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Strong Law for the Depth of Circuits

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Report LSI–96–18–R

UPC
Facultat d'Informàtica
de Barcelona - Biblioteca
15 MAR. 1996

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Abstract

This paper studies uniformly distributed size n circuits (labelled acyclic graphs) consisting of binary gates and a constant number of inputs. We show that their depth is asymptotically $2e \ln n$ in probability. To this end, we use coupling to bound circuit depth from above and below by those of two alternative processes, and then establishing the result by embedding them in suitable continuous time branching processes.

1 The Problem

A circuit is a directed acyclic graph whose nodes are labelled from a given set: the nodes with in-degree 0 are called inputs and all the remaining nodes are gates. Circuits with binary gates (unbounded fanout) are a widely used model for parallel computations by giving appropriate labels to nodes: for example, if inputs are labelled by Boolean variables and gates by binary Boolean functions, then Boolean circuits are obtained (see for e.g. [BDG95, page 106]). The size of a circuit is the number of its nodes, corresponding to the cost of the structure, while its depth is the length of a longest path, corresponding to its computation time. The present paper looks circuits as labelled directed acyclic graphs of in-degree 2 (not necessarily connected). Alternatively, a size n circuit with a inputs is a description $(\alpha_1, \ldots, \alpha_a, \alpha_{a+1}, \ldots, \alpha_{a+n})$ where each α_i with $i > a$ is a pair (α_j, α_k) with $j, k < i$. We then distribute them uniformly at random, and show that the depth of size n circuits (with a constant number of inputs) converges to $2e \ln n$ in probability, i.e. for any $\varepsilon > 0$, the probability that it is between $(2e - \varepsilon) \ln n$ and $(2e + \varepsilon) \ln n$ tends to 1 as $n \to \infty$.

Díaz *et al.* [DSS+94] use the following incremental process to generate at random a circuit $C_{a,n}$ beginning with a inputs. Generally, $C_{a,n}$ is a circuit on the set of $n + a$ nodes $1, \ldots, n + a$. $C_{a,1}$ consists simply of a input nodes, labelled by $1, \ldots, a$. To obtain $C_{a,n+1}$ from $C_{a,n}$, the gate $n + 1$ is joined by edges to two nodes chosen from $\{1, \ldots, n\}$ uniformly at random with replacement. In other terms, $C_{a,n}$ is chosen uniformly at random among

*This research was supported by ESPRIT Long Term Research Project 20244-ALCOM-IT.

all $((n+a-1)!/a!)^2$ size n circuits with a inputs. We refer to these circuits as *random recursive circuits*. In [DSS+94], the authors proved that, for any constant a, the depth of $C_{a,n}$ is between $e \ln n$ and $4e \ln n$, in probability. The present paper decides the constant, i.e. the depth of $C_{a,n}$ converges to $2e \ln n$ in probability.

2 Proof Outline, Definitions and Notations

Instead of handling a random recursive circuit $C_n := C_{1,n}$, we consider alternative incremental processes B_n and D_n which are easier to analyze, and such that their depths bound that of C_n from above and below. (We only consider the case $C_{1,n}$ since similar arguments hold also for $C_{a,n}$.) We then prove that their depths converge to $2e \ln n$. Following, we give an alternative notation for circuits and then a more precise sketch of the proof. For a circuit C, $depth(i)$ denotes the depth of the node i , i.e. the length of a longest path from an input to it. Since we concern depth, we may look a size n circuit C as a string of integers $depth(1)depth(2)\cdots depth(n)$, and $C(i)$ denotes the (possibly repeated) value of the ith smallest bit. For example, if $C = 0121$ then $C(1) = 0$, $C(2) = C(3) = 1$ and $C(4) = 2$ (if $i < j$ and $C(i) = C(j)$ then the ith smallest bit precedes the jth smallest bit.) Then C_{n+1} decides its $(n+1)$th bit as $\max\{C_n(i), C_n(j)\} + 1$, where i, j are chosen from $\{1, \ldots, n\}$ uniformly at random with replacement, i.e. according to the distribution $Pr[i = x] = M_{nx} := (2x-1)/n^2$. Thus, C_n is a growing string of integers generated by the following discrete time process. It begins from 0 at time $t = 1$. At time $t = n$, it decides the nth bit according to the distribution (M_{nx}). We say that C_n is an incremental string with transition distribution $M = (M_{nx})$.

In order to establish an upper bound of $max(C_n) := max_i C_n(i)$ (corresponding to the depth of C_n), we propose an alternative incremental string B_n such that $max(B_n)$ (stochastically) dominates $max(C_n)$ i.e. $Pr[max(B_n) \geq i] \geq Pr[max(C_n) \geq i]$ for any i. The transition distribution of B_n is given by L: $L_{nx} = 1$ if $n/2 \leq x \leq n$ and $L_{nx} = 0$ otherwise. So, the $(n+1)$th bit is $B_n(x) + 1$, where x is chosen from the half largest bits of B_n uniformly at random. Clearly, L dominates M, i.e. for any n and i, $\sum_{i \leq x \leq n} M_{nx} \leq \sum_{i \leq x \leq n} L_{nx}$. Therefore, intuitively, the intensity of growth of $max(C_n)$ is smaller than or equal to that of $max(B_n)$ for each discrete time $t = n$. Moreover, since they start from the same state 0, $max(B_n)$ may dominate $max(C_n)$ at any time. This intuition is formally proved due to the existence theorem of a coupling for a pair of general Markov chains (see for e.g. [Lin92, pp 127–131].) We will give, however, an elementary proof of this theorem for incremental strings: first, for the completeness of the paper; secondly, our simple case may illuminate the general theorem (i.e. Strassen's theorem [Str65]) with a short proof; and finally, we actually exhibit an algorithm which inputs a pair of transition distributions and outputs a coupling.

To show a lower bound on $max(C_n)$, we consider the incremental string $D_n^{(k)}$, with a parameter $k = 1, 2, \ldots$, whose transition distribution is the following k-step staircase matrix $N^{(k)}$: $N_{nx}^{(k)} = 2(i+0.5)/kn$ if $in/k < x \leq (i+1)n/k$ (when n is a multiple of k.) Then, for any k, M dominates $N^{(k)}$, hence $max(C_n)$ dominates $max(D_n^{(k)})$.

We prove, for any $\varepsilon > 0$, with probability one, that $(2e+\varepsilon)\ln n$ and $(2e-\varepsilon)\ln n$ are an upper and lower bound for B_n and $D_n^{(k)}$ (for sufficiently large k), respectively. These bounds imply that both $max(C_n) \to 2e\ln n$ and $max(B_n) \to 2e\ln n$ in probability. To show this, we follow Pittel's proof for the height of random recursive trees. A size n recursive tree has n nodes labelled by $1,\ldots,n$. The root is 1, and the labels increase along every path leading away from 1. Several measures of (uniformly distributed) random recursive trees are well understood, including depth of nodes [Szy90], [Mah91] and the height [Pit94], which we use in our proof. Pittel proves the strong law for $max(R_n)$: $max(R_n) \to e\ln n$ in probability. He considers a continuous time birth process R_t. The initial ancestor is born at time $t = 0$, producing its children according to a Poisson process with rate 1 (a pure birth process with independent $Exp(1)$ inter-birth times); each child mimics its parents, producing their children independently according to the (time shifted) Poisson process. Consider the chronological sequence of birth time $t_1 < t_2 < \cdots$ so that t_n is the moment when the nth child is born ($t_1 := 0$). A remarkable thing is that, for each n, R_{t_n} has the same distribution as R_n, and therefore Kingman's theorem [Kin75] is applicable to R_n. As a string of depths, R_t grows according to the following continuous time process: it begins from 0 at time $t = 0$; Generally, R_{t_n} is a string of length n, and $R_t = R_{t_n}$ while $t_n \leq t < t_{n+1}$. During this interval, each bit of R_t produces its child according to a Poisson process with rate $I_{nx} := 1$ and the $(n+1)$th bit is $R_{t_n}(x)+1$ if the xth smallest bit of R_{t_n} produces the $(n+1)$th child. We say that R_t is a (continuous time) incremental string with birth rate $I = (I_{nx})$. Similarly, B_n and $D_n^{(k)}$ are embedded in continuous time processes as incremental strings B_t and $D_t^{(k)}$ whose birth rates are given by J and $K^{(k)}$, respectively: $J_{nx} = 1$ if $x \geq n/2$ and 0 otherwise; $K_{nx}^{(k)} = (i+0.5)/(k-0.5)$ if $in/m < x \leq (i+1)n/k$.

Neither B_t nor $D_t^{(k)}$ is a birth process (as opposed to R_t), since their birth rates depend on both population and position (n and x). We have, however, a natural coupling of each of them with $R_{t'}$ for some $t' > t$. More generally, let X_t and Y_t be two incremental strings with birth rates P and Q, respectively, such that $P_{nx} \leq Q_{nx}$ for all n, x. Then we observe that X_t and a substring of Y_t, referred to as $(X,Y)_t$, are equally distributed by the following process. We produce children according to birth rate Q of Y_t, but eliminate some of them. The initial ancestor is counted. At the moment when $(n+1)$th child is born, say from the xth smallest bit, we flip the coin and decide, with probability $1 - P_{nx}/Q_{nx}$, to eliminate it (and all of its descendants.) Let t and t' be the moment when the nth child is born in B_t and $(B,R)_t$, respectively. Due to the coupling $(B,R)_t$, we have that $max(B_{t'})$ is dominated by $max(R_{t'})$. Then we apply Pittel's limit theorem for the height of a random recursive tree, which derives $max(R_{t'}) \to 2e\ln n$ in probability. The proof for the lower bound uses the coupling $(D^{(k)}, R)_t$, as well as Szymański's result about the probability distribution of depths of the nodes in a random recursive tree.

3 Domination by Coupling

In this section we prove the dominations for B_n, C_n and $D_n^{(k)}$.

Theorem 1 *$max(B_n)$ dominates $max(C_n)$, and $max(C_n)$ dominates $max(D_n^{(k)})$.*

In fact, this theorem is a corollary of the next proposition.

Proposition 1 *Let X_n, Y_n be two random strings with transition distributions P, Q, respectively. If Q dominates P, then $max(Y_n)$ dominates $max(X_n)$ for each n.*

It is sufficient to construct, for each n, a finite set of triples (p_i, s_i, s_i'), where $p_i \in [0,1]$ and s_i, s_i' are strings of length n, such that

(a) $\sum_i p_i = 1$,

(b) $Pr[X_n = i] = \sum_{\{s_j = i\}} p_j$,

(c) $Pr[Y_n = i] = \sum_{\{s_j' = i\}} p_j$,

(d) $max(s_i) \leq max(s_i')$.

A set of these triples is called a *coupling* of X_n and Y_n with respect to the order given by max. In other terms, a coupling P of X and Y is a distribution on $\{1, \ldots, n\} \times \{1, \ldots, n\}$ such that, for any s, s', the distributions of X and Y are $P(\cdot, s')$ and $P(s, \cdot)$, respectively, and $P(\{(s, s') : max(s) \leq max(s')\}) = 1$. Since the order given by max is too dense to construct a coupling, we define another one which implies the order max. We take $s \preceq s'$ iff s and s' has the same length and $s(i) \leq s'(i)$ for all i. And then, we construct a coupling for this order, referred to as condition (d') $s_i \preceq s_i'$.

Let us first give an elementary proof of the Strassen's theorem for random variables taking a finite number of values.

Lemma 1 *Let X, Y be two random variables, taking values in $\{1, \ldots, n\}$. If Y dominates X, i.e. $Pr[Y \geq i] \geq Pr[X \geq i]$ for any i, then there is a coupling of X and Y with respect to the standard order of integers.*

Proof. Let $P = \emptyset$. We increase P by putting triples (p_i, k_i, k_i') in it, where $p_i \in [0,1]$ and $k_i, k_i' \in \{1, \ldots, n\}$, until obtaining a desired coupling. First, we put all the following triples: $(Pr[X = i], i, i)$ if $Pr[X = i] \leq Pr[Y = i]$ and $(Pr[Y = i], i, i)$ otherwise. Let

$$rest(P, j) = \begin{cases} Pr[X = j] - Pr[Y = j] - \sum_{k_i = j} p_i & \text{if } Pr[X = j] \geq Pr[Y = j] \\ \\ Pr[X = j] - Pr[Y = j] + \sum_{k_i' = j} p_i & \text{otherwise} \end{cases}$$

If $rest(P, i) = 0$ we say that P fills i. The construction finishes when P fills all $i = 1, \ldots, n$. ¿From the construction of P,

(e) $rest(P, j) \geq 0$ if $Pr[X = j] \geq Pr[Y = j]$,

(f) $rest(P, j) \leq 0$ if $Pr[X = j] < Pr[Y = j]$.

Since Y dominates X,

(g) $\sum_{j\leq i} rest(P,j) \ \geq\ 0$ for any i,

(h) $\sum_{j\geq i} rest(P,j) \ \leq\ 0$ for any i.

Newly added triples must not break any of these conditions. Let i be such that P has not filled it yet. Without loss of generality, we may assume that P fills all $j = 1,\ldots,i-1$. Then P is going to feed more triples to fill also i. Note that $rest(P,j) = 0$ for $j < i$ and $rest(P,i) \neq 0$. We see that $rest(P,i) > 0$ by (g), and also

$$rest(P,i) + \sum_{\{j>i,rest(P,j)<0\}} rest(P,j) \ \leq\ 0$$

by (h). Thus, there are indices $i < k_1 < k_2 < \cdots < k_l$ such that

$$rest(P,k_j) \ \leq\ 0 \quad \text{for all } k_j,$$

$$rest(P,i) + \textstyle\sum_{j=1}^{l-1} rest(P,k_j) > 0, \ \text{ and}$$

$$a := rest(P,i) + \textstyle\sum_{j=1}^{l} rest(P,k_j) \ \leq\ 0.$$

Hence, without breaking any of (e)-(h), we add the following triples, which finally fill i:

$$(-rest(P,k_j),i,k_j), \quad j = 1,\ldots,l-1 \quad \text{and} \quad (-a,i,k_l).$$

In this way, all $i = 1,\ldots,n$ are filled with at most $2n$ triples. □

We apply Lemma 1 for the transition distributions on $\{1,\ldots,n\}$ given by the nth rows of P and Q of Proposition 3, obtaining their coupling $\left\{(p_i^{(n)}, k_i^{(n)}, k_i'^{(n)})\right\}$.

We now finish the proof of Proposition 3 by constructing a coupling of X_n and Y_n for $\preceq$ that satisfies (a)-(c) and (d'). The construction is by induction on n. For $n = 0$, the coupling is $\{(1,0,0)\}$, because both X and Y start from the same state 0. Suppose that we have a coupling $\{(p_i,s_i,s_i')\}$ for X_{n-1} and Y_{n-1}. Since $s_i \preceq s_i'$ and $k_j^{(n)} \leq k_j'^{(n)}$, we see

$$s_{ij} := s_i s_i(k_j^{(n)}) \preceq s_{ij}' := s_i' s_i'(k_j'^{(n)})$$

for any i,j. Therefore, the coupling of X_n and Y_n is the set of triples: $(p_i p_j^{(n)}, s_{ij}, s_{ij}')$ for all i,j. Note that the size of our coupling of X_n and Y_n is at most $2^n n!$. □

4 The depth of circuits

In this section we establish a strong law for the depth of circuits.

Theorem 2 *The depth of random recursive circuits satisfies*

$$\frac{max(C_n)}{\ln n} \ \to\ 2e \quad \textit{in probability.}$$

We establish this by proving an upper and lower bound of $max(B_n)$ and $max(D_n^{(k)})$ given by Lemma 2 and Lemma 3, respectively.

4.1 Upper Bound

Lemma 2 *For any $\varepsilon > 0$,*

$$max(B_n) \leq (2e + \varepsilon) \ln n \quad \textit{with probability one.}$$

Proof. We work on continuous time branching processes R_t and $(B, R)_t$. Recall that t'_n is the moments when nth child is born in $(B, R)_t$. We know that $max(R_{t'})$ dominates $max(B_{t'})$, so it suffices to prove the upper bound for $H_n := max(R_{t'_n})$.

We first compute t'_n. The birth rate of $(B, R)_t$ is given by J. Therefore, its transition rate at any time when the length of a string is n equals

$$\sum_{i=1}^{n} J_{ni} = n/2.$$

Hence, by closely following Pittel's proof, we derive

$$\frac{t'_n}{\ln n} \longrightarrow 2 \text{ in probability.} \tag{1}$$

Now we compute H_n. Let $T(k)$ be the moment when the first member of the kth generation is born. Pittel proves that

$$\frac{T(k)}{k} \longrightarrow e \text{ in probability.} \tag{2}$$

by using Kingman's theorem. Since

$$T(H_n) \leq t'_n \leq T(H_n + 1),$$

dividing by H_n and letting $n \to \infty$ implies, due to (2),

$$\frac{H_n}{t'_n} \longrightarrow e \text{ in probability.}$$

Therefore, combining with (1) we conclude

$$\frac{H_n}{\ln n} \longrightarrow 2e \text{ in probability.}$$

□

4.2 Lower Bound

Lemma 3 *For any $\varepsilon > 0$ there is an integer k such that*

$$max(D_n^{(k)}) \geq (2e - \varepsilon) \ln n \quad \textit{with probability one.}$$

Fix k. We use parameters δ, γ and l, where the former two ones are sufficiently small positive constants, and the last is an appropriately fixed integer specified in the proof. We modify $D_n^{(k)}$ in such a way that it simulates R_n until the $n^{2\delta}$th child is born, and after that moment it simulates $D_n^{(k)}$. Note that the modified process is dominated by the original one in the sense of Proposition 1. Let t_n and t'_n be the moments when nth child is born in R_t and $(D^{(k)}, R)_t$, respectively. We recall the stochastical process $(D^{(k)}, R)_t$: the process produces children as R_t, but eliminating some of them. All the children from the first up to the $n^{2\delta}$th are counted. At the moment when n'th child, $n' > n^{2\delta}$ is born, say from the xth smallest bit, we flip the coin and decide, with probability $1 - N_{n'x}^{(k)}$ to eliminate it. Thus, for any n', the n'th child born from the n'/k largest bits is always counted (since $N_{n'x}^{(k)} = 1$ for $x \geq n'/k$.) The effect of this elimination is so small that the growth of $max((D^{(k)}, R)_t)$ may be the same with that of $max(R_t)$. This intuition is formally proved in each of l equally divisions of time interval $[0, t'_n)$. More precisely, for $i = 1, \ldots, l$, let

$$\delta_1 = 0, \delta_2 = \delta, \quad (1-\gamma)\delta_{i+1} = \delta_i, \quad n_i = n^{2\delta_i}, \quad \tilde{t}_i = t_{n_i}$$

and $H_i = max((D^{(k)}, R)_{\tilde{t}_i})$. The next claim shows that $max((D^{(k)}, R)_t)$ grows as $max(R_t)$ during each interval $[\tilde{t}_i, \tilde{t}_{i+1})$.

Claim 1 *Let v_i denote the largest bit in $(D^k, R)_{\tilde{t}_i}$ (thus its value is H_i) and E_i be the following event:*

(i) $H_i \;\geq\; 2(1-\gamma)\delta_i e \ln n$ and

(j) None of the descendants of v_i which are born during $[\tilde{t}_i, \tilde{t}_{i+1})$ is eliminated.

Then, for each $i = 1, \ldots, l$, $Pr[E_i] \;\rightarrow\; 1$ as $n \rightarrow \infty$.

Proof. The proof is by induction on i. $Pr[E_1] = 1$ from definition. We suppose that E_{i-1} holds with probability 1, and prove that also does E_i. We first show that (j) holds with probability one. Suppose that, with non-zero probability, some descendant of v_i is eliminated at some time $t \in [\tilde{t}_i, \tilde{t}_{i+1})$. Since $R_{n_{i+1}}$ and $R_{\tilde{t}_{i+1}}$ have the same distribution, we see that, with non-zero probability, the (n_i/k)th largest bit of $R_{n_{i+1}}$ is greater than or equal to H_i. Therefore, $depth(n_{i+1})$, which is the value of the tail bit of $R_{n_{i+1}}$, must satisfy

$$Pr[depth(n_{i+1}) \;\geq\; 2(1-\gamma)e\delta_i \ln n] = \Omega(\frac{1}{kn_{i+1}^{\gamma}}).$$

This contradicts Fact 1 (see below).

Now we show that (i) also holds with probability one. The descendants of v_i which are born during $[\tilde{t}_i, \tilde{t}_{i+1})$ are distributed as R_t with $t \in [0, \tilde{t}_{i+1} - \tilde{t}_i)$. For recursive trees it has been proven that

$$\frac{t_n}{\ln n} \;\rightarrow\; 1 \text{ in probability,} \qquad (3)$$

so in our terms,
$$\frac{\tilde{t}_{i+1}-\tilde{t}_i}{\ln n} \rightarrow 2(\delta_{i+1}-\delta_i) \text{ in probability.} \qquad (4)$$
As in the upper bound case, combining (2), (4) and
$$\tilde{t}_{i+1}-\tilde{t}_i \leq T(max(R_{\tilde{t}_{i+1}-\tilde{t}_i})+1)$$
derives
$$max(R_{\tilde{t}_{i+1}-\tilde{t}_i}) \geq 2(1-\gamma)(\delta_{i+1}-\delta_i)e\ln n$$
with probability 1. Therefore, we conclude
$$H_{i+1} \geq H_i + 2(1-\gamma)(\delta_{i+1}-\delta_i)e\ln n \geq 2(1-\gamma)\delta_{i+1}e\ln n$$
with probability 1. □

Let D_n be the depth of the nth node of a random recursive tree. Szymański proves that
$$Pr[D_{n+1}=i] = \frac{1}{n!}\left[\begin{matrix} n \\ i \end{matrix}\right]$$
where $\left[\begin{matrix} n \\ i \end{matrix}\right]$ is the ith coefficient of $x(x+1)(x+2)\cdots(x+n-1)$, called the ith Stirling number of the first kind of order n. Since this number is bounded from above by
$$\left[\begin{matrix} n \\ i \end{matrix}\right] \leq \frac{((1+o(1))\ln n)^i}{n\cdot i!},$$
the distribution of D_n satisfies the next fact.

Fact 1 *For any $\varepsilon > 0$ there is some $\delta > 0$ such that*
$$Pr[D_{n+1} \geq (1-\delta)e\ln n] = O(n^{\varepsilon-1})$$

Now we finish the proof of Lemma 3. The transition rate of the process $(D^{(k)}, R)_t$ at any time when a string has length $n \geq n^{2\delta}$ is
$$\sum_{x=1}^{n} K_{nx}^{(k)} = r_k\cdot n,$$
where $r_k := k/2(k-0.5)$, and consequently
$$\frac{t'_n}{\ln n} \rightarrow 2\delta + (1-2\delta)/r_k \text{ in probability.} \qquad (5)$$
We choose l to satisfy $\tilde{t}_{l-1} \leq t'_n < \tilde{t}_l$, and use (3) and (5) to derive
$$(1-\gamma)a(k,\delta) \leq \delta_l \leq a(k,\delta)/(1-\gamma)+\gamma, \qquad (6)$$

where $a(k,\delta) := 2(1-\frac{1}{r_k})\delta + \frac{1}{r_k}$. The right-hand side of (6) gives an upper bound of l, which depends only on k, γ and δ (not on n), so in (*) we can apply Claim 1 for $i = l-1$, obtaining

$$max((D^k, R)_{t'_n}) \geq \max(H_{l-1}) \overset{*}{\geq} 2(1-\gamma)^2\delta_l e \ln n \overset{**}{\geq} (1-\gamma)^3 a(k,\delta) e \ln n,$$

where (**) is by the left-hand side of (6). Since $(1-\gamma)^3 a(k,\delta) \rightarrow 2$ as $k \rightarrow \infty$ and $\gamma, \delta \rightarrow 0$, Lemma 3 follows by taking k sufficiently large and both γ, δ sufficiently small. □

Acknowledgments

We wish to thank Josep Díaz for encouragment and many comments. We also thank Maria Serna and Hosam Mahmoud for helpful discussions.

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