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telecos BCN

Speed estimation of a Formula Student vehicle using Kalman Filters

Degree Thesis
submitted to the Faculty of the
Escola Tècnica d'Enginyeria de Telecomunicació de Barcelona
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Alexis Navarro Vaquera

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Nomenclature

Symbols	Units	Meaning
$\mathbf{A}$	-	System matrix
$\mathbf{B}$	-	Input matrix
$\mathbf{C}$	-	Output matrix
$\mathbf{D}$	-	Feed-through matrix
x	-	States vector
y	-	Measurement vector
u	-	Input vector
X_{kp}	-	Predicted states
X_{kM}	-	Measured states
K	-	Kalman Gain
P_{kp}	-	Predicted covariance matrix
$\mathbf{Q}$	-	Process noise covariance matrix
$\mathbf{R}$	-	Measurement noise covariance matrix
f	-	State update function
h	-	Measurement function
$\dot{\psi}$	rad/s	Yaw rate
$\ddot{\psi}$	rad/s^2	Yaw acceleration
F_x	N	Longitudinal force
F_y	N	Lateral force
F_z	N	Vertical foce
M_z	Nm	Aligning moment
m	kg	Vehicle mass
v_x	m/s	Longitudinal velocity
v_y	m/s	Lateral velocity
γ	rad	Camber angle
α_{ij}	rad	Tyre sideslip angle (front or rear, right or left)
β	rad	Vehicle sideslip angle
C_x	N/rad	Longitudinal cornering stiffness
C_y	N/rad	Lateral cornering stiffness
C_{ai}	N/rad	Adaptative cornering stiffnes (front or rear wheel)
λ	-	Longitudinal slip
l_i	m	Length of the vehicle (front or rear)
t_i	m	Width of the vehicle (front or rear)
δ	rad	Steering angle
$v_{x,wh}$	m/s	Longitudinal velocity of the wheels
$v_{y,wh}$	m/s	Lateral velocity of the wheels
a_x	m/s^2	Longitudinal acceleration
$\dot{v}_x$	m/s^2	Longitudinal acceleration
a_y	m/s^2	Lateral acceleration
$\dot{v}_y$	m/s^2	Lateral acceleration

Abbreviations

CAN Controller Area Network

COG Center of Gravity

DV Driverless

EBS Emergency Brake System

EKF Extended Kalman Filter

ETSEIB Escola Tècnica Superior d'Enginyeria Industrial de Barcelona

ETSETB Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona

GNSS Global Navigation Satellite System

IMD Isulation Monotoring Device

IMU Inertial Measurement Unit

INS Inertial Navigation System

KF Kalman Filter

LVS Low Voltage System

PU Process Unit

RES Remote Emergency System

SAE Society of Automotive Engineers

TS Tractive System

UPC Universitat Politècnica de Catalunya

Abstract

In this thesis the technology of virtual sensing is introduced. Understanding the information provided by the different sensors, having a great awareness of your vehicle dynamics and the set up of the estimators plays a vital role when modelling a vehicle and may end up improving the prototype performance.

As a consequence, two different models are presented: Bicycle and Four-Wheel Model. Furthermore, in order to fulfill with the estimations required two estimators are studied, Kalman Filter and Extended Kalman Filter. In order to check the correct behaviour of these several algorithms, some basic simulations with Matlab/Simulink have been carried out.

Lastly, a more accurate approach is done by the software IPG CarMaker, from IPG Automotive, that allows to obtain data directly from a simulated vehicle and verify the performance of the designed Extended Kalman Filter.

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Name	
Alexis Navarro Vaquera	
Arnau Dòria Cerezo	

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Name	Alexis Navarro Vaquera	Name	Arnau Dòria Cerezo
Position	Project Author	Position	Project Supervisor

1 Introduction

1.1 Statement of purpose

This thesis is a result of participating in Formula Student with the team BCNeMotorsport. Formula Student is an engineering competition which a car is designed, manufactured and set-up by university students. Formula Student is a international competition in which universities from around the world compete against each other. This competition is a marvellous opportunity to take part in a real engineering project and grow up as an engineer as well as in personal skills.

Taking part in Formula Student means continuously evolution. This evolution is paced by the rules itself, as it is mandatory to build a new car if you have participated in one competition, as well as for the new solutions that brings the technology. New solutions can bring a new aero-package, a lighter chassis, a new accumulator or new algorithms to take profit from all the power from the battery and optimize it through the different four electric motors that the team have already introduced.

Evolution also means going faster, and going faster means that there is a vital necessity in accuracy from all the sensors in order to monitor all the data obtained. Sensors are becoming greater but there are some signals that are not really optimal or with an error that make them no reliable. For example, our Torque Vectoring algorithm needs an input of lateral velocity so as to calculate the slip angle of the car. Nevertheless, this value is not good enough in order to have the accuracy that is desired.

Hence, state estimations are made. Not only in order to satisfy the optimization of the Torque Vectoring, as having a good knowledge of velocities would allow the team to localize the car in the track with less error and enable the Xaloc and CAT14x to have a significantly better performance in terms of velocity as there would be less error integrated in the trajectory, ending up with more probability to finish the different dynamic events.

1.1.1 Objectives

The main objective of this project is the development of an Extended Kalman Filter in order to estimate velocities for a Formula Student vehicle. This estimator will help the team to make a step forward in terms of performance. To reach this goal, some specific objectives must be achieved:

- To develop the vehicle models for the Kalman Filter design.
- To design a Kalman Filter and an Extended Kalman Filter for the speed estimation of the vehicle.
- To build the Simulink models to validate the algorithms.
- To implement (using the required programming language) and test the obtained algorithms using IPG CarMaker vehicle simulator.

1.2 Requirements and specifications

Project Requirements

- Build an estimator with longitudinal velocity, lateral acceleration and yaw rate as entries and with longitudinal acceleration, delta (steering angle), lateral force discomposed from longitudinal force and aligning moment as inputs, to be capable of estimate longitudinal velocity, lateral velocity and yaw rate.
- The estimator should provide reasonable estimations with data provided from a software simulator.
- Validate the system with different data and inputs.

Project Specifications

- The estimator might be an Extended Kalman.
- The design of the estimator and model of the vehicles should be done in Simulink.
- Validation done by Simulink, software simulator (IPG CarMaker) and might be used real data provided by an IMU and GPS sensors.

1.3 Methods and procedures

This project is based on the PhD thesis 'Virtual Sensing for Vehicle Dynamics' [1], written by Sebastiaan van Aalst. This project study if it is suitable to estimate different states of our car, CAT14x and design a suitable Extended Kalman Filter.

The main ideas are provided by the supervisor as he has given the tools and the documentations so as to carry on all these studies.

All the software implementation with Matlab/Simulink has been entirely done as well as all the simulation and data obtained by IPG.

1.4 Work plan, milestones and Gantt diagram

1.4.1 Work Plan Packages

Project: Kalman Filters	**WP ref:** WP1
Major constituent: Redaction and creation	Sheet 1 of 6
Short description: Once the Kalman have been introduced and studied, the next step would be building them and prove that they are working correctly. Previously, linearization and discretization have been explained so as to make easier the introduction of the filter.	**Planned started date**: 17/09/2021 **Planned end date**: 11/11/2021

Internal task T1: Get to known Kalman Filters and write the theory down. **Internal task T2**: Design Kalman Filter for Bicycle Model. **Internal task T3**: Design Kalman Filter for Four-Wheel Model.	**Deliverable**: Theory of Kalman Filters and the design of them.

Project: Vehicle Dynamics	**WP ref:** WP2
Major constituent: Vehicle modelling	Sheet 2 of 6
Short description: Build a solid model so as to prove the different Kalman Filters. Several parameters must be taken into account as it has to be as real as possible.	**Planned started date**: 15/10/2021 **Planned end date**: 5/11/2021
Internal task T1: Bicycle Model. **Internal task T2**: Four-Wheel Model.	**Deliverable**: Vehicle modelling.

Project: Complete Simulink	**WP ref:** WP3
Major constituent: Tuning Kalman Filters	Sheet 3 of 6
Short description: Put together the different vehicle models and its Kalman Filters so as to tune them and check the first results.	**Planned started date**: 1/12/2021 **Planned end date**: 16/12/2021
Internal task T1: Set up Kalman Filter for Bicycle Model. **Internal task T2**: Set up Kalman Filter for Four-Wheel Model.	**Deliverable**: Model and estimator working together and obtain the first estimations.

Project: Software	**WP ref:** WP4
Major constituent: Estimator verification	Sheet 4 of 6
Short description: Test the behaviour of the estimator with different inputs and data. Vehicle could be simulated on a software as well as simulate the different sensors that are placed in the car. Finally, real data could be considered.	**Planned started date**: 17/12/2021 **Planned end date**: 27/12/2021
Internal task T1: Add different inputs in the complete Simulinks. **Internal task T2**: IPG CarMaker simulator.	**Deliverable**: Several simulations and estimations.

Project: Sensor and data analyzed	**WP ref:** WP5
Major constituent: Collect data and documentation	Sheet 5 of 6
Short description: Which sensors do we have and which type of information collect. Once we have the information, how we analyzed it.	**Planned started date**: 28/11/2021 **Planned end date**: 1/12/2021

Internal task T1: Sensors and actuators. **Internal task T2**: Gems Data Analyzer. **Internal task T3**: IPG CarMaker introduction.	**Deliverable**: Information about the sensors and the programs used to analyze the data.

Project: Documentation.	**WP ref:** WP6
Major constituent: Document process	Sheet 6 of 6
Short description: Develop procedures and reviews to keep track of the project.	**Planned started date**: 6/10/2021 **Planned end date**: 10/01/2022
Internal task T1: Project Proposal and Workplan. **Internal task T2**: Project Critical Review. **Internal task T3**: Final Review.	**Deliverable**: Documents requested by the university.

1.4.2 Milestones

WP	Task	Short title	Milestone/Deliverable	Date(week)
1	All	Kalman Filter documentation	Documentation	Mid October
2	1	Bicycle Model	Basic vehicle modelling	Last October
2	2	Four-Wheel Model	Complete vehicle modelling	First November
3	All	Tuning Kalman Filters	First results	Mid November
4	All	Estimator verification	Estimator results	First December
5	All	Sensor and data analysis	Documentation	First December
6	3	Final Review	Project deliverables	First January

1.4.3 Gantt Diagram

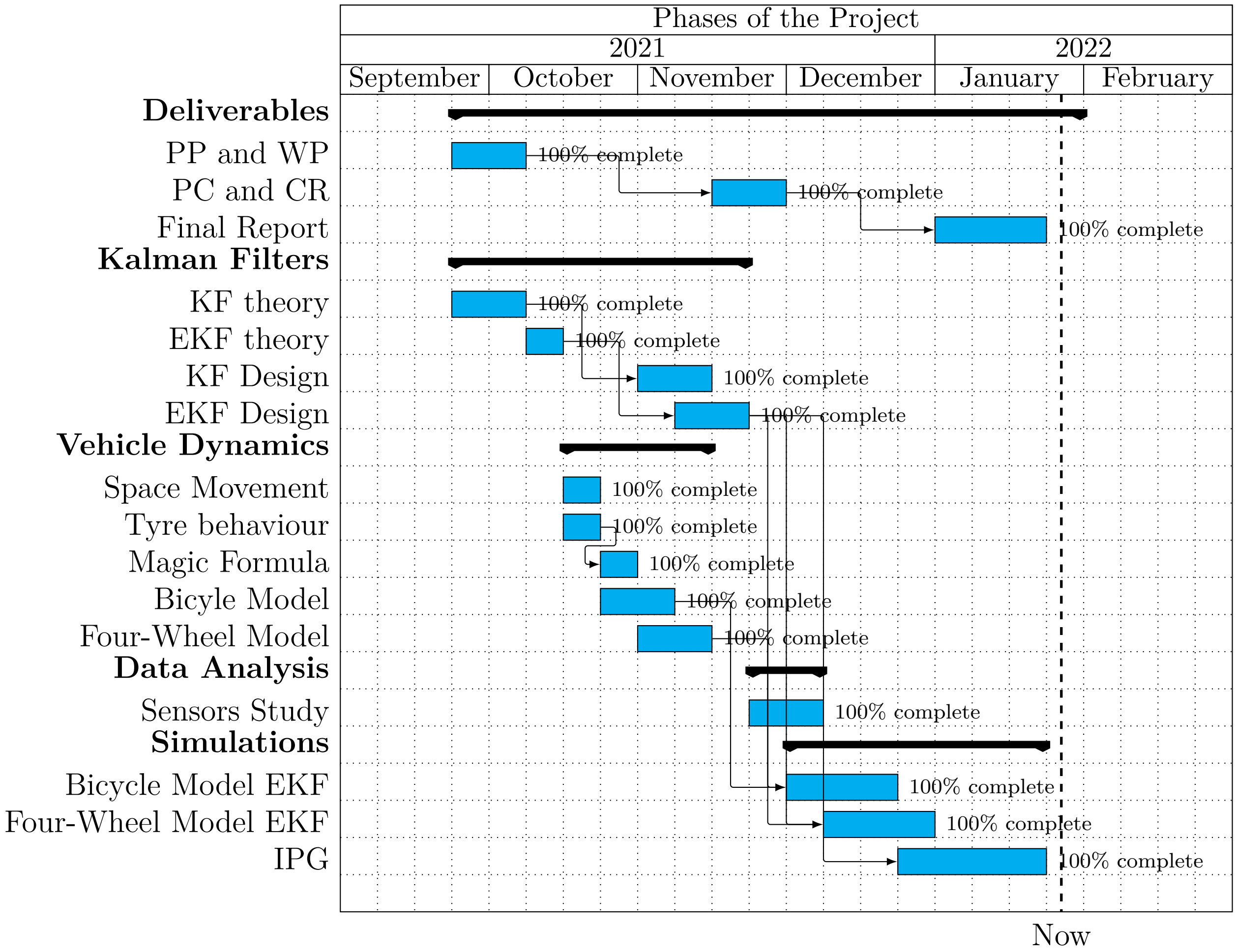

Figure 1.1: Gantt diagram of the project

1.5 State of the art

Nowadays there is a great variety of sensors which provides almost all the necessary useful data in order to achieve an appreciable control of the vehicle. Nevertheless, some of these sensors are quite expensive and not every team can afford buying some of them. The same happens with automotive brands, because of economic reasons they can not provide some sensors to their vehicles.

However, although a large number of signals can be monitored, some measurements would be obtained with an uncertainty produced by noise or any type of calibration. As a consequence for this reason, the accuracy of the state vector that is trying to get fit correctly the state space form of the system will be affected.

In order to correct or indirectly measure those states of interest, a solution widely proposed is virtual sensing [1]. Virtual sensing uses information available from other measurements and process parameters to calculate an estimate of the quantity of interest. Even though, this technology is not significantly cheap, and the required knowledge of the tyre forces is not trivial as well.

All these applications of vehicle state estimation are based on kinematic and dynamic models. The kinematic models do not depend on the forces. Hence, the tyre parameters will be omitted and as a consequence, the dynamics of the specific vehicle too. On the other hand, the dynamic model depends on the knowledge of the tyres so as to obtain its forces, those forces allow modelling the dynamic of the vehicle.

As the tyres are the only element that are in contact with the floor is vital to have characterized this element [2]. Depending on its understanding, we would be able to extract precisely its forces and its limits. Hence, the better is the knowledge, the greater would be the accuracy of the dynamic model. Moreover, this model also provides a satisfactory robustness against measurement errors compared with a kinematic model.

The observers that are extensively used in virtual sensing are the Kalman Filter and the Extended Kalman Filter. Kalman Filter could be a considerable solution for street vehicles as they tend to have a linear behaviour. Nonetheless, due to the fact of the features of the circuits and the typical behaviour of a Formula Student prototype, high nonlinearities must be taken into account and because of that Extended Kalman Filters are used.

1.6 Project overlay

Chapter 1 - Introduction

The first chapter is an introduction to virtual sensing and the necessity of this technology.

The objectives, requirements and specifications, methods and procedures that have been followed as well as the work plan, milestones and Gantt diagram that have helped organizing this thesis are also shown.

Chapter 2 - Formula Student

The second chapter introduces the Formula Student competition. From where and when it was born to its organization, technical inspections and its several static and dynamic events.

At the end is presented our team, BCN eMotorsport, and the plans for the future with our new prototype CAT14x.

Chapter 3 - Vehicle Dynamics

In this third chapter the vehicle dynamics and tyre modelling are introduced. The vehicle and tyre frames are defined. An introduction to the tyre is done because of the direct importance of this element to the performance of the vehicle as well as the performance of our observer. Tyre behaviour is explained through longitudinal and lateral forces and the Magic Formula, due to that a dynamic vehicle models will be used and is key to have a great understanding of these forces.

Those dynamic models of the vehicle are as well presented. The Four-Wheel Model takes into the account each wheel of the vehicle with any simplification. On the other hand, the Bicycle Model simplifies the vehicle in two wheels, one per each axle. Both of them are thought on a planar motion.

Chapter 4 - Methodology

The fourth chapter introduces firstly the nonlinear system and linearization as a Formula Student Vehicle is contemplated to work with high nonlinearities. Secondly, working with sensors involves the discretization of the signals. Having introduced these basic tools, Kalman's observers are presented.

Finally is described how the first simulations are taken with the software chosen. It joins the models introduced in Chapter 3 with the observers in order to test if our models and Kalman Filters are correctly set.

Chapter 5 - Vehicle Data Analysis

The fifth chapter relies on the awareness of the several sensors that our prototype is equipped with. The sensors that plays an important role in this project are presented.

Previously to work with the real prototype and its sensors, some virtual simulations are done in order to check the functioning of several algorithms. This is done with the help of IPG Automotive software.

Finally, once the algorithms are proven in the vehicle, is vital to process and understand the signals acquired through the sensors. Hence, the software Gems Data Analysis Pro is introduced as it does help us in these corroborations.

Chapter 6 - Experiments and results

In the sixth chapter, once it has been set the observer previously, the Extended Kalman Filter is tested with inputs provided by the IPG CarMaker. IPG CarMaker allow us to simulate a real track and a more accurate behaviour of our prototype. Hence, high nonlinearities will appear and will be managed so as to finish the set up of the EKF.

Chapter 7 - Budget

In the seventh chapter a cost of this project is approximated.

Chapter 8 - Conclusions

In the eighth chapter, ends up with the conclusions taken from the different simulations done and its results.

Chapter 9 - Future work

Last but not least, the ninth chapter contains the following steps that will be taken after finishing this project.

2 Formula Student

2.1 Brief introduction to Formula Student

Formula Student is a student engineering competition where the teams from universities around the world design, build and set-up a small-scale formula style racing car. All these cars are based on a series of rules specified in [3] written by the main competition from Germany.

The competition was born in 1987 in the E.E.U.U thanks to the Society of Automotive Engineers (SAE) under the name of Formula SAE. It was not until 1998 that a brand new competition, Formula Student, arrived to the UK and was organized with the Institution of Mechanical Engineers (IMeche). The inaugural event, co-sanctioned by SAE was a fantastic success. The event has evolved throughout the years to be in the latest engineering trends and challenging engineering students at top level. It started with combustion prototypes. In 2010 Formula Student Electric was established, so the vehicle was required to be fully electrically powered. Later, in 2017, the Driverless category was born, where a car without a driver and with cameras, sensors and actuators had to be able to finish a series of events. In 2019 a change in the rules was proposed. This change implies the integration of the combustion and electric modalities with the driverless one. Hence, the team will have the opportunity to participate in a new category: The Driverless Cup. In this category, the prototypes will participate in the dynamics events as usual in combustion or electric, but with the same vehicle will have to participate in autonomous dynamic events.

Figure 2.1: FSG 2021. [4]

This competition also takes in consideration other aspects from engineering, as could be having a knowledge of the cost of your prototype, present and argument your own design or create a business plan so to explain the viability of your project. All these points are also evaluated on the competition in the static events. In a nutshell, it is evaluated a complete engineering project, since what has been designed to how fast your car is.

2.2 Formula Student Event Organization

2.2.1 Participation

There is a maximum number of teams that are allowed to participate in each competition organized by different countries. In order to determinate the teams that will participate in the competition, an exam is done per each country. Hence, the teams with the best results gain a place in the specified Formula Student competition. All the exams are taken on the same day and every country has its own hour to take the exam.

2.2.2 Static Events

As it has been explained, Formula Student evaluates a complete engineering project. Thus, it is important to have great sense of the cost of your vehicle, be able to show and argument the solutions you have made on the car and present a plan that would verify the viability and the reason of your project. All of these points make up the Static Events, which is split up in three parts:

- **Business Plan Presentation**: The judges will evaluate that your business model could become a profitable business.
- **Cost and Manufacturing**: It will be evaluated if the team have understood the manufacturing process and costs related to the construction of their prototype.
- **Engineering Design Event**: Present to the judges the different solutions that you have adopted in your car so as to achieve the desired goals and fulfil with the rules.

2.2.3 Dynamic Events

The main goal of a formula race car is to be quicker than the other cars. Nevertheless, previously to put the cars on the tracks, a series of inspection must be passed. These inspections are held because of controlling that the prototypes follow the regulation and guarantee that the vehicles are safe.

Technical Inspections

Those inspections are divided in the following parts regarding the Driverless Cup category with electric powertrain:

- **Preinspection**: The team must present the driver equipment and the set of tyres which are going to be used during the event.
- **Accumulator Inspection**: The accumulator, its charger and all the necessary equipment in order to manipulate these devices will be checked.
- **Electrical Inspection**: The vehicle bust be electrically safe and rules compliant. The insulation resistance between the TS and LVS will be measured.

- **Mechanical Inspection**: The aim of this this inspection is to determine if the car is mechanically safe and rules compliant. Several items are revised, as could be the monocoque, quick jack and push bar, the cockpit distances with the tallest driver of the team, etc.
- **Autonomous System Inspection**: The aim of this inspection is to determine if the car is safe in autonomous mode and rules compliant. Sensors must meet with local legislation, RES functionality is also checked as well as having the pertinent tools so as to manipulate all the systems, among others.
- **Tilt Test**: The vehicle with a driver is placed on a platform which rotates 60 degrees around the longitudinal axis of the car. The test is carried out to ensure if the vehicle can manage a lateral acceleration of around 1.7g and there are no liquid leaks.
- **Vehicle Weighing**: In order to have a control of the weight of the vehicle, the prototype will be weighted with all its liquids filled to the maximum. This weight may be checked again lately and if there is a significant variation the team will be penalized.
- **Brake test**: The driver must lock all four wheels and stop the vehicle in a straight line at the end of the acceleration run (75 m) specified by the officials using only the mechanical brake.
- **EBS test**: The test will be performed in a straight line marked with cones similar to acceleration. During the brake test, the vehicle must accelerate in autonomous mode up to at least 40 km/h within 20 m. From the point where the RES is triggered, the vehicle must come to a safe stop within a maximum distance of 10 m.
- **Rain Test**: The aim of this test is to verify the insulation of the vehicle. The test is passed if the IMD is not triggered while water is sprayed at the vehicle for 120 s and 120 s after the water spray has stopped.

Dynamic Events

- **Skid Pad and DV Skid Pad**: In this event the driver has to turn right twice and turn left another two times following the layout of an eight. The time that the team gets is the mean of both second turns. The layout for Driverless vehicles is the same, although the score in this category takes into account the cars that have finished this event and all the times must be under 25 s.
- **Acceleration and DV Acceleration**: In this event the driver, or the vehicle, must complete 75 m in the shortest time possible.
- **Autocross**: The objective is to drive around a track defined by the organization which has some limitations specified in the rules, using the minimum lap time.
- **Trackdrive**: It consists on completing 10 laps in autonomous mode. The length of one lap is approximately 200 m to 500 m.
- **Endurance**: This is the most important dynamic event. It is about a resistance event where the car must complete 22 km. It is an outstanding challenge for the

reliability of the vehicle as well as for the pilot as he or she must manage the accumulator energy without downgrading too much the pace.

- **Efficiency**: Through an official data logger the consumption of energy is measured during the Endurance. This consumption is evaluated. The most efficient vehicle will win this event.

2.3 BCN eMotorsport

BCN eMotorsport is a formula student team formed by students from ETSETB and ETSEIB. BCN eMotorsport is a result of fusing ETSEIB Motorsport and Driverless UPC due to the changes introduced in the rules.

ETSEIB Motorsport was born in 2007 with a combustion prototype. The team was formed by 13 brave students. The project kept growing, and in 2012, the first electric Formula Student prototype was made under the name of CAT05e, becoming the first of the country. Lately, in 2019, Driverless UPC appeared on the map manufacturing as well the first Driverless Formula Student vehicle in Spain. It was such a success as in the first year that the team participated in competitions, all the dynamic events were finished. The histories were written separately until 2020 as mentioned before, but the teams joined forces to compete as an unique team and with an unique car in 2022 in order to participate in the Driverless Cup.

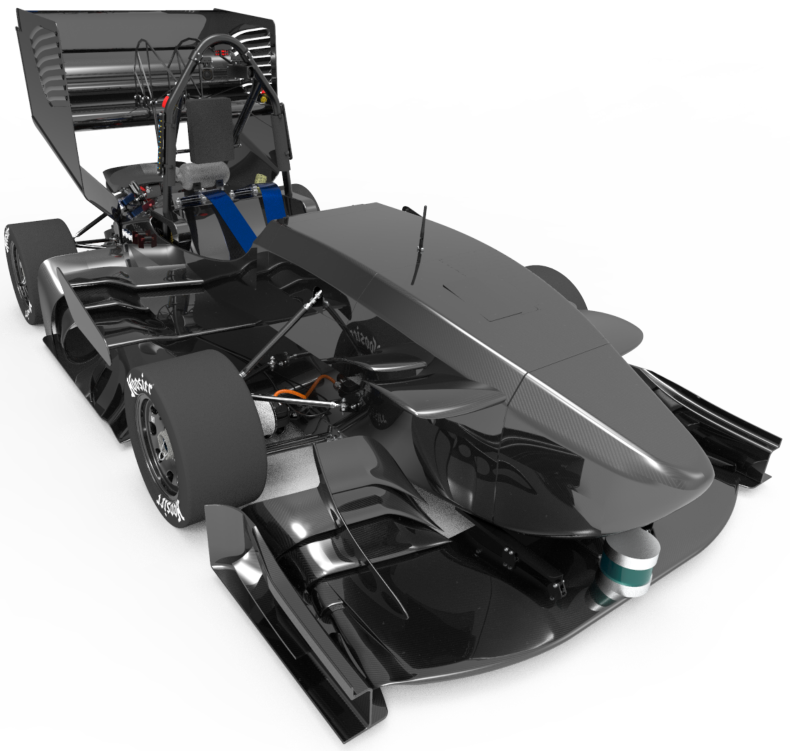

Figure 2.2: BCN eMotorsport: CAT14x.

3 Vehicle Dynamics

3.1 Vehicle Space Movements

The following axes are considered to describe the rigid movement of the vehicle chassis. It is originated in the center of gravity (COG) of the vehicle body. Its x-axis is along the longitudinal axis of the vehicle body, its y-axis along the side axis and to the left and its z-axis is pointed upwards. A car itself has three important movements in space: yaw, pitch and roll, one affecting each axis.

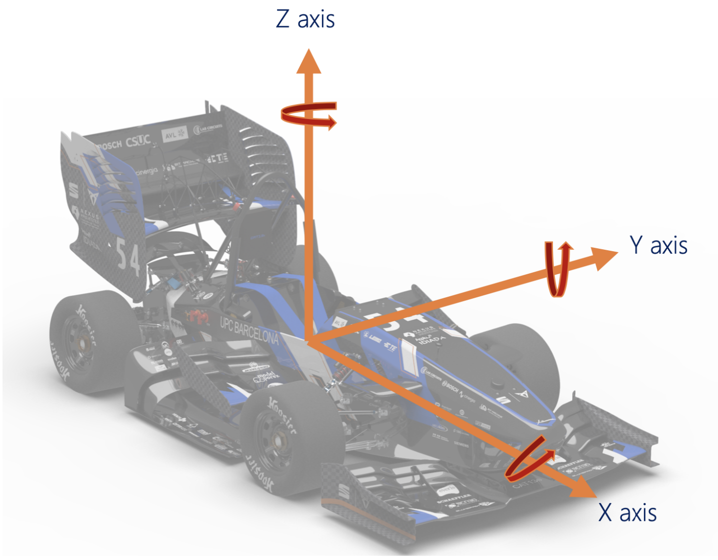

Figure 3.1: Yaw, Pitch and Roll axes.

- **Yaw**: Yaw is the movement around the Z Axis. Yaw rate is the vehicle's angular velocity about its vertical axis in degrees or radians per second in order to determine the orientation of the vehicle as it hard-corners or threatens to roll-over.
- **Pitch**: Pitch is the movement of the front and rear parts of the vehicle around the Y Axis. As a result of accelerating or braking forces.
- **Roll**: Roll is the movement of the left and right parts of the vehicle around the X Axis as a result from lateral accelerations while cornering.

3.2 Tyre behaviour

Wheels are a vital part of the vehicle and must be taken into accounts as is the only part that is in contact with the ground. Having as much as possible information is crucial so as to have a good model and a good performance on the track.

The interaction between the wheels and the ground makes the vehicle capable of moving forward due to the forces produced. These forces are extremely difficult to measure as they depend on several parameters as could be the material, profile or stiffness.

In order to describe the tyre road interaction a three-axis system is used attached to the centre of the contact patch.

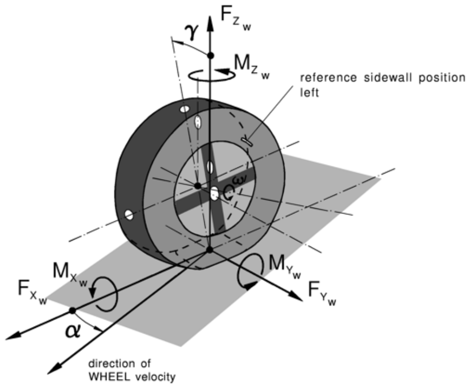

Figure 3.2: Tire forces and moments. [5]

The tyre orientation is defined by the camber angle γ, representing the inclination of the tyre plane according to the x-axis, and the sideslip angle α, defining the z-axis rotation of the velocity vector and the x-axis. It can be estimated by:

$$\tan \alpha = \frac{v_y}{v_x} \tag{3.1}$$

Where v_x is the longitudinal velocity and v_y the lateral velocity, both coordinates of the velocity vector v.

A planar motion will be considered, so the forces that we are taking into consideration for our studies are the following:

- $\mathbf{F_x}$: Longitudinal or tractive force. While the car is accelerating and a torque is applied, this force is defined as positive. While braking, negative.
- $\mathbf{F_y}$: Lateral or cornering force. While left-cornering is positive. While right-cornering is negative.
- $\mathbf{M_z}$: Self aligning moment. Is a moment which tends to re-align tires. Is a restoring moment that attempts to return the wheels to a zero slip angle state.

3.2.1 Longitudinal Force

In order to create the force between the road and the tire, the tire needs some longitudinal slip known as slip ratio. Longitudinal force allows the car to accelerate or brake and it also depends on the vertical force.

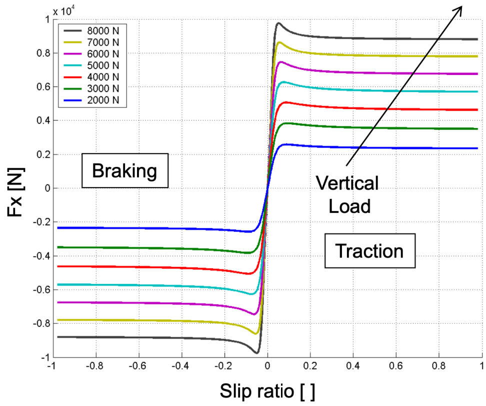

Figure 3.3: Longitudinal force vs slip ratio for different normal loads. [6]

Longitudinal slip ratio is generally defined as:

While accelerating,

$$\lambda = \frac{R_w w - v_x}{R_w w} \tag{3.2}$$

While braking,

$$\lambda = \frac{v_x - R_w w}{v_x} \tag{3.3}$$

Defining v_x as the longitudinal velocity, R_w is the wheel radius and w is the wheel angular speed.

From the Figure 3.4 and the wheel dynamic equation:

$$T_{a,b} - R_w F_x = \dot{w} I_w \tag{3.4}$$

Defining T as the wheel torque, R_w the wheel radius, I_w inertia of the wheel, $\dot{w}$ is the wheel angular acceleration and F_x is the longitudinal force. It can be seen how the slip ratio depends on the torque delivered and the F_x.

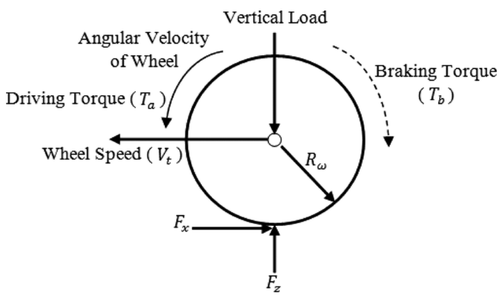

Figure 3.4: Longitudinal tyre dynamics. [7]

3.2.2 Lateral Force

Lateral or cornering force is produced by a vehicle wheel during cornering. The forces that appear while turning depend on the vertical force as well as on the slip angle. Slip angle is defined as the angle between the direction in which a wheel is pointing and the direction in which it is actually traveling.

It is shown that longitudinal and lateral forces present a linear behaviour for small slip ratio and small slip angle, respectively. Because of the characteristics of the tracks where there are a lot of bends, in Formula Student cars high slip angles are present and as a result of that, this approximation can not be ideal for this type of vehicles.

In the following picture, it can be seen the behaviour described above for different normal loads, as lateral force depend on that as well. The orange one is the one with greater normal load, and the blue one with the less.

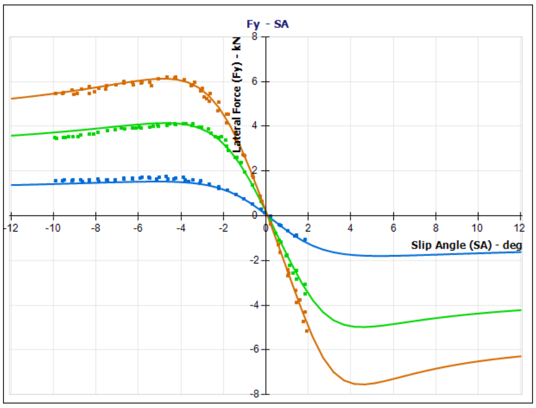

Figure 3.5: Lateral force vs slip angle for three different vertical loads. [8]

3.2.3 Magic Formula

There are several mathematical equation so as to model the previous forces. The most used one is the Pacejka, also known as the Magic Formula. The Magic Formula tyre model was first created by Hans Baastian Pacejka.

Pacejka's tire model describes braking force, lateral force, and self-aligning torque in terms of slip ratio and slip angle as a semi-empirical tire model described with the so-called magic formula. The expression

$$Y(x) = D\sin(C\arctan(Bx - E(Bx - \arctan(Bx)))) + S_v \tag{3.5}$$

outputs the shape of cornering force for a certain vertical load.

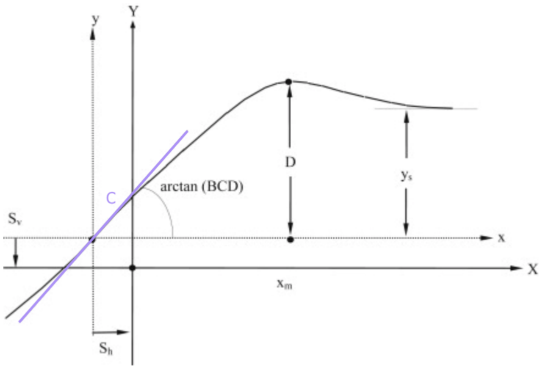

Figure 3.6: Magic Formula graphical representation. [9]

Where B is the stiffness factor, C the shape factor (Cornering stiffness), D the peak value, E the curvature factor and S_v the vertical shift.

Cornering stiffness of a tire is its ability to resist deformation in the shape of a tire while the vehicle corners. The more flexible the tire is the less stiffer it is. For a given tyre, the load and inflation pressure are the main factors affecting the cornering stiffness. When a vehicle is moving at high speed, lateral force acting on tires increases the possibility for vehicle to get into a critical situation.

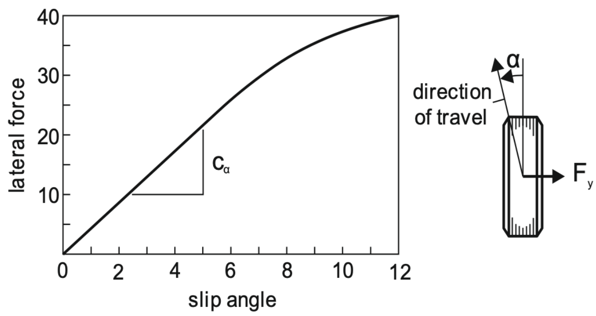

Figure 3.7: Basic functional relationship of lateral force and slip angle. [10]

The linear approximation that has been described for longitudinal force and lateral force are described by these simplified equations:

$$F_x = C_x \lambda \tag{3.6}$$

$$F_y = C_y \alpha \tag{3.7}$$

3.3 Four-Wheel Model

It has already been presented a model so as to represent the behaviour of the tyres. Moving on, shall we present a model in order to represent the behaviour of the vehicle as a whole. The four wheel model take into account the longitudinal and lateral velocities of each wheel, their slip angle and the effect of their forces. A planar motion is considered.

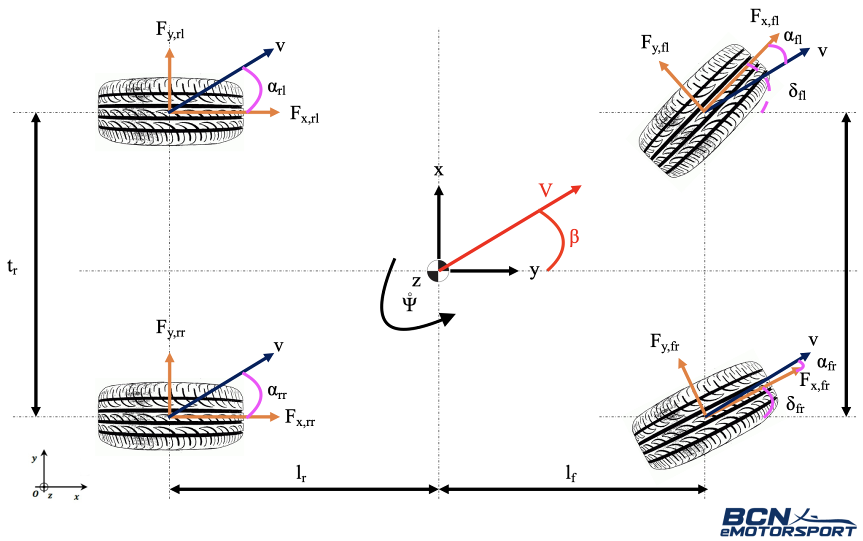

Figure 3.8: Four-Wheel Model representation.

In this model, as all the wheels are being considered, the two different tracks of the car must be known. Thus, the lateral load would be differentiated from both sides resulting in a more accurate approximation. Hence, having a great understanding of the tyres plays a vital role in this dynamic model.

The equations of motion of this model are:

$$\begin{aligned} m\dot{v}_x =& F_{x,fl}\cos(\delta_{fl}) + F_{x,fr}\cos(\delta_{fr}) - F_{y,fl}\sin(\delta_{fl}) - F_{y,fr}\sin(\delta_{fr}) \\ &+ F_{x,rl} + F_{x,rr} + mv_y\dot{\psi} \end{aligned} \tag{3.8a}$$

$$\begin{aligned} m\dot{v}_y =& F_{y,fl}\cos(\delta_{fl}) + F_{y,fr}\cos(\delta_{fr}) + F_{x,fl}\sin(\delta_{fl}) + F_{x,fr}\sin(\delta_{fr}) \\ &+ F_{y,rl} + F_{y,rr} - mv_x\dot{\psi} \end{aligned} \tag{3.8b}$$

$$\begin{aligned} I_{zz}\ddot{\psi} =& l_f F_{x,fl}\sin(\delta_{fl}) + \frac{t_f}{2}F_{y,fl}\sin(\delta_{fl}) + l_f F_{y,fl}\cos(\delta_{fl}) - \frac{t_f}{2}F_{x,fl}\cos(\delta_{fl}) \\ &+ l_f F_{x,fr}\sin(\delta_{fr}) - \frac{t_f}{2}F_{y,fr}\sin(\delta_{fr}) + l_f F_{y,fr}\cos(\delta_{fr}) + \frac{t_f}{2}F_{x,fr}\cos(\delta_{fr}) \\ &- l_r F_{y,rl} - l_r F_{y,rr} + \frac{t_r}{2}F_{x,rr} - \frac{t_r}{2}F_{x,rl} \end{aligned} \tag{3.8c}$$

As has been told, having a considerable knowledge of the tyre will represent an outstanding study and representation of the tyres and their parameters.

In a nutshell, an appreciable model of the tyres would lead on a noteworthy model of the whole car.

The following equations represents the longitudinal and lateral velocity of each wheel. Notice that i denotes front or rear wheels, and j denotes right or left wheels.

$$\begin{bmatrix} v_{x,w_h,ij} \\ v_{y,w_h,ij} \end{bmatrix} = \begin{bmatrix} \cos(\delta_{ij}) & \sin(\delta_{ij}) & 0 \\ -\sin(\delta_{ij}) & \cos(\delta_{ij}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} \pm l_i \\ \pm \frac{t_j}{2} \\ 0 \end{bmatrix} \right) \tag{3.9}$$

These velocities will enable us to study the lateral forces that are being used in the equations of motion. Moreover, a linear approximation of the tyre's cornering stiffness should be taken in consideration.

$$F_{y,ij} = C_i \alpha_{ij} \tag{3.10}$$

Where the sideslip angle of each wheel is defined as:

$$\alpha_{ij} = \arctan\left(\frac{v_{y,w_h,ij}}{v_{x,w_h,ij}}\right) \tag{3.11}$$

If wheel suspension is contemplated or more information of this model is required, see 2.1.2 Vehicle Dynamics section of [1].

3.4 Bicycle Model

Four-Wheel Model already studied, shall we introduce a simpler but effective model: The Bicycle Model. This model simplifies the axes of the car in just one wheel. Hence, front and rear tracks will be neglected and as a result the lateral load force will be ignored as well. Furthermore, it implies having the same sideslip angle on right and left tyres.

This model relies on the longitudinal and lateral vehicle dynamics. It does well at capturing the motion of a vehicle in normal driving conditions and it is useful for extracting some first results, as it has been used in this report in order to correct the equations of the EKF.

The Figure 3.9 shows a representation of the Bicycle Model.

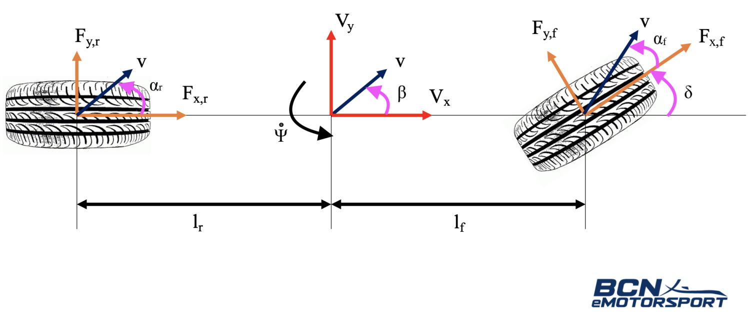

Figure 3.9: Bicycle Model representation.

The equations of motion are:

$$\dot{v}_x = \frac{1}{m}(F_{x,f}\cos(\delta_f) - F_{y,f}\sin(\delta_f) + F_{x,r}) + v_y\dot{\psi} \tag{3.12a}$$

$$\dot{v}_y = \frac{1}{m}(F_{y,f}\cos(\delta_f) - F_{x,f}\sin(\delta_f) + F_{y,r}) - v_x\dot{\psi} \tag{3.12b}$$

$$\ddot{\psi} = \frac{1}{I_{zz}}(l_f F_{x,f}\sin(\delta_f) + l_f F_{y,f}\cos(\delta_f) - l_r F_{y,r}) \tag{3.12c}$$

As well as in the four wheel model, the tyres velocities are described as:

$$\begin{bmatrix} v_{x,w_h,i} \\ v_{y,w_h,i} \end{bmatrix} = \begin{bmatrix} \cos(\delta_i) & \sin(\delta_i) & 0 \\ -\sin(\delta_i) & \cos(\delta_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} \pm l_i \\ 0 \\ 0 \end{bmatrix} \right) \tag{3.13}$$

Just considering front traction and again a linear approximation of the cornering stiffness, the lateral force per each wheel (front and rear) is defined as:

$$F_{y,i} = C_i\alpha_i, \tag{3.14}$$

being i front or rear.

Finally, the sideslip angle is determined by:

$$\alpha_i = \arctan\left(\frac{v_{y,w_h,i}}{v_{x,w_h,i}}\right) \tag{3.15}$$

4 Methodology

4.1 Theoretical framework

4.1.1 Nonlinear systems and linearization

A nonlinear system is a system in which the change of the output is not proportional to the change of the input. These systems are describing changes that may seem unpredictable, contrasting with much simpler linear systems.

Therefore, nonlinear system are inherently difficult to test and to interpret the results correctly. Hence, we should take time so as to cover all possible input regimes in order to understand the correct behaviour of the system and check that is suitable working with different conditions as sweep directions or initial conditions for example.

Nonlinear systems equations in continuous time are generally described as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{4.1a}$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x}, \mathbf{u}) \tag{4.1b}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the input vector, $\mathbf{y} \in \mathbb{R}^p$ is the output vector, and functions $\mathbf{f}, \mathbf{h}$ relate them.

On the other hand we have the linear systems, which are based on the use of a linear operator. A system is linear if and only satisfies the additive and homogeneity properties without restrictions.

Additive property:

$$F(x_1 + x_2) = F(x_1) + F(x_2) \tag{4.2}$$

Homogeneity property:

$$F(ax) = aF(x) \tag{4.3}$$

Linear systems equations in continuous time are generally described as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{4.4a}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{4.4b}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the state matrix, $\mathbf{B} \in \mathbb{R}^{n\times m}$ is the input matrix, $\mathbf{C} \in \mathbb{R}^{p\times n}$ is the output matrix, and $\mathbf{D} \in \mathbb{R}^{p\times m}$ is the feed forward matrix.

As nonlinear dynamical equations are difficult to solve, nonlinear systems are commonly approximated by linear equations (linearization).

Linearization allows us to use tools for studying linear systems to analyze the behaviour of a nonlinear function near a given point, is the process of taking the gradient of a nonlinear function with respect to all variables and creating a linear representation at that point.

$$\frac{\partial \mathbf{x}}{\partial \mathbf{t}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{4.5}$$

The linear approximation of a function is the first order term of its Taylor expansion around the point of interest, x*.

$$\frac{\partial \mathbf{x}}{\partial \mathbf{t}} = \mathbf{f}(\mathbf{x},\mathbf{u}) \approx f(\mathbf{x}^*,\mathbf{u}^*) + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{\mathbf{x}^*,\mathbf{u}^*}(\mathbf{x}-\mathbf{x}^*) + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}|_{\mathbf{x}^*,\mathbf{u}^*}(\mathbf{u}-\mathbf{u}^*) \tag{4.6}$$

Hence:

$$\dot{\mathbf{x}} - \dot{\mathbf{x}^*} = \mathbf{A}(\mathbf{x}-\mathbf{x}^*) + \mathbf{B}(\mathbf{u}-\mathbf{u}^*) \tag{4.7a}$$
$$\mathbf{y} = \mathbf{C}(\mathbf{x}-\mathbf{x}^*) + \mathbf{D}(\mathbf{u}-\mathbf{u}^*) \tag{4.7b}$$

where,

$$\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{\mathbf{x}=\mathbf{x}^*,\mathbf{u}=\mathbf{u}^*}, \mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}}|_{\mathbf{x}=\mathbf{x}^*,\mathbf{u}=\mathbf{u}^*}, \mathbf{C} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}|_{\mathbf{x}=\mathbf{x}^*,\mathbf{u}=\mathbf{u}^*}, \mathbf{D} = \frac{\partial \mathbf{h}}{\partial \mathbf{u}}|_{\mathbf{x}=\mathbf{x}^*,\mathbf{u}=\mathbf{u}^*} \tag{4.8}$$

4.1.2 Discretization of continuous systems

Discretization is the process of transferring continuous functions, models, variables, and equations into discrete counterparts. This process is usually carried out as a first step toward making them suitable for numerical evaluation and implementation on digital computers as our Process Unit (PU).

Consider the continuous time system in state space that we want to discretize:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{4.9a}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{4.9b}$$

An approximation with a series of substitution known as *Backward Euler Approximation* for state space models will be made.

We will proceed with finding an approximation of the time derivative of $x(t)$. We can approximate $\dot{x}(t) := \lim_{\epsilon \to 0} \frac{x(t)-x(t-\epsilon)}{T_s}$, where $T_s \equiv$ sampling time, as:

$$\dot{x}(t) \approx \frac{x(t) - x(t-T_s)}{T_s} \tag{4.10}$$

for small enough values of T_s. Since we are interested in the approximation only when $t = kT_s$, defined $x(kT_s) := x_k$, we substitute $t = kT_s$ into the previous expression (4.10) obtaining:

$$\dot{x}(kT_s) \approx \frac{x_k - x_{k-1}}{T_s} \tag{4.11}$$

Finally, defining $u_k := u(kT)$ and $y_k := y(kT)$ and substituting (4.11) into the sampled version of the continuous time state space system (4.9), we have:

$$\dot{\mathbf{x}}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_k \tag{4.12a}$$
$$\mathbf{y}_k = \mathbf{C}_{k-1}\mathbf{x}_{k-1} + \mathbf{D}_{k-1}\mathbf{u}_k \tag{4.12b}$$

Where,

$$\mathbf{A}_{k-1} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{\mathbf{x}=\mathbf{x}_{k-1},\mathbf{u}=\mathbf{u}_k}, \mathbf{B}_{k-1} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}}|_{\mathbf{x}=\mathbf{x}_{k-1},\mathbf{u}=\mathbf{u}_k} \tag{4.13}$$

As we apply this approximation for linear and nonlinear systems we obtain the following results.

Linear systems:

$$\frac{\mathbf{x}_k - \mathbf{x}_{k-1}}{T_s} \approx \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_k \rightarrow \mathbf{x}_k = T_s\mathbf{A}\mathbf{x}_{k-1} + T_s\mathbf{B}\mathbf{u}_k + \mathbf{x}_{k-1} \tag{4.14}$$

Nonlinear systems:

$$\frac{\mathbf{x}_k - \mathbf{x}_{k-1}}{T_s} = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k) \rightarrow \mathbf{x}_k = T_s\mathbf{A}_{k-1}\mathbf{x}_{k-1} + T_s\mathbf{B}_{k-1}\mathbf{u}_k + \mathbf{x}_{k-1} \tag{4.15}$$

4.2 Kalman Filter

A KF is an iterative mathematical process that uses a set of equations and consecutive data inputs to quickly estimate the true value, position, velocity, etc. of the object being measured, when the measured values contain unpredicted or random error, uncertainty, or variation.

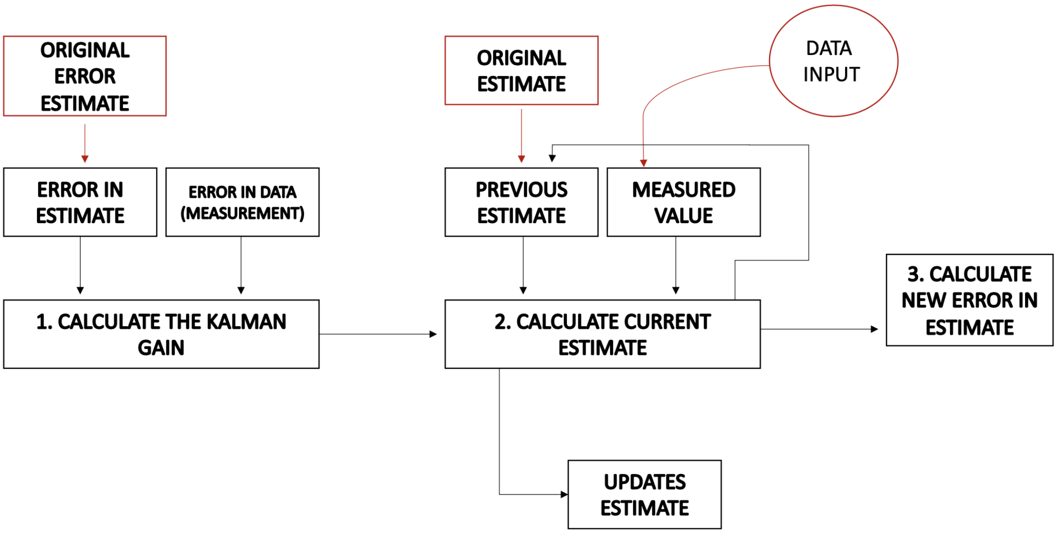

Figure 4.1: Kalman Filter schematic.

The KF is based on the linear state equations (4.4).

The equations (4.4) are not taking into account the noise processes, which are typically represented by $\mathbf{w}_s$ and $\mathbf{v}_s$, meaning model and measurement noise. They are assumed to behave as Gaussian white noise. Its covariance matrices are:

$$\mathbf{Q} = E[\mathbf{w}\mathbf{w}^T] \tag{4.16}$$

$$\mathbf{R} = E[\mathbf{v}\mathbf{v}^T] \tag{4.17}$$

KF is based on prediction and correction. The prediction step has the goal to estimate the state and calculate the error covariance matrix using the data from the previous state. It is a predicted state based on physical model and previous state.

$$\hat{\mathbf{x}}_{kp} = \mathbf{A}\hat{\mathbf{x}}_{k-1} + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \tag{4.18}$$
$$\hat{\mathbf{P}}_{kp} = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \mathbf{Q}_k \tag{4.19}$$

The following step is update and correct. The measurement inputs are considered as well as the Kalman Gain so as to correct and update our state space estimation. The Kalman Gain is used so as to put more or less weight on the measurement or on the predicted state based on physical model. Furthermore, the process covariance matrix is updated.

$$K = \frac{\hat{\mathbf{P}}_{kp}\mathbf{H}}{\mathbf{H}\hat{\mathbf{P}}_{kp}\mathbf{H}^T + \mathbf{R}} \tag{4.20}$$
$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{kp} + K[\mathbf{Y} - \mathbf{H}\hat{\mathbf{x}}_{kp}] \tag{4.21}$$

and finally:

$$\hat{\mathbf{P}}_k = (\mathbf{I} - K\mathbf{H})\hat{\mathbf{P}}_{kp} \tag{4.22}$$

KF needs an initial value for the state and an error covariance defined so as to start operating.

Moreover, the system has to be observable in order to correctly apply the KF. That means that, for any possible evolution of state and control vectors, the current state can be estimated using only the information from outputs (information obtained by sensors).

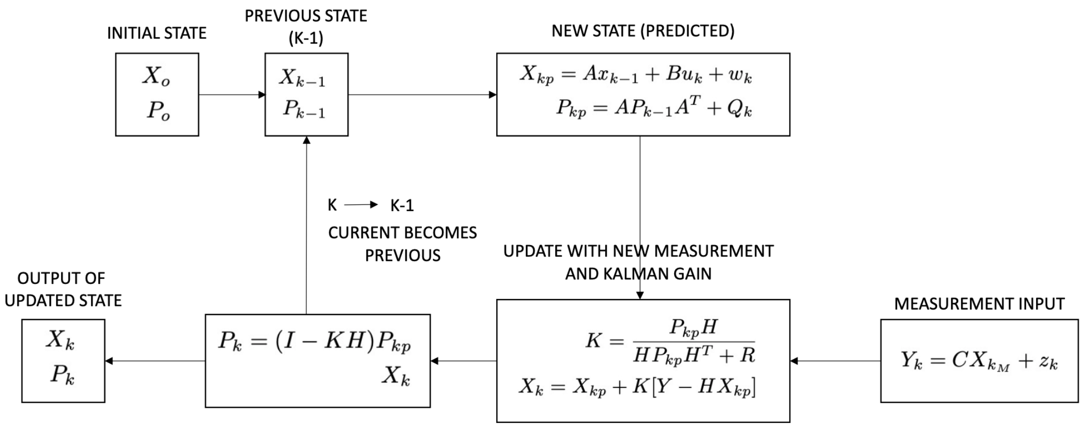

Figure 4.2: Kalman Filter equations schematic.

4.2.1 Dynamic Model with Kalman Filter

Having described all the necessary parts in order to simulate a vehicle and introduced the Kalman observer, the next step would be joining those algorithms so as to find out if our system is observable and could give us the estimations deserved.

The software that will be used with the aim of obtaining the longitudinal velocity, lateral velocity and yaw rate is Matlab/Simulink. It has been chosen as it gives such facilities when it comes to describe a system mathematically, make any operations, export and import data as well as IPG CarMaker is also based in this software, and so on.

The dynamic model describes the motion of a vehicle that result from forces and torques. Those forces and torques are produced by the tyres.

As it has been described previously in Section 3, Formula Student prototypes tend to have a high nonlinear behaviour due to its characteristics and the features of the tracks where different dynamic events are placed.

Consequently, being the KF a linear system, it was not considered so as to get the necessary estimations. Moreover, the system was not observable when yaw rate reached the value 0 and the lateral velocity diverges. Furthermore, working with sensors involves make all the required manipulations in discrete time making it more suitable for an EKF. Thus, because of all these reasons the KF was rejected.

4.3 Extended Kalman Filter

The EKF is a nonlinear system. At each sample instant, around the current operating point a linearization is done and then the same procedure is followed as in a KF.

The discretized space state is generally represented by:

$$\hat{\mathbf{x}}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k \tag{4.23}$$

$$\hat{\mathbf{y}}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k \tag{4.24}$$

The first thing that has to be done in order to build an EKF is discretization as linearization would take place around those points obtained. Hence, the state space should be:

$$\dot{x} \approx \frac{x_k - x_{k-1}}{T_s} \rightarrow x_k \approx \dot{x} - T_s + x_{k-1} \tag{4.25}$$

$$y = h(x, u) \tag{4.26}$$

Linearization allow us to transform A and C matrices as follows:

$$\mathbf{A}_k = \frac{\partial \mathbf{x}_k}{\partial \mathbf{x}}|_{x=x_{k-1}, u=u_k} \tag{4.27}$$

$$\mathbf{C}_k = \frac{\partial \mathbf{y}}{\partial \mathbf{x}}|_{y=y_k, u=u_k} \tag{4.28}$$

The filter keeps divided in two different parts. Prediction step would be:

$$\hat{\mathbf{x}}_{kp} = \mathbf{f}(\hat{\mathbf{x}}_{k-1}, \mathbf{u}) \tag{4.29}$$

$$\hat{\mathbf{P}}_{kp} = \mathbf{A}_k \mathbf{P}_{k-1} \mathbf{A}_k^T + \mathbf{Q} \tag{4.30}$$

And finally, the correction step:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{kp} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{h}(\hat{\mathbf{x}}_{kp}, \mathbf{u}_k)) \tag{4.31}$$

$$\hat{\mathbf{P}}_k = (\mathbf{I} - K_k\mathbf{C}_k)\hat{\mathbf{P}}_{kp} \tag{4.32}$$

$$k_k = \frac{\hat{\mathbf{P}}_{kp}\mathbf{H}_k^T}{\mathbf{H}_k\hat{\mathbf{P}}_{kp}\mathbf{H}_k^T + \mathbf{R}} \tag{4.33}$$

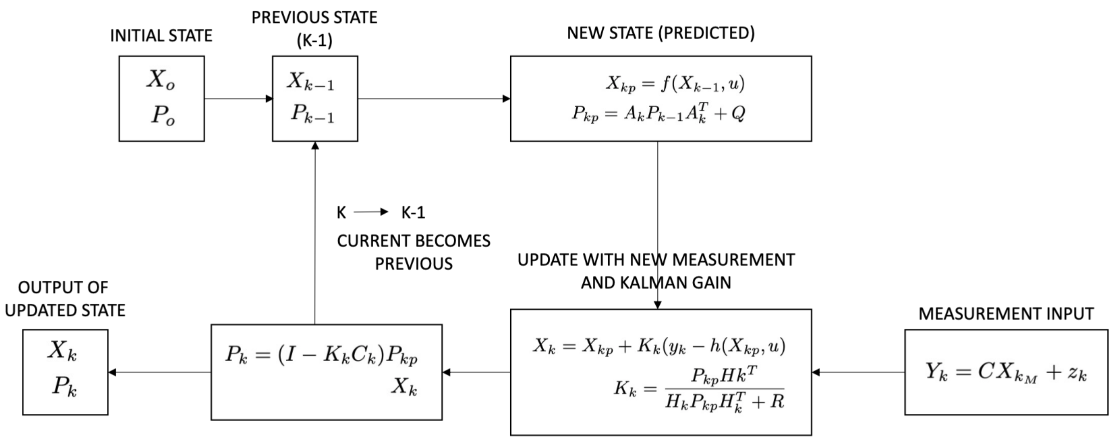

Figure 4.3: Extended Kalman Filter equations schematic

4.3.1 Dynamic Model with Extended Kalman Filter

As a result of the properties mentioned above the EKF was chosen. In addition, it does not require a high computational cost as might have the Unscented Kalman Filter [11].

The EKF is based on nonlinear system (4.12) where:

$$\begin{aligned}
\mathbf{x} &= [v_x, v_y, \dot{\psi}, C_{af}, C_{ar}]^T \\
\mathbf{u} &= [a_x, \delta, F_{y,Fx}, M_{z,Fx}]^T \\
\mathbf{y} &= [v_x, a_y, \dot{\psi}]^T
\end{aligned}$$

and vectors $\mathbf{f}$ and $\mathbf{h}$ are, respectively:

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} a_x + v_y\dot{\psi} \\ -\frac{C_{af}+C_{ar}}{mv_x}v_y - (\frac{C_{af}l_f - C_{ar}l_r}{mv_x} + v_x)\dot{\psi} + \frac{C_{af}}{m}\delta + \frac{F_{y,Fx}}{m} \\ -\frac{C_{af}l_f - C_{ar}l_r}{I_{zz}v_x}v_y - \frac{C_{af}l_f^2 - C_{ar}l_r^2}{I_{zz}v_x}\dot{\psi} + \frac{C_{af}l_f}{I_{zz}}\delta + \frac{M_{z,Fz}}{I_{zz}} \\ 0 \\ 0 \end{bmatrix} \tag{4.34}$$

and

$$\mathbf{h}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} v_x \\ -\frac{C_{af}+C_{ar}}{mv_x}v_y - \frac{C_{af}l_f - C_{ar}l_r}{mv_x}\dot{\psi} + \frac{C_{af}}{m}\delta + \frac{F_{y,Fx}}{m} \\ \dot{\psi} \end{bmatrix} \tag{4.35}$$

Measurements that are specified in $\mathbf{y}$ are the several information collected by the sensors. Those measurements correspond to the GPS and the IMU described in Section 5. It is known that sensors introduce a level of uncertainty because of the Gaussian noise, bias or orientation, among others. Thus, we may end up with bad measures or not as accurate as desired. All of that, introduce uncertainty to the state.

In order to simulate those measurements, the equations (3.8) and (3.12), provided by the Four-Wheel Model and the Bicycle Model respectively, have been taken. With the aim of simulating the uncertainty of the sensors, Gaussian blocks have been added to the longitudinal and lateral velocities, and yaw rate signals resulted from the output of the two models.

As it can be seen in the state vector, cornering stiffness is also estimated, updated. Tyre behaviour may change during a run (nonlinear tyre behaviour, road friction changes, etc.) so an adaptive linear tyre model is introduced [1]. Hence, as it has been remarked in this thesis, a good model of the tyre is crucial for the performance of this algorithm.

The inputs from the vector $\mathbf{u}$ are obtained by directly with sensors or calculated in the model, as could be the $F_{y,Fx}$ and $M_{z,Fx}$.

In order to implement the EKF the steps described in Subsection 4.1 must be done.

Bicycle Model

The first step so as to check if our observer would work, or we might correct anything, is done by introducing the outputs provided by the Bicycle Model Subsection 3.4. Should we observe that the mathematical equations in the model that should provide us with each state are not the same as the ones we have in the EKF.

From the outputs of the model are obtained the signals simulating the sensors as well as obtaining the parameters known by the car.

In this first attempt for checking the behaviour of our observer a quite simple model is considered. The Bicycle Model is contemplated as just front-wheel drive. The necessary inputs for this model are the steer angle (δ) and the longitudinal forces from the tyres, but as there is just traction in the front wheel, the longitudinal rear force is neglected.

As well as with the models, it is started with a simple simulated maneuver and then a more complex one is added. It allows us to make sure that there are not major problems. Firstly, a steering wheel stroke is simulated using a pulse generator block. The input δ is this pulse generator and the longitudinal force is simulated by a PI over the longitudinal velocity state. Finally, a sinusoidal maneuver is made so as to check if the observer responds quickly to these changes of direction.

The first simulation is done with the parameters described above and emulating a steering wheel stroke maneuver.

In this first simulation we can see a very correct estimation of v_x. For this signal, we have a good reading of the sensors and the uncertainty is low. The error seen is caused by the white Gaussian noise.

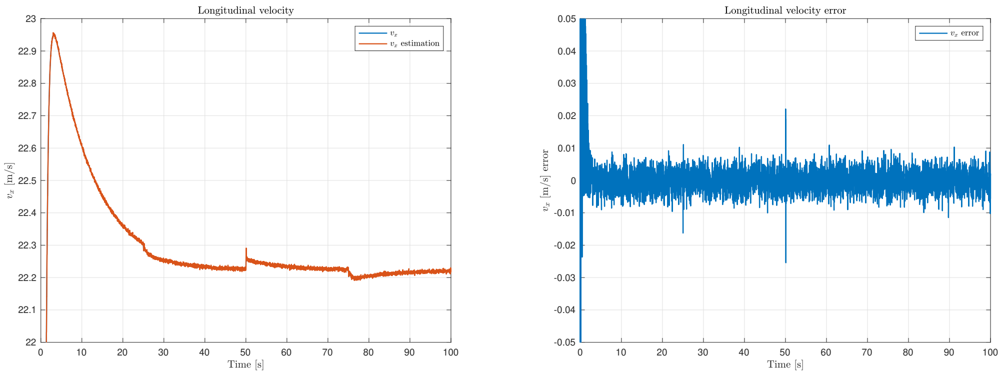

Figure 4.4: Longitudinal velocity. Bicycle Model.

Lateral velocity was the critical point of the simulation as it is where the greatest degree of uncertainty lies. As it can be seen, there are some peaks in the estimator when there are sudden changes in direction. The observer converges very quickly and suggests that it is well-formulated and well-tuned. The error is caused by the noise and peaks mentioned.

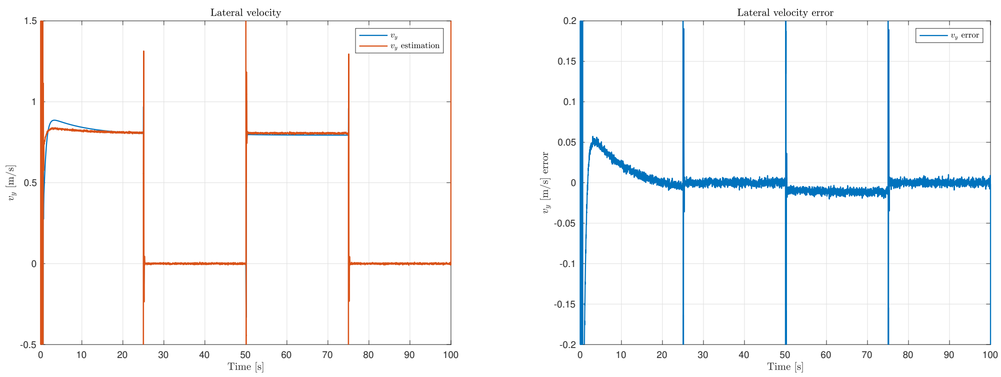

Figure 4.5: Lateral velocity. Bicycle Model.

Yaw rate has the same degree of uncertainty as longitudinal velocity but is also sensitive to changes such as v_y. The error is due to the same factors mentioned before.

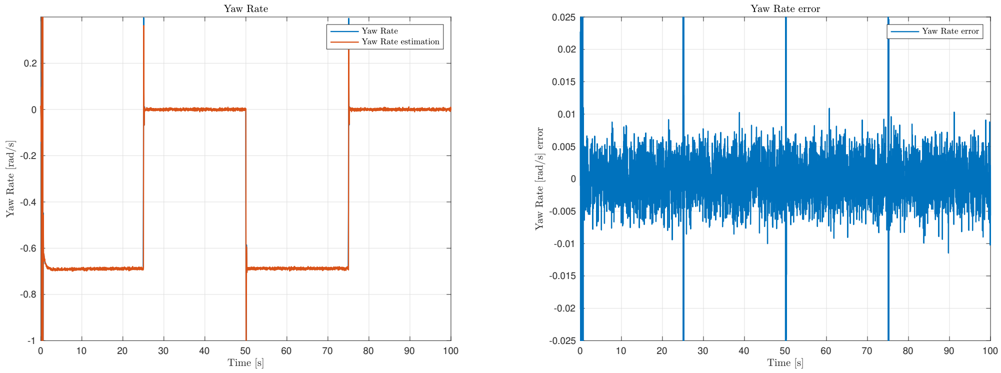

Figure 4.6: Yaw rate. Bicycle Model.

Now shall we emulate a sinusoidal steering wheel maneuver so as to make it more complex. For longitudinal velocity we obtain the same results as with the simplest simulation.

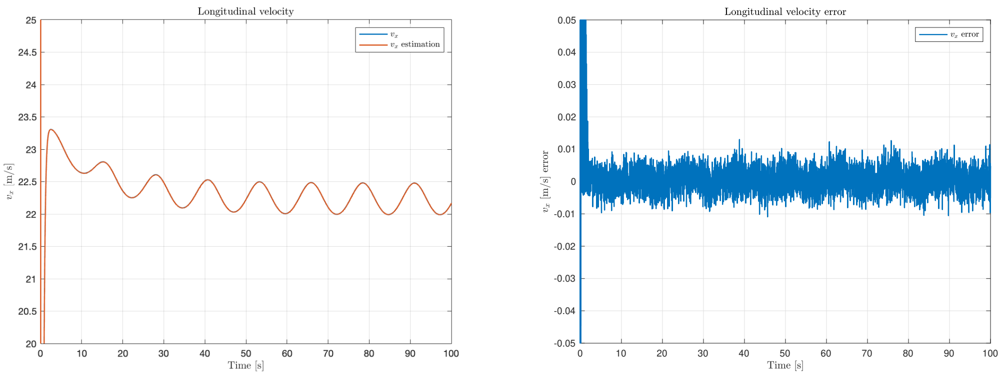

Figure 4.7: Longitudinal velocity. Bicycle Model.

Talking about lateral velocity, it can be seen in Figure 4.8 a remarkable offset at the beginning. This offset decrease in the following turns but do not converge to the reference signal. This offset might be caused by the suddenness of the movement as the simulated model reaches great lateral velocities.

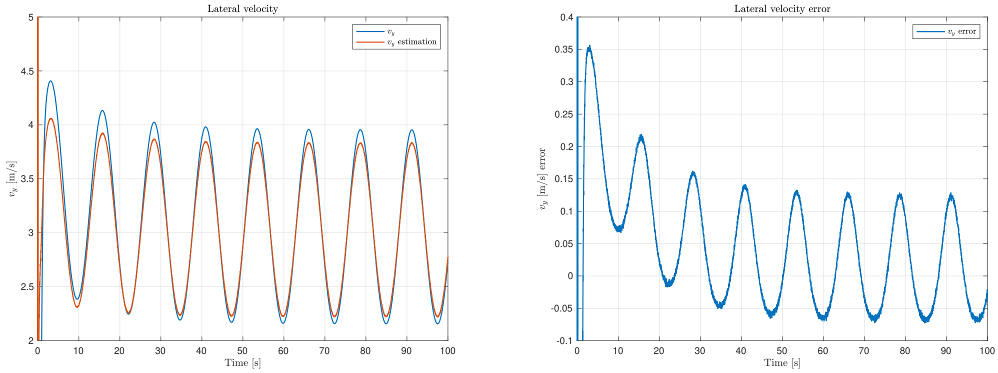

Figure 4.8: Lateral velocity. Bicycle Model.

Lastly, for yaw rate the same conclusions as in the last simulation can be taken.

Hence, it is considered that the model and the EKF seems to be working as desired and a more complex model could be introduced.

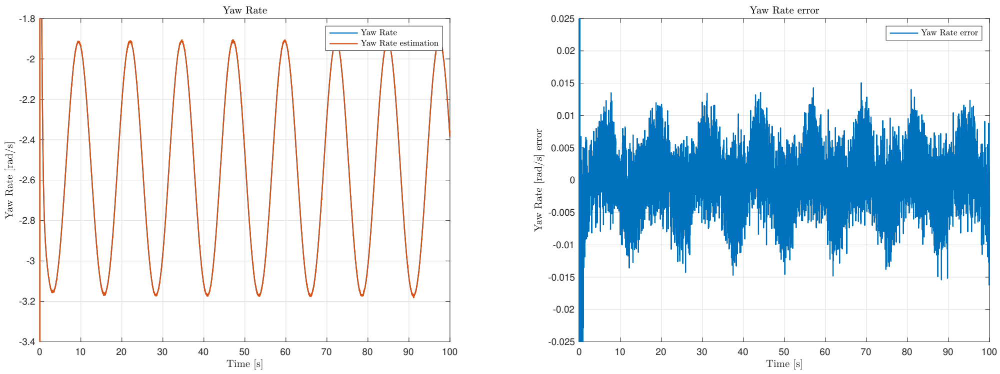

Figure 4.9: Yaw rate. Bicycle Model.

Four-Wheel Model

In this model we follow the same philosophy described in the Bicycle Model.

The differences we find with the previous simulation are that in this model we introduce all-wheel drive. That is, the longitudinal forces of both axes are taken into account. In addition, the angle of rotation of the two front wheels are considered.

These inputs, that are needed to introduce the movement to the model, are simulated in the same way as previously done in Bicycle Model.

Once the changes have been introduced, simulations will be run in the same order as before. Once again, it will be started with a steering wheel stroke maneuver simulation.

Despite the mentioned changes that the model has suffered, v_x is significantly good and no accuracy has been lost.

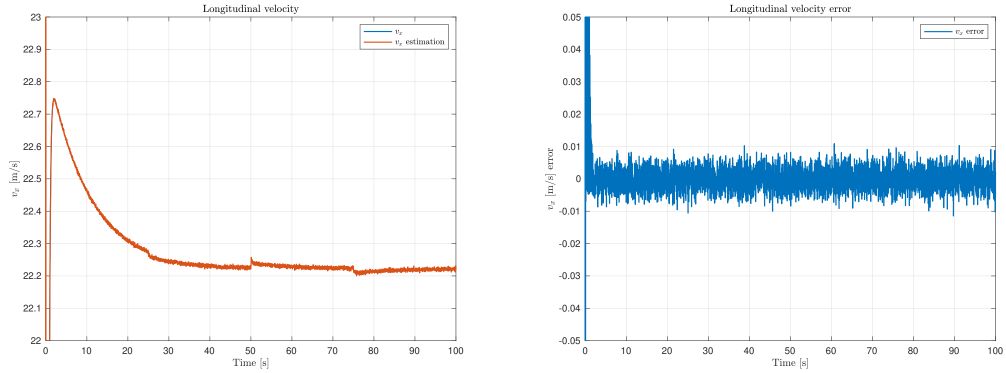

Figure 4.10: Longitudinal velocity. Four-Wheel Model.

For v_y we also obtain fantastic results.

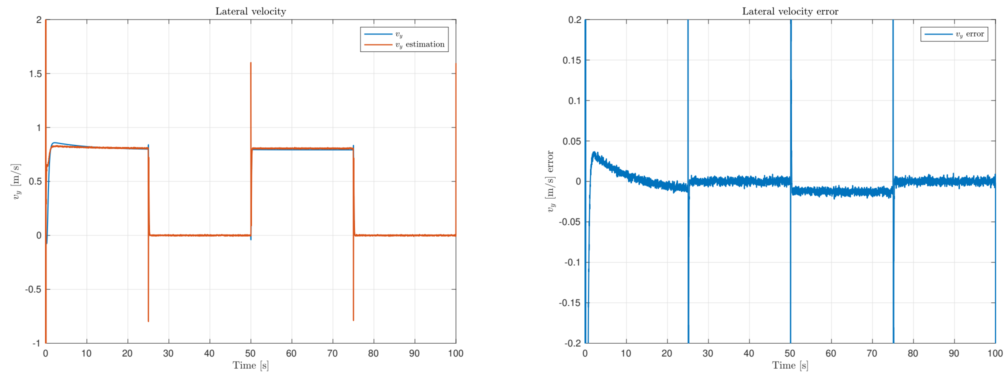

Figure 4.11: Lateral velocity. Four-Wheel Model.

With the yaw rate it is confirmed that the first simulation with the Four-Wheel Model has given outstanding results.

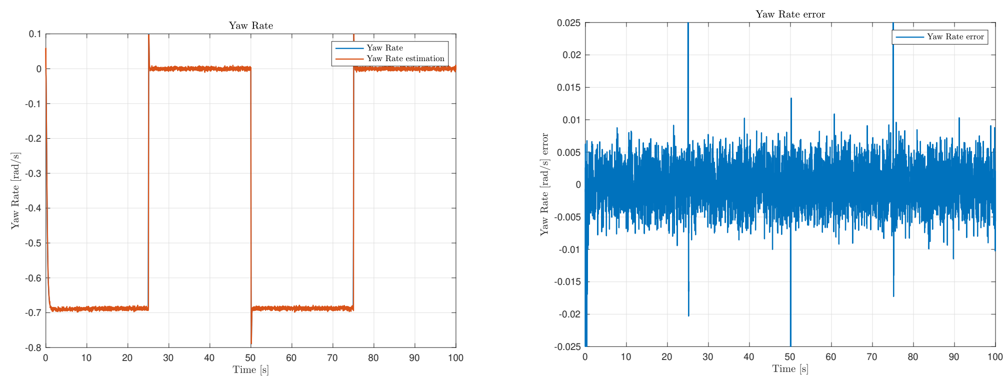

Figure 4.12: Yaw rate. Four-Wheel Model.

Emulating a sinusoidal steering wheel maneuver, fantastic results are obtained for longitudinal speed. The error is only produced by the Gaussian white noise in its great majority.

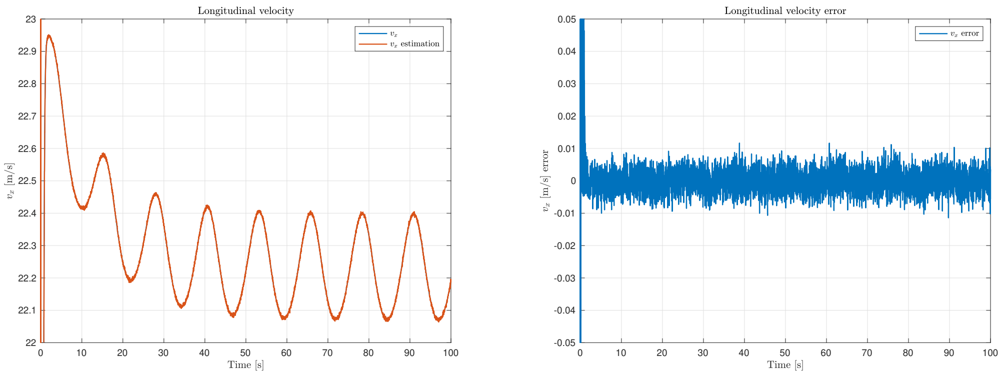

Figure 4.13: Longitudinal velocity. Four-Wheel Model.

Moving on to lateral velocity, the results are good enough, but the first offset is striking and then the observer is able to correct. However, it is replicated to a lesser extent in the following minor amplitudes. This error may be introduced by the divergence in the model and EKF formulas. Furthermore, lateral velocity reaches great values with sudden changes, this maneuver could be useful for calibration but might be impossible in reality.

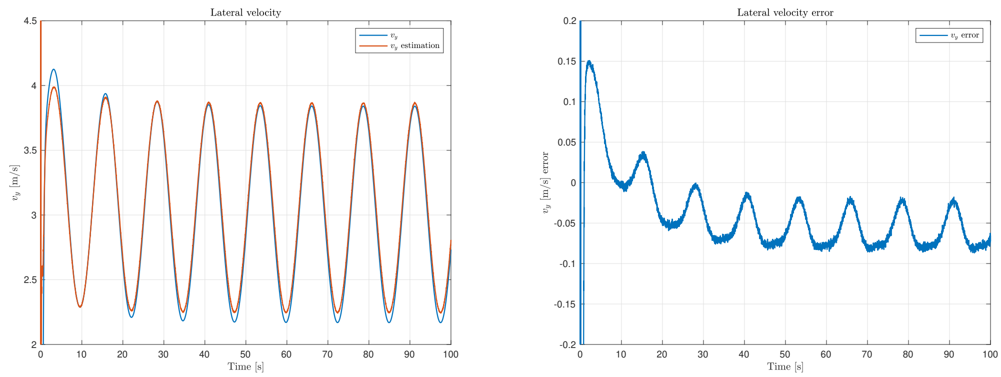

Figure 4.14: Lateral velocity. Four-Wheel Model.

With the following great estimation of the yaw rate we can end the modelling of the vehicle and the tuning of the EKF. We seem to have a good basis for testing the observer with data extracted from IPG CarMaker.

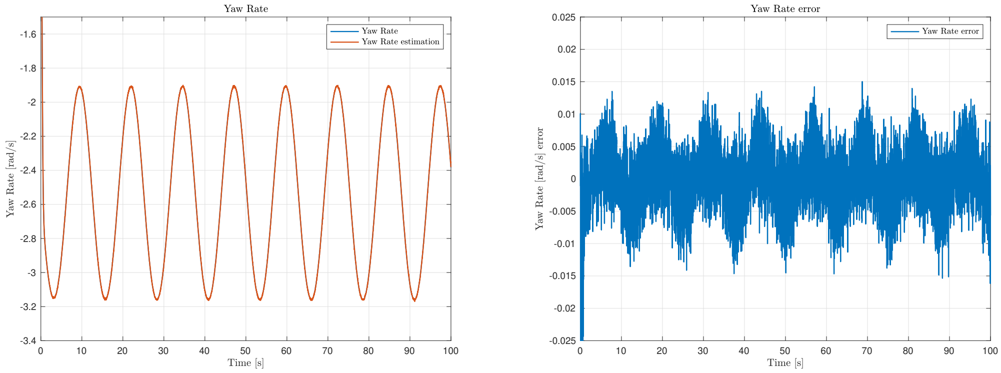

Figure 4.15: Yaw rate. Four-Wheel Model.

5 Vehicle Data Analysis

In order to proof that every upgrade that it is done in the car works as expected, a lot of data has to be taken, checked and compared in order to see if the results are positive. In motorsport, sensors and actuators play a vital role on the performance of the car. Without them, we would never know with certainty how much progress have been done with all the upgrades that members of the team have been working on for so long. Hence, gather all the information possible is crucial to stop as soon as possible the stopwatch.

5.1 Data Acquisition

So as to transmit all the information collected from the sensors the Controller Area Network (CAN) communication protocol is used. Moreover, so as to keep all these data provided from the different sensors and allow the team to analyse all the information offline, the car is equipped with a data logger. Furthermore, the team also has a real time telemetry module in which every member of the team can personalise the information that wants to consult.

All the sensors that are used in the car have both analytic and controllable purposes. Due to the specific characteristics of this type of cars, a reasonable study has to be done in order to find out which sensors are more suitable to give us a particular information. Additionally, the price must be taken into account as sensors are a significant inversion and agreements with companies are crucial so as to achieve them.

5.2 Sensors and actuators

Inertial Navigation System - INS

The INS that our team is using is the Ellipse 2-N from SBG Systems. This sensor is placed below the seat of the driver so as to be as closer as possible from the centre of gravity of the car, as all the calculations are done from this reference point. Ellipse 2-N provides us with some basic but essential information as are the $\dot{\psi}$, v_x, v_y, a_x and a_y. Furthermore, it is also equipped with an internal GNSS (Global Navigation Satellite System) receiver so as to determine the position of the car inside the track.

Figure 5.1: SBG: Ellipse 2-N

DUAL GNSS/INS

The VN-300 is a miniature, high-performance Dual Antenna GNSS-Aided Inertial Navigation System (Dual GNSS/INS) that combines MEMS inertial sensors, two high-sensitivity

GNSS receivers, and advanced Kalman filtering algorithms to provide optimal estimates of position, velocity, and attitude.

Due to the union of the driverless and electric manual team, we can count on solutions and sensors that were used in the two previous prototypes. Because of that, we will also equip a VN-300 DUAL GNSS/INS from Vectornav. It allows us to measure the velocity before 15 km/h are reached, the GPS frequency update is better as well as the accuracy because of the dual antenna.

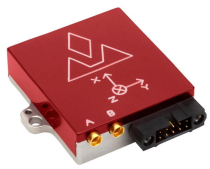

Figure 5.2: VECTORNAV VN-300 DUAL GNSS/INS

Inertial Measurement Unit – IMU

The IMU which is installed in the car is provided by ACEINNA. The ACEINNA MTLT305D is a dynamic tilt sensor, 3D accelerometer and 3D Rate Sensor (Gyro). It is used to accurate the process of data analysis.

Figure 5.3: ACEINNA MTLT305D

Linear Displacement Sensor

Linear Displacement Sensor model CLS 0952 from Active Sensors. This type of sensor is used to determine the elongations of the suspension springs, the measurement of the steering rack position to know the steering wheel angle and measure the torque command from the driver through the pedal pedal position.

Figure 5.4: Linear Displacement Sensor: Active Sensors CLS 0952

Resolver

Resolver RE-15 model from LTN Servotechnik. This sensor is capable of measuring the electric motor angular speed, vital for slip ratio calculations or traction control algorithm among others.

Figure 5.5: Resolver RE-15

Electric motor

From CAT12e our vehicles are equipped with 4 electric motors. The team chose the Fischer electric motors which are specifically designed for Formula Student vehicles. Its peak power is 35 kW with a maximum available torque of 29 Nm and maximum speed of 18000 rpm. Nevertheless, any team is allowed to reach more than 80 kW for more than 500 ms due to competition rules.

Inverter

Our vehicle is equipped with two MOBILE DCU 60/60 double inverters for the control of the electric motors. One inverter per axle.

Figure 5.6: LENZE MOBILE DCU 60/60

5.3 Data analysis software: Gems Data Analysis Pro

As it is vital to get as much data as possible, it is also crucial to know how to interpret the information taken from the different sensors. A great knowledge of the signals received makes an important step forward in order to understanding the behaviour of the vehicle and identify quickly and precisely any problem that could be happening.

Thankfully, for the purpose of studying and verifying all these data, the team can rely on the software Data Analysis Pro (GEMS Performance Electronics). It allows to import and treat the logged data from the vehicle and organize that data depending on their purposes. Hence, we can graph any information in the way which is more important for us or organize several signals depending on the section that we are interested in.

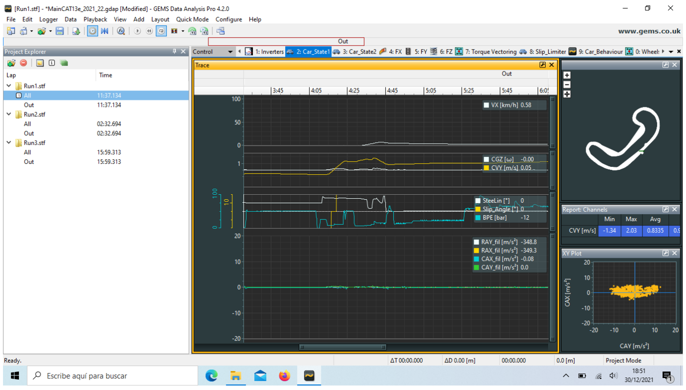

Figure 5.7: GEMS Data Analysis Pro

The figure depicts the specific windows for Vehicle Controls Department.

5.4 Data simulation software: IPG CarMaker

Nevertheless, if any algorithm is being developed and it has finished the first proofs in Matlab environment, we own the possibility to tune this algorithm with a more complex model provided by IPG. IPG CarMaker is a software that enable us to define mathematically our car, from the behaviour of the wheels, suspension parameters, unsprung masses and so on. Furthermore, we can simulate the different sensors that are set in the real car and obtain measures from these ones directly. Hence, we can simulate a real situation and verify that our algorithm works as expected, and if not, tune it so as to obtain the behaviour desired.

Having this tool correctly set is key as it enable us to verify several algorithms without putting in danger a pilot. Moreover, meanwhile the car is being built, our department, Vehicle Controls, can keep working on different implementations in order to get ready when the car is ready to run, as well as other departments may use it to proof its algorithms too.

Briefly, IPG CarMaker allow us to obtain direct measurement from an accurate model with the aim of proving our algorithms.

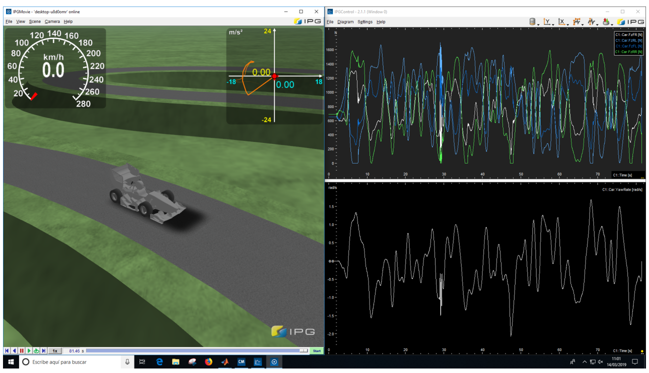

Figure 5.8: IPG CarMaker simulation.

6 Experiments and results

One of the principal objectives of this report was to design an estimator able to work with data obtained from IPG CarMaker so as to compare the results with the information given by the mentioned software as well. Because of this comparison, a more accurate tuning of EKF could be achieved as the simulated model is undoubtedly more complex than the models introduced in Section 3.

Shall we mention that the measurements and inputs taken in order to obtain an estimation have been realized under another Matlab/Simulink based on IPG CarMaker and exported to the script that has already been used in previous simulations. The software allows us to have direct measurements from the longitudinal forces of the wheels, steering wheel angle, longitudinal acceleration, self aligning moment, longitudinal velocity, lateral acceleration and yaw rate.

Nevertheless, all the features of our car introduced in IPG CarMaker were lost at the beginning of this season due to a computer problem. The team has been working hard so as to establish the main basis in order to run simulations. However, the tyre model used by the software is still different from the one the team uses, as well as the suspension and other elements. Moreover, the sensors that could be simulated have a different frame of reference and we are still understanding how to extract the information as we did the last season.

On the other hand, this experiment could give as an extraordinary base to work on the future once we correctly adjust the software and check if our estimate is able to work with high nonlinearities.

The layout of the circuit where all the data has been extracted is the following:

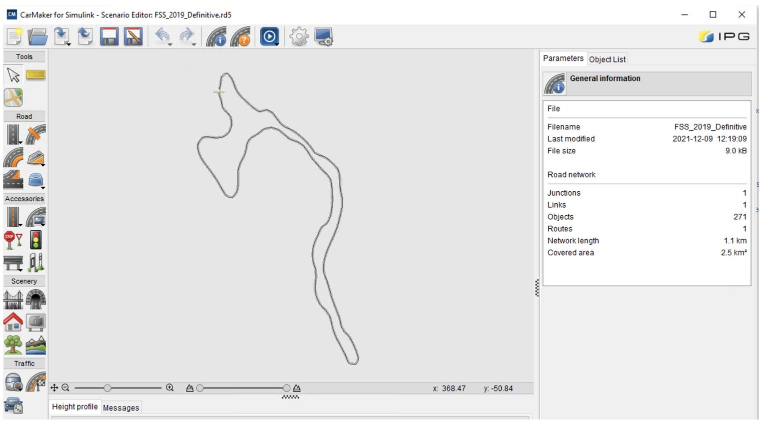

Figure 6.1: Data taken from this circuit layout. FSS 2019.

6.1 Longitudinal velocity

As it has been introduced at the beginning of this chapter, several problems have been faced so as to obtain the correct measures from the software. In order to verify that the longitudinal velocity signal was correctly given by CarMaker, a very simple circuit was previously designed so as to check easily if the velocity obtained was logic. Once it was verified, this signal was taken as a known state of the vehicle provided by the sensors described in Section 5, any noise was added to the Simulink.

Afterwards, matrix **Q** and **R** were adjusted to obtain the more accurate estimation possible:

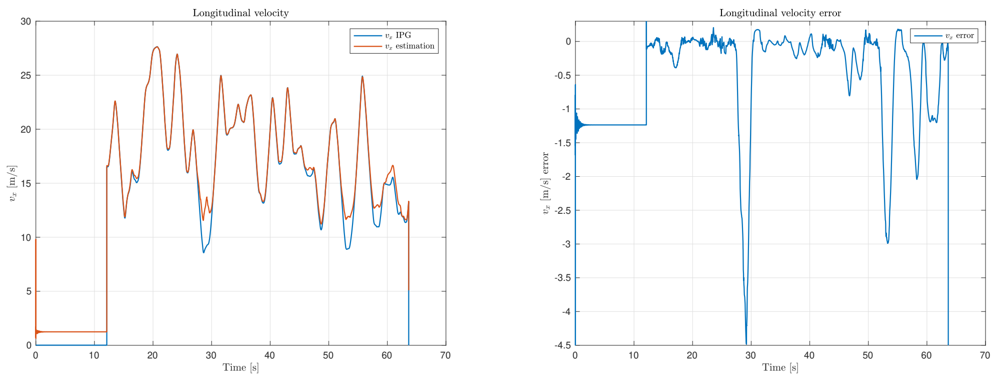

Figure 6.2: v_x obtained from IPG compared with v_x estimated and its error.

6.2 Lateral velocity

Lateral velocity is the more sensitive signal in relation with the discrepancies in tyre model and the global parameterization of the vehicle. Despite all the singularities that we have cope with, the results obtained shows a similar reactions to the data obtained by the software, which invite us to believe that our estimate is capable of obtaining a great estimation while the knowledge of the tyres is accurate. The results would be better if there was not a major discrepancy between the tyre model of the CarMaker and the model introduced in the EKF.

The lateral velocity signal is useful for estimating the slip angle of the car, needed for tuning the Torque Vectoring. Furthermore, if the signal is outstandingly precise it can be used for localizing the vehicle in the circuit. Finally, it could be seen through a bend if the vehicle has a stable cornering if the values are constant.

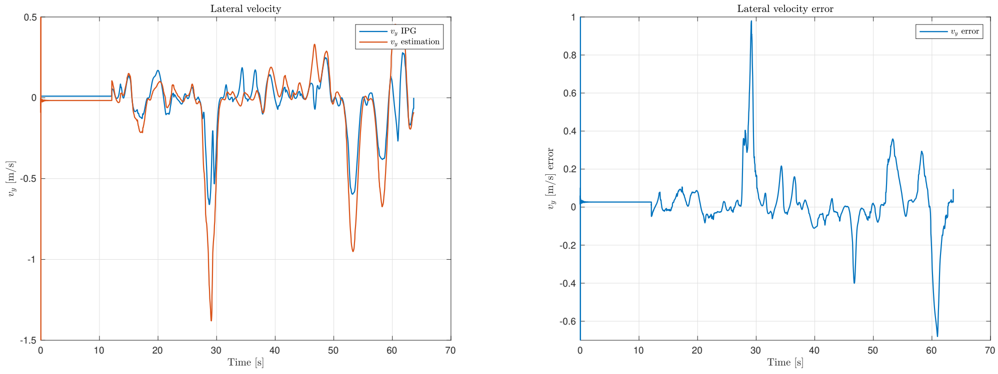

Figure 6.3: v_y obtained from IPG compared with v_y estimated and its error.

6.3 Yaw rate

As well as in longitudinal velocity, yaw rate is measured by the sensors and it is known to have a great accuracy. Hence, the matrices **Q** and **R** were adjusted particularly so as to obtain the following results:

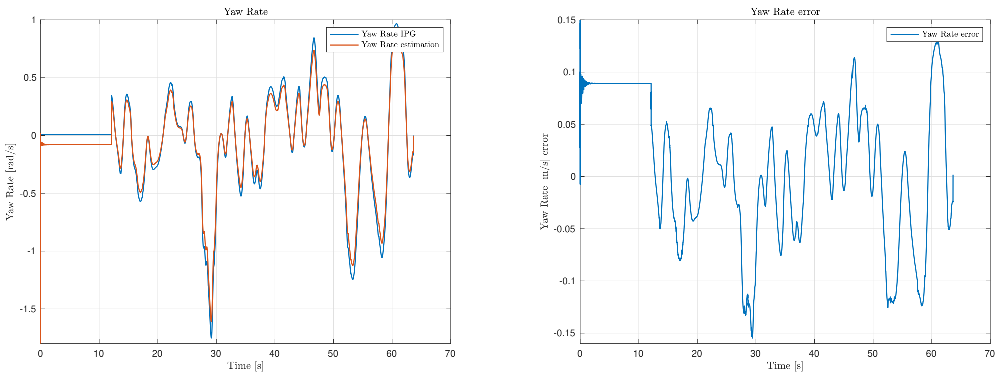

Figure 6.4: Yaw rate obtained from IPG compared with yaw rate estimated and its error.

Yaw rate is used in some way to change the behaviour of the car through an optimization problem. If it is detected that the car does not enter in the bend as desired, e.g if it has understeer, monitoring the yaw rate and adjusting a constant it is possible to modify the yaw motion and sort the understeer out.

7 Budget

This chapter brings a high-level economic analysis of the whole project. The tables 7.1 and 7.2 contain the analysis of the main tasks done during the realization of this thesis and the principal resources needed to carry it out.

Becoming a member of a Formula Student team involves self-learning. The great majority of knowledge needed is far away from the scope of the degrees done. Moreover, Vehicle Controls department is one of the newest of the team. Therefore, the basis of this part of the team are not firmly set and becomes a fantastic challenge to build them and may indicate a way on how things should be done. As a consequence, hundreds of hours have been devoted in order to understand, test, fail and keep on repeating the same steps so as to finally get some potential great results. Hence, several meetings must be done with the professor associated in order to organize many ideas and learn new algorithms or skills as well as to ensure that everything was understood.

Thankfully, we can count on an outstanding building where we have many amenities as all the computers, softwares which are provided by huge and important companies and the help of our team-mates. The models have been designed with Matlab/Simulink and IPG CarMaker was used in order to verify with more complex inputs the behaviour of the observer.

The total cost of the project is 20.699,30€.

Project analysis	Hours spent	€/h	Total (€)
Study and understanding	160	20	3.200,00€
First presentations	8	20	160,00€
Recording Documents	10	20	200,00€
Explanation of supervisor	15	20	300,00€
Research	200	20	4.000,00€
Design of bicycle model	20	20	400,00€
Design of four-wheel model	28	20	560,00€
Implementation of EKF	70	20	1.400,00€
Simulations	30	20	600,00€
Import and extract data from IPG	40	20	800,00€
Validation	30	20	600,00€
TOTAL	**611**		**12.220,00€**

Table 7.1: Cost of the project analysis.

Project resources	Total Months	€/Month	Total (€)
Laptop	4	187.5	750,00€
MATLAB/SIMULINK	4	35	140,00€
IPG CarMaker	4	333.3	1.333,30€
LaTeX	4	14	56,00€
Office computer	4	50	200,00€
Office	4	1500	6.000,00€
TOTAL		**2.119,80€**	**8.479,30€**

Table 7.2: Cost of the project resources.

8 Conclusions

The main goals of this project were to develop different models so as to verify the Kalman Filters' design and estimate longitudinal and lateral velocity as well as the yaw rate with data from IPG CarMaker.

Firstly, understanding the technology of virtual sensing and its necessity was important so as to realize how could be approached and its constraints, as all the systems are not observable.

Once we had the main base was studied, the observers KF and EKF were introduced. Being KF a linear system, allowed us to understand easier the main idea of this type of observer and learn its limitations. From here, we were able to devote our time into a nonlinear system as EKF is.

In order to verify the design of our observers, we previously had to study the fundamental vehicle dynamics as well as the tyre's behaviours for the purpose of modelling a vehicle. This objective was approach through two different models, the Bicycle Model and the Four-Wheel Model. The first one is a simplification of a vehicle that allow to check the observers implementation in an easier way, for finally prove them in a more sophisticated system. There are more complex models, but as this thesis is focused on planar motion, were not considered.

The first approach for corroborating the correct design of the observers was carried out with simple inputs that were done by simulating a simple bend or emulating a sinusoidal trajectory through Matlab/Simulink. The errors of this estimates did not get over the value of 0.15.

Finally, with the help of IPG CarMaker a significant more complex model of a vehicle was used. The simulation consisted on this model taking a lap in a real circuit of Formula Student. This simulation provided the necessary information in order to compare the results as well as the inputs so as to be introduced in the EKF. Due to the discrepancy in the tyre model between the model of IPG and the one used in the observer, lateral velocity estimation did not converge with the accuracy desired. Nevertheless, it has been proven that the EKF's design can be useful once exists a correlation between those models.

9 Future Work

For the upcoming months, the team will keep on working on the modelling of the IPG CarMaker and designing a clear interface for the new PU so as to detect issues easier and identify the signals of interest to be shown in GEMS Data Analysis Pro.

Before keeping on working in the algorithm presented in this thesis, is vital to understand how our model and the sensors used are working. This exercise will allow us to identify better our principal drawbacks and how to manage the different signals provided by the sensors.

As a consequence of this work, the design of the EKF would be able to present a more suitable results in IPG CarMaker and lastly, to be proven in the car.

Moreover, this thesis has also provided a methodology in order to prove future algorithms. Having a clear methodology of how new updates should be tested will also help us to be more efficient and faster.

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