Zero speed sensorless scheme for PMSM under decoupled sliding mode control

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Abstract—Position estimation at low/zero speed for permanent magnet synchronous machines is commonly achieved with the injection of high frequency signals. This paper presents a new sensorless scheme that accurately observe the rotor angle at low speeds, including standstill condition, without injecting high frequency signals. The algorithm is built upon a previous sliding mode current controller and uses the switching functions slopes inaccuracies produced by the saliency effect to determine the rotor position and speed. Thanks to the equivalent control concept and together with a sliding mode based PLL observer defined by a complex switching function, the algorithm provides both rotor position and speed in a compact form. The paper includes the main working principle and defines the operational range of the proposed scheme. Finally, the feasibility of the method is corroborated through experimental results in sensorless configuration.

Index Terms—PMSM, sliding mode control, sensorless control

I. INTRODUCTION

THE permanent magnet synchronous machine (PMSM) is widely used in fields as mobility, electric vehicles, industrial equipments or domestic appliance due to its low inertia, high power density ratio and good efficiency [1]. In all of these applications, the so-called sensorless schemes (operate without position/speed sensor) have gained the interest by the researchers in the last years. Removing the position sensor from the system reduces the weight and cost of the drive and also increases the system reliability. The sensorless operation requires the design of observers or estimators to know the rotor position, since, as a synchronous machine, a high performance machine control needs the mechanical position and speed in the feedback loops.

State of the art of the sensorless techniques for PMSM can be divided in two main families: saliency-based and model-based approaches. In [2] a very complete summary of that techniques and their main characteristics are detailed. Following that work, the classification shown in Figure 1 depicts the current state of the art.

The saliency-based approaches are focused on detecting the variations of the stator inductances with the rotor position (saliency), and allows to achieve sensorless operation at low speeds. The two main methods are based on a high frequency signal injection or a fundamental PWM excitation (FPE). The signal injection methods [3]–[12] employ different patterns of high frequency components (sinusoidal, pulsating,...) which are injected, commonly, through the $\alpha\beta$ voltages. That voltages, and their corresponding generated currents, are used to observe the rotor angle using a simplified high frequency saliency motor model. It should be highlighted that the process to extract the high frequency currents and the subsequent estimation implies complicated signal processing stages, being one of its main drawbacks. Similarly, the FPE technique modifies the space vector modulation adding some signals test, evaluating later the effect in the currents and computing the estimation from that information [13]–[18]. Some of these methods require additional sensors or considerable computing power for processing the information. Besides, all of them have to deal with the signal distortion and in some cases with acoustic noise problems because of the modification in the switching pattern.

The model-based approaches use the state space model of the PMSM to design state observers for the induced electromagnetic force (EMF). The open loop techniques reconstruct the EMF voltages and, using the model equations, estimate the rotor position and speed. These techniques are sensible to the power stage non-linearities or parameter drifts and the observed values can be seriously affected. Other observers are based on closed loop schemes alleviating the inaccuracies produced by unknown dynamics. Examples include Luenberger observers [19], Kalman filters [20], adaptive methods [21]–[23], or sliding mode [24]–[29] and higher-order sliding mode observers [30]. The main drawback of those techniques is that EMF voltages decrease as speed does and disappears at zero speed, degrading and losing the estimation, respectively.

In this paper a position and speed estimation for a PMSM working at low/zero speed is proposed. With respect to the...
aforementioned methods, this algorithm is not excessively complex, does not inject high frequency signals, and does not require of the EMF voltage (not model-based). From the Sliding Mode Control (SMC) algorithm designed in [31] a new approach for sensorless operation arises. The SMC in [31] was designed under the assumption of no saliency in the machine, thus allowing the application of a decoupling technique. However, this simplification, makes the switching function slopes to be lightly coupled, with variations that depend on the system state, including the rotor position as a function of the motor saliency. In [32] a theoretical analysis of how this information can be used to estimate the rotor position was presented, but just some simulation results in open loop operation were provided. Moreover, the estimation was performed using trigonometric properties, which always increases the complexity in the implementation stage. Besides, several issues were not included in [32], such as the development of an speed observer and the experimental evaluation of sensorless schemes. In this work a novel observer algorithm is presented, the so-called Decoupled-Based Sliding Mode PLL (DB-SMPLL), with the following properties:

- The algorithm works at low speed, including standstill condition, neither injecting signal, nor modifying the modulation.
- The rotor position and speed are obtained from the switching function slopes and the discontinuous controls. The observer only needs to know the bus voltage and the saliency level.
- The algorithm includes a sliding mode PLL, which directly provides the observed values of rotor position and speed in a compact form.

Moreover, the paper presents a theoretical analysis of the admissible speed and load torque working range of the algorithm.

The paper is structured as follows: the motor model and the decoupled control scheme are presented in Section II. In Section III, the relation between the rotor position and switching function slopes, and its dependency with the speed and load torque, are analysed. The methodology proposed in order to obtain the position and speed of the motor is developed in Section IV. Finally, the Section V explains the implementation details and contains the experimental results obtained in an off-the-shell PMSM.

II. DECOUPLED SLIDING MODE CURRENT CONTROL

The motor current dynamics of a distributed windings star-connected PMSM can be described by [1]:

\[
L \frac{di}{dt} = -(R I + \frac{dL}{dt}) i - e(\omega_c, \theta_e) + v_{bus} B u \tag{1}
\]

where \( i = [i_a, i_b, i_c]^T \) is the stator currents, \( e(\omega_c, \theta_e) \) are the EMF voltages, \( R \) is the stator resistance, \( v_{bus} \) and \( u = [u_a, u_b, u_c]^T \) are the bus voltage and the discontinuous controls of the voltage source inverter (VSI), respectively. \( \omega_c \) and \( \omega_e \) are the position and rotor speed, respectively. The matrix \( B \) is

\[
B = \frac{1}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \tag{2}
\]

and the inductance matrix is defined as

\[
L = \begin{bmatrix} L_a & L_{ab} & L_{ac} \\ L_{ab} & L_b & L_{bc} \\ L_{ac} & L_{bc} & L_c \end{bmatrix}, \tag{3}
\]

where \( L_a, L_b \) and \( L_c \) are the stator windings self-inductances and \( L_{ab}, L_{bc} \) and \( L_{ac} \) the mutual inductances, given by:

\[
\begin{align*}
L_a &= L_m + \Delta L \cos(x), \\
L_b &= L_m + \Delta L \cos(y), \\
L_c &= L_m + \Delta L \cos(z), \\
L_{ab} &= -L_m/2 + \Delta L \cos(y), \\
L_{bc} &= -L_m/2 + \Delta L \cos(z), \\
L_{ac} &= -L_m/2 + \Delta L \cos(x), \\
\end{align*}
\]

being \( \Delta L \) the saliency factor and \( x = 2\theta_e, y = 2\theta_e - 2\pi/3 \) and \( z = 2\theta_e + 2\pi/3 \). This factor, from a control point of view, is often neglected since \( L_m >> \Delta L \).

The SMC presented in [31] ensured that the motor currents tracked their references by controlling two of them, since in star-connected PMSM \( i_a + i_b + i_c = 0 \) holds. The designed switching surfaces were:

\[
S = [s_a, s_b, s_n]^T = [i_a^* - i_a, i_b^* - i_b, \int (v_n^* - v_n) \, dt]^T, \tag{5}
\]

where the extra degree of freedom was used to regulate the average value of voltage \( v_n \) (voltage at the PMSM star connection) to zero. The values \( i_a^*, i_b^* \) are the current references for the SMC, which are generated from a \( dq \) to \( abc \) transformation (see Figure 2). Similarly, the voltage reference \( v_n^* \) is the desired average value of the voltage \( v_n \), which in general is set to zero. This voltage can be approximated by [1]:

\[
v_n \approx \frac{v_{bus}}{3} [u_a + u_b + u_c]. \tag{6}
\]

The time derivatives of the switching functions in (5) reveals that the system is coupled. In order to decouple the control actions, a decoupling method was used and a new switching function vector \( \sigma \) was defined as:

\[
\sigma = MS, \tag{7}
\]

where \( M \) is the decoupling matrix. In the design presented in [31], the saliency factor was neglected (\( \Delta L = 0 \)) and, hence, \( L_a = L_b = L_c = L_m \). Under this assumption, the time derivatives of (7) yield:

\[
\dot{\sigma} = Mf_1 - v_{bus}MCu \tag{8}
\]

where

\[
C = \frac{1}{3L_m} \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad f_1 = \begin{bmatrix} i_a^* + \frac{R}{L_m} i_a + \frac{e_a}{L_m} \\ i_b^* + \frac{R}{L_m} i_b + \frac{e_b}{L_m} \\ v_n^* \end{bmatrix}
\]

and being \( L_m = 3/2L_m \). Finally, selecting \( M = C^{-1} \), (8) becomes:

\[
\dot{\sigma} = Mf_1 - v_{bus}Iu, \tag{9}
\]

being \( I \) the 3x3 identity matrix. In [31] is proved that SM control laws ensure finite time convergence of \( \sigma \) and \( S \) to zero, thus implying that the motor currents track the reference ones.
The vector considered, the decoupled switching function derivatives become:

\[ \dot{\delta}_i = f_{2i} - v_{bus} u_i + v_{bus} (A_1 \delta_i + A_2 \delta_{2i}) , \]  

(10)

with \( i \in \{a, b, c\} \) and \( A_1 = \frac{2 \Delta L L_m}{3(k_m^2 - \Delta_k^2)} , A_2 = \frac{\Delta L^2}{3(k_m^2 - \Delta_k^2)} \). The vector \( f_2 = [f_{2a}, f_{2b}, f_{2c}]^T \) contains all the terms that do not depend on the control, so it is related with slow system dynamics or with the equivalent control [33]. The functions \( \delta_i \) and \( \delta_{2i} \) are shown in Table I.

As mentioned above, the switching function derivatives given in (10) are the source of information for the DB-SMPLL. More specifically, functions \( \delta_i \) contain the information of the rotor position through its dependency on \( 2\theta_c \). This information can be extracted using the equivalent control concept [33]. Imposing \( \dot{\delta}_i = 0 \) one gets:

\[ f_{2i} = v_{bus} u_{ieq} - v_{bus} (A_1 \delta_{ieq} + A_2 \delta_{2ieq}) , \]  

(11)

where \( u_{ieq} \) are equivalent control signals and \( \delta_{ieq} , \delta_{2ieq} \) arise when \( u_a , u_b , u_c \) are replaced by their corresponding equivalent control values in \( \delta_i , \delta_{2i} \), respectively, as shown in Table I.

Furthermore, replacing (11) in (10) one gets:

\[ \dot{\delta}_i = v_{bus} (u_{ieq} - u_i) + v_{bus} \alpha_i , \]  

(12)

with

\[ \alpha_i = A_1 (\delta_i - \delta_{ieq}) + A_2 (\delta_{2i} - \delta_{2ieq}) . \]  

(13)

From Table I it can be verified that the defined functions \( \alpha_i \) depend on the discontinuous controls and the rotor position \( 2\theta_c \). Moreover, \( \alpha_i \) also relies on the terms \( \delta_{ieq} \) and \( \delta_{2ieq} \). These terms can be calculated from the equivalent controls \( u_{ieq} \). Using (11) it is straightforward to deduce:
where

\[ u_{a_{eq}} = \frac{1}{v_{bus}} \left( f^*_a + \sqrt{3} \Delta L \left( \frac{d i^*_a}{dt} \sin(x) - \frac{d i^*_a}{dt} \sin(y) \right) + 2 \sqrt{3} \omega_c \Delta L \left( i^*_b \cos(x) - i^*_a \cos(y) \right) \right), \]

\[ u_{b_{eq}} = \frac{1}{v_{bus}} \left( f^*_b + \sqrt{3} \Delta L \left( \frac{d i^*_b}{dt} \sin(y) - \frac{d i^*_a}{dt} \sin(z) \right) + 2 \sqrt{3} \omega_c \Delta L \left( i^*_b \cos(y) - i^*_a \cos(z) \right) \right), \]

\[ u_{c_{eq}} = \frac{1}{v_{bus}} \left( f^*_c + \sqrt{3} \Delta L \left( \frac{d i^*_c}{dt} \sin(z) - \frac{d i^*_a}{dt} \sin(x) \right) + 2 \sqrt{3} \omega_c \Delta L \left( i^*_b \cos(z) - i^*_a \cos(x) \right) \right), \]

Finally, the expressions in (13) can be updated by previously determining the terms \( e_{\delta_{eq}} \) and \( \delta_{2_{eq}} \) by replacing the control signals by their equivalent counterparts in Table I:

\[ \alpha_a = A_1 \delta_a + A_2 \delta_{2a} - \Gamma \sin(\theta_e - \phi), \]

\[ \alpha_b = A_1 \delta_b + A_2 \delta_{2b} - \Gamma \sin(\theta_e - \phi - \frac{2\pi}{3}), \]

\[ \alpha_c = A_1 \delta_c + A_2 \delta_{2c} - \Gamma \sin(\theta_e - \phi + \frac{2\pi}{3}), \]

where

\[ \Gamma = \frac{\Delta L}{v_{bus}} \sqrt{2 \omega_c^2 \left( \frac{L_m + \Delta L}{L_m - \Delta L} \right)^2 + R^2 I^2 \left( \frac{L_m + \Delta L}{L_m - \Delta L} \right)^2} \]

and

\[ \varphi = \arctan \left( \frac{RI(\Delta L - L_m)}{\lambda \omega_e (\Delta L + L_m)} \right) \]

have been defined.

Summarizing, each \( \alpha_i \) signal has three terms, namely:

1) The first term, \( A_1 \delta_1 \), depends on \( 2\delta_e \).
2) The second term, \( A_2 \delta_{2i} \), is an offset that depends on the saliency.
3) The third term mainly varies with the motor speed and the load through \( \phi \) and \( \Gamma \).

The third term can be neglected under certain conditions, which are analysed in the following. First of all, it should be remarked that the disturbance amplitude is the same for all \( \alpha_i \) in each control combination. Notice that \( \Gamma \) depends on the motor speed, being \( A_1 \) independent of the working conditions. Therefore, in order to evaluate the impact of the speed in the \( \alpha_i \) signals, a maximum disturbance factor is defined as the ratio between \( A_1 \) and \( \Gamma \). From the definition of \( A_1 \) and (18) this factor can be determined as:

\[ \rho = \frac{3}{4v_{bus}} \sqrt{2 \omega_c^2 \left( 1 + \frac{\Delta L}{L_m} \right)^2 + R^2 I^2 \left( 1 - \frac{\Delta L}{L_m} \right)^2} \]

Figure 4 shows the values of the maximum disturbance factor for a PMSM as a numerical example. The used parameters are shown in Table V and for speed values ranging from standstill to a speed of 300 rad/s and load of 0 A, 5 A and 10 A. It can be concluded that the relation between \( \rho \) and the speed is almost linear, \( \rho < 0.4 \) for all the evaluated speed and load scenarios and \( \rho < 0.1 \) when the machine operates below 80 rad/s. Since the defined disturbance factor considers the worst operation case (the disturbance is in phase with the position signal), the designer can use this information to bound the impact of the speed in the observed rotor position. Therefore, within that speed range one can approximate (17) by:

\[ \alpha_a \approx A_1 \delta_a + A_2 \delta_{2a} \]
\[ \alpha_b \approx A_1 \delta_b + A_2 \delta_{2b} \]
\[ \alpha_c \approx A_1 \delta_c + A_2 \delta_{2c} \]

Remark 1. The expression obtained in (10) holds for the motor matrix inductance shown in (3), which corresponds to the standard distributed windings model. As a consequence, the proposed estimation algorithm is useful for machines responding to that model. For other machines models with non-distributed windings, non-linearities or saturation effects, the estimation principle is also applicable, but some adaptations may be necessary.

IV. DECOUPLED-BASED SLIDING MODE PLL (DB-SMPLL)

Figure 5 shows the scheme of the proposed observer. The algorithm can be divided in two operational blocks: a block that generates a three-phase balanced signal using a
TABLE II
VALUES OF $\delta_i$, $\delta_2$ AS A FUNCTION OF THE CONTROL

<table>
<thead>
<tr>
<th>$u_a$, $u_b$, $u_c$</th>
<th>$\delta_a$</th>
<th>$\delta_2a$</th>
<th>$\delta_b$</th>
<th>$\delta_2b$</th>
<th>$\delta_c$</th>
<th>$\delta_2c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1, -1, -1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1, -1, 1</td>
<td>2cos($z$)</td>
<td>2</td>
<td>2cos($x$)</td>
<td>2</td>
<td>2cos($y$)</td>
<td>-4</td>
</tr>
<tr>
<td>-1, 1, -1</td>
<td>2cos($y$)</td>
<td>2</td>
<td>2cos($z$)</td>
<td>-4</td>
<td>2cos($x$)</td>
<td>2</td>
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<tr>
<td>-1, 1, 1</td>
<td>-2cos($x$)</td>
<td>4</td>
<td>-2cos($y$)</td>
<td>-2</td>
<td>-2cos($z$)</td>
<td>-2</td>
</tr>
<tr>
<td>1, -1, -1</td>
<td>2cos($x$)</td>
<td>-4</td>
<td>2cos($y$)</td>
<td>2</td>
<td>2cos($z$)</td>
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</tr>
<tr>
<td>1, -1, 1</td>
<td>-2cos($y$)</td>
<td>-2</td>
<td>-2cos($z$)</td>
<td>4</td>
<td>-2cos($x$)</td>
<td>-2</td>
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<tr>
<td>1, 1, 1</td>
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<td>-2</td>
<td>-2cos($x$)</td>
<td>-2</td>
<td>-2cos($y$)</td>
<td>4</td>
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<tr>
<td>1, 1, 1</td>
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<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

$x = 2\theta_e, \quad y = 2\theta_e - 2\pi/3, \quad z = 2\theta_e + 2\pi/3$

A. Three-phase balanced signals generator

From (12), the values of $\alpha_i$ can be computed from the system under sliding motion measuring $\hat{\delta}_i$, $u_{ieq}$ (just low pass filtering the discontinuous control signals) and knowing (or measuring) the value of $u_{bus}$, as:

$$\alpha_i = \frac{\hat{\delta}_i}{u_{bus}} - (u_{ieq} - u_i).$$

(22)

Once these signals are known, they should be properly processed to extract the information of $\theta_e$. From equation (21) and Table I, it is straightforward to evaluate the resulting signals as a function of the discontinuous control. Table II shows the values for each different state of the control vector. Notice that all control combinations (with the exception of the zero vectors $(-1, -1, -1)$ and $(1, 1, 1)$) contain $\cos(x), \cos(y)$ and $\cos(z)$ in $\delta_i$, see Table II. Hence, using the discontinuous control signals from the SMC, it is possible to generate the three-phase balanced signal

$$\mu(\theta_e) = 2A_1 \begin{bmatrix} \cos(2\theta_e) \\ \cos(2\theta_e - 2\pi/3) \\ \cos(2\theta_e + 2\pi/3) \end{bmatrix}$$

(23)

for any control combination through a multiplexed selector. Let us show the specific case $(-1, -1, 1)$ as an example. From the Table II, (21) results in:

$$\begin{align*}
\alpha_a & \approx 2A_1 \cos(z) + A_1 \frac{\Delta L}{L_m} \\
\alpha_b & \approx 2A_1 \cos(x) + A_1 \frac{\Delta L}{L_m} \\
\alpha_c & \approx 2A_1 \cos(y) - 2A_1 \frac{\Delta L}{L_m},
\end{align*}$$

(24a) (24b) (24c)

where $A_2$ has been expressed in terms of $A_1$. Equations in (23) are essentially the signals in (24) when the offsets terms are cancelled. In the multiplexor stage the offset cancellation (right terms in (24)) is performed, assuming known the saliency. This cancellation is based on the estimated value of parameter $A_1$, setting $\hat{A}_2 = A_1 \frac{\Delta L}{L_m}$, see Figure 5.

Remark 2. The zero vector combinations $(1, 1, 1)$ and $(-1, -1, -1)$, do not allow to extract the rotor information since they do not apply voltage to the motor. In these cases, the algorithm should keep the last obtained three-phase balanced signal until an active vector appears again.

B. Sliding mode PLL algorithm

The second stage consists in extracting the phase angle from the three phase signal $\mu(\theta_e)$. The most intuitive solution is to apply trigonometric properties to $\mu(\theta_e)$, as often proposed in the $\alpha\beta$ injection algorithms [3]–[5]. However, the result of the arctangent function should be adapted for being used in the system, thus hindering the algorithm. Additionally, for the speed observer, a time derivative should be included which always leads to noise problems in real systems.

An alternative is to design an algorithm able to extract the rotor angle from $\mu(\theta_e)$. This work proposes a scheme based on complex sliding modes PLL that allows the design and analysis in a compact form [34]. The algorithm does not require the knowledge of parameter $A_1$ and the speed is extracted without performing time derivatives. In the following, both position and speed observers are detailed.

1) Position observer: Transformation from three-phase variables to a complex variable is equivalent to the well-known $abc$ to $\alpha\beta$ change of coordinates [35]. The complex signal, $\mu(\theta_e)$, is obtained by means of the complex vector,

$$T = \frac{2}{3} \begin{bmatrix} 1 & -1 & j\sqrt{3} \\ -2 & -1 & -j\sqrt{3} \end{bmatrix}$$

(25)

as

$$\mu(\theta_e) = T \mu(\theta_e) = 2A_1 e^{j2\theta_e}.$$  

(26)

The sliding mode PLL consists of:

$$\begin{align*}
\hat{A}_1 &= -k_R \text{sign}(s_{cR}) \\
\hat{c}_e &= -k_I \text{sign}(s_{cI}),
\end{align*}$$

(27a) (27b)

where $\hat{A}_1$ and $\hat{c}_e$ are the observed values of the signal amplitude and the rotor angle, respectively, $k_R, k_I > 0$ are high gains to ensure the convergence properties and $s_{cR}, s_{cI}$ are the real and imaginary parts of the complex switching function

$$s_e = 2\hat{A}_1 - \mu_e e^{-j2\hat{c}_e}.$$  

(28)

Notice that, since $s_e = s_{cR} + js_{cI}$, the real and imaginary parts of the complex switching function can be written as:

$$\begin{align*}
s_{cR} &= 2\hat{A}_1 - 2A_1 \cos(2\theta_e - 2\hat{c}_e) \\
s_{cI} &= -2A_1 \sin(2\theta_e - 2\hat{c}_e).
\end{align*}$$

(29a) (29b)

Sliding modes on the imaginary axis are given by

$$\theta_e - \hat{c}_e = \frac{n\pi}{2},$$

(30)
where \( n \in \mathbb{Z} \). Moreover, sliding motion occurs if \( s_{cI}\dot{s}_{cI} < 0 \). Then, from (29b) with (27b) one gets
\[
s_{cI}\dot{s}_{cI} = -s_{cI}4A_1 \cos(2\theta_c - 2\dot{\theta}_c)(\omega_c + k_I \text{sign}(s_{cI}))
\]
and, since \( A_1 > 0 \), the sliding motion is guaranteed if \( k_I > |\omega_c| \) and
\[
\frac{(4n-1)\pi}{4} < \theta_c - \dot{\theta}_c < \frac{(4n+1)\pi}{4}.
\]
It is straightforward to show that the dynamics are given by
\[
\dot{\theta}_c = -k_I \quad \text{for} \quad n\pi < \theta_c - \dot{\theta}_c < \frac{(2n+1)\pi}{2}
\]
and by
\[
\dot{\theta}_c = k_I \quad \text{for} \quad \frac{(2n-1)\pi}{2} < \theta_c - \dot{\theta}_c < n\pi.
\]
Hence the sliding motion on \( \theta_c - \dot{\theta}_c = 0 \) is achieved whenever \( |\theta_c - \dot{\theta}_c| < \pi/2 \).

On another hand, once the sliding motion in \( s_{cI} = 0 \) is reached, in the real axis one has that \( s_{cR}\dot{s}_{cR} = 0 \), using (30), simplifies in
\[
s_{cR}\dot{s}_{cR} = -2s_{cR}k_R \text{sign}(s_{cR}).
\]
The sliding motion is guaranteed for any \( k_R > 0 \) and implies \( \dot{A}_1 = A_1 \). As \( A_1 \) is a constant, the observed value \( \dot{A}_1 \) will be used for the offset cancellation in the three-phase signal generation block.

2) Speed observer: The observed rotor speed is defined as
\[
\hat{\omega}_c = \dot{\theta}_c
\]
and, from (27b), is straightforwardly calculated by filtering the switching action \( k_I \text{sign}(s_{cI}) \) under sliding motion,
\[
\tau \ddot{\omega}_c = -\dot{\omega}_c - k_I \text{sign}(s_{cI})
\]
where \( \tau \) is the low pass filter time constant, see Figure 5.

Thanks to the observer structure, the estimated speed is directly obtained without additional calculations and time derivatives. The tuning of low-pass filter should be designed as the discontinuous signal is properly filtered and the dynamics of the overall controlled system is preserved. In next Section additional comments can be found regarding its implementation.

Remark 3. It should be highlighted that the method does not require the knowledge of the inductance or the amplitude of the signal (23), since the rotor information is in the frequency of that signal. The PLL extracts the rotor position for any value of \( A_1 \). Only the offset adjustment assumes known the saliency level \( \Delta L/L \).

3) Speed control stability analysis: When the scheme shown in Figure 2 is configured for a PMSM speed control, three different closed-loops can be observed, namely, the SMC of the currents, the SMC of the DB-SMPPLL and the outer PI speed controller. Regarding the inner loops, the following considerations can be made:

- The current control loop, based on sliding modes, reaches the reference in, approximately, 200 microseconds, approximately. This transient can be neglected when compared to the motor mechanical time constant \((J/B \approx 0.5 \text{ s})\), see Table III.
- The DB-SMPPLL estimator is composed by two different blocks. The first one, the three-phase balanced signals generator, is an algebraic processor without any additional dynamics. The second block, the sliding mode PLL algorithm, uses the complex switching function (28) and only adds the free dynamics given by (32) and (33). Selecting the value of the SMC parameter \( k_I \) large enough guarantees the convergence to the switching surface in few milliseconds, which can be also neglected compared with the mechanical dynamics. Finally, it has to be remarked that the speed is estimated using the first order filter in (36), which can not be neglected.

Therefore, considering the mechanical dynamics of the PMSM [1], the low pass filter of the estimator and the PI controller, the closed-loop dynamics can be described by the following equations:
\[
J\dot{\omega}_c = K_T(k_p\epsilon + k_i\zeta) - B\omega_c,
\]
\[
\dot{\zeta} = \omega^*_c - \hat{\omega}_c,
\]
where \( k_p, k_i > 0 \), are the proportional and the integral gains of the PI controller, respectively. Finally, the closed-loop speed control of the PMSM with the proposed estimator is asymptotically stable when the following condition from the application of the Routh Stability Criterion to (37), is fulfilled:

\[
(J + B\tau)(B + k_pK_T) > J\tau k_iK_T.
\]

(37c)

V. EXPERIMENTAL RESULTS

The DB-SMPLL algorithm, together with the decoupled SMC, have been digitally implemented using the microcontroller F28335 from Texas Instruments. In order to execute all the controllers in a single device, the tasks have been divided in two routines. The main routine basically contains the decoupled SMC control implemented through hysteretic controllers. In that routine, the measurements of the switching functions slopes and the low pass filters for the acquisition of the equivalent control are included. The evaluation of these expressions is performed with a sampling period of 6 \( \mu s \). The second routine is executed with higher sampling time, and includes the speed control for the reference generation to the SMC, the rest of the DB-SMPLL (multiplexor, SMC-PLL,...) and the variable hysteresis block, which ensures a fixed-switching frequency operation [31]. This routine is executed at 80 \( \mu s \). Both control routines have different execution priorities. Specifically, the fast routine has high priority and can not be preemttable while the slow one can be preemttable. With regard to the equivalent controls, first-order discrete time filters with a cut-off frequency tuned a decade above the maximum speed of the experiments have been used.

For the experiments, an off the shelf 3-phase PMSM (Unimotor 115E2 SM-PMSM) has been used. The machine can be seen in Figure 6, where the VSI, the electronics including the F28335 and the DC motor for applying torque to the motor shaft are also shown.

| TABLE III |
| EXPERIMENTAL DATA |

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMSM Drive details</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated Power, Torque, Speed</td>
<td>( P, T_e, \omega_e )</td>
<td>2.54 kW, 8.1 N-m, 3000 rpm</td>
</tr>
<tr>
<td>PMSM parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_T, K_e )</td>
<td>0.93 N-m/A, 57 Vkrpm</td>
<td></td>
</tr>
<tr>
<td>Pair poles</td>
<td>( p )</td>
<td>3</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>( L_a, L_b, L_c )</td>
<td>1.5 ( \pm ) 0.25 mH</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>( R )</td>
<td>0.36 ( \Omega )</td>
</tr>
<tr>
<td>Inertia</td>
<td>( J )</td>
<td>4.57 ( \cdot ) ( 10^{-3} ) kg-m(^2)</td>
</tr>
<tr>
<td>Friction</td>
<td>( B )</td>
<td>8.75 ( \cdot ) ( 10^{-3} ) N-m-s</td>
</tr>
<tr>
<td>DC bus</td>
<td>( \pm v_{bus} )</td>
<td>( \pm 135 ) V</td>
</tr>
</tbody>
</table>

| DB-SMPLL and PI Speed controller parameters | | |
| DB-SMPLL gains | \( k_p, k_i \) | 1000, 1000 |
| DB-SMPLL filter time constant | \( \tau \) | 0.1306 |
| PI Speed controller gains | \( k_p, k_i \) | 0.015, 0.027 |

The tests have been carried out in a full sensorless operation with a standard PI speed controller to evaluate: 1- Speed control with different reference profiles, including reversal test; 2- Zero speed conditions; 3 - Load transients with non-zero speed control. Table III shows the values of the control parameters used in the experimentation. The low pass filter time constant has been set to a quarter of the time constant of the PMSM mechanical part, whereas the PI controller gains has been calculated to ensure an overdamped transient response.

A. Speed control test

The first test is a speed reversal from 100 rad/s to -100 rad/s. The Figure 7 shows the estimated and the real speed and motor position. The signals \( \hat{\theta}_e, \theta_e \) are scaled as 2.5 V corresponds to \( 2\pi \) radians. Notice how the estimated speed perfectly tracks the real one, thus confirming the good observer performance. The figure also presents the error in the angle estimation. Specifically, the maximum estimation error is ten degrees, which corresponds to the 3% of the full range.

![Fig. 6. Workbench with an off the shelf 3-phase Unimotor 115E2 SM-PMSM mechanically coupled with a DC machine, the micro-controller and the VSI.](image_url)

![Fig. 7. Speed reversal test from 100 rad/s to -100 rad/s. Channel 1 (blue) \( \hat{\theta}_e \), \( \pi/1.25 \) rad/div. Channel 2 (red) \( \theta_e \), \( \pi/1.25 \) rad/div. Channel 3 (green) \( \dot{\omega}_e \), 250 rad/s div. Channel 4 (orange) \( \dot{\omega}_e \), 250 rad/s div.](image_url)
In the second test, shown in Figure 8, a speed profile is applied speeding up the machine up to 300 rad/s (electrical), which corresponds to 30% of the nominal speed. Notice how in the start-up the observed position converge to the real one. It should be noticed that one of the benefits of the algorithm is that can operate at the low and zero speed region, however this test demonstrates that the system can also work at medium speed levels as it was analysed in Section III. The maximum estimation error is again of 3%. Moreover, from the figure it can be seen how the error remains constant along the different speeds of the test. This is because the proposed algorithm does not use filters for the angle estimation, and the angle error does not depend on the machine working point.

Figure 9 shows the results of an extremely low speed reversal test, specifically a speed which corresponds to 0.25% of the nominal speed. The test validates that the system controllability is also guaranteed in such low speed. The oscillations that appear in the motor position which are attributed to non-linearities of the mechanical part. In the figure, the rotor angle error and the full range for the speed have been detailed. This results confirms that the proposed algorithm is able to estimate the rotor position at very low/zero speeds, where the model-based algorithms fail and, up to now, high frequency signal injection was required.

B. Zero speed control test

In this test a speed profile, including zero speed, is applied. The table shown over the Figure 10 presents the reference speed at any moment. Again, from Figure 10, the expected behaviour of the observer is confirmed.

Figure 11 shows the effect of different load transients when the reference speed is set to zero. In the test, beside the estimated angle, it is shown the \(i_q\) and the \(i_d\) currents. The \(i_d\) current is processed in the micro-controller and send it out through a Digital to Analog Channel (DAC) for visualization, where 1 V represents 2.5 A, approximately. This test confirms that the system controllability is preserved in these conditions.

C. Load impact test

Finally, load impacts with a speed reference of 50 rad/s are shown in Figure 12. The test validates the good behaviour of the estimated speed with smooth transients when load variations are applied to the mechanical shaft. The \(i_q\) current is processed through a DAC of the F28335 for, again, visualization purpose where 1 V represents 2.5 A. Moreover, the angle error behaves again within the ±10 degrees during all the test. This test was performed for negative speeds obtaining the same good behaviour.
Fig. 11. Zero speed control test with several load impacts. Channel 1 (blue): $\theta_e$, $\pi$/1.25 rad/div. Channel 2 (red): $\dot{\theta}_e$, $\pi$/1.25 rad/div. Channel 3 (green): $i_0$, 20 A div. Channel 4 (orange): $i_d$, 2.5 A div.

Fig. 12. Load impact at steady-state speed of 50 rad/s. Channel 1 (blue): $\theta_e$, $\pi$/1.25 rad/div. Channel 2 (red): $\dot{\theta}_e$, $\pi$/1.25 rad/div. Channel 3 (green): $i_0$, 2.5 A div. Channel 4 (orange): $\omega_e$, 50 rad/s div.

VI. CONCLUSIONS AND FUTURE RESEARCH

In this paper a full sensorless scheme has been proposed for a PMSM under a sliding mode current control. The method is based on the decoupling inaccuracies produced in the switching functions by the machine saliency. The paper shows, through a mathematical analysis, how the rotor angle information appears in the switching functions slopes and how it can be extracted from those measurements, observing both the rotor position and speed. For that purpose, a sliding mode PLL based on a complex switching function algorithm have been proposed. The main benefits of the presented solution is that the sensorless control can be maintained with zero speed without neither injecting signal, nor modifying the modulation nor using any model of the machine. The experimental results have confirmed the correct behaviour of the sensorless system under low and zero speeds conditions, keeping the controllability of the machine during speed reversals and load impacts, thus validating the designed algorithm.

ACKNOWLEDGEMENTS

This work was supported by the Government of Spain through the Agencia Estatal de Investigación Project DPI2017-85404-P and by the Generalitat de Catalunya through the AGAUR Project 2017 SGR 872.

REFERENCES


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