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Efficient Analysis of Sheets with Nonzero Thickness

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***Abstract*— The conventional scattering analysis of perfectly conducting plates neglects the scattering contribution of the rim in the discretization of the Electric-Field Integral Equation. This so-called thin-plate scheme manages many less unknowns than the full approach, arising from modelling the whole plate, with acceptable accuracy in many practical applications. A recent approach, so-called thick-plate, has proved to show similar accuracy as the full scheme, also in those cases where the thin-surface fails; namely, the scattering analysis of thick enough plates, especially under oblique incidences, with low grazing angles. In this paper, we reveal how the thick-plate scheme shows improved computational times, especially in large scale computations, as compared to the full approach. Also, we discuss how the thick-plate analysis is amenable to parallelization, thereby leading to computational times comparable with the thin-surface approach.**

Keywords—Integral Equations, Method of Moments.

I. Introduction

The electromagnetic scattering analysis of perfectly conducting (PEC) plates with very small thickness is normally undertaken through the thin-plate approximation [1]. This scheme derives from the definition of the Electric-Field Integral Equation (EFIE) as the plate thickness tends to zero. In the limit, this Integral Equation degenerates into a problem where the unknowns involve the sum of the currents over the top and bottom faces of the plate [2]. The plate is then modelled as an open surface, thereby discarding the scattering contributions due to the rims. The thin-plate approximation becomes very advantageous because of the drastic reduction of unknowns with respect to the full modelling of the plate as a closed surface. In general, the thin-plate approximation can be adopted in many practical applications, with little sacrifice of accuracy. However, in cases where the effect of the plate rim on the scattering pattern becomes critical, such as the scattering analysis of thick enough plates, with low-angle grazing incidences, the thin-plate approximation becomes inaccurate. Recently, a new so-called thick-surface scheme has proved to show accurate results, also for those cases where the thin-plate scheme fails [3],[4]. In this paper, we discuss on the computational efficiency of the thick-plate scheme as compared with the full method-of-moment (MoM) approach. We reason the suitability of the thick-plate scheme in the concurrent analysis of plates with large electrical dimensions.

II. Thick Plate

Consider the scattering analysis of a free-standing square PEC plate with nonzero thickness. The faces and rim of the plate are discretized with rectangular facets, thereby giving rise to a mesh with N_E edges. The current is expanded with the divergence-conforming constant-normal linear-tangential rooftop basis functions, $\{\boldsymbol{t}_n\}$ [3], defined over pairs of adjacent rectangles.

The approximated electric-field boundary condition over the surface boundary S of the plate yields

$$\left[\tilde{\boldsymbol{E}}^s\right]_{\tan,\boldsymbol{r}\in S} = -\left[\boldsymbol{E}^{inc}\right]_{\tan,\boldsymbol{r}\in S} \tag{1}$$

where $\boldsymbol{E}^{inc}$ and $\tilde{\boldsymbol{E}}^s$ denote, respectively, the incident field and the approximated scattered field, which is defined as

$$\left[\tilde{\boldsymbol{E}}^s\right]_{\tan,\boldsymbol{r}\in S} = \begin{bmatrix} -jk\eta\sum_{n=1}^{N_E} c_n \iint_{Q_n} G(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{t}_n(\boldsymbol{r}')dS' \\ -j\dfrac{\eta}{k}\sum_{n=1}^{N_E} c_n \iint_{Q_n} \nabla G(\boldsymbol{r},\boldsymbol{r}')\nabla'\cdot\boldsymbol{t}_n(\boldsymbol{r}')dS' \end{bmatrix}_{\tan,\boldsymbol{r}\in S} \tag{2}$$

where $\{c_n\}$ stand for the set of unknowns, whereas G , k and η denote the free-space Green's function, wavenumber and impedance. The matrix system results from the rooftop testing over the rectangular tessellation over S of the condition in (1).

Our thick-plate scheme applies the condition in (1) only over the N_R duplets of rectangles sharing mesh edges that lie inside the plate rim or that match the outer boundary line of the rim (see Fig. 1-(a)). The subset of functions $\{\boldsymbol{t}_1,...,\boldsymbol{t}_{N_R}\}$ ($\subset\{\boldsymbol{t}_1,...,\boldsymbol{t}_{N_E}\}$) are required for testing purposes over the rim mesh. The thick-plate scheme does not make use of the field condition (1) as such over the edges arising in the top- or bottom-face meshes of the plate. Instead, two different electric-field conditions are established over the so-called mid-surface S_M, located inside the plate (see Fig. 1 (a)),

$$\tilde{\boldsymbol{E}}^s(\boldsymbol{r})\Big|_{\tan,\boldsymbol{r}\in S_M} = -\boldsymbol{E}^{inc}(\boldsymbol{r})\Big|_{\tan,\boldsymbol{r}\in S_M} \tag{3}$$

$$\frac{\partial}{\partial z}\left(\tilde{\boldsymbol{E}}^s(\boldsymbol{r})\right)\Big|_{\tan,\boldsymbol{r}\in S_M} = -\frac{\partial}{\partial z}\left(\boldsymbol{E}^{inc}(\boldsymbol{r})\right)\Big|_{\tan,\boldsymbol{r}\in S_M} \tag{4}$$

where the variable z evolves normally with respect to S_{M} (z=0). The field conditions in (3) and (4) are tested with a set of rooftop basis functions $\left\{ \boldsymbol{t}_1^M ... \boldsymbol{t}_{N_M}^M \right\}$ straddling the N_M interior edges arising from the rectangular tessellation of S_M.

The thick-plate MoM-system then yields

$$\iint_{Q_r} \boldsymbol{t}_r \cdot \tilde{\boldsymbol{E}}^s(\boldsymbol{r})\, dS = -\iint_{Q_r} \boldsymbol{t}_r \cdot \boldsymbol{E}^{inc}(\boldsymbol{r})\, dS \qquad (5)$$

$$\iint_{Q_m^M} \boldsymbol{t}_m^M \cdot \tilde{\boldsymbol{E}}^s(\boldsymbol{r})\, dS = -\iint_{Q_m^M} \boldsymbol{t}_m^M \cdot \boldsymbol{E}^{inc}(\boldsymbol{r})\, dS \qquad (6)$$

$$\iint_{Q_m^M} \boldsymbol{t}_m^M \cdot \frac{\partial}{\partial z}\left(\tilde{\boldsymbol{E}}^s(\boldsymbol{r})\right) dS = -\iint_{Q_m^M} \boldsymbol{t}_m^M \cdot \frac{\partial}{\partial z}\left(\boldsymbol{E}^{inc}(\boldsymbol{r})\right) dS \qquad (7)$$

$$1 \le r \le N_R, \quad 1 \le m \le N_M$$

where Q_m^M represents the pair of rectangular facets that share the m-th interior edge at the mesh over S_M.

Furthermore, our thick-plate scheme relies on the rearrangement of the rooftop-expanded current contributions in terms of their z-even or z-odd mid-surface symmetric contributions [6]. Namely; the "sum" and "dif" contributions, as we call them, arise from summing or subtracting, respectively, pairs of mirror contributions over the upper and lower halves of the rectangular mesh around the plate (see Fig. 2) [3]. This allows the transformation of the matrix system in (5) , (6) and (7) into two decoupled systems with roughly half the unknowns of the original full problem. The z-even / z-odd matrix systems arise from the application of the field conditions in (6) / (7) and the summation / subtraction of the N_B mirror pairs of rim-tested conditions in (5) [3]. The self-symmetric N_S rim-tested contributions in (5) straddling the top and bottom halves are appended to the corresponding matrix system, with shared symmetry. Therefore; whereas the full MoM-system handles N_E unknowns ($2xN_M+2xN_B+N_S$), the uncoupled systems manage either N_M+N_B or $N_M+N_B+N_S$ unknowns. This is computationally more efficient, especially for a big amount of unknowns, and can be easily parallelized.

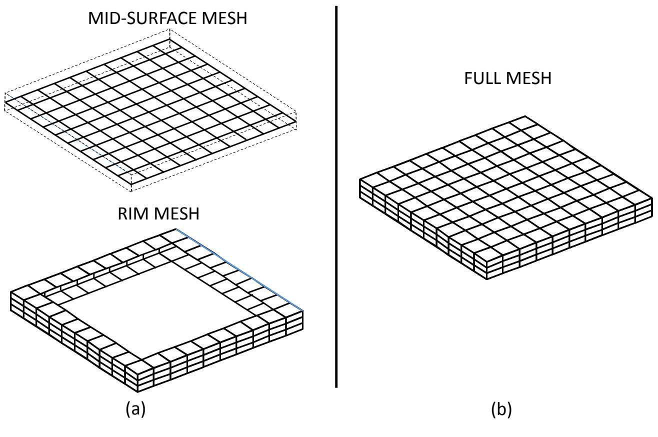

Fig. 1. Rectangular meshes adopted for the following MoM-schemes: (a) Thick-plate; (b) Full.

III. Computational Effort

We have shown the accuracy of our thick-surface approach in the scattering analysis of flat or slightly curved surfaces with nonzero thickness [3],[4]. Our thick-plate scheme reaches very similar accuracy as the full MoM-scheme in cases, with big enough thickness or under small-angle grazing incidences, where the thin-surface fails completely. As regards the computational effort, the thick-plate scheme clearly outperforms the full MoM-scheme in the following aspects:

- Mesh generation: A symmetric rearrangement for the full MoM-scheme, where the whole structure is meshed (see Fig. 1 (b)), involves grouping top-bottom pairs of mirror facets over the whole mesh. In contrast, the thick-plate approach may be constructed from a simpler mesh, with separate meshes for the rims and the mid-surface (see Fig. 1 (a)). The search of the top-bottom symmetric facets then needs to be carried out only over the rim mesh. This becomes advantageous for big electrical dimensions of the thick plates, since the amount of face-contributions becomes bulky. With the thick-surface approach, though, the search of these symmetric contributions over the top and bottom face meshes is circumvented because one single mesh, over the mid-surface, is adopted.
- Computation of the solution: the sequential solution of the two matrix systems arising in the thick-surface scheme, with roughly half the number of unknowns, is less time-consuming than the solution of the single matrix of the full MoM-approach. This is more so in large scale computation, for electrically big plates with nonzero thickness. If the solution of the systems arising in the thick-surface approach is undertaken concurrently, the computational time becomes even comparable to the thin-surface approach.

IV. Results

In Fig. 2, we show the direct solving times for the full and thick-plate schemes in the scattering analysis of a PEC square plate with thickness 0.1 m and several sides (1m; 2m; 3m; 4m; 5m) and λ=1m. This corresponds with the direct solution of matrix systems handling, respectively, 864, 2080, 4320, 7360 and 11200 unknowns if the full MoM-scheme is adopted. Congruently, the number of unknowns managed by the two subsystems in the thick-surface approach are 456+408 (1λ); 1080+1000 (2λ); 2220+2100 (3λ); 3760+3600 (4λ); 5700+5500 (5λ). Clearly, the serial solution of the thick-plate scheme is less time-consuming than the full scheme. Moreover, an estimate for the parallel solution will produce about half solve-time.

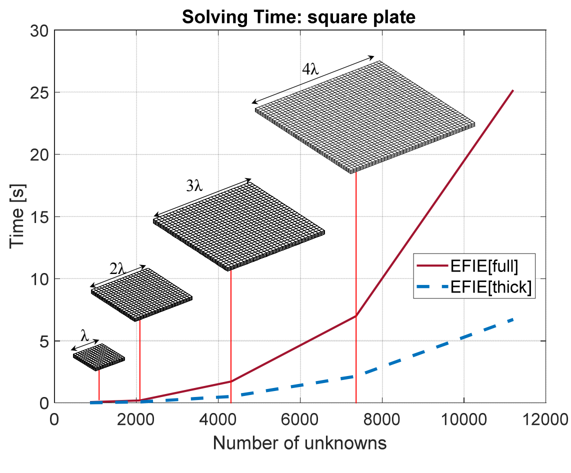

Fig. 2. Direct solving times for several square PEC-plates (1λx1λ; 2λx2λ; 3λx3λ; 4λx4λ; 5λx5λ) and thickness of 0.1m

The thin-surface approach for the same plates in Fig. 2 makes use of 264 (1λ); 760 (2λ) ; 1740 (3λ); 3120 (4λ); 4900 (5λ) unknowns, which translates into direct solving times from 0.0185 s (1λ) and 0.139 s (3λ) up to 2.25 s (5λ). These, as expected, are smaller than the computational times shown in Fig. 2, but compare well with the direct solving-time estimates required in the concurrent solution with the thick-plate approach. Nonetheless, the thin-strip scheme shows great inaccuracies in the scattering analysis of such plates under low-angle grazing incidences. See for example Fig. 3, where the full and thick-plate schemes show very similar RCS-accuracy in the analysis of the plate with side 4λ, thickness 0.1m and 2º grazing incidence.

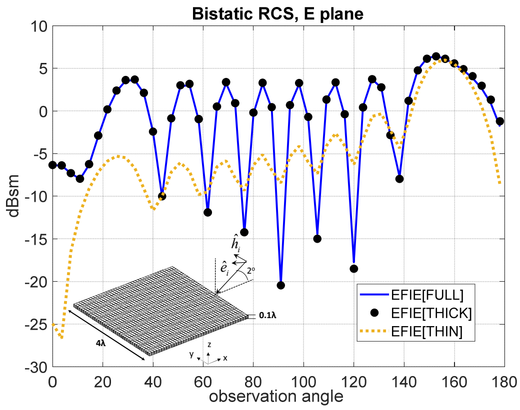

Fig. 3. E-plane RCS for the full, thick-strip and thin-strip schemes for a plate with side 4 m and thickness 0.1m under a 2º-angle incidence. The discretization defines 7360 edges and λ=1m.

V. Conclusion

The thick-plate approach excels as a MoM-scheme well-suited for the scattering analysis of plates with nonzero thickness because it provides very similar accuracy as the full-MoM approach and with lower solving times in cases where the thin-plate approximation fails.

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References

[1] B. M. Kolundzija and A. R. Djordjevic, Electromagnetic Modeling of Composite Metallic and Dielectric Structures, Boston: Artech House, 2002.

[2] E. H. Newman and M. R. Schrote, "On the Current Distribution for Open Surfaces," *IEEE Trans. Antennas Propag.*, vol. AP-31, no. 3, pp. 515-518, May 1983

[3] E. Ubeda, I. Sekulic, and J. M. Rius, "Efficient and accurate electromagnetic scattering analysis of perfectly conducting thick plates," presented at the IEEE Int. Symp. Antennas and Propagation, Atlanta, GA, USA, Jul. 7–12, 2019.

[4] E. Ubeda, and J. M. Rius, "Efficient analysis of thick curved metal sheets," presented at 15th European Conference on Antennas and Propagation (EuCAP), Dusseldorf , Germany, March 22-26, 2021

[5] R. Graglia, D. R. Wilton , and A. F. Peterson, "Higher order interpolatory vector bases for computational electromagnetics," *IEEE Trans. Antennas Propag.*, vol. 45, no. 3, March 1997

[6] R. McWeeny, Symmetry: An Introduction to Group Theory and Its Applications, Mineola, New York: Dover Publications, Inc., 2002.