Optimal $\mathcal{H}_\infty$ Control Design of MMC-based HVDC Links

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***Abstract*—Modular multilevel converter (MMC) has emerged as the preferred choice for Voltage Source Converter (VSC)-based High Voltage Direct Current (HVDC) systems due to its low losses, low harmonic distortion, modularity, and redundancy. These advantages come at the expense of a complex control system with the strong coupling among its control variables, which complicates the design procedure of both its individual control loops and the DC link controllers. In fact, the performance improvement of a control loop could lead to degraded performance of other feedback loops. Hence, to deal with such a complex dynamic system, this article suggests adopting multivariable $\mathcal{H}_\infty$ optimal control techniques in order to ensure stability and optimized performance of a VSC-HVDC link. First, a full-order, centralized multi-input-multi-output (MIMO) controller is derived based on $\mathcal{H}_\infty$ optimisation and used as a benchmark for the system performance level. Then, a fixed-structure, decentralized MIMO controller is synthesized to make a compromise between the optimal performance and feasibility of practical implementation. Finally, simulations are conducted to evaluate and compare the performance degradation due to the migration from a high-order, centralized controller towards a low-order, decentralized controller.**

***Index Terms*—H-infinity optimization, HVDC, MMC, VSC, robust multivariable control, robustness.**

I. Introduction

Voltage Source Converter (VSC)-based High Voltage Direct Current (HVDC) technology is currently being used for various applications, such as offshore wind power integration or long distance power transmission [1], [2]. For these applications, the preferred converter topology is the Modular Multilevel Converter (MMC), as it not only facilitates high voltage operation, but also offers additional benefits such as low losses, reduced harmonic distortion, and scalability [3].

With the introduction of VSCs into HVDC systems, the controllability has improved significantly, enabling a flexible control of the system variables, including independent active and reactive power control [4], [5]. Furthermore, the MMC structure has also increased the degrees of freedom of the control system compared to the conventional 2-level VSC structures [6]. Although such controllability offers more control possibilities [7], it poses new challenges to the control design problem as more variables should be coordinated to ensure an adequate system performance.

Particularly, in the MMC-based VSC-HVDC applications, it has been shown that the system control loops are highly coupled and that interactions among them can deteriorate system overall dynamics [8], [9]. In fact, the optimal performance of one control loop could come at the expense of the performance degradation of other control loops [8]. Therefore, in such a multivariable, heavily coupled control system, an optimal control design technique is required to simultaneously design the control loops of an MMC-based HVDC link [10]. In an optimal control design, the multivariable controller is often derived in an iterative fashion to minimize the maximum errors of the control variables. These errors are commonly measured via $\mathcal{H}_2$-norm or $\mathcal{H}_\infty$-norm of the closed-loop system [11].

Several studies have used $\mathcal{H}_2/\mathcal{H}_\infty$ optimal control in the field of power electronics and HVDC systems. In [12]–[15], the $\mathcal{H}_2/\mathcal{H}_\infty$ optimal control design has been adopted for a 2-level VSC. The application of multiple 2-level VSCs has been addressed in [16], where a convex optimization method is suggested minimizing the $\mathcal{H}_2/\mathcal{H}_\infty$-norm of the errors. It aims at the performance optimization of parallel grid-connected VSCs. Using $\mathcal{H}_\infty$ optimal control, the overall performance of an HVDC link based on back-to-back line-commutated converters (LCC) has been optimized in [17]. The control loops of the rectifier and the inverter are synthesized simultaneously and the robustness of the controller is demonstrated for uncertainties related to the AC grid. However, few studies have applied $\mathcal{H}_2/\mathcal{H}_\infty$ design practice on an MMC. In [18], $\mathcal{H}_\infty$ design principles has been used for an MMC. The dynamics of the AC and DC sides are not considered, and only proportional controllers are designed for the state feedback. Recently, $\mathcal{H}_2$ optimal control has been applied to an MMC-based HVDC system in [19], focusing on the DC voltage regulation dynamics, without including the dynamics of the slave MMC.

To the best of the authors' knowledge, there is no previous work in the literature on the implementation of $\mathcal{H}_\infty$ design for the performance optimization of a complete MMC-based HVDC link. Within this context, this article adopts $\mathcal{H}_\infty$ optimal design to obtain a multi-input multi-output (MIMO) control structure for the DC voltage, active power and internal energy of both MMCs. First, using $\mathcal{H}_\infty$ optimal design a controller is obtained in order to study the potential of the full-order MIMO-based structure. Then, aiming to obtain a simpler controller for real implementation, a fixed-structure reduced-order controller design is performed, considering that the structure and the order of the MIMO controller are the key design parameters that influence the controller complexity and the optimal performance level. It will be shown how the system performance is affected due to the migration from a high-order, centralized controller towards a low-order, decentralized controller via frequency analysis and time-domain simulations.

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II. SYSTEM DESCRIPTION

The structure of an MMC-based HVDC link is shown in Fig. 1. For such an interconnector, typically one of the two MMCs (master) regulates the DC voltage of the link, while the active power flow across the link is controlled by the other MMC (slave). Reactive power can be regulated independently from the active power via both MMCs [20].

Fig. 1. An MMC-based HVDC link

The typical structure of a grid-connected MMC is shown in Fig. 2. This converter has six arms, each of them consists of N_{arm} half-bridge submodules with a capacitance C_{SM} and a series arm reactor. A submodule (SM) can be controlled separately to either insert its capacitor in the circuit or to bypass it, enabling a controllable source operation per arm [21]. The voltages of the six arms can be regulated to achieve the desired power exchange between DC and AC sides [6].

Fig. 2. Modular Multilevel Converter structure

The overall control system of an MMC is presented in Fig. 3. As detailed in [6], the control system is a cascaded structure in which the inner loops regulate the AC grid current (i_s), and the DC and AC additive currents (i_{sum}). The outer loops generate references for the inner loops and they might be adapted depending on the control mode as:

- DC voltage or active power control: either the DC voltage control loop (master MMC) or the active power loop (slave MMC) generates the active current reference for the AC current control (i_s^{q*}).
- Reactive power control: the reactive current reference (i_s^{d*}) can be either directly calculated from the system reactive power requirements or might be provided by an outer reactive power/AC voltage controller.
- Total energy control: the total stored energy in the MMC's submodules, (W_t) is affected by the mismatch between the DC and AC power. It should be maintained at its rated value, given by
$$W_t^* = 6\frac{1}{2}\frac{C_{SM}}{N_{arm}}\left(V^{\mathrm{dc}*}\right)^2 \quad (1)$$
The output of this control loop is the reference for the zero-sequence component of the DC additive current $i_{\mathrm{sum}}^{0\mathrm{dc}*}$.
- Arms and legs energy balancing: in order to achieve sustained operation, the energy differences between two legs ($W_{a\to b}$, $W_{a\to c}$) and the energy differences between upper and lower arms of three legs ($W_{u\to l}^a$, $W_{u\to l}^b$, $W_{u\to l}^c$) are maintained at zero by five control loops. The outputs of the energy control loops generate the additive current references ($i_{\mathrm{sum}}^{\alpha\beta\mathrm{dc}*}$ and $i_{\mathrm{sum}}^{\alpha\beta\mathrm{ac}*}$).

Fig. 3. Overall structure of an energy-based control system

Note that $i_{\mathrm{sum}}^{\alpha\beta\mathrm{dc}}$ and $i_{\mathrm{sum}}^{\alpha\beta\mathrm{ac}}$ are only used for the MMC internal energy balancing, while $i_{\mathrm{sum}}^{0\mathrm{dc}}$ determines the DC power exchange of the MMC. The component $i_{\mathrm{sum}}^{0\mathrm{ac}}$ should be regulated to 0 to avoid line frequency currents on the DC link. Since the MMC operates in grid-following mode, a Phase Locked Loop (PLL) is used to track the AC voltage at the point of common coupling (PCC) (u^{qd}). It is also assumed that a Nearest Level Modulation (NLM) approach is used to implement the switching strategy of the converter [21].

The PI controllers of i_s and i_{sum} are designed to achieve a fast closed-loop response and the PI coefficients are commonly determined based on the desired time constant (few milliseconds) and MMC arm impedance [6]. However, the design of the outer control loops are more challenging as there is a strong coupling among total energy (W_t), DC voltage (V^{dc}), and active power (P) [9], [22], [23]. To cope with this challenge, this article assumes an adequate performance of the inner current controllers and focuses on the $\mathcal{H}_\infty$ optimal design of the outer controller ($K(s)$). The suggested structure for $K(s)$ is shown in Fig. 4. It has two diagonal and two off-diagonal partitions. The latter represents a coupling between the MMCs, i.e., the errors from the slave MMC need to be sent to the controller to synthesize the control actions for the master MMC.

Fig. 4. Structure of MIMO controller $K(s)$

III. System Modeling

For the purpose of the $\mathcal{H}_\infty$ design, two system models are needed: a detailed, non-linear model that contains all control loops as indicated in Fig. 3, and an overall linear model which is derived by interconnecting the state-space equations of the master and slave MMCs, their control systems, and the HVDC cable. The non-linear model is built in MATLAB/Simulink and is used for the validation of the linear model.

An Arm Average Model (AAM) is used to represent the MMC converter, which can be seen in the lower left part of Fig. 2. The state-space equations of an MMC are more comprehensible if formulated by differential and additive variables, instead of using upper and lower arms variables [6], [8]. Referring to Fig. 2, the following variable changes are commonly defined:

$$\begin{cases} v_{\text{diff}}^j \triangleq \frac{1}{2}\big(-v_u^j + v_l^j\big) \\ v_{\text{sum}}^j \triangleq v_u^j + v_l^j \\ i_{\text{sum}}^j \triangleq \frac{1}{2}\big(i_u^j + i_l^j\big) \end{cases} \qquad \begin{cases} R \triangleq R_s + R_g + \frac{R_a}{2} \\ L \triangleq L_s + L_g + \frac{L_a}{2} \end{cases} \tag{2}$$

where v_{diff}^j, v_{sum}^j, and i_{sum}^j, are, respectively, differential voltage, additive voltage, and additive current of phase (leg) j, ($j = a, b, c$). Using variables in (2) and assuming the DC and AC sides of the MMC are balanced, the following state-space equations in the abc-frame are derived:

$$v_{\text{diff}}^{abc} - v_g^{abc} = R\,\boldsymbol{\mathcal{I}}\,i_s^{abc} + L\,\boldsymbol{\mathcal{I}}\,\frac{di_s^{abc}}{dt} \tag{3}$$

$$v_{\text{sum}}^{abc} - V^{\text{dc}}[1\ 1\ 1]^T = -2R_a\,\boldsymbol{\mathcal{I}}\,i_{\text{sum}}^{abc} - 2L_a\,\boldsymbol{\mathcal{I}}\,\frac{di_{\text{sum}}^{abc}}{dt} \tag{4}$$

where V^{dc} is the DC side voltage and $\boldsymbol{\mathcal{I}}$ is a 3×3 identity matrix. Equations (3) and (4) represent the dynamics of the MMC in the abc-frame. As indicated in Fig. 3, it is more convenient to transform (3) into a rotating qd-frame, and (4) into a stationary $\alpha\beta$-frame [6], [8]. However, the transformation and linearization of (4) is challenging due to the simultaneous presence of AC and DC terms in the additive current i_{sum}^{abc}. In [24], the additive current is transformed into several synchronous frames to achieve a very accurate linear model of the MMC. However, it has been reported that a linear model with an acceptable accuracy can be obtained even if the AC terms of the additive current are ignored [6], [25]. In fact, the AC terms of the additive currents are only needed for analysis of unbalanced AC voltage sags or DC pole imbalances. Hence, in the linear model, only the zero component of DC additive current $i_{\text{sum}}^{0\text{dc}}$ is considered, meaning that the internal energy differences between arms and legs are assumed to be zero, and only the total stored energy (W_t) needs to be controlled at the rated value.

Fig. 5. Overall state-space linear model of an MMC-based HVDC link

As a result, the linear state-space equations of an MMC electrical circuit have three state variables: i_s^q, i_s^d (by applying Park transformation on (3)), and $i_{\text{sum}}^{0\text{dc}}$. Also, one additional linear differential equation is obtained from the linearization of the total energy:

$$\frac{dW_t}{dt} = -\frac{3}{2}v_{\text{diff}}^{qd}\ i_s^{qd}\ +\ 3i_{\text{sum}}^{0\text{dc}}\ v_{\text{sum}}^{0\text{dc}} \tag{5}$$

In addition to the MMC electrical circuit, the control loops related to i_s^{qd}, $i_{\text{sum}}^{0\text{dc}}$, W_t, V^{dc} or P, and PLL are expressed in state-space equations to build a linear model of an MMC.

With regard to the HVDC cable, it is modeled using a lumped parameters model relying on vector fitting, which was developed in [26]. This modeling method implements a number of parallel RL branches to accurately reproduce the cable frequency response.

Finally, the overall linear model of the complete MMC-based HVDC link is shown in Fig. 5. The transformation matrix T_c^{qd} is given in the Appendix. Note that the dynamics of the master and slave MMCs interact through the HVDC cable. Since the focus of optimal control is to design the four outer control loops regulating V_1^{dc}, P_2, W_{t1}, and W_{t2}, the overall linear model is rearranged into the system indicated in Fig. 6. The inner current control loops, i_s^{qd} and $i_{\text{sum}}^{0\text{dc}}$, thus become part of the open loop model $G(s)$ as seen by the outer control loops. The computational delay in converter control loops is modelled via a Padé approximation of order three.

IV. $\mathcal{H}_\infty$ Design Setup

In this section, a procedure to formulate the HVDC system optimal performance as an $\mathcal{H}_\infty$ problem is presented. The system general model, including the appropriate weighing functions, is derived, and based on this model, an optimal $K(s)$ is computed.

Fig. 6. Overall linear model of the system used as the core of $\mathcal{H}_\infty$ design

A. System General Model

The system general model is shown in Fig. 7, which is an expansion of the open loop model $G(s)$ as described by

$$Y = G(s)\ [D\ \ U]^T \tag{6}$$

The output vector Y contains the measured variables to be regulated at their reference values. The AC grid voltages are considered as disturbances to the system and are included in the disturbance vector D. The vector U contains control actions, which are in fact, the inner current control loop references. The output vector Y is subtracted from the command (reference) vector R in order to generate the error vector ($E = R - Y$) to be sent to the controller $K(s)$. These vectors are given by

$$\begin{cases} R = [V_1^{\mathrm{dc}*}\ W_{t1}^*\ P_2^*\ W_{t2}^*]^T \\ Y = [V_1^{\mathrm{dc}}\ W_{t1}\ P_2\ W_{t2}]^T \end{cases} \quad \begin{cases} D = [v_{g1}^q\ v_{g1}^d\ v_{g2}^q\ v_{g2}]^T \\ U = [i_{s1}^{q*}\ i_{\mathrm{sum1}}^{0\mathrm{dc}*}\ i_{s2}^{q*}\ i_{\mathrm{sum2}}^{0\mathrm{dc}*}]^T \end{cases}$$

Note that the model is in a small-signal formulation, but, for the sake of simplicity, the symbol Δ has been removed. The plant model $G(s)$ needs to be expressed in the per-unit system, which is important as the inputs/outputs of $G(s)$ have different units. Thus, diagonal matrices R_{base}, D_{base}, U_{base}, and E_{base} are used to derive the per-unit model $G_{pu}(s)$ (see Fig. 7) and they contain the base values for the vectors R, D, U, and E, respectively.

Fig. 7. $\mathcal{H}_\infty$ design setup: $G(s)$ is the system open loop transfer function matrix, and $K(s)$ is the controller

B. Weighing Functions

The frequency-dependant Weighing functions are indispensable to a meaningful $\mathcal{H}_\infty$ design. In fact, the engineering knowledge about the plant and the system performance requirements are included in the $\mathcal{H}_\infty$ design through the weighing functions selection. Note that these functions are included in the optimization formulation, so they directly influence the final controller performance. Next, the different weighing functions are detailed together with a basic design criteria selection for each of them.

1) Error weight function - $F_e(s)$: A basic requirement of the control system is the ability to maintain zero steady-state errors for control variables. This can be achieved by highly penalizing the DC values of e, which is equivalent to setting a high DC gain (k_e) for $F_e(s)$. However, this high gain has to decrease after a certain frequency (w_e) in order to give the errors a certain freedom to fluctuate. In fact, with the help of $F_e(s)$, not only the DC errors are removed, but also the maximum deviation of Y from R is limited. The desired frequency response of $F_e(s)$ is sketched in Fig. 8(a).

$$F_e(s) = k_e\ \frac{\omega_e}{s + \omega_e} \tag{7}$$

2) Disturbance weight function - $F_d(s)$: The disturbance vector d, which contains the AC network voltages, is penalized with a constant gain (k_d) up to the cut-off frequency of w_d that is set at the double fundamental frequency of the AC grid. This selection accounts for any imperfection in the PLL operation that may impose AC ripples over the qd components [27]. Beyond w_d, the gain could be reduced assuming that no control action is needed beyond that frequency in normal operation.

$$F_d(s) = k_d\ \frac{\omega_d}{s + \omega_d} \tag{8}$$

3) Control action weight function - $F_u(s)$: The weighing function related to the control actions, $F_u(s)$, has a frequency response that is opposite to that of $F_e(s)$ and $F_d(s)$. Up to w_a, which is equal to the controller bandwidth, $F_u(s)$ has a very low gain, meaning that the control actions are not penalized within the controller bandwidth. However, after w_a, the gain increases immediately to penalize any control actions beyond the controller bandwidth. The frequency response of $F_u(s)$ is sketched in Fig. 8(b).

$$F_u(s) = k_u\ \Big(\frac{s + \omega_a}{s + \omega_b}\Big)^2 \tag{9}$$

4) Reference weight function - $F_r(s)$: It is a common practice for system operators to avoid step changes in the references values (vector r) as it may excite undesired high-frequency modes. Hence, the references are considered to go through the weighing function $F_r(s)$, which is often a low-pass filter with unity as gain by

$$F_r(s) = \frac{\omega_r}{s + \omega_r} \tag{10}$$

Referring to Fig. 7, the final general model of the system used for the purpose of $\mathcal{H}_\infty$ design is described as

$$\begin{aligned} \dot{x} &= Ax + B_1[\tilde{r}\ \ \tilde{d}]^T + B_2 u \\ e &= C_1 x + Z_{11}[\tilde{r}\ \ \tilde{d}]^T + Z_{12} u \\ [\tilde{e}\ \ \tilde{u}]^T &= C_2 x + Z_{21}[\tilde{r}\ \ \tilde{d}]^T + Z_{22} u \end{aligned} \tag{11}$$

Fig. 8. Desired frequency response of the weighing functions, (a) error and disturbance weighing functions $F_e(s)$ and $F_d(s)$, (b) control action weighing function $F_u(s)$

which can be expressed as the transfer function matrix $G_w(s)$ relating the output vector $[\tilde{e}\ \tilde{u}\ e]^T$ to the input vector $[\tilde{r}\ \tilde{d}\ u]^T$, calculated by

$$G_w(s) = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} (s\mathcal{I} - A)^{-1} [B_1\ B_2] \tag{12}$$

C. $\mathcal{H}_\infty$ Design Principles

The $\mathcal{H}_\infty$ optimal control problem is to find a stabilizing controller $K(s)$ that minimizes the $\mathcal{H}_\infty$-norm of the closed-loop system considering the defined weighing functions [11]. The closed-loop system is obtained by partitioning $G_w(s)$ as

$$\begin{bmatrix} \tilde{e} \\ \tilde{u} \\ e \end{bmatrix} = G_w(s) \begin{bmatrix} \tilde{r} \\ \tilde{d} \\ u \end{bmatrix} \Rightarrow \underbrace{G_w(s)}_{12\times 12} = \begin{bmatrix} \underbrace{g_{11}(s)}_{8\times 8} & \underbrace{g_{12}(s)}_{8\times 4} \\ \underbrace{g_{21}(s)}_{4\times 8} & \underbrace{g_{22}(s)}_{4\times 4} \end{bmatrix} \tag{13}$$

$$[\tilde{e}\quad \tilde{u}]^T = g_{11}(s)\ [\tilde{r}\quad \tilde{d}]^T + g_{12}(s)\ u \tag{14}$$

$$e = g_{21}(s)\ [\tilde{r}\quad \tilde{d}]^T + g_{22}(s)\ u \tag{15}$$

and absorbing $K(s)$ into the plant model

$$u = K(s)\ e \tag{16}$$

The closed-loop model referred to as Linear Fractional Transformation (LFT) is computed as

$$\mathscr{F}_l(G_w, K) = g_{11}(s) + g_{12}(s)\ K(s)\big(\mathcal{I} - g_{22}(s)\ K(s)\big)^{-1} g_{21}(s) \tag{17}$$

Note that the $\mathscr{F}_l$, which connects the weighted general model G_w and the controller K, relates the inputs $[\tilde{r}\quad \tilde{d}]^T$ to the outputs $[\tilde{e}\quad \tilde{u}]^T$. Then, the $\mathcal{H}_\infty$-norm of $\mathscr{F}_l$, which is the peak of the maximum singular values of $\mathscr{F}_l$ over the entire frequency range of the system can be calculated as

$$||\mathscr{F}_l(G_w, K)||_\infty = \max_w\ \bar{\sigma}\big(\mathscr{F}_l(G_w, K)(jw)\big) \tag{18}$$

Therefore, by minimizing this peak, in fact, the deviation of the error vector from zero (equally interpreted as limiting the peak closed-loop gain) is minimized.

In order to compute $K(s)$, it is assumed that the $\mathcal{H}_\infty$-norm of the closed-loop system is smaller than an arbitrary value given as

$$||\mathscr{F}_l(G_w, K)||_\infty\ <\ \gamma \tag{19}$$

where $\gamma \in \mathbb{R}$ is the optimal performance level of the system. The optimization algorithm [28] uses a γ-iteration technique that reduces γ value iteratively to approach an optimal solution. In other words, the algorithm attempts to reduce the peak gain of the closed-loop system as much as possible while maintaining system stability. For each value of γ, there exists a stabilizing $K(s)$ that satisfies the condition of (19), if and only if the following three conditions hold [29]:

1) $X_\infty \geq 0$ is a solution to the algebraic Riccati equation:

$$A^T X_\infty + X_\infty A + C_1^T C_1 + X_\infty(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty = 0$$

2) $Y_\infty \geq 0$ is a solution to the algebraic Riccati equation:

$$A Y_\infty + Y_\infty A^T + B_1 B_1^T + Y_\infty(\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty = 0$$

3) $\rho(X_\infty Y_\infty) < \gamma^2$

where $\rho(\cdot)$ denotes the largest eigenvalue. The algorithm stops once γ reaches a certain minimum relative error (typically around one percent). Once X_∞ and Y_∞ are found at each γ-iteration, the MIMO controller $K(s)$ can be defined as

$$K(s) = B_2^T X_\infty (s\mathcal{I} - A_\infty)^{-1} Y_\infty C_2^T (\mathcal{I} - \gamma^{-2} Y_\infty X_\infty)^{-1} \tag{20}$$

D. Controller order and structure for real applications

The controller $K(s)$, which is called a full-order controller, is a transfer function matrix with the same order as the general plant model $G_w(s)$. Although such a full-order controller can achieve the optimal solution, it is usually a high-order controller that is practically complicated to implement in real systems. Moreover, in case of an HVDC link, a full-order controller would imply a centralized control system with communication, since it needs the error signals from both master and slave MMCs, to synthesize the control actions. These two challenges of the full-order controller, i.e. the high order and centralized nature, can be addressed by defining a fixed control structure with a lower order, referred to as a fixed-structure controller. In this method, the control designer has the freedom to set some elements of the controller matrix $K(s)$ to zero, implying a decentralized controller, and simultaneously, limit the order of other matrix elements. However, the trade-off is that the fixed-structure controller may not be able to achieve the same level of performance as the full-order controller.

Once the optimal controller $K(s)$, either full-order or fixed-structure, is obtained, its effectiveness is confirmed by observing the singular values of two closed-loop systems: (i) the sensitivity transfer function $S(s)$, which is a transfer function matrix from the disturbance vector d to the measured variable vector y, and (ii) the complementary sensitivity function $T(s)$, which is a transfer function matrix from the reference vector r to y. These two transfer function matrices are related to the closed loop performance of the system and do not include the weighing functions (weighing functions are only used during the design procedure to shape the frequency response of the controller).

Ideally, the singular values of $T(s)$ should fall within the gray area shown in Fig. 9(a) to ensure an ideal reference tracking [30]. For frequencies up to the control system bandwidth, the maximum and minimum singular values should be confined to a narrow band (approximately around 0 dB equivalent to 1 in absolute magnitudes), meaning that y is tracking r. The maximum deviation of y from r is defined by the upper and lower limits. Above the bandwidth, the singular values should have a standard attenuation of -20 dB/decade to suppress possible noises on the reference signals. In contrast to $T(s)$, the singular values of $S(s)$ have an attenuation of 20 dB/decade within the bandwidth to achieve desired disturbance rejection. Beyond the bandwidth, the singular values should not be allowed to exceed the 0 dB threshold for one decade as presented in Fig. 9(b).

Fig. 9. Ideal singular values of (a) complementary sensitivity $T(s)$, and (b) sensitivity $S(s)$. The gray area shows the permissible location for singular values

V. $\mathcal{H}_\infty$ Design and Discussion

In this section, the $\mathcal{H}_\infty$ optimal control is used to design both full-order and fixed-structure control systems for the MMC-based HVDC link with a cable length of 100 km and specifications provided in Table I and II of the Appendix.

The corner frequencies of the weighing functions are dictated by the bandwidth of $K(s)$. According to the basic rules of the cascaded control design, the response of $K(s)$ (outer loops) has to be slower than the response of the inner loops. It is considered that the inner current loops typically have a time constant of only 1 ms, i.e, they have a bandwidth of 1000 rad/s. Considering the time constant of $K(s)$ to be ten times slower, the bandwidth of $K(s)$ is set at 100 rad/s. With this assumption, the following initial values are defined for the weighing functions:

$$\begin{cases} k_e = 10 \\ w_e = 10 \text{ rad/s} \end{cases} \quad \begin{cases} k_d = 1 \\ w_d = 629 \text{ rad/s} \end{cases} \quad \begin{cases} k_u = 0.1 \\ w_a = 100 \text{ rad/s} \\ w_b = 1000 \text{ rad/s} \end{cases}$$

The selection of these parameters would avoid excessively stressing the controller within the control bandwidth as well as preventing that the controller acts beyond the defined frequency bandwidth, ensuring an adequate overall performance.

A. Full-Order Controller

Applying the $\mathcal{H}_\infty$ optimal design on the $G_w(s)$, a stabilizing $K(s)$ is found that achieves a minimum γ of 0.33. The singular values of $T(s)$ and $S(s)$ are plotted in Fig. 10. The upper and lower limits for the singular values of $T(s)$ are set at 0.82 dB and -0.91 dB, respectively. This corresponds to a $\pm 10\%$ of deviation of y. In terms of reference tracking, the full-order controller is adequately tracking the references within the bandwidth since there is no gain limit violation. In terms of attenuation beyond the bandwidth, the performance of the system is very close to the ideal form, albeit some of the singular values have marginally less attenuation than -20 dB/decade. The attenuation of the voltage disturbances is also below the limit of 20 dB/decade, as shown in Fig. 10(b). This system performance can be regarded as a benchmark for the control design since it is obtained by a full-order, centralized controller. In fact, $\mathcal{H}_\infty$ design has the full freedom to shape the singular values of this system.

Fig. 10. Singular values of (a) $T(s)$, and (b) $S(s)$ with the full-order, centralized controller

The elements of $K(s)$, $k_{ij}(s)$, have an order of 81 (equal to the order of $G_w(s)$), and it is a centralized controller since the off-diagonal partitions have non-zero elements. The transient response of the system for a power change (P_2) from 0 to 1 pu is given in Fig. 11. The maximum deviation belongs to V_1^{dc}, which is well under 1% of nominal value.

Fig. 11. Transient response of the control variables for the full-order, centralized controller; P_2 is increased from 0 to 1 pu.

A sweep on the weighing gains is presented in Fig. 12, evaluating the system performance as affected by the weighing function gains. The $\mathcal{H}_\infty$-norm of $T(s)$ cannot be further reduced by penalizing the errors by k_e higher than 10. Also, an increase in k_d does not reduce the $\mathcal{H}_\infty$-norm. Moreover, if k_u is selected to be smaller than 0.2, the $\mathcal{H}_\infty$-norm would exhibit less sensitivity towards the gains k_e and k_d.

Fig. 12. The sensitivity of the $\mathcal{H}_\infty$ design to the gains of the weighing functions, (a) sweep of k_e and k_u in case k_d is constant at 1, and (b) sweep of k_d and k_u in case k_e is constant at 10

As mentioned earlier, $K(s)$ has an order of 81 (i.e., it has 81 states) which makes the practical implementation of this controller complicated. However, not all states contribute equally to the system behaviour. In the next section, we will show to which extent the order of $K(s)$ can be reduced.

B. Reduced-order Centralized Controller

The energy of the each state of $K(s)$ is commonly measured by the Hankel singular values (HSV). The states with high energy contribute substantially to the overall behavior of the controller. So, the behaviour of $K(s)$ can be accurately reproduced by discarding the states with the lower energy, and altering the remaining states to preserve the DC gain of the system. As it is shown in Fig. 13, after state 21, the energies of the states significantly drop, meaning that with only 21 out of 81 states, the behavior of the full-order $K(s)$ can be approximated. This is of particular importance as it shows the $\mathcal{H}_\infty$ design for full-order controller typically uses a large, and sometimes practically unnecessarily, high order.

Fig. 13. Hankel singular values of $K(s)$

Even reducing the order to 21, the practical implementation of $K(s)$ is still complicated. Keeping the order aside, this centralized controller requires long-distance fast communication. Therefore, in the next sections, a trade-off between performance and ease of practical implementation is obtained by defining the fixed-structure controller.

C. Fixed-structure Decentralized Controller

The centralized $K(s)$ is made decentralized by setting the off-diagonal partitions of $K(s)$ to zero, i.e., $k_{13}(s)$, $k_{14}(s)$, $k_{23}(s)$, $k_{24}(s)$, as well as, $k_{31}(s)$, $k_{32}(s)$, $k_{41}(s)$, and $k_{42}(s)$. Once again, the $\mathcal{H}_\infty$ optimal design is applied on the $G_w(s)$, but this time, $K(s)$ has a fixed structure (decentralized) and its order is initially limited to 21, based on previous experience.

The singular values of $T(s)$ and $S(s)$ are shown in Fig. 14. Although the singular values of $T(s)$ do not exceed the limits within the bandwidth, their attenuation cannot be maintained at -20 dB/decade for higher frequencies (singular values violate the boundary). This suggests that, compared with the full-order controller, the fixed-structure controller provides a lower damping at frequencies higher than bandwidth. This performance deterioration is due to moving from a centralized to a decentralized controller.

Fig. 14. Singular values of (a) $T(s)$, and (b) $S(s)$ with the fixed-structure, decentralized controller

The results obtained from the singular value analysis can be also observed from the transient response of the system, which is shown in Fig. 15. While no variables exceed the $\pm 10\%$ from the nominal value, the system damping is lower as compared to the damping of the full-order controller.

Fig. 15. Transient response of the control variables for the fixed-structure, decentralized controller with the order of 21

At this stage of design, we continue reducing the order of the decentralized controller from 21 to only 1 to study the extent of performance deterioration due to the lower order controller. Computing $K(s)$ for each order via $\mathcal{H}_\infty$ design principles, the maximum singular values of $T(s)$ and $S(s)$ are presented in Fig. 16. Note that only maximum singular values are shown for the sake of clarity. Even with a first-order controller, which corresponds to a PI compensator, the controller manages to confine the over/undershoot within the limits; however, the damping of the maximum singular values is significantly reduced (they exceed the attenuation of -20 dB/dec). Also, it can be seen that if the order of the controller is increased to a second order, the damping is improved. Furthermore, increasing it to a third order provides a damping approximately equal to the damping of the 21-order controller.

Fig. 16. Maximum singular values of (a) $T(s)$, and (b) $S(s)$ with the fixed-structure, decentralized controller with order from 21 to 1

Based on the previous control design analysis, it is reasonable to conclude that a fixed-structure, decentralized, third-order controller is able to make a trade-off between optimal performance level and the ease of practical implementation.

VI. Conclusion

In this paper, an $\mathcal{H}_\infty$ optimal design method has been implemented to derive a MIMO controller that regulates the internal energy of the master and slave MMCs, the DC voltage, and the active power flow of an MMC-based HVDC link. A general state-space model of the entire system suitable for $\mathcal{H}_\infty$ design has been developed, and the weighing functions as the indispensable parts of a meaningful $\mathcal{H}_\infty$ design have been detailed. First, a full-order, centralized controller that obtains near ideal performance level for the system has been designed. The practical implementation of this controller has been challenging since (i) it requires communication link between master and slave MMCs, and (ii) the order of the controller is high. So, the fixed-structure controller has been suggested to obtain a trade-off between the optimal performance and the ease of practical implementation. In the fixed-structure controller, each converter station uses its local variables and the order of the controller is limited in the design stage. In addition, the impact of the controller order on the system performance has been studied with the objective of finding more simple and implementable structures. Based on the conducted analysis, it can be stated that a fixed-structure, decentralized, third-order controller is able to obtain a good performance level ensuring a simple practical implementation.

APPENDIX

Transformation (21) is used to calculate the feedback measures i_s^{qdc} and u^{qdc}, whereas the inverse transformation (22) is adopted in v_{diff}^{qdc} to obtain v_{diff}^{qd},

$$\Delta x^{qdc} = \mathbf{T}_\mathbf{c}^{qd}(\Delta x^{qd}, \Delta e_\theta)^T \tag{21}$$

$$\Delta x^{qd} = \mathbf{T}_\mathbf{c}^{qd^{-1}}(\Delta x^{qdc}, \Delta e_\theta)^T \tag{22}$$

$$\mathbf{T}_\mathbf{c}^{qd} = \begin{bmatrix} \cos(e_{\theta 0}) & -\sin(e_{\theta 0}) & -\sin(e_{\theta 0})x_0^q - \cos(e_{\theta 0})x_0^d \\ \sin(e_{\theta 0}) & \cos(e_{\theta 0}) & \cos(e_{\theta 0})x_0^q - \sin(e_{\theta 0})x_0^d \end{bmatrix}$$

MMCs and AC grids parameters are given in Table I, while the HVDC cable parameters are provided in Table II.

TABLE I
MMC AND AC GRID PARAMETERS

Parameter	Symbol	Value	Units
Rated (base) active power	P_N	500	MW
Rated (base) AC-side voltage	U_N	320	kV
Rated (base) DC-side voltage	V_N^{dc}	±320	kV
Grid short-circuit ratio	SCR	10	-
Coupling impedance	R_s+jL_s	0.01+j0.2	pu
Arm reactor impedance	R_a+jL_a	0.01+j0.2	pu
Converter submodules per arm	N_{arm}	400	-
Average submodule voltage	V_{SM}	1.6	kV
Submodule capacitance	C_{SM}	8	mF

TABLE II
CABLE PARAMETERS

Symbol	Value	Units	Symbol	Value	Units
r_1	0.1265	Ω/km	l_1	0.2644	mH/km
r_2	0.1504	Ω/km	l_2	7.2865	mH/km
r_3	0.0178	Ω/km	l_3	3.6198	mH/km
c	0.1616	μF/km	g	0.1015	μS/km

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