Perceptual Maps to Aggregate Information from Decision Makers

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Abstract. Understanding different perceptions of human being when using linguistic terms is a crucial issue in human-machine interaction. In this paper, we propose the concept of perceptual maps to model human opinions in a group decision-making context. The proposed approach considers a multi-granular structure using unbalanced hesitant linguistic term sets. An illustrative case is presented in the location decisions made by multinationals enterprises of the energy sector within the European smart city context.

Keywords. Fuzzy systems, hesitant linguistic terms, multi-criteria decision-aiding.

1. Introduction

A better interaction between humans and intelligent systems needs being able to capture some human abilities such as asking questions or constructing explanations. Humans consider real world situations from different perspectives. And considering this heterogeneity could revert in better group decision making.

Multiple-criteria group decision making (MCGDM) is used when a group of experts or decision makers (DMs) express their assessments or preferences on a set of attributes (or criteria) for a set of alternatives and an optimal representative or solution is needed to solve the problem [3]. Many practical applications have used hesitant fuzzy linguistic term sets (HFLTSs) to deal with the linguistic information involved in MCGDM problems. Most of the GDM applications found in the literature, which are framed as MCDM problems with linguistic assessments modelled by means of HFLTSs, are assumed to be built over a uniform and symmetrically distributed linguistic term set (LTS). However, there exist many GDM situations where attributes relate to qualitative characteristics that need to be assessed by linguistic terms represented by unsymmetrical or not uniformly distributed LTSs, i.e., unbalanced LTS, such as for example, the evaluation of creditworthiness and credit risk quality of bonds [2] or factors affecting the comfort of passengers [4]. Similarly, it is also very common to find GDM situations

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with DMs having different backgrounds or knowledge and this also needs to be modelled by different LTS. In this paper, these differences between the DMs' semantics of the linguistic term set are represented via the concept of perceptual maps. This concept, together with the idea of the gap measurement in the proposed distance, represent a step forward with respect to the state-of-the-art due to its flexibility to model not only hesitancy but also discrepancies between DMs' assessments.

The main goal of this paper is to show the feasibility and practicability of a fuzzy decision-aiding approach, using multi-granular and unbalanced hesitant fuzzy linguistic term sets in a real world multi criteria group decision making situation involving several experts who elicitate their opinions with linguistic assessments. An illustrative application is presented in this paper, framed in the scheme of location decisions made by multinationals enterprises (MNEs) of the energy sector within the European Smart city context.

The rest of the paper is organized as follows. Section 2 introduces preliminary concepts. Section 3 presents the illustrative case and the decision-aiding approach considering perceptual maps to aggregate information from decision makers. Finally, the conclusions and future work are presented in Section 4.

2. Preliminaries

Hesitant fuzzy linguistic term sets were introduced in [10]. They are useful to capture the human way of reasoning using linguistic expressions involving different levels of precision. Based on this concept, in this section, a formal introduction to perceptual maps and projected space to aggregate decision makers' opinions is presented.

Let S be a finite totally ordered set of linguistic terms, $\mathcal{S} = \{s_1, \ldots, s_n\}$, with $s_1 < \ldots < s_n$.

Definition 1 *([10])* A *hesitant fuzzy linguistic term set (HFLTS)* over $\mathcal{S}$ is a subset of consecutive linguistic terms of S, i.e. $\{x \in \mathcal{S} \mid a_i \leq x \leq a_j\}$, for some $i, j \in \{1, \ldots, n\}$ with $i \leq j$. We also consider the *empty HFLTS:* $\{\} = \emptyset$, and the *full HFLTS:* $\mathcal{S}$.

Hereafter, the non-empty HFLTS $H = \{x \in \mathcal{S} \mid a_i \leq x \leq a_j\} = \{a_i, a_{i+1}, \ldots, a_j\}$ is also denoted by $[a_i, a_j]$. If $i = j$, $[a_i, a_i]$ is the singleton $\{a_i\}$. The set of all HFLTSs over S is denoted by $\mathbb{S}_n$: $\mathbb{S}_n = \{[a_i, a_j] \mid i, j \in \{1, \ldots, n\}, i \leq j\} \cup \{\emptyset\}$, being $\mathbb{S}_n^* = \mathbb{S}_n - \{\emptyset\}$ the set of non-empty HFLTSs. The binary operation, *connected union,* $\sqcup$, of two HFLTSs was defined in [6] as the least element of $\mathbb{S}_n$, based on the subset inclusion relation $\subseteq$, that contains both HFLTSs. The intersection, $\cap$, of HFLTSs is a closed binary operation on the set $\mathbb{S}_n$. In ([6]) it is proved that $(\mathbb{S}_n, \sqcup, \cap)$ is a non-distributive lattice.

Next, we consider the concept of a normalized measure over a linguistic term set S, which may not be balanced: the perceptual map.

2.1. Perceptual map

Definition 2 *[8]* Let S be a totally ordered finite LTS, $S = \{s_1, s_2, ..., s_n\}$. Let μ' denote a measure over S such that $\mu'(s_i) > 0$, $\forall i \in \{1, 2, \ldots, n\}$. Then, the *perceptual map*, μ, induced by μ', is a function $\mathbb{S}_n^* \to [0, 1]$ defined as:

$$\mu(H_S) = \frac{\sum\limits_{s_i \in H_S} \mu'(s_i)}{\sum\limits_{i=1}^{n} \mu'(s_i)} \tag{1}$$

for any $H_S \in \mathbb{S}_n^*$.

The perceptual map μ provides a normalized measure on $\mathbb{S}_n^*$ and, as highlighted in [9], there exists a bijective function between the set of perceptual maps over a set S of granularity n and the set of partitions of $[0, 1]$ with n non-empty subintervals. Indeed, given a partition of the interval $[0, 1]$ defined by the strictly ordered n-tuple of real numbers $P = \{\alpha_0, \cdots, \alpha_i, \cdots, \alpha_n\}$ such that $0 = \alpha_0$ and $\alpha_n = 1$, there exist a perceptual map μ defined over a LTS with granularity n, with $\mu(s_i) > 0$ and $\alpha_i - \alpha_{i-1} = \mu(s_i)$, $\forall i \in \{1, \cdots, n\}$.

Note that there are several methods, either supervised or non-supervised, to define the landmarks of the partition and consequently the perceptual map underlying different decision making styles.

An extended lattice of HFLTSs $\overline{\mathbb{S}_n} = \mathbb{S}_n^* \cup \mathcal{A} \cup (-\mathbb{S}_n^*)$ was defined in [6], considering the set $\mathcal{A}$ of null HFLTSs and the set $-\mathbb{S}_n^*$ of negative HFLTSs. Note that, intuitively, null HFLTSs correspond to landmarks and negative HFLTSs modelize the gap between a pair of disjoint HFLTSs. This extension allows us to consider a distance between HFLTSs that takes into account the gap between two HFLTSs if they do not overlap.

2.2. A perceptual-based distance for unbalanced HFLTSs

Given any perceptual map, μ, the definition of width of a HFLTS $H_S \in \overline{\mathbb{S}_n}$ and the distance between two HFLTSs H_S^1, $H_S^2 \in \overline{\mathbb{S}_n}$ are defined in [8] as follows:

$$W_\mu(H_S) = \begin{cases} \mu(H_S), & H_S \in \mathbb{S}_n^*; \\ 0, & H_S \in \mathcal{A}; \\ -\mu(-H_S), & H_S \in (-\mathbb{S}_n^*). \end{cases}$$

Let H_S^1, $H_S^2 \in \overline{\mathbb{S}_n}$, then:

$$D_\mu(H_S^1, H_S^2) = W_\mu(H_S^1 \sqcup H_S^2) - W_\mu(H_S^1 \sqcap H_S^2) \tag{2}$$

provides a distance in the lattice $(\overline{\mathbb{S}_n}, \sqcup, \sqcap)$, were $\sqcup$ and $\sqcap$ are the extended union and extended intersection respectively [8]. Note that this distance considers the gap between two HFLTSs if they do not overlap.

2.3. A projected space for multi-perceptual GDM

In order to compare and operate with unbalanced HFLTSs based on different perceptual maps, a projected space is defined to project linguistic assessments built over different perceptual maps onto a projected linguistic structure [8], specifically:

Definition 3 *([8])* Let $\{P_{\mu_j} \mid j \in 1, \ldots, k\}$ be the set of partitions associated to the set of perceptual maps $\{\mu_j \mid j \in 1, \ldots, k\}$ and the set of LTS $\{S_{\mu_j} \mid j \in 1, \ldots, k\}$. Each P_{μ_j} is a partition of the unit interval defined by $\{\lambda_0^j, \lambda_1^j, \lambda_2^j, \cdots, \lambda_{n_j}^j\}$, with $\lambda_0^j = 0$, $\lambda_{n_j}^j = 1$ and n_j denotes the cardinality of each S_j. The *projected partition* associated to $\{P_{\mu_j} \mid j \in 1, 2, \ldots, k\}$ is P_p, defined by $\bigcup_{j=1}^k \bigcup_{l=0}^{n_j} \{\lambda_l^j\}$.

Definition 4 *([8])* Let P_p be the projected partition of the set $\{P_{\mu_j} \mid j \in 1, 2, \ldots, k\}$ defined by $\{\lambda_0, \lambda_1, \ldots, \lambda_{n^*}\}$. We define the *projected LTS*, S^*, as the set that contains the *projected basic labels*, s_α^*, i.e., $S^* = \{s_\alpha^* \mid \alpha \in 1, 2 \ldots, n^*\}$, where n^* is the cardinality of the set $\bigcup_{j=1}^k \bigcup_{l=0}^{n_j} \{\lambda_l^j\}$ *; and the projected normalized measure over S^*, μ_*' induced by this partition as:*

$$\mu_*'(s_\alpha^*) = \lambda_\alpha - \lambda_{\alpha-1}, \alpha \in 1, 2, \ldots, n^* \tag{3}$$

where $s_\alpha^* \in S^*$.

Note that the projected basic labels are only considered for computational purposes and the semantics that apply to each S_j do not apply for S^*. From the above concepts, we lastly introduce the concept of projected perceptual map:

Definition 5 *([8])* Let S^* be a projected LTS, $S^* = \{s_1^*, s_2^*, ..., s_{n^*}^*\}$ and μ_*' its projected normalized measure. Then, the *projected perceptual map* μ_* is the perceptual map induced by μ_*' in $\mathcal{H}_{S^*}^*$ (as defined in equation 1).

Note that the previous definitions not only deal with unbalanced LTS but are also adapted to contexts of multi-granularity when the LTS used by each DM, S_j, are of different cardinality.

The previously described perceptual map and projected algebraic structure, will allow us to deal with MCGDM problems where each decision maker has its own qualitative reasoning approach. In this direction, an adaptation of the Fuzzy TOPSIS method was defined in [8,7] to rank different alternatives when considering a framework where DMs are allowed to use different perceptual maps.

3. Illustrative Case: Energy MNEs locations

Energy multinational enterprises (MNE) provide green energy services and products to the city they are located. In addition, as the rest of MNEs, they generate jobs and stimulate local economy. Understanding the multi-criteria decision making process followed by these energy enterprises in their location decisions would facilitate local governments to attract them.

In this paper, an extension of the study presented in [7] is conducted considering different perceptual maps. We use the concepts defined in the previous section to design a framework for ranking criteria and sub-criteria governing the energy MNEs strategic location decisions. The challenge of understanding the process that energy MNEs follow in their location decisions is studied using extended fuzzy multi-perceptual linguistic TOPSIS method.

The framework is build based on extracting the relative importance of 27 sub-criteria by asking the opinion to ten experts of the field. Subcriteria were extracted from literature review and a workshop with academics and practitioners. Hereinafter, let be $S = \{N :$ not important, $L :$ low importance, $S :$ somewhat important, $V :$ very important, $E :$ extremely important$\}$. Figure 1 shows the linguistic expressions given by the 10 experts based on the linguistic term set of five elements S.

Sub-Criteria	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
Home-Host Country Distance	V	N, L	V	V	N	L	V	L	N	S
Host country GDP per capita	L	L, S	L	S	L	L	S	L	N	S
Host country level of welfare state	L	S, V	V	L	L	S	S	L	N	S
Host country political stability perception	S	V	E	V	S	V	V	S	N	V
Host country's corruption perception	L	V	E	V	L	E	V	S	N	V
The city size	V	S	V	L	L	L	L	L	S	V
City's cultural and language distance perception	S	N	L, S	S	L	S	S	S	L	N
City's climate characteristics	N	V, E	N, L	E	S	L	V	S	S	L
City's connectivity—infrastructural features	V	L	S, V	V	L	V	V	S	N	S
City's reputation, image and prestige	S	L	S, V	S	S	S	S	L	N	S
City government degree of transparency	L	V	V, E	E	?	E	V	V	N	V
City government bureaucracy level	L	E	S, V	V	L	V	V	V	E	V
Access to financial support provided by city government	V	V	S, V, E	S	S	S	V	V	E	V
City government support to public-private partnerships (PPP)	V	E	V, E	S	?	S	V	V	V	V
City GDP per capita	S	S	S	L	?	L	S	L	N	V
Municipal economic budget	S	S, V	V, E	L	S	L	V	L	L, S	S
City R&D expenditure	S	S	S, V	L	L	L	N	L	L, S	V
The service economy of the city	S	L	L, S, V	V	S	V	V	L	L, S	S
Stakeholders' pressure in the city	S	S, V	V, E	V	L	S	V	S	L, S	V
Citizens' environmental awareness	L	V	V, E	E	E	S	L	L	L, S	V
City's air quality	L	V	S	S	V	S	N	L	L	V
Degree of city transition to renewables	L	V, E	S	E	N	L	V	V	E	V
Competition intensity in the city	V	S, V	V, E	L	L	S	S	L	N	S
Pool of skilled labor in the city	V	S, V	S, V, E	L	V	V	E	L	N, L	V
Access to needed suppliers	V	S	S, V, E	S	S	S	V	S	L, S	V
City's potential customers	V	V	V, E	E	V	V	S	S	S	E
City's degree of know-how, innovation and technological exchanges	S	L	S, V	S	E	S	N	L	S, V	V

Figure 1. Linguistic expressions given by the ten experts in relation to the importance of each sub-criteria [7].

Three different initial assumptions with respect to the type of perceptual-map owned by each of the ten experts involved in the group decision-aiding situation are considered. For each scenario, corresponding to each of these assumptions,
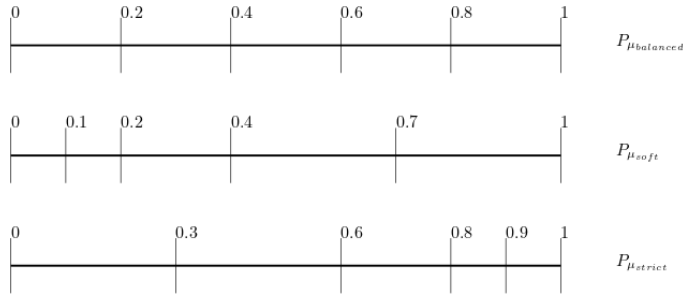
Figure 2. Partitions corresponding to the different perceptual maps assumed to model experts' opinions in situation A, situation B and situation C.

the evaluation results were computed. The three different assumptions (starting-points) based on the existence of different perceptual-maps within the group of experts are the following:

- **Situation A.** The setting with respect to the perceptual maps is assumed to be $\mu_l(s_i) = 0.2$, $\forall\, i \in \{1,2,3,4,5\}$ and $\forall\, l \in \{1,2,\ldots,10\}$. This means that the qualitative reasoning process of all experts can be modelled by means of an equally and symmetrically distributed LTS. The partition associated to this perceptual map, $\mu_{balanced}$, is shown in figure 2.
- **Situation B.** The setting with respect to the perceptual maps is assumed to be $\mu_l(s_1) = \mu_l(s_2) = 0.3$, $\mu_l(s_3) = 0.2$, $\mu_l(s_4) = \mu_l(s_5) = 0.1$, $\forall\, l \in \{1,2,3,4,5\}$ and $\mu_l(s_i) = 0.2 \;\forall\, i \in \{1,2,3,4,5\}$, $\forall\, l \in \{6,7,8,9,10\}$. This represents a situation where the first five experts have a qualitative reasoning process that could be considered 'strict' or 'perfectionist', owing a perceptual map, μ_{strict}, illustrated in figure 2. The rest of the experts are assumed to elicit their opinions based on $\mu_{balanced}$.
- **Situation C.** The setting with respect to the perceptual maps is assumed to be $\mu_l(s_1) = \mu_l(s_2) = 0.1$, $\mu_l(s_3) = 0.2$, $\mu_l(s_4) = \mu_l(s_5) = 0.3$, $\forall\, l \in \{1,2,3,4,5\}$ and $\mu_l(s_i) = 0.2 \;\forall\, i \in \{1,2,3,4,5\}$, $\forall\, l \in \{6,7,8,9,10\}$. In this situation, the first five experts are assumed to be 'generous' or 'soft' when eliciting their opinions. This is modeled with a perceptual map, μ_{soft}, whose associated partition is also illustrated in figure 2. The rest of the experts are assumed to elicit their opinions based on $\mu_{balanced}$.

Starting-points A, B and C are graphically illustrated in figure 3.

The projected LTS, S^* and the projected perceptual map, μ_* are computed for each situation:

- **Situation A.** The projected LTS is $S^*_A = \{s^*_1, s^*_2, s^*_3, s^*_4, s^*_5\}$ and $\mu^A_* = \mu^A_*(s^*_i) = 0.2 \;\forall\, i \in \{1,2,3,4,5\}$.
- **Situation B.** The projected LTS is $S^*_B = \{s^*_1, s^*_2, s^*_3, s^*_4, s^*_5, s^*_6, s^*_7\}$ with $\mu^B_* = \mu^B_*(s^*_i) = 0.2 \;\forall\, i \in \{1,4,5\}$ and $\mu^B_* = \mu^B_*(s^*_i) = 0.1 \;\forall\, i \in \{2,3,6,7\}$.
- **Situation C.** The projected LTS is $S^*_C = \{s^*_1, s^*_2, s^*_3, s^*_4, s^*_5, s^*_6, s^*_7\}$ with $\mu^C_* = \mu^C_*(s^*_i) = 0.2 \;\forall\, i \in \{3,4,7\}$ and $\mu^C_* = \mu^C_*(s^*_i) = 0.1 \;\forall\, i \in \{1,2,5,6\}$.

The corresponding projected partitions are illustrated in figure 4.
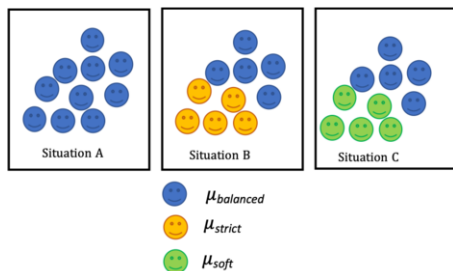
Figure 3. An illustration of the three different starting-points, based on the different perceptual-map hypothesis considered
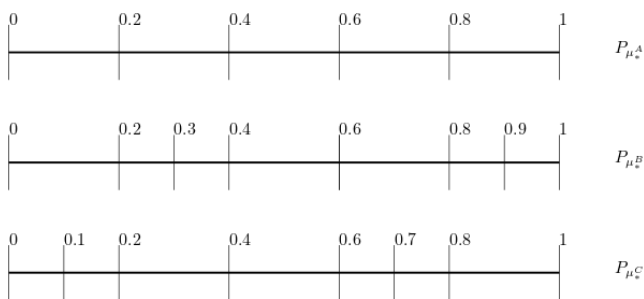

Figure 4. The projected partitions resulting of situation A, B and C

Then, for each situation A,B and C, using the projected perceptual map of Definition 5, each individual linguistic assessment is mapped onto the corresponding projected space.

The individual projected assessments are aggregated, for each sub-criterion, by considering a possibility function computed by normalizing the frequencies obtained from DMs' opinions.

Then, following an adapted version of the Fuzzy TOPSIS method [7,8], the positive and negative ideal solutions are identified, and the distances of each sub criteria to the positive and negative ideal solutions are calculated, respectively, by using the distance of Subsection 2.2. Based on these distances the closeness coefficient is obtained for each subcriteria. Based on the relative closeness coefficients' values, the partial weight of each sub-criterion within each criteria group is distributed, i.e., the weight percentages of each criteria group sum up to 100%. The subcriteria are then ranked within each criteria group. Combining the average weights of the main criteria with the relative importance of each sub-criteria within each group, a final ranking is obtained. The ranking is illustrated in Table 1.

According to the results of Table 1, the most relevant factor is 'City's potential customers', regardless of the perceptual-map hypothesis. Besides, the relevant

Sub-criteria	Sit. A	Sit. B	Sit.C
City?s potential customers	8.07%	8.59%	9.06%
Access to financial support provided by city government	7.96%	7.37%	7.76%
City government support to public-private partnerships	7.93%	7.20%	8.21%
City government degree of transparency	7.27%	8.26%	7.27%
City government bureaucracy level	6.84%	7.15%	6.75%
Stakeholders? pressure in the city	6.14%	4.03%	6.25%
Degree of city transition to renewables	6.09%	4.46%	5.24%
Access to needed suppliers	5.51%	4.22%	4.70%
Pool of skilled labor in the city	5.40%	5.41%	4.97%
Citizens? environmental awareness	4.10%	5.20%	5.60%
City?s degree of know-how, innovation and...	3.94%	3.54%	2.29%
Host country political stability perception	3.88%	2.87%	3.19%
The service economy of the city	3.86%	2.89%	2.78%
Host country?s corruption perception	3.44%	3.69%	3.18%
City?s air quality	2.81%	3.33%	2.15%
City?s connectivity?infrastructural features	2.75%	2.53%	3.04%
Home-Host Country Distance	2.29%	2.49%	2.30%
Municipal economic budget	2.09%	2.68%	2.39%
Competition intensity in the city	2.09%	3.22%	3.97%
City?s climate characteristics	1.69%	1.83%	2.24%
City?s reputation, image and prestige	1.52%	1.39%	1.07%
City GDP per capita	1.38%	2.33%	1.55%
City?s cultural and language distance perception	1.05%	0.63%	0.77%
The city size	1.00%	1.60%	0.86%
City R&D expenditure	0.52%	2.04%	1.00%
Host country level of welfare state	0.39%	0.93%	1.31%
Host country GDP per capita	0.00%	0.00%	0.00%

Table 1. Sub-criteria overall relative weight, for each situation considered

percentage weight of this sub-criterion is quite similar in the three situations: 8.0671 % , 8.590% and 9.06% for situations A, B and C, respectively. The rest of the sub-criteria related to market conditions for energy firms, which are access to needed suppliers, pool of skilled labor, city's degree of know-how and competition intensity in the city are placed in the 8th, 9th, 11th, 19th positions of the rank (situation A), respectively. In the case of situation B and C, these sub-criteria positions are: 9th, 6th, 12th, 14th and 10th, 9th, 18th and 11th, respectively. It is important to highlight that customers and suppliers' environments are more relevant than the factor of competition intensity in the city, regardless of the hypothesis.

As expected, the least valued sub-criterion is 'Host Country GDP per capita' in all situations.

It is also relevant to notice that the resulting TOP 5 sub-criterion are all the same, regardless of the hypothesis. Moreover, without considering the first ranked sub-criterion which belongs to market conditions, the following four sub-criteria all belong to the group of 'City's government and its policies'.

4. Conclusions

This paper proposes the concept of perceptual maps to model human opinions in a group decision-making context. The proposed approach considers a multi-granular projected structure using unbalanced hesitant linguistic term sets.This multi-perceptual framework allow us to deal with MCGDM problems where each decision maker has its own qualitative reasoning approach.

An illustrative case is presented in the location decisions made by multinationals enterprises of the energy sector within the European smart city context. The illustrative case is an extension of a previous study presented in [7] by considering different perceptual maps. In this multi-perceptual framework, criteria and sub-criteria governing the energy MNEs strategic location decisions are ranked.

Acknowledgments

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