part of the Equation 2 and divided again by the number of users, the following was obtained:

\[
\frac{M \cdot c_t}{\lambda R} = \frac{2L \cdot ct}{Hv \cdot \lambda_d R} + \frac{2w \cdot ct}{3v} + \frac{2v \cdot c_t}{a} + \tau' \cdot c_t
\]

Inverse to \( \lambda_d \) Independent from \( \lambda_d \)

The extra distance for picking and dropping points in the same street block in Equation 4 was omitted for sake of simplicity as it was in the previous equations.

The access time for a DRT is 0 and for the waiting time the behavior is the same than for the conventional line which is completely independent from the demand of the service:

\[
\lambda R \cdot \beta_t \cdot \frac{(A)}{\lambda R} = 0
\]

\[
\lambda R \cdot \beta_t \cdot \frac{(W)}{\lambda R} = \beta_t \cdot \frac{1}{2} \left( H + \frac{S_H^2}{H} \right)
\]

Independent from \( \lambda_d \)

Finally, for the in-vehicle travel time and following Equation 6 and Equation 11 is possible to obtain the next expression:

\[
\lambda R \cdot \beta_t \cdot \frac{(IVTT)}{\lambda R} = \beta_t \cdot \frac{l}{v} + \beta_t \cdot \frac{Hl}{3} \cdot \frac{\lambda_d Rw}{La} + \beta_t \cdot \frac{\lambda_d RvHl}{La} + \beta_t \cdot \frac{\lambda_d RlH \tau'}{2L}
\]

Independent from \( \lambda_d \) Proportional to \( \lambda_d \)

Thus, the Zt/pax equation can be rewritten in three parts, the first one is the one that is inverse to \( \lambda_d \) the second one is independent and the third the one that is proportional to \( \lambda_d \) as follows:
\[
\frac{Z}{pax} = \left[ \frac{2L \cdot cd}{H} + \frac{2L \cdot ct}{Hv} \right] \cdot \left( \frac{1}{\lambda_d R} \right) + \left[ \frac{2w}{3} \cdot c_d + \frac{2w \cdot ct}{3v} + \frac{2v \cdot ct}{a} + \tau' \cdot c_t + \beta_t \cdot W + \beta_t \cdot \frac{l}{v} \right] + \left[ \frac{w}{3v} + \frac{v}{a} + \frac{\tau'}{2} \right] \cdot \lambda_d R \beta_t \frac{Hl}{L}.
\]

In a comparable way to the conventional bus, for the DRT the diseconomies of scale are given only from the user cost. That is, it is the portion of the vehicle travel time price (IVTT) that increases as demand increases and which consequently generates the diseconomies of scale. For the original model it is once understood that the diseconomies of scale are reflected only in the user's cost and that the operator does not see these cost increases reflected in his costs as demand increases.

The access time is zero and the waiting time is independent of demand, so it is a constant cost regardless of whether there are many or few users. The factor that adds to the economy of scale is the cost of the operator, both for time and distance, since this is relatively constant with few or many passengers, if it is divided among more users the cost is lower.

### 4.2. Fleet-Dependent Model

#### 4.2.1. Conventional Bus Lines

As it could be seen in section 3.3.1 at the time of setting up the equations for the fixed line model, depending on the fleet (M) there is a variable that makes the equations take one direction or another, this variable is the Se or number of effective stops per direction. For the study of diseconomies of scale, it will be assumed that the number of boarding and alighting is lower than the number of predefined stops spaced at s spacing meaning that it will be assumed that:

\[
2\lambda_d LwH < \frac{2L}{s}
\]

Therefore, when convening the first expression of the Equation 2 and the expression for Q in Table 2 and diving it by \( \lambda R \) (The total number of passengers per hour) to obtain the cost per passenger, the result in the following:
\[
\frac{Q \times c_d}{\lambda_d R} = \frac{c_d}{\lambda_d R} \left( Mv - \frac{2\lambda_d R v^2}{a} - \lambda_d R v' \tau' \right)
\]

Which can be expressed as well in the following way:

\[
\frac{Q \times c_d}{\lambda_d R} = \frac{c_d vM}{\lambda_d R} - c_d \left( \frac{2v^2 + v' a}{a} \right)
\]

Then, considering the second part of the Equation 2, a fix number of fleet \( M \) and divided again by the number of users, the following was obtained:

\[
\frac{M \times c_t}{\lambda_d R}
\]

Finally, the operator cost equation could be written as follows:

\[
\frac{Z_a}{\lambda_d R} = \frac{c_d vM}{\lambda_d R} + \frac{c_t M}{\lambda_d R} - \frac{c_d \left( 2v^2 + v' a \right)}{a}
\]

Inverse to \( \lambda_d \)  Independent from \( \lambda_d \)

For the fleet-dependent model, as can be seen, the cost of the operator contributes mainly to economies of scale, despite having a factor that is constant regardless of the increase and decrease in demand, the major contribution of this factor is to the reduction of total price per person when demand increases

The access time is expressed with the equation in Table 2, therefore:

\[
\lambda R \cdot \beta_t \left( A \right) = \beta_t \frac{s + w}{2 \cdot v_w}
\]

Independent from \( \lambda_d \)

And for the waiting time, the expression in Table 2 was took into consideration as follows, where it all represented a diseconomy of scale:
\[ \lambda_d R \ast \beta_t \left( \frac{W}{\lambda_d R} \right) = \beta_t \frac{L}{Mv - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau'} \]

Proportional to \( \lambda_d \)

Finally, the in-vehicle travel time per person can be expressed by taking into consideration once again the equation in Table 2 as follows:

\[ \lambda R \ast \beta_t \left( \frac{IVTT}{\lambda R} \right) = \beta_t \frac{2l \cdot \lambda_d wL}{Mv - \frac{2\lambda_d Rv^2}{a} - \lambda_d Rv \tau'} \left( \frac{v + \tau'}{2} \right) + \beta_t \frac{l}{v} \]

Proportional to \( \lambda_d \) \quad Independent from \( \lambda_d \)

For the user cost per passenger most of the equations represent diseconomies of scale or constant terms with demand such as access time. The access time once again, is independent from the amount of user. Since there is now a limited fleet, waiting time does affect as demand increases. This is because the recovery time of the vehicle once it has made a trip causes the headway to increase, with a larger headway the waiting time is longer. As far as the IVTT is concerned, it is mostly increased as demand increases, mainly because the number of stops increases (acceleration and deceleration time increases) and the time to get on and off the bus also increases. The time to travel “l” at a speed “v” is constant and independent of demand.

Finally, in order to calculate the cost per passenger the above equations are used to calculate the Za and Zu. The sum of these to and the division by \( \lambda_d R \) will give the cost per passage in the following way:

\[ \frac{Z_T}{\lambda_d R} = \frac{c_d \nu M}{\lambda_d R} + \frac{c_l M}{\lambda_d R} + \beta_t \left( W + IVTT - \frac{l}{v} \right) + \beta_t \left( \frac{l}{v} + \frac{s + w}{2v_w} \right) - c_d \left( \frac{2v^2 + v \tau' a}{a} \right) \]

Inverse \( \lambda_d \) \quad Proportional to \( \lambda_d \) \quad Independent from \( \lambda_d \)

However, by finding the point where diseconomies of scale start to "gain ground" over economies of scale we may determine the precise demand at which the conventional line system starts to deteriorate total cost/passenger wise. To find when the slopes of the two equations
intersect, we must derive the equation for economies of scale and diseconomies of scale and define the point of intersection. Given the complexity and length of the equations, a simplification of the terms to be derived is made and expressed as follows:

In this way, the economies of scale can be expressed as follows:

\[
E(\lambda_d) = \frac{c_d v M}{\lambda_d R} + \frac{c_t M}{\lambda_d R}
\]

Following the abbreviations of Table 3:

\[
E(\lambda_d) = \frac{T}{\lambda_d}
\]

Deriving this expression:

\[
E'(\lambda_d) = -\frac{T}{\lambda_d^2}
\]

Then, the diseconomies of scale will have the following expression:

\[
D(\lambda_d) = \beta_t \left( \frac{L}{M v - \lambda_d \left( \frac{2 R v^2 + a R v \tau'}{a} \right)} \right) + \beta_t \left( \frac{2 l \lambda_d R}{M v - \lambda_d \left( \frac{2 R v^2 + a R v \tau'}{a} \right)} \right) \left( \frac{v + \tau'}{2} \right) - \beta_t \frac{l}{v}
\]
Following the abbreviations of Table 3:

\[
D(\lambda_d) = \frac{\sum + P\lambda_d}{(Mv - \lambda_d\Pi)} - \beta_t \frac{l}{v}
\]

\[
D'(\lambda_d) = \frac{\Pi\Sigma + MPv}{(Mv - \lambda_d\Pi)^2}
\]

The intersection of both curves can be expressed as follows:

\[
E'(\lambda_d) = D'(\lambda_d)
\]

\[
\lambda_d = \frac{\Pi\Sigma Mv - \sqrt{\Pi M^2 v^2 (\Pi\Sigma + MPv)}}{\Pi^2 T - MPv - \Pi\Sigma}
\]

4.2.2. Demand Responsive Lines

For a DRT line, when convening the first expression of the Equation 2 and Equation 16 and diving it by \(\lambda R\) (The total number of passengers per hour) to obtain the cost per passenger, the result in the following:

\[
\frac{Q * c_d}{\lambda_d R} = \frac{vM - \lambda_d R \left(2v^2 + v\tau' a\right)}{\lambda_d R} * cd
\]

Which can be expressed in the following way:

\[
\frac{Q * cd}{\lambda_d R} = \frac{c_d vM}{\lambda_d R} - \frac{c_d \left(2v^2 + v\tau' a\right)}{a}
\]

Then, considering the second part of the Equation 2 a fix number of fleet M and divided again by the number of users, the following was obtained:

\[
\frac{M * c_t}{\lambda_d R}
\]
Finally, the operator cost equation could be written as follows

$$\frac{Z_\alpha}{\lambda_d R} = \frac{c_d v M}{\lambda_d R} + \frac{c_t M}{\lambda_d R} - c_d \left( \frac{2v^2 + vv' + a}{\lambda_d R} \right)$$

Inverse to $\lambda_d$  \hspace{1cm} Independent from $\lambda_d$

For the fleet-dependent model, as can be seen, the cost of the operator contributes mainly to economies of scale, despite having a factor that is constant regardless of the increase and decrease in demand, the major contribution of this factor is to the reduction of total price per person when demand increases

The access time for a DRT is 0 and for the waiting time the expression in Equation 17 was used as follows:

$$\lambda R \cdot \beta_t \left( \frac{A}{\lambda R} \right) = 0$$

$$\lambda_d R \cdot \beta_t \left( \frac{W}{\lambda_d R} \right) = \beta_t \left( \frac{L}{v \left[ M - \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a + \tau'} \right) \right]} - \frac{S_H^2 \cdot v \left[ \lambda_d R \left( \frac{2w}{3v} + \frac{v^2}{a + \tau'} + \frac{\tau}{a} \right) \right]}{4L} + \frac{S_H^2 \cdot v M}{4L} \right)$$

Proportional $\lambda_d$  \hspace{1cm} Proportional$^1$ $\lambda_d$  \hspace{1cm} Independent from $\lambda_d$

It is important to take into consideration that the whole first expression is reciprocal to the demand of the system. This is because in the first term the demand density is in the denominator with negative sign so that as the whole expression increases when $\lambda_d$ increases. In the same way, for second term, the demand is in the numerator to it will increases when $\lambda_d$ increases.

---

$^1$ This expression is proportional to $\lambda_d$ but the negative sign makes the overall cost falls as $\lambda_d$ rises: diseconomies of scale. For simplicity, it is not written in the final equation, but it is taken into account in the results.
Finally, the in-vehicle travel time per person can be expressed by taking into consideration 
Equation 6 and Equation 18 as follows:

\[
\frac{\lambda R \cdot \beta_t (IVTT)}{\lambda R} = \beta_t \left( \frac{2 \lambda_d R w}{3} - \frac{2 L}{\nu M - \lambda_d R \left( \frac{2 w}{3} + \frac{v^2}{a} + \tau' \right)} \right) \left( \frac{1}{\lambda_d R} + \frac{\tau' \lambda_d}{2 L} + \frac{2 \lambda_d R \left( \frac{2 w}{3} + \frac{v^2}{a} + \tau' \right)}{M - \lambda_d R \left( \frac{2 w}{3} + \frac{v^2}{a} + \tau' \right) a} \right) + \frac{1}{\nu}
\]

Proportional $\lambda_d$  
Independent from $\lambda_d$

The access time once again, is zero for DRT. Since there is now a limited fleet, waiting time 
does affect as demand increases. This is because the recovery time of the vehicle once it has 
made a trip causes the headway to increase, with a larger headway the waiting time is longer. 
As far as the IVTT is concerned, it is mostly increased as demand increases, mainly because 
the number of stops increases (acceleration and deceleration time increases) and the time to get 
on and off the bus also increases. The time to travel “l” at a speed “v” is constant and independent of demand.

Finally, in order to calculate the cost per passenger the above equations are used to calculate 
the $Z_a$ and $Z_u$. The sum of these to and the division by $\lambda_d R$ will give the cost per passenger in the following way:

\[
\frac{Z_T}{\lambda_d R} = \frac{c_d v M}{\lambda_d R} + \frac{c_t M}{\lambda_d R} + \beta_t \left( W - S_h^2 \cdot \frac{v M}{4 L} + IVTT - \frac{1}{v} \right) + \beta_t \left( \frac{1}{v} + S_h^2 \cdot \frac{v M}{4 L} \right) - c_d \left( \frac{2 v^2 + v \tau' a}{a} \right)
\]

Inverse $\lambda_d$  
Proportional to $\lambda_d$  
Independent from $\lambda_d$

By plotting these terms separately depending on demand, the point where the diseconomies of 
scale win out over economies of scale can be found. That is, when those terms that decrease 
with demand are less than the terms that grow with demand It may be claimed that the demand 
for this intersection occurs after the system has already failed, since operating prices have 
already begun to rise in response to demand.

However, by finding the point where diseconomies of scale start to "gain ground" over 
economies of scale we may determine the precise demand at which DRT system starts to
deteriorate cost wise. To find when the slopes of the two equations intersect, we must derive the equation for economies of scale and diseconomies of scale and define the point of intersection. Given the complexity and length of the equations, a simplification of the terms to be derived is made and expressed as follows:

Table 4. Expression of Diseconomies and Economies of Scale of DRT

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Original Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Π</td>
<td>( R \left( \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right) )</td>
</tr>
<tr>
<td>P</td>
<td>( \frac{\beta_t L}{v} )</td>
</tr>
<tr>
<td>Σ</td>
<td>( \frac{2\beta_t R w}{3v^2} )</td>
</tr>
<tr>
<td>T</td>
<td>( \frac{2\beta_t R w t' l}{3v} )</td>
</tr>
<tr>
<td>Υ</td>
<td>( \frac{2\beta_t R l}{a} )</td>
</tr>
<tr>
<td>Φ</td>
<td>( \tau' l \beta_t )</td>
</tr>
<tr>
<td>Χ</td>
<td>( \frac{c_d v M + c_t M}{R} )</td>
</tr>
<tr>
<td>Ψ</td>
<td>( \frac{S_H^2 \ast v \ast \beta_t}{4L} )</td>
</tr>
</tbody>
</table>

In this way, the economies of scale can be expressed as follows:

\[
E(\lambda_d) = \frac{c_d v M}{\lambda_d R} + \frac{c_t M}{\lambda_d R} - \frac{\beta_t \ast S_H^2 \ast v}{\lambda_d R} \left[ \frac{2w}{3v} + \frac{v^2}{a} + \tau' \right]
\]

Following the abbreviations of Table 4:

\[
E(\lambda_d) = \frac{X}{\lambda_d} - \Psi \ast \Pi \ast \lambda_d
\]

Deriving this expression:

\[
E'(\lambda_d) = - \frac{X}{\lambda_d^2} - \Psi \ast \Pi
\]
The diseconomies of scale will have the following expression:

\[
D(\lambda_d) = \beta_t L \frac{2L}{v\left[M - \lambda_d R\left(\frac{2w}{3v} + \frac{v^2}{a} + \tau'\right)\right]}
\]

\[
+ \beta_t \left\{ \frac{2\lambda_d R w}{3v} \left[ M - \lambda_d R \left(\frac{2w}{3v} + \frac{v^2}{a} + \tau'\right) \right] \right\} \left(1 + \frac{\tau' l \lambda_d}{2L v} \right)
\]

\[
+ \frac{2\lambda_d R l}{\left[ M - \lambda_d R \left(\frac{2w}{3v} + \frac{v^2}{a} + \tau'\right) a \right]} + \tau' l \lambda_d \left[ M - \lambda_d R \left(\frac{2w}{3v} + \frac{v^2}{a} + \tau'\right) a \right]
\]

Following the abbreviations of Table 4:

\[
D(\lambda_d) = \frac{P}{M - \Pi \lambda_d} + \frac{\Sigma}{M - \Pi \lambda_d} + \frac{T \lambda_d^2}{M - \Pi \lambda_d} + \frac{Y * \lambda_d}{M - \Pi \lambda_d} + \Phi \lambda_d
\]

\[
D'(\lambda_d) = \frac{\Pi(P + \Sigma - T \lambda_d^2) + M(2T \lambda_d + Y)}{(M - \Pi \lambda_d)^2} + \Phi
\]
5.  Externalities

In this chapter a new concept that was not included in the previous models is analyzed. This is the externalities generated by the operation of the system. Depending on the number of kilometers traveled and the number of vehicles in operation, the number of emissions increases or decreases. This chapter aims to explain how these externalities are calculated, equations used and dependent variables. The objective of this chapter is to compare how the operation in similar scenarios of a conventional line and a DRT line varies. Given that previously only a total cost has been taken into account without including that the operation of these services can affect not only the operator and the users but also has an impact on the entire community where these services transit.

Three externalities of passenger transportation were calculated: Type A emissions, CO2 emissions and accidents. Type A emissions include two polluting gases expelled by gasoline motor vehicles: Oxides of Nitrogen (NOx), Volatile Organic Compound (VOC) and Energy Consumption (EC or FC). Type A emissions calculations are carried out separately from the CO2 and the SO2 gas, these emissions are depending on the amount of the fuel consumption. Moreover, the last externality took into consideration was the cost of outwardness due to road accidents.

These externalities are calculated for DRT vehicles and conventional buses. It is important to take into consideration that for the DRT vehicle two scenarios where considered: a van of eight seats is taken in addition to the driver's seat and a minibus of less than 15 tones. In the case of the conventional bus, ones again two scenarios where considered: a vehicle with more than 8 passengers and a maximum weight of 15 tons and medium bus once again for more than 8 passengers and with a weight between 15 and 18 tons. According to (OMM, 2021) Euro V is the most used technology in Barcelona and the second most used in Spain after Euro VI, the same author assures that except for Madrid, Diesel is the most used fuel in Spain. Additionally, (ICCT, 2021) assures that in Europe (EU-28) by 2020, 67% of new passenger cars registrations were Euro 6d-Temp and that Diesel car represent the biggest share in the type of fuel of vehicles. Therefore, and to be conservative, Euro V-Diesel technology is assumed for both vehicle models in this study.
The total cost of externalities can be expressed as:

\[ Z_{\text{Externalities}} = Z_{\text{Type\,A}} + Z_{\text{CO}_2} + Z_{\text{SO}_2} + Z_{\text{Accidents}} \]

(19)

5.1. Emission Type A

The procedure followed for the calculation of the Emissions A is the one recommended by the European Environment Agency (2019) in their 13th Report. First, the speed dependent emission factor (EF) is calculated with the following equation:

\[ EF_{\text{NOx, VOC, EC}} = \frac{\alpha V^2 + \beta V + \gamma + \delta}{\varepsilon V^2 + \zeta V + \eta} [g/\text{veh} - \text{km}] \]

(20)

The constants for the pollutant NOx and VOC and for the Energy Consumption (EC) are shown below:

<table>
<thead>
<tr>
<th>Constant</th>
<th>Diesel - 8 Seat Van</th>
<th>Diesel - Bus &lt; 15 ton</th>
<th>Diesel - Bus 15-18 ton</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.00007</td>
<td>0.00074</td>
<td>0.00070</td>
</tr>
<tr>
<td></td>
<td>0.00038</td>
<td>0.00001</td>
<td>-0.00047</td>
</tr>
<tr>
<td></td>
<td>0.06216</td>
<td>0.00002</td>
<td>-0.00120</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.01138</td>
<td>0.03648</td>
<td>0.01317</td>
</tr>
<tr>
<td></td>
<td>-0.03423</td>
<td>-0.00268</td>
<td>0.04402</td>
</tr>
<tr>
<td></td>
<td>1.92418</td>
<td>0.00082</td>
<td>0.09632</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.94595</td>
<td>-0.05083</td>
<td>13.71178</td>
</tr>
<tr>
<td></td>
<td>1.31598</td>
<td>0.24894</td>
<td>4.05807</td>
</tr>
<tr>
<td></td>
<td>4.05807</td>
<td>-7.65453</td>
<td>0.11704</td>
</tr>
<tr>
<td>(\delta)</td>
<td>1.92361</td>
<td>0.07632</td>
<td>0.04473</td>
</tr>
<tr>
<td></td>
<td>4.34673</td>
<td>1.45389</td>
<td>7.40159</td>
</tr>
<tr>
<td></td>
<td>4.80469</td>
<td>1.94829</td>
<td>5.66182</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>-0.00005</td>
<td>1.57275</td>
<td>-0.00055</td>
</tr>
<tr>
<td></td>
<td>0.00006</td>
<td>-0.00159</td>
<td>-0.00009</td>
</tr>
<tr>
<td></td>
<td>0.02151</td>
<td>0.00036</td>
<td>-0.00011</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>0.00426</td>
<td>0.45398</td>
<td>0.11848</td>
</tr>
<tr>
<td></td>
<td>0.00376</td>
<td>0.11810</td>
<td>0.00967</td>
</tr>
<tr>
<td></td>
<td>-0.07150</td>
<td>0.01068</td>
<td>0.01063</td>
</tr>
<tr>
<td>(\eta)</td>
<td>1.00000</td>
<td>0.30754</td>
<td>1.64789</td>
</tr>
<tr>
<td></td>
<td>0.07250</td>
<td>7.06805</td>
<td>0.21734</td>
</tr>
<tr>
<td></td>
<td>0.02906</td>
<td>0.94548</td>
<td>0.15779</td>
</tr>
</tbody>
</table>

The emission factor is expressed in g/veh-km. Multiplying this by the total number of km traveled in one hour (Q) is possible to calculate the number of grams emitted in an hour. The Handbook of the External Costs of Transport, CE Delft (2019) has a robust list of cost of emission expressed in €/kg by country. The average prices for EU-28 are the followings:
The expression of the cost of emissions type A per hour is the following:

\[
\frac{Z_{Type\ A}}{h} = (EF_{NOx} \times \frac{21.3}{1000} + EF_{VOC} \times \frac{1.2}{1000}) \times Q
\]  

(21)

### 5.2. CO₂ – Emissions

The procedure to calculate the grams of CO₂ emitted by the operation of the bus is differs from the one described in the previous point. For this, Equation 22 found in (EEA, 2019) is used to calculate the mass od CO₂ emitted by vehicles. Here, the Fuel Consumption (FC or EC) of CO₂ calculated in the last section is considered. For Diesel fuel, the ratio of hydrogen to carbon (\(r_{HC}\)) is 1.89 and the ratio of oxygen to carbon (\(r_{OC}\)) is 0.0.

\[
E_{CO_2} = 44.011 \times \frac{FC}{12.011 + 1.008 \times r_{HC} + 16.000 \times r_{OC}}
\]  

(22)

Multiplying this by the total number of km traveled in one hour (Q) is possible to calculate the number of grams emitted in an hour. Considering that the bus is going to be operated by diesel and the cost of one ton of CO2 is 100 € the ton according to (Delft, 2019), the constants to calculate CO₂ is the one showed in Table 7.

### Table 6. Cost of Pollutant NOx and VOC EU -28

<table>
<thead>
<tr>
<th>Pollutant</th>
<th>NOx</th>
<th>VOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>€/kg</td>
<td>21.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

### Table 7. Parameter and Calculation of CO₂ Emissions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>CO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_{HC})</td>
<td>1.8</td>
</tr>
<tr>
<td>(r_{OC})</td>
<td>0</td>
</tr>
<tr>
<td>€/ton</td>
<td>100</td>
</tr>
</tbody>
</table>

The expression of the cost of emissions of CO₂ per hour is the following:
### 5.3. SO2 – Emissions

The grams of SO2 emitted by the operation of the bus is calculated with Equation 24 found in (EEA, 2019) which is based in the amount of Fuel Consumption o (FC or EC) and the weight related Sulphur in fuel \( k_s \) \( \text{g fuel} \). EEA established an amount of 40 ppm for Diesel fuel, 1 ppm represents \( 10^{-6} \) g/g fuel.

\[
E_{SO_2} = 2 \times k_s \times FC
\]

(Multiplying this by the total number of kilometers traveled in one hour \( Q \) is possible to calculate the number of grams emitted in an hour. Considering that the bus is going to be operated by diesel and the cost of SO2 in EU28 is 10.9 €/kg (Delft, 2019), the calculation of the cost per hour of SO2 is quite forward.

<table>
<thead>
<tr>
<th>Table 8. Parameter and Calculation of SO2 Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>k_s (g/g fuel)</td>
</tr>
<tr>
<td>€/kg</td>
</tr>
</tbody>
</table>

The expression of the cost of SO2 per hour is the following:

\[
\frac{Z_{SO_2}}{h} = E_{SO_2} \times Q \times \frac{10.9}{1000}
\]
5.4. Accidents

The calculation of road accidents caused by transportation, is quite simple. According to (Delft, 2019) it is necessary to define the type of vehicle, in this case, Bus/Coach and then the type of road; assuming an Urban Road the following parameter was considered:

<table>
<thead>
<tr>
<th>Parameter and Calculation of Accidents</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bus-Coach</strong></td>
</tr>
<tr>
<td>€-cent per pkm</td>
</tr>
<tr>
<td>0,8</td>
</tr>
</tbody>
</table>

Dividing this amount by the number of passengers transported in the buses/vans in an hour ($\lambda_d R$) And multiplying it by the number of kilometers traveled in one hour (Q) is possible to calculate the cost per hour of accidents in both cases DRT and conventional bus.

The expression of the cost of accidents per hour is the following:

$$\frac{Z_{Accidents}}{h} = \frac{0,8}{\lambda_d R} * Q \tag{26}$$
6. Results Analysis

6.1. Diseconomies of Scale

As previously stated, the total cost of the service per person was divided into three parts: one that increased with demand growth, one that decreased with demand growth, and one that remained constant regardless of the demand behavior. Various scenarios were assessed to understand how diseconomies differed with wider service regions, changing vehicle capacities, a restricted fleet, different demand, and variable operating costs, among other things.

6.1.1. Headway Dependent Model

In this case, a series of similar conditions were defined for conventional and DRT service, as follows: l=3,5 km, L=5 km, w=1 km, v=25 km/h, a=0,85 m/s², and a fixed headway of 0,25 h or 15 minutes. The only three conditions that varied from one model to another are the cost per distance (conventional: 1,51 €/veh-km and DRT: 0,81 €/veh-km), cost per time (conventional: 52,2 €/veh-km and DRT: 47,44 €/veh-km) and vehicle capacity (conventional: 77 and DRT: 22). In this way, a conservative position is taken when analyzing the prices of the DRT operation, which are considered to be lower than those of a conventional service. The results obtained for both cases are as follows:

![Figure 3. Conventional Line- Diseconomies of Scale with H constant](image)

![Figure 4. DRT Line- Diseconomies of Scale with H constant](image)
The first thing to note in Figure 3 and Figure 4 is the total cost per passenger (blue line, left axis). For the case of a conventional bus, the total cost per passenger decreases as demand increases. The opposite happens for the DRT line, where there is an exceedingly high total cost increase after a demand of approximately 11 pax/km$^2$. To understand this increase compared to conventional line, it is necessary to understand the economies (red line, right axis) and diseconomies of scale (green line, left axis) since the constant cost (yellow line, left axis) has relatively the same value for conventional as for DRT systems. The economies of scale, it can be seen that in the case of a conventional bus are higher at the beginning (with low demands they exceed the constant price) compared to those of DRT, but ultimately, with higher demands both curves have remarkably similar values and decreasing behaviors that tend to zero.

From the foregoing, it is clear that the difference in total price per person is attributable to scale inefficiencies. The DRT, on the other hand, appears to be considerably more expedited, going from dwindling numbers of 1 €/pax to seven times this price for a demand ten times bigger (50 pax). For a demand of 50 passengers, the diseconomies for a regular bus are not even 2 €/pax.

In this case the model allows the fleet to grow freely as demand increases (purple line and right axis of the graphs). However, there is a much faster growth in the case of DRT service compared to conventional service. In the second case there are only two fleet increases from 2 to 3 vehicles and then to 4 vehicles with the highest demand since the capacity is higher and the Q is lower which means that the distance traveled or to be traveled is less. On the other hand, the DRT increases its fleet with each increase in demand.

For this comparison there is an especially important note to take into consideration. When doing the division of the components of "M" we are obviating a “rounding up” that the objective function does when the value of the fleet number has a number with decimals. Therefore, by not taking rounded up and exact fleet values, the total costs are lower than the actual ones calculated by the objective function. This is a limitation of this analysis, and it is recommended for future work to consider that for the fleet in particular. More in-depth studies should be done where this rounding up is taken into consideration and that the cost graphs can reflect these cost increases due to fleet acquisition.
6.1.2. Fleet Dependent Model

In order to set a fleet to certain DRT operating conditions it was necessary to have an overview of a minimum fleet required depending on the size of the demand. The expression in Equation 15 was taken as a reference to define a minimum number of fleet necessary to operate in a given area. This was done by setting the denominator of this equation greater than zero, moreover, with given initial conditions the minimum M can be defined for a pre-established demand. The speed used for this analysis was 25 km/h, the acceleration was 0.85 m/s\(^2\) and the boarding and alighting time was 5 seconds, the w, and L and therefore R were varied for each case.

![Figure 5. Amount of Fleet Required for a Certain Demand in Different Service Areas](image)

The first thing to notice in this graph is that the configuration that tolerates the most demand with a small fleet is the 1 km\(^2\) square area, which is the smallest area under study. As the service area increases the number of vehicles required for smaller demands increases. The curve with the steepest slope and therefore the one that requires the most vehicles with the smallest demand is the 9 km\(^2\) square area, i.e. same L and w of 3 km. The extended version of this graph can be consulted in section 8.3 where more user demands are taken into account.

This graph was used to acquire a better understanding of how to pick a fixed fleet for a specific area and how to alter demand up to a certain point to understand its diseconomies. It would
have been more difficult to find a fleet and the level of demand that would cause the system to collapse or become more expensive without this pre-analysis.

For example, for the following case, it was considered reasonable to study an area of 5 km$^2$ rectangular (L=5 km, w=1 km) with the characteristics defined above. By consulting Figure 5, it can be noted that in this particular case and with a fleet of about 5 vehicles could be analyzed up to 25 pax/km$^2$-h which represents 125 passengers per hour before the system collapses. The following results for the economies and diseconomies of scale were obtained:

In both graphs above, the blue line shows the total costs (Z$_a$+Z$_u$) of the DRT service. In the left graph the total cost is broken down into the other three lines, the diseconomy of scale (in green), the economy of scale (in red) and the constant costs (in yellow). The latter, is a relatively low cost compared to the total costs for any demand, representing always less than 10% of the total cost. Moreover, as can be seen the diseconomies of scale has a fairly low value with small demand but starts to grow rapidly after about 15 pax/km$^2$-h.

In the case of the graph on the right, the total costs are divided only into user cost (in green) and operator cost (in red). The user cost increases as demand increases, while the operator cost decreases as demand increases. By comparing both graphs it is possible to see the similarity between the diseconomies - user cost and the economies - operator cost. From this it can be
derived that the diseconomies of scale are governed mainly by the user cost, which means that when demand increases, the agent that is really affected is the user himself and not the service operators. In order to understand where this rise in user costs and the fall in operator costs comes from, the following graph was made:

According to Equation 16, the number of kilometers driven by the fleet in a given unit of time (Q), decreases as the demand increases when the fleet is fixed. This is because as the number of users increases, the vehicle is forced to make smaller tours in order to be able to pick up more people. That is, if there is little demand dispersed throughout the area, the vehicle can go from one point to another in the service area, making the route longer. But with more densified demand, the bus will seek to pick up additional people on shorter routes, making the mileage less. This will represent a savings of cost for the operators.

On the other hand, by having to pick up more people on the way with the same amount of fleet, travel and waiting times per passenger increase (yellow and blue line). This explains more graphically how, when operating a DRT service with a fixed fleet, the increase in the total cost is borne solely by the passengers. Operators experience only an economy of scale in their costs,
and as demand increases, the service only lose attractiveness to users because the time it takes to get from origin to destination becomes longer.

As seen in Figure 6 there is a point where the curve of diseconomies meets the curve of economies and this represents the point where the diseconomies have already surpassed the economies and the system is more expensive with the increase in demand. However, this does not represent the point where the diseconomies begin to gain ground over the economies, to find this point it is necessary to calculate the derivative of both curve and find the intersection. This point would represent the exact demand where the system starts to become more expensive per passenger and the system ceases to be “total cost profitable”. With this demand it is also possible to calculate the optimal occupancy of the vehicles, which means that it is possible to find the number of people that the DRT apps should allow per vehicle in order not to exceed the travel time of the passengers and increase the total price of the service. In other words, this will provide the optimal occupancy.

Next, the intersections of the derivatives and the economies and diseconomies are presented in order to show how both points vary depending on the service area. The initial conditions are the same as explained at the beginning of this chapter with a W of 1 km for all cases and a L varying between 10, 5 and 2,5 km with a fix fleet of 5 vehicles.

The previous graphs show, first and foremost, that as the service area shrinks, the system supports more users, which is the same pattern outlined in Figure 5. In addition, when the
service area grows, the distance between the derivatives and the intersection of the economics and scale diseconomies becomes closer. This indicates that while serving a broader area, the transition from economically viable to non-economically viable occurs more quickly. This is also seen on the right axis of the graphs where the slope of the economies and diseconomies increases as the service area increases, indicating that the curve increases in convexity.

Now, by taking the intersection between the derivatives it is possible to calculate the occupancies for the critical demands for each service area. This will result in the maximum occupancy that should be taken per vehicle at critical demand in order not to increase user costs so much that the system is no longer profitable. The following table shows the occupancies for a DRT and a conventional service at critical demands. This exercise was repeated for the same capacity of 22 passengers but for different fleets of 5, 10 and 15 vehicles.

Table 10. Optimal Occupancies for DRT and Conventional Service

<table>
<thead>
<tr>
<th>Scenario</th>
<th>λd Critical DRT (pax/km^2-h)</th>
<th>λd Critical Conv (pax/km^2-h)</th>
<th>Pax-h DRT</th>
<th>Pax-h Conv</th>
<th>Occupancy (pax) DRT</th>
<th>Occupancy (pax) Conventional Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>M=5 Cap = 22</td>
<td>2,5 km^2</td>
<td>28,5</td>
<td>47,4</td>
<td>72</td>
<td>118,4</td>
<td>3,2</td>
</tr>
<tr>
<td></td>
<td>5 km^2</td>
<td>16</td>
<td>22,4</td>
<td>80</td>
<td>112,1</td>
<td>3,91</td>
</tr>
<tr>
<td></td>
<td>10 km^2</td>
<td>8,2</td>
<td>10,3</td>
<td>82</td>
<td>103,1</td>
<td>4,08</td>
</tr>
<tr>
<td>M=10 Cap = 22</td>
<td>2,5 km^2</td>
<td>48,5</td>
<td>98,0</td>
<td>121,3</td>
<td>244,9</td>
<td>2,81</td>
</tr>
<tr>
<td></td>
<td>5 km^2</td>
<td>28,5</td>
<td>47,4</td>
<td>142,5</td>
<td>236,9</td>
<td>3,73</td>
</tr>
<tr>
<td></td>
<td>10 km^2</td>
<td>16</td>
<td>22,4</td>
<td>160</td>
<td>224,2</td>
<td>4,68</td>
</tr>
<tr>
<td>M=15 Cap = 22</td>
<td>2,5 km^2</td>
<td>65</td>
<td>148,8</td>
<td>162,5</td>
<td>372,0</td>
<td>2,34</td>
</tr>
<tr>
<td></td>
<td>5 km^2</td>
<td>39</td>
<td>72,6</td>
<td>195</td>
<td>363,1</td>
<td>3,16</td>
</tr>
<tr>
<td></td>
<td>10 km^2</td>
<td>23</td>
<td>34,8</td>
<td>230</td>
<td>348,3</td>
<td>4,29</td>
</tr>
</tbody>
</table>

As can be seen in the table above, occupancies are well below vehicle capacity for the DRT services, nevertheless, for the conventional line are higher and in some cases above capacity. This suggests that it is possible to operate DRT services with much smaller vehicles and the opposite for the conventional line. The vicious cycle explained at the beginning of this work explained that as demand for a DRT service increases, costs rise so much that there would be a tendency to switch to a conventional service. The results in the table above justify this principle.
since, as can be seen, the conventional service "tolerates" much more demand than the DRT service.

When the service area is small the occupancy of each vehicle is higher than the capacity. This indicates that at full vehicle load the operator would obtain the lowest cost scenario within the feasible occupancies, but that a larger vehicle size would be better because it would allow the system to achieve configurations with lower cost per trip. In these cases, it is advisable to increase the vehicle capacity. Even in the other scenarios with larger service areas, which are also very close to full occupancy, for passenger comfort reasons, the vehicle capacity can also be increased. These results justify those conventional lines operate with larger vehicles. For the cases studied, a vehicle with a capacity of 22 passengers is very large for a DRT service but very small for a conventional line.

As can be seen in the table above, the tendency as the fleet grows is to increase the critical demand and therefore the occupancy of each vehicle. In order to better visualize the cut-off points between economies and diseconomies with different fleet sizes, the following graph shows the behavior of both curves and how vehicle occupancy increases as demand increases.

Figure 12. Occupancy, Economies and Dis-economies of Scale for M=5,10,15
Figure 12 indicates that as the vehicle fleet grows, the cut-off points are given at greater demands for the same service area (5 km$^2$), implying that more people can be carried with a higher M. Naturally with a fixed fleet, occupancy increases with each increase in demand. The smaller the number of vehicles, the faster the maximum capacity of the vehicle (22 passengers) is reached, this you can see in the right axis of the graph by the dotted lines. Another interesting fact of this cost division (economies-diseconomies and constant costs) is that the diseconomies are practically independent of the service area. That is, the dark blue, orange and dark green lines are very similar for service areas of 10 and 15 km$^2$ (see Annex 8.3.) This indicates that if a DRT bus operator has a fixed fleet, and operates in different areas, its "savings" or economies of scale will always be the same, the only thing that will change will be the diseconomies of scale which increase with the service area (see Annex 8.3.).

To round out this subchapter, we'll look at the original model's tolerance level with a fixed fleet. For this purpose, a hypothetical case of 10 pax/km$^2$-h in an area of 2.5 km$^2$ with a vehicle capacity of 22 passengers was optimized. All other variables were kept constant to the ones of the examples given above. As a result, it was obtained to operate with a fleet of 2 vehicles and a headway of 11 minutes. Then, with the same variables and a fixed fleet, the demand was increased progressively in order to see when the model was no longer “total cost-economically” viable.
As can be seen in the graph above, the "current" demand is 10 pax/km²-h, meaning that the optimization was made for this demand. The point where diseconomies of scale overtake the economies of scale is 15 pax/km²-h and it is at 13 pax/km²-h that the system starts to become more expensive for the users as demand increases. Since it was analyzed an area of 2.5 km², it can be said that in this particular case the model is only tolerating 7 extra passengers, which means that when performing the optimization, the fleet suggested by the model is very low and sensitive to changes in demand. This would cause the DRT system to collapse rapidly if there is a slight increase in the number of users of the system and a bigger fleet would have to be considered to be able to sustain the increase in demand.
The graph above illustrates the derivative of the overall cost of the service to help clarify where the point of 7 more passengers comes from. It can be seen that the cut-off point of this derivative with the X-axis is at 13 pax/km²-h. This is the lowest point of total costs and after this demand the total cost per passenger starts to increase and the derivative starts to be negative. Multiplying the 13 pax/km²-h by the service area (2.5 km²) gives a total of 32 passengers per hour. Since the design demand is 10 pax/km²-h which corresponds to 25 passengers per hour, the difference between the critical point is only 7 passengers per hour.

6.2. Externalities

6.2.1. Test Instance Description

The model used for this chapter was the headway dependent model. This is because, being a model that optimizes according to the total price, once the demand increases sufficiently, the fleet will also increase so as not to increase the user's costs. In this chapter we want to check how this increase in fleet represents an increase in externalities.

The analysis presented below was based on an urban rectangular-shaped area where a corridor varies from L=10, 5 and 2.5 and a constant w = 1 which would be served by an on-demand
service or a traditional service. This is in order to study different service areas and understand how externalities increase or decrease depending on how dispersed the demand is in a particular area.

This corridor would serve a demand varying from 5 to 100 pax/km²-h which would cause the fleet to vary from 1 to 54 vehicles for the DRT model and from 1 to 19 vehicles for the conventional model depending on the area served. The maximum capacity used for the buses was 8 for the 8-passenger van model, 22 for the minibus and 70 for the 15–18 tons conventional bus. Other common parameters used are: $\beta_t = 10 \, \text{€/pax-h}$, $v_w = 2.5 \, \text{km/h}$, $v = 25 \, \text{km/h}$, $l = 3.5 \, \text{km}$, $a = 0.895 \, \text{m/s}^2$, $s_{\text{min}} = 0$, $\theta = 1.1 \, \text{€/trip}$, $\tau' = 5 \, \text{sec}$, $H_{\text{min}} = 3 \, \text{min}$ and $S^2_{\text{H}} = 0.003 \, \text{h}$.

For the calculation of externalities only 2 variables are needed which are $Q$ (veh-km/h) and the operating speed $v_c$. These were extracted from the model and the sum of the 5 externalities explained before (Nox, VOC, CO₂, SO₂ and Accidents) per hour were calculated to obtain a total hourly $Ze$.

### 6.2.2. Same Vehicle Comparison

When an area currently has a bus system in place and wants to replace it with DRT service by changing fleets due to the high expenses of operating with the same vehicles, it's critical to understand the additional costs that this switch would incur for the environment and the community.

For this purpose, in this sub-chapter it was assumed the autobus service area of 5 km² with demand varying from 5 to 50 pax/km²-h and a vehicle of less than 15 tons. Table 11. shows how, using the same vehicle, the grams of pollutants per hour vary for a conventional service (Conv) and a demand responsive transport (DRT).
The first thing to analyze is that the pollutant most present in passenger transport is CO₂, with very considerable differences with respect to NOX, VOC and SO₂. Carbon dioxide is one of the main emissions contributing to global warming. The pollutant least present in passenger transport for this model is SO₂.

Perhaps the most interesting thing to analyze is that using the same vehicle, DRT service accounts for more grams of pollutants than conventional service. This is because the DRT being a door-to-door service increases the mileage to take users to their destination and pick them up where requested. Likewise, the model increases the number of vehicles (M) in service in order...
to avoid penalizing the overall cost with passenger waiting or travel time. This combination of variables results in the DRT service emitting more emissions than a traditional service, which increases as demand grows, as seen in the table above.

Table 12. Comparison of Ze/Revenue - DRT and Conventional

<table>
<thead>
<tr>
<th>Demand λd (pax/km²)</th>
<th>Scenario</th>
<th>Total Cost of Externalities (€/h)</th>
<th>Revenue (€/h)</th>
<th>Ze/Revenue Δ%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,00</td>
<td>Conv</td>
<td>3,42</td>
<td>27,50</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>5,26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10,00</td>
<td>Conv</td>
<td>5,90</td>
<td>55,00</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>10,03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15,00</td>
<td>Conv</td>
<td>8,32</td>
<td>82,50</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>14,91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,00</td>
<td>Conv</td>
<td>8,92</td>
<td>110,00</td>
<td>8%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>17,57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25,00</td>
<td>Conv</td>
<td>9,56</td>
<td>137,50</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>22,46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30,00</td>
<td>Conv</td>
<td>11,70</td>
<td>165,00</td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>27,21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35,00</td>
<td>Conv</td>
<td>12,44</td>
<td>192,50</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>32,27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40,00</td>
<td>Conv</td>
<td>14,46</td>
<td>220,00</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>32,76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45,00</td>
<td>Conv</td>
<td>14,90</td>
<td>247,50</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>39,53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50,00</td>
<td>Conv</td>
<td>15,60</td>
<td>275,00</td>
<td>6%</td>
</tr>
<tr>
<td></td>
<td>DRT</td>
<td>41,91</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Externalities have a very high price at the societal level and are usually blamed on the direct emitters, in this case, the bus operators. The table above summarizes the price of the externalities of emissions plus the accident rate for the DRT and a conventional operation. The fourth column shows the revenue per hour depending on the amount of demand and the fifth
column shows the percentage of all the revenue that would be invested only in covering the externalities. Thus, this percentage is always higher for the DRT but decreases as the demand increases since the revenue increases at a faster rate.

Finally, the last column is the difference between using a DRT and a conventional service, i.e., for a demand of 5 pax/km² using a conventional service instead of a DRT service, the operator would be saving 7% of its revenue only in externalities. This proportion rises as demand rises, implying that by transitioning from a traditional operation to a DRT without replacing the fleet, the amount of money the corporation might save by retaining the previous operation rises in tandem with the number of customers.

From this section it can be concluded that in terms of externalities it really does not make much sense to change the operation from a conventional to a DRT if the same vehicle fleet will be used. Not only does the amount of emissions increase, but the overall cost of externalities (including accidents) is always higher for a door-to-door service than for a conventional one, especially since the number of vehicles in operation (M) and the mileage per hour (Q) are increased.

6.2.3. Different Kind of Vehicle Comparison

Moving from a traditional to a DRT operation with the same vehicle fleet was shown in the preceding section to have no practical advantage in terms of externalities, but instead, causes them to increase dramatically as demand rises. This section will analyze what happens to the cost of externalities (Ze) when the service area and the type of vehicle varied. It's worth repeating that in this scenario, the original optimized model is employed with no fleet or headway constraints in order to illustrate how the externalities change when the fleet and mileage are increased freely as demand grows.
Figure 15. Sensitivity Analysis of Ze to $\lambda d$ and L for DRT (8-van) Conventional (<15 t)

Figure 15. Sensitivity Analysis of Ze to $\lambda d$ and L for DRT (8-van) Conventional (<15 t) shows a comparison between a DRT service operated by an 8-person van plus driver, and a traditional service operated by a "mini" bus of less than 15 tons. The demand was varied from 5 to 100 pax/km\(^2\) and 3 rectangular areas of a bandwidth (w) equal to 1 km and an L of 10 Km, 5 km and 2.5 km are compared. Ze takes into consideration the externalities: NOX, VOC, CO\(_2\), SO\(_2\) and accidents.

As can be seen in this case, the results agree with the work of (Diana, Quadrifoglio, & Ponello, 2007) where the DRT has externalities usually lower than the conventional service. These decreases occur despite the considerable difference in fleet size between one service and the other, which can be seen in the axis on the right. A dotted line indicates the difference in vehicle numbers, which increases as demand grows. The difference in vehicles in a 10 km\(^2\) area at the highest density analyzed (100 pax/km\(^2\)-h) is 35 vehicles, 18 vehicles in a 5 km\(^2\) area, and 9
vehicles in a 2.5 km² region, with the DRT fleet always being bigger. Minor differences in the number of vehicles can be seen at lower demand levels, below 15 pax/km²-h.

The price changes are seen in the left axis, where there are large price fluctuations based on demand; as demand rises, the price of externalities rises for both scenarios (DRT and Conventional). Nevertheless, the conventional service always having bigger cost of externalities. These changes are much greater as the service area increases. For example, a difference of 30 €/h occurs between the two services for a demand of 90 pax/km²·h in a service area of 10 km², whereas the difference is roughly 15 €/h in a service area of 5 km². For a 2.5 km² service area, the difference is much less. With smaller demand, these disparities are less pronounced; for example, around 5 pax/h, the price of externalities does not exceed 10 €/h in any region of operation.

Some of the DRT companies that operated (or currently operate) with vehicles with capacities as low as the one studied in the previous case are Kutsuplus (9 seats), Breng flex (8 seats) and Northern Beaches (5-8 seats). However, nowadays most operators work with larger vehicles such as ArrivaClick (15 seats) and Shottle-Sant Cugat (15 seats) and Flexi Go (17 seats). For this reason, here is analyze the case where the capacity of the vehicles can be up to 17 seats or minibuses of less than 15 tons. This will be compared to a conventional bus route with a fleet of 15–18-ton buses, which is slightly larger than in the previous case.
In Figure 16 it’s possible see how the behavior of the graphs change a lot compared to Figure 15. First of all, now the DRT service always has higher prices for externalities than the conventional service, except in the case where there is a very low demand (2-3 pax/km2-h) and with a smaller service area (2.5 km²). Again, there is always a positive and increasing difference between the number of DRT vehicles and a conventional line, which is shown on the right axis. The price differences in externalities increase as demand increases. For low demand (15 pax/km²-h) the externality price is less than 20 €/h for any type of vehicle and operation.

From the last graph, it can also be concluded that at the level of externality costs, a conventional service (Fleet: 15-18 t) has a very similar behavior to a DRT (Fleet: <15 t), but if it would operate in an area twice as large. This is supported by the light blue-red and orange-green lines in the center of the graph. This basically means that a DRT service pollutes twice as much per person as a conventional service. This is because of the mileage and the number of vehicles in
operation and undoubtedly because of the change in vehicle technology. Now that the vehicles in operation are closer in size, the number of externalities of a DRT increases a lot compared to a van-8 passengers.

6.3. Case Study: Sant Cugat

6.3.1. Test Instance Description

Sant Cugat de Vallès is a municipality that belongs to Vallès Occidental and includes the districts of Mira-Sol, La Floresta and Les Planes. According to the Metropolitan Area of Barcelona (2022) Sant Cugat de Vallès has a population of approximately 88,921 inhabitants and a surface area of 48.2 km². Its main economic activity is services followed by industry and finally by construction with 3.2%. With respect to its land use, in 2011 it was mainly residential, in second place was the industrial zone and in third place parks and equipment.

The first implementation of a DRT in this city was in 2017 where a line connecting the Can Barata area with the train station was set into operation. Thanks to the success of this first line, two years later the Placa del Coll- Les Planes line was implemented which would replace the conventional line L5. In 2019 a pilot test of a bus on demand was launched for the sector of Can Trabal which connected these sectors with the center of Sant Cugat. This line started with 22 stops of which 10 belonged to the old conventional bus line L10. Today, all three sectors are in operation and this year a stop was added to provide accessibility to people in "Sol i Aire".

This on-demand bus was previously evaluated by the MultiDEPART (2021) project which was a project funded by the EIT Urban Mobility between 2021 and 2022 and which aims to model competitive designs of on-demand public transport services. The first phase of this design, which will be analyzed in this chapter, is based on the model of (Estrada, Salanova, Medina-Tapia, & Robusté, 2021) i.e., the optimization of a service with variable headway. These lines are operated by Shotl, the following figures show the three areas that operate today with DRT systems taken directly from the app and the possible routes of each area taken from the MultiDEPART project:
The three lines routes studied were based on the fact that stops could be made at any of the three geofences presented below:

The project MultiDEPART evaluated three on-demand scenarios which were conservative (2.94 pax/km²-h), medium (4.9 pax/km²-h) and critical (8.82 pax/km²-h). For this evaluation it was required to compare the design proposed by this initiative and once we have the number of vehicles for the "optimal" design, submit the same lines to the model with limited fleet. This is in order to understand the tolerance of the service to increases in demand when the fleet is...
limited, and the operator simply cannot acquire more vehicles quickly when demand increases. The input parameters used to optimize the three routes are the following, which once again, were took from (MultiDEPART, 2021)

**Table 13. Input Variables for Sant Cugat DRT System**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Plaça del Coll – L5</th>
<th>Can Trabal – L10</th>
<th>Can Barata</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip length, l (km)</td>
<td>2,38</td>
<td>7,28</td>
<td>6,475</td>
</tr>
<tr>
<td>Route length, L (km)</td>
<td>3,4</td>
<td>10,4</td>
<td>9,25</td>
</tr>
<tr>
<td>Band width, w (km)</td>
<td></td>
<td>0,5</td>
<td></td>
</tr>
<tr>
<td>Demand density per hour, ld (pax/km²-h)</td>
<td>4,9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vehicle capacity, C (pax)</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cruising speed, v (km/h)</td>
<td>20,40</td>
<td>24,96</td>
<td>35,8</td>
</tr>
<tr>
<td>Acceleration rate, a (m/s²)</td>
<td>0,85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boarding and alighting τ’ (s/pax)</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Walking speed, v_w (km/h)</td>
<td>2,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Headway variance, s²_H (h²)</td>
<td>0,003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit distance cost, cd (€/veh-km)</td>
<td>0,7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit temporal cost, ct (€/veh-h)</td>
<td>33,41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fare, q (€/trip) in service a) and b)</td>
<td>1,1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of time, bt (€/pax-h)</td>
<td>10,00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimal headway (min)</td>
<td>3,00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Street spacing, D_s (km)</td>
<td>0,25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the model explained above and for the case of Placa del Coll-Les Planes, it was obtained that for the average and maximum demand evaluated, the amount of fleet needed is 1 vehicle. For the L10 Can Trabal and Can Barata lines, the fleet needed for the average demand was 2 vehicles and the fleet needed for the maximum demand was 3 vehicles. These were the parameters (fleet) used to calculate the tolerance of the system before the costs increase per passenger. It is important to mention that indeed the study highlights that 22-passenger vehicles are too large for the demand in the area and that it could easily operate with 10-passenger vehicles to lower costs. However, for the results shown below, a 22-passenger vehicle is evaluated assuming that demand will increase and that the ultimate objective is to understand to what extent the DRT system can support this growth in passengers with the same fleet.
6.3.2. Economies and Diseconomies of Scale and Externalities for Sant Cugat Lines

As can be seen in the following graph, the cut-off point of the derivatives of the economics and diseconomies has not yet been reached for a demand exceeding the capacity of the vehicle under study (22 pax). This means that although the diseconomies have already exceeded the economies, the system as such is still profitable and the price per passenger has not yet started to increase. For this particular zone, there would be no problem operating with the same fleet at critical demand (8,82 pax) because even for 8,5 passengers the system has not started to become more expensive. For this particular region, the use of a DRT system is considered positive.

![Figure 21. Economies and Diseconomies Placa de Coll Les Planes -L5](image)

In the case of the Can Barata line, as can be seen in the graph below, the cut between the derivatives before reaching the maximum capacity of the vehicle can be seen. The cut-off is at 8,8 pax/km²-h which means that the service starts to show losses when there are 29 passengers or more in an hour requesting service at Can Barata. This does not really leave a very large margin of demand growth over time since the average demand case (4,9 pax/km²-h) was analyzed, which means that from the average demand, the number of users that could be increased is 13 passengers per hour, otherwise the system starts to become expensive for the
passengers since the travel and waiting time increases considerably. However, the occupancy for a demand of 10 pax/km²-h is only 7 passengers, so a 22-passenger capacity vehicle is definitely too large to serve this area. As suggested in MultiDEPART, this reaffirms that it is indeed possible to operate with 10-passenger vehicles to save costs and reduce externalities.

![Figure 22. Economies and Diseconomies Can Barata](image)

Finally, for the L10 Can Trabal line, the result is similar to the previous one. As can be seen in the following graph, the cut-off point is 6.4 pax/km²-h, which only gives a margin of 6 passengers per hour from the average demand for the system to remain profitable.
Therefore, this line as showed in the last graph would not support the critical demand of 8,82 pax/km²-h being the line designed in Sant Cugat that is most at the limit of becoming non-cost viable with a small change in demand.

As mentioned above, the occupancies for optimum demand do not exceed 10 passengers in any of the cases. Therefore, the following is a study of the impact on externalities when operating a vehicle with a capacity of 10 passengers versus a vehicle with a capacity of 22 passengers.
The figure above shows how for all three lines the number of externalities for 22-passenger vehicles are much higher than for 10-passenger vehicles. Although there is always an increase in the price of externalities as demand increases, the 10-passenger vehicles do not exceed €1 euros per hour of operation. On the other hand, the externalities costs per hour for 22-passenger vehicles range from €2,5 to almost €6 euros per hour of operation. These differences represent almost double the costs of Nox emissions, VOC, CO₂, SO₂ and accidents for all lines in operation in Sant Cugat.
7. Conclusions and Recommendations

This thesis is mainly a demand responsive transport evaluation of a fleet dependent model that analyzed the challenge of unprofitability and environmental impact with increasing demand. This was made by a study of three aspects of DRT systems: the economies and diseconomies of a headway-dependent model, the economies and diseconomies of a fleet-dependent model and the externalities generated for a fleet with optimal occupancies. The main idea of this work is to understand when and under what conditions the DRT system should be implemented instead of a conventional system considering the operator cost, the user cost, and the environmental cost. This chapter is structured in such a way that the reader can find the conclusions of each of the above-mentioned topics and linkages between them in between.

As proved in the paper (Estrada, Salanova, Medina-Tapia, & Robusté, 2021) this work reaffirms that for this headway dependent model the total cost per passenger for a conventional bus service decreases as demand increases, this behavior is constant. This is contrary to an on-demand bus, where there is a cost reduction for small demands, but at a certain demand (critical demand) the behavior of the total cost changes and there starts to be an increase in cost per passenger, this growth in the total cost per passenger is continuous for any increase in demand. Furthermore, for small passenger demands the total cost per passenger of an on-demand bus is lower than that of a conventional bus, after the DRT point of increase in total cost per passenger, the on-demand bus will always have total costs above that of a conventional bus.

For the headway-dependent model, the increases in fleet with increasing demand are smaller for a conventional line than for a DRT line. For both services, the increase in M only affects the economies of scale and the constant cost, so it can be concluded that the increase in fleet with increasing demand does not negatively affect the agency's cost per passenger as demand keeps increasing. The diseconomy of scale of this model is given exclusively by the in-vehicle travel time which has a constant agent (the travel time along l) for a DRT and a conventional line. Then, for the conventional line the IVTT increases exclusively with the acceleration and deceleration of at each stop and the time taken for boarding and alighting of each passenger at each stop. In the case of the DRT the diseconomy of scale present in the IVTT is given by three terms, the first one is the transversal distance and detours of the route, and like the conventional route, the other two terms are: the acceleration and deceleration at each stop as well as the
boarding and alighting times. The latter two, naturally increase as demand increases because each increase in passengers represents more stops and boarding and alighting times.

Since it was assumed that the conventional bus would have the same number of stops as the number of users (not more than one user will get on/off the vehicle in every stop) and that the same densities are being evaluated for both cases, it can be concluded that the IVTT of an on-demand bus is higher than that of a conventional bus because of the detour distance and extra miles traveled in each trip and not because of the number of accelerations, decelerations, boardings and alightings of each trip. Finally, it is concluded that for the headway-dependent model, the diseconomy of scale is given exclusively by the extra mileage compared to the conventional route. For the DRT system the total cost per passenger increases only because the user cost per passenger increases. The agency cost per passenger in both cases only presents economies of scale and costs independent of demand. In terms of the operator, there will always be economies of scale, even if the number of vehicles in operation increases, because this cost is offset by the decrease in the system's cost per passenger.

Finally, the demand-independent costs of both systems under equal conditions are practically the same. From this it can be concluded that the access time (which is independent of demand), which is present only in the conventional system, does not significantly affect the total system cost per passenger compared to the total cost per passenger of the DRT.

For the fleet-dependent model, a different behavior in the diseconomy of scale can be noticed in comparison to the headway-dependent model. In this case, instead of having a linear behavior as in the previous case, it grows more slowly for low demands and in a more accelerated way for higher demands. From this, is possible to conclude that for a DRT service with a fixed fleet, losses due to diseconomies will be felt more "suddenly" than for a fixed headway model when the demand increases.

Both models are similar in their diseconomies of scale in the sense that these are exclusively represented by the users' cost, and the operators' cost is more represented by the economies of scale. Therefore, it can be concluded, once again, that the operation of a DRT service with a fixed fleet at high demands will not really mean losses for the operator but only for the user.
From the analysis of the headway-dependent model and the fleet-dependent model, it can be concluded that the vicious cycle discussed above for a DRT service is reflected in the total costs. However, it is not reflected in the operator's cost, i.e., an operator will not feel losses for operating a service where demand continues to grow. On the contrary, it will feel losses when users consider that travel time has increased so much that they would prefer a conventional bus service and stop using the DRT service. If users are not given more options, there is a danger that users will switch from using public transport to using their own private vehicles, representing a threat not only to the transport system but also to the welfare of communities.

These increasing user costs and decreasing operator costs occur for varied reasons. The concept of economies of scale lies in the fact that if there is a relatively constant price divided by an increasing number of users, the cost per passenger will decrease. The operator's cost depends on two factors, Q and M. Since M is constant in this model, the only thing left to evaluate is what happens with the number of vehicle-km/h of trips. As seen in the Results chapter, Q also decreases as demand increases, so this explains the accelerated decrease in operator costs per passenger. In the case of user costs, this reflects a diseconomy of scale, which means that although the cost is increasingly divided by a greater number of users, this does not compensate for the growth of the other factor. This is because both the in-vehicle travel time and the waiting time increase as demand increases for a DRT service, so that the cost per passenger always increases as demand increases.

The point of intersection of the derivatives of the economies and diseconomies of scale represents the demand where the price per passenger is the lowest, in other words, the point at which the cost per passenger starts to increase. It can be concluded that by having smaller service areas and the same fixed fleet, this point of intersection between economies and diseconomies of scale occurs at higher demands. This is because the in-vehicle travel time will depend on the amount of demand and the service area. By having to serve more people in a larger area the travel time per person increases, increasing the cost to the user and therefore increasing the diseconomies of scale.

It is recommended when implementing a DRT with a limited fleet to find this critical point and compare it with the average demand with which it is operating. If the demand is higher than this intersection, this means that the system is becoming more expensive with each increase in demand and that it is advisable to serve fewer people or increase the vehicle fleet, operating a
DRT service represents a tradeoff between the amount of fleet in operation and the number of users it can serve.

Optimal occupancy increases as the area of operation increases. This means that the tradeoff discussed in the previous paragraph becomes more permissive as the service area increases. However, the increase in fleet does not have a significant impact on the optimal occupancy, which means that if demand increases, this forces the system to increase the fleet to keep remarkably similar occupancy levels. The increase in fleet does not mean an increase in occupancy, so if the number of users increases, there would be no positive repercussion if the operator were to increase the capacity of the vehicles, given that the optimal occupancy is constant for any number of vehicles operating.

The original model's allowability for demand increase with the same fleet depends on the size of the service area and the amount of fixed fleet it has, with more fleet there is more allowability. However, for the hypothetical case and the three lines of the case study there was not much margin of increase in demand per hour. From this it can be concluded that in case of using the original optimization model, demand projections should be made to know how much growth is expected in the service and if it is feasible to buy the necessary vehicles to be able to service this growing demand or induce demand the service will create.

The externalities produced by bus transport increase as the vehicle capacity increases, the service area increases and therefore, as the demand increases as well. By having exactly the same conditions and optimizing the original headway dependent model, DRT service pollutes more than conventional service since the mileage and number of vehicles needed to operate increases.

As seen in the results analysis, a vehicle with a capacity of 22 passengers is extremely limited for a conventional line. However, it is an extremely large vehicle for DRT service since this type of service requires low occupancy to ensure that passenger travel times do not increase and ridership costs (and diseconomies) do not exceed operator costs. It is recommended to analyze the adequate capacity for both DRT and conventional service when there is a limited fleet before implementation. If not, there is a risk of underutilized vehicles and excessively amount of externalities generated by the fleet. The methodology studied in this thesis is a good
approach to estimate the right vehicle sized that should be operated in certain area for a DRT and conventional lines.

Since the optimal occupancies for DRT service are relatively low, it is recommended to operate DRT services with small vehicle capacities to reduce operating costs and externalities generated by the system. For the cases studied in Sant Cugat, a bus with a capacity of 10 passengers would be sufficient to operate the three lines optimally without incurring extra costs for the users/operator.

One aspect not sufficiently explored in this study was the social welfare of DRT systems. As seen in section 2.3, to know how much social well a transport service does to the community, it is not only necessary to know the total cost for the operators and the externalities, but also the consumer surplus generated by using this service instead of a conventional one. It is recommended for future work to generate a model that expresses the increase or decrease in demand as the price of the service varies. This in order to be able to calculate the consumer surplus depending on particular fare cases and to make a more complete estimate of the social welfare of a DRT service and a conventional bus service. This will allow a decision to be made with a more robust justification of whether to move to this service or not.

The model of Estrada et al (2021) used for this thesis assumes that the demand is constant not throughout the whole service area and time. Therefore, this thesis does not considered fluctuation during the day (since it is analyzed on an hourly basis) for any of the models under study. It is recommended that for future work, an optimization of this factor be done since, as seen in the subchapter 2.3, DRT services may fail because there are exceedingly long periods during the day where no service is provided. This model and the one created in this thesis do not consider this factor and bases the success of failure of the system only by studying one hour of operation.

A limitation of the original model, and therefore of the fleet-dependent model, is that both always consider an hourly constant H (headway) for the DRT service. Practically speaking, this does not really sound plausible because the whole concept of an on-demand bus is that it only runs when a new passenger boards. Having a constant headway during an hour of operation is a conceptual simplification of the operation of an on-demand bus in order to make the modeling comparable to that of a conventional bus. For future work, a model could be made which
represents the reality of an on-demand bus where there is not a fixed headway per hour but several random departures with different time gaps between each departure. This would naturally represent more of what happens with this service, however, it is understood that it represents a noticeably big challenge due to the complexity of the randomness of the demand and the operation times.

A flat rate does not reflect the service provided by a DRT system. Especially because it provides a door-to-door service like a cab, but in the case of Barcelona, for example, it is charged like a normal bus service where there is access time. Although no fares were analyzed in this work, as a reflection it is recommended to consider flexible fares that depend on the mileage traveled. This will not only benefit the operator in most cases but will also reflect in the price the negative externalities generated by the trip.

As a general conclusion about DRT services, it can be said that they are very susceptible to small variations in demand. It is an important challenge that transport service operators take on when operating a DRT since, in spite of serving more coverage, the time on the vehicle can increase considerably, thus deteriorating the level of service. The main limitation of these services at the user level is the reliability of the service, where it is exceedingly difficult to guarantee the user an arriving time when there is a constantly changing demand.

As a last recommendation for future studies, the following proposal is presented. To use the maximum occupancy for DRT vehicles proposed in this work and the methodology used to arrive at it in order to generate a response model of maximum user travel times. This in order to be able to communicate to the users in advance the estimated time of duration of their trip and thus increase the reliability of the service. In this way, the cost per IVTT is perceived by the user taking into consideration a new concept of non-fixed hourly headway and non-uniform demand in the service area. This travel time can be linked to the cost of the service and adjust this model to a variable fare depending on the estimated travel time.
Bibliography


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8. Annexes

8.1. Thesis Placement

8.1.1. Name of workplace, description, and projects

CARNET is a research center which was funded by SEAT S.A., Volkswagen Group Innovation, and the Universitat Politècnica de Catalunya - Barcelona Tech (UPC). CARNET is a center that brings together academic and industrial partners on the topics of sustainable urban mobility and innovation. This company is based in Barcelona but has made a name for itself through partnerships with a diverse group of organizations in several European cities, including local authorities, corporate and governmental companies, start-ups, and academic institutions.

Among the many fields that CARNET works on are transport sustainability, new mobility concepts, micromobility, multimodality, MaaS, last mile logistics, demand responsive transport, transport-oriented development, acceptability of citizen of new mobility solutions, pricing technics etc. CARNET focuses on innovative urban transportation solutions that bridge the gap between academic research and industrial innovation.

Among the most recent projects highlighted by CARNET are: SmartHubs (a project that strengthens and complements existing mobility hubs and together with AMB incorporates Bicibox to ensure bicycle safety), Raptor EIT (a partnership with Tel Aviv and CityZone to address traffic congestion), LogiSmil (A work co-funded by EIT Urban Mobility to test autonomous urban delivery robots, ADD), ECOSWAP (a project that encourages the swappable battery ecosystem in order to accelerate the recharging of clean energy motorcycles) and MultiDEPART (a project with three tools to help public transport operators implement DRT services), among many others.

8.1.2. Duration of the position, and work level

My position in CARNET took place from 01/03 to 13/07 of the year 2022. Period in which I conceptualized and carried out my thesis work. The level of work involved in this document was high and demanding. Thanks to CARNET and UPC and due to space limitations, me and my EIT colleagues were provided with a working room close to the CARNET office facilities.
in order to be able to draft the thesis there. The work took place in this office during the 5 days of the week of the period mentioned above excluding 2 days of Holy Week (April 13 and 14). One day per week, I visited the CARNET office where I had the opportunity to discuss with my colleagues about problems that arose in the thesis and the correct way of leading it. I was always guided in the right way thanks to their great experience and knowledge in the field.

8.1.3. Relationship of the outplacement to the thesis

The general idea of this thesis, and the approach to the vicious circle of DRT was conceptualized together with my coordinator at CARNET, Laia Pages, my thesis director, Hugo Badia and the UPC master professor Miquel Estrada. In which we considered the problem that affects the operators of this technology and its profitability when facing high demands. Especially Laia has vast experience with this technology since her doctoral thesis was on this topic.

CARNET is one of the main partners of MultiDEPART project, which as mentioned earlier in the document bases its first phase on the (Estrada, Salanova, Medina-Tapia, & Robusté, 2021) model. The same model was used for the elaboration of this thesis. In this way, using the results of the first phase of the MultiDEPART project and the closeness I had to it due to this placement, I was able to enrich the results and conclusions obtained.

Moreover, since I was doing research on Demand Responsive Transport services and models, during my time at CARNET I had the opportunity to have a more holistic approach of this technology by taking part in the writing of a proposal for EIT Urban Mobility related to DRT last mile logistics called DREAMS. It aims to use existing DRT technologies for public transport and incorporate them into the last mile delivery, in order to speed up the delivery of goods and reduce the negative impact of logistics vehicles and operation within urban cities.

8.1.4. Name and title of mentor during outplacement

Ph.D. Laia Pages Giralt: Executive and research manager of CARNET
laia.pages@carnetbarcelona.com
8.2. Agency and User Cost Comparisons for Headway Dependent Model

The following graphs show a comparison of the different agents within the headway-dependent model. The first graph compares how agency costs decrease with demand and how, conversely, IVTT increases with demand growth. The second graph shows the behavior of the agency cost per passenger and the user cost per passenger. In which they have similar behaviors to the economies and diseconomies explained above and where the DRT has a much more aggressive growth in the user cost than the conventional service.

*Figure 25. Agency and IVTT Comparison for a Conv and DRT Line in Same Conditions*
8.3. Diseconomies of Scale for Different Areas of Service for the Fleet Dependent Model

The following graphs are a continuation and expansion of the economics and diseconomies results for the demand-dependent model. The same results are presented with the exception that now more study areas are added for each amount of fixed fleet (2.5, 5 and 10 km²).
Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit

Mariana Montero Vega

Figure 28. Close up to the Diseconomies of Scale for Various R and M and their Grow %

Figure 29. Derivate of the Economies and Diseconomies of Scale for Different Service Area and M=5
Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit
Mariana Montero Vega

Figure 30. Amount of Fleet Required for a Certain Demand in Different Service Areas (Extended Version)

The following graph shows the economics and diseconomies for the three Sant Cigat lines studied and how they evolve with growth in demand. It can be seen that all have shortfalls before 10 passengers per square kilometer per hour.

Figure 31. Economies and Diseconomies of Scale for Sant Cugat DRT Lines
8.4. Externalities Comparison.

The following is an extension of the tables presented above on externalities results. You will find the graphs by service area and by vehicle type and the price per pollutant or externalities as well as the total price represented by a blue line, which is the sum of all externalities. The last three graphs are a comparison of total externality costs and how they vary as the demand being served varies. Different vehicle sizes of vehicles are compared and a single area of service of 10 km² is analyzed in these graphs. The rest of the areas was described in the section 6.2.3.

![Externalities for a 8 Passenger Van in a 10km² Area of Service](image)

*Figure 32. Externalities for a 8 Passenger Van in a 10km² Area of Service*
Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit
Mariana Montero Vega

Figure 33. Externalities for a 8 Passenger Van in a 5 km² Area of Service

Figure 34. Externalities for a 8 Passenger Van in a 2.5 km² Area of Service
Figure 35. Total Cost for DRT and Conventional Lines in a 10km² Area of Service

Figure 36. Total Cost for DRT Lines in a 10km² Area of Different Fleet
Comparison of Operating Strategies for Bus Services: from Fixed Routes to Demand Responsive Transit
Mariana Montero Vega

Figure 37. Total Cost for DRT and Conventional Lines in a 10km² Area of Different Fleet

Figure 38. Total Cost for DRT and Conventional Lines in a 10km² Area of Same Fleet