# **Online Action Recognition**

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#### **Abstract**

Recognition in planning seeks to find agent intentions, goals or activities given a set of observations and a knowledge library (e.g. goal states, plans or domain theories). In this work we introduce the problem of Online Action Recognition. It consists in recognizing, in an open world, the planning action that best explains a partially observable state transition from a knowledge library of first-order STRIPS actions, which is initially empty. We frame this as an optimization problem, and propose two algorithms to address it: Action Unification (AU) and Online Action Recognition through Unification (OARU). The former builds on *logic unification* and generalizes two input actions using weighted partial MaxSAT. The latter looks for an action within the library that explains an observed transition. If there is such action, it generalizes it making use of AU, building in this way an AU hierarchy. Otherwise, OARU inserts a Trivial Grounded Action (TGA) in the library that explains just that transition. We report results on benchmarks from the International Planning Competition and PDDLGym, where OARU recognizes actions accurately with respect to expert knowledge, and shows real-time performance.

#### Introduction

The prediction of the most likely actions, plans or goals of an agent, has been a topic of interest in the planning community since the work by Ramírez and Geffner (2009), which posed the recognition task via planning given a domain theory and a set of observations. Other work adopted this convention of recognition as planning for related tasks, i.e. goal recognition and environment design (Keren, Gal, and Karpas 2014), production and recognition of context-free grammars (Segovia-Aguas, Jiménez, and Jonsson 2017a), classification of planning instances in generalized plans (Segovia-Aguas, Jiménez, and Jonsson 2017b), counter-planning (Pozanco et al. 2018), model recognition (Aineto et al. 2019) and recognition with noisy observations (Sohrabi, Riabov, and Udrea 2016; Aineto, Jimenez, and Onaindia 2020).

In this paper, we address the problem of *Online Action Recognition* (OAR), which consists in recognizing the actions performed by an agent in an *open-world*, given a *state* 

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### Online Action Recognition through Unification **Action Unification Hierarchy** action-96 **Action Unification** params: X0, X1, X2, X3, X4 pre: at(X3,X2), clear(X1), is-nongoal(X2), is-player(X3), location(X1), location(X2), move-dir(X1,X2,X0), move-dir(X2,X1,X4), thing(X3) action-100 add: at(X3,X1), clear(X2) params: X0, X1, X2, X3, X4 del: at(X3.X2), clear(X1) pre: at(X3,X2), clear(X1), is-player(X3), location(X1), location(X2), move-dir(X1,X2,X0), move-dir(X2,X1,X4), thing(X3) **Trivial Grounded Action** action-99 add: at(X3,X1), clear(X2) pre: at(player-01,pos-5-6), clear(pos-6-6), is-goal(pos-5-6), is-nongoal(pos-6-6), is-player(player-01), location(pos-5-6), location(pos-6-6), del: at(X3,X2), clear(X1) **Transition** move-dir(pos-6-6,pos-6-6,dir-right), move-dir(pos-6-6,pos-5-6,dir-left), thing(player-01) add: at(player-01,pos-6-6)?, clear(pos-5-6) ### 井 del: at(player-01,pos-5-6), clear(pos-6-6) 3

Figure 1: Illustration of AU and OARU in sokoban.

transition and a knowledge library of first-order STRIPS actions (also known as domain theory). We restrict OAR to symbolic inputs and deterministic action effects. However, we set an initially empty knowledge library that must be inductively filled from partially observable transitions. We describe an algorithm called Action Unification (AU), related to syntactic logic unification (Snyder 2001). AU generalizes two actions using an encoding to Weighted Partial MaxSAT (WPMS). We also propose Online Action Recognition through Unification (OARU), an algorithm that makes repeated use of AU to merge ad-hoc explanations of the transitions into its action library. In the following, we use action as a generic category; schema or model for parameterized actions; grounded action for parameter-less actions; and Trivial Grounded Action (TGA) for ad-hoc actions derived

directly from observations.

Example 1. Figure 1 uses sokoban (Junghanns and Schaeffer 1997) to illustrate AU and OARU. The whole AU hierarchy, which summarizes the history of merged actions, is in the top. We zoom in one part, showing: (1) action-96, a schema already present in OARU's library; (2) a TGA action-99, constructed from the transition depicted graphically at the bottom; and (3) a 5-ary schema action-100 produced by AU, which generalizes the previous two actions. The transition corresponds to the character moving to the right. AU has determined that the distance between action-96 and action-99 is 3. Later, we will see that this distance is calculated in terms of relaxed precondition predicates. Some predicates have a question mark indicating that they are uncertain due to the open world setting. We will also see how this uncertainty can be dispelled through AU. Overall, the example shows how OARU surmised from the past that the player could not move from a goal cell, and corrects this assumption when shown otherwise.

The contributions of this work are: (1) a formal method for generalizing two planning actions with the AU algorithm, which deals with an *NP-Hard* problem; (2) a scalable, accurate and suitable for real-time usage algorithm to recognize and acquire general schemata from partially observable state transitions, named OARU; and (3) an evaluation of acquired knowledge libraries with expert handcrafted benchmarks from the International Planning Competition (IPC) (Muise 2016) and PDDLGym (Silver and Chitnis 2020).

#### **Related Work**

The problem of OAR relates to plan, activity and intent recognition (PAIR) (Sukthankar et al. 2014) and learning action models (Arora et al. 2018). The recognition in PAIR is defined as a prediction task of the most plausible future, i.e. the most probable plan or goal an agent will pursue (Ramírez and Geffner 2009; Ramírez and Geffner 2010), while our recognition task serves as an explanation of the past (Chakraborti et al. 2017; Aineto et al. 2019; Aineto, Jimenez, and Onaindia 2020). Also, model-based approaches for recognition assume a knowledge library with goals, plans or domain theories is known beforehand. We release the problem from this assumption, where the knowledge is acquired in an incremental online fashion.

We refer to learning the representation of the world dynamics as the problem of learning action models, which has been a topic of interest for long time (Gil 1994; Benson 1995; Wang 1995). STRIPS-like actions have been learned with algorithms such as ARMS (Yang, Wu, and Jiang 2007) with a weighted MaxSAT; SLAF (Amir and Chang 2008) with an online SAT solver that computes the CNF formula compatible with partial observations; LOCM (Cresswell, McCluskey, and West 2009; Cresswell and Gregory 2011; Gregory and Cresswell 2015) computing finite state machines of object sorts; FAMA (Aineto, Jiménez, and Onaindia 2018; Aineto, Celorrio, and Onaindia 2019) learning from minimal observability with an off-the-shelf classical planner; and Representation Discovery (Bonet and Geffner

2020) that learns with a SAT solver from plain graphs. In the latter, only action labels are known, while in the rest of approaches, the name and parameters of each declarative action are known, which strictly constraints the learning problem. In our case, no prior labels and parameters bound the problem, like in Suárez-Hernández et al. (2020), but inductively learned with every new partially observable state transition

The work by Amado et al. (2018) proposes the LatPlan algorithm for goal recognition. It first learns grounded action models in the latent space from images, such as AMA2 (Asai and Fukunaga 2018). However, grounded actions are generated without soundness guarantees, may not generalize to other instances, and the learning method requires observing all possible non-symbolic transitions, while OARU inputs are symbolic, computes actions which do not require to observe all state transitions and generalize to other instances.

### **Preliminaries**

Let us first introduce preliminary notation on first-order logic and unification, and weighted partial MaxSAT.

### **First-order logic**

The formal language to describe relations between constant symbols is known as *first-order logic* or *predicate logic*. *Propositional logic* is subsumed by *first-order logic* in that propositions are specific interpretations of relations over constant symbols. The **syntax** of a first-order language consists of a (possibly) infinite set of *logical terms* (mathematical objects) and *well-formed formulae* (mathematical facts) denoted as T and  $\mathcal{F}$ , respectively.

**Definition 1** (First-Order Logical Term). A first-order logical term  $t \in T$  is recursively defined as  $t = c|v|f_n(t_1,\ldots,t_n)$  where t is either a constant symbol  $c \in \mathcal{C}$ , a variable symbol  $v \in \mathcal{V}$ , or a functional symbol  $f_n$  with n arguments s.t.  $t_i \in T$  for all  $1 \le i \le n$ .

**Definition 2** (Well-Formed Formula). A well-formed formula (wff)  $\varphi \in \mathcal{F}$  is inductively defined as a predicate (or atomic formula)  $\varphi = p_n(t_1, \ldots, t_n)$ ; a negation  $\neg \varphi$ ; a logical connective conjunction  $\varphi \land \psi$ , disjunction  $\varphi \lor \psi$ , implication  $\varphi \to \psi$ , or biconditional  $\varphi \leftrightarrow \psi$  s.t.  $\psi$  is also a wff; or a quantifier  $\forall_v \varphi$  or  $\exists_v \varphi$ , for universal and existential quantification of a wff  $\varphi$  and variable  $v \in \mathcal{V}$ , respectively.

The first-order **semantics** is a structure which consists of an interpretation function  $\mathcal{I}$  and a *universe*  $\mathcal{D}$  with the nonempty set of all objects. The set of objects is used to ascribe meaning to terms and formulae with an interpretation function but, while terms are interpreted into objects, predicates and other formulae have boolean interpretations. The *interpretation over a logical term*  $\mathcal{I}(t)$  is an assigned function to term t which maps a tuple of already interpreted arguments of n objects in  $\mathcal{D}^n$  to a single object in  $\mathcal{D}$ . Hence, the interpretation of constant terms is  $\mathcal{I}(c):\mathcal{D}^0\to\mathcal{D}$ , from variables is  $\mathcal{I}(v):\mathcal{D}^1\to\mathcal{D}$ , and functional symbols is  $\mathcal{I}(f):\mathcal{D}^n\to\mathcal{D}$ , which maps 0,1 and n objects into one, respectively. The interpretation of an n-ary predicate is  $\mathcal{I}(p):\mathcal{D}^n\to\{\text{true},\text{false}\}$ . The *interpretation over a well-formed formula*  $\mathcal{I}(\varphi)$  consists of: (1) the interpretation of

variables and functionals given a set of objects  $\mathcal{D}$ ; and (2) the boolean evaluation of predicate interpretations, and logical connectives. In this paper we only focus on constant, variables and first-order predicate symbols such as follows:

**Example 2.** In Figure 1, there are 7 predicate symbols  $\{at_2, clear_1, is-player_1, location_1, move-dir_3, thing_1\}$ , 5 variables  $\{X0, X1, X2, X3, X4\}$  and 5 constants  $\{player-01, pos-5-6, pos-6-6, dir-left, dir-right\}$ . In the first state shown, the player is in position pos-5-6, so  $\mathcal{I}(at_2(player-01, pos-5-6)) = true$ . In the second one, it is no longer there, so the same interpretation is false. Predicates with variable arguments such as at(X1, X0) require an interpretation of variables at(X1, X0) require an interpretation of variables at(X1, X0) before evaluation (known as grounding), e.g. at(X1, X0) = at(X1, X1) = at(X1) =

### **First-Order Unification**

The problem of first-order unification (unification for short) consists in making both sides of a set of equations syntactically equal (Snyder 2001). Then, the unification problem is defined as a set of potential equations  $\mathcal{U}=\{l_1 \doteq r_1,\ldots,l_n \doteq r_n\}$  where each left and right-hand sides are first-order logical terms  $l_i,r_i\in T$ .

A substitution  $\sigma: \mathcal{V} \to T$  is a function that maps variables into terms, and its notation is  $\{v_1 \to t_1, \dots, v_n \to t_n\}$  where each  $v_i \in \mathcal{V}$  and  $t_i \in T$ . An instance of a term t is  $t\sigma$  and expanded as  $t\{v_1 \to t_1, \dots, v_n \to t_n\}$ , where all variable substitutions are applied simultaneously. Two terms l and r of a potential equation  $l \doteq r$  are syntactically equal if exists a substitution  $\sigma$  such that  $l\sigma \equiv r\sigma$ , where  $\equiv$  stands for syntactically equivalent.

**Example 3** (Substitution). Given a pair of constants player,  $loc \in \mathcal{C}$  and variables  $v_1, v_2 \in \mathcal{V}$ , the potential equation  $at(v_1, loc) \doteq at(player, v_2)$  is syntactically equivalent when the substitution is  $\sigma = \{v_1 \rightarrow player, v_2 \rightarrow loc\}$ .

The substitution of terms is also named a *unifier*. A *solution* to  $\mathcal{U}$  is a *unifier*  $\sigma$  if  $l_i\sigma \equiv r_i\sigma$  for all  $1 \leq i \leq n$ . In the context of this paper, we are interested only in the unification of predicates without nested arguments (i.e. no functions).

#### **Weighted Partial MaxSAT (WPMS)**

A literal l is a boolean variable l=x or its negation  $l=\neg x$ . A clause c is a disjunction of literals, and a weighted clause (c,w) extends the clause c with a natural number  $w\in\mathbb{N}$  (or  $\infty$ ) which represents the cost of not satisfying (c,w). In WPMS, a weighted clause is either hard if the weight is infinite, and soft otherwise.

In contrast to (partial Max) SAT where the input formula is described in conjunctive normal form (conjunctions of clauses), the input to a WPMS problem is a set of weighted clauses  $\Phi = \{(c_1, w_1), \ldots, (c_n, w_n)\}$ , which may be divided into  $\Phi_s = \{(c_1, w_1), \ldots, (c_m, w_m)\}$  and  $\Phi_h = \{(c_{m+1}, \infty), \ldots, (c_n, \infty)\}$ , for soft and hard weighted clause sets, respectively. Then,  $\Phi = \Phi_s \cup \Phi_h$ .

Let  $vars(\Phi)$  be the set of all boolean variables in  $\Phi$ . A *truth assignment* is the interpretation of boolean variables

into 0 or 1, i.e.  $\mathcal{I}: vars(\Phi) \to \{0,1\}$ . Then, the truth assignment for a set of weighted clauses  $\Phi$  is:

$$\mathcal{I}(\Phi) = \sum_{i=1}^{m} w_i (1 - \mathcal{I}(c_i)). \tag{1}$$

Therefore, WPMS is the problem of finding an *optimal* assignment  $\mathcal{I}^*(\Phi)$  (Ansótegui and Gabas 2013), such that the hard clauses are satisfied and Equation 1 is minimized:

$$\mathcal{I}^*(\Phi) = min\{\mathcal{I}(\Phi)|\mathcal{I}: vars(\Phi) \to \{0, 1\}\}. \tag{2}$$

The optimal assignment of Equation 2 could be inferred from  $\mathcal{I}^*(vars(\Phi))$ . Also, note that Equation 1 only iterates over soft clauses, since falsifying a hard clause would result in infinite cost and the solution would be invalid.

### **STRIPS Action Model**

This work aims at recognizing the action models  $\mathcal{A}$  in a domain represented with the STRIPS fragment of the Planning Domain Definition Language (PDDL) (McDermott et al. 1998; Haslum et al. 2019) and negations. An *action schema*  $\alpha \in \mathcal{A}$  is defined by a 3-tuple  $\alpha = \langle head_{\alpha}, pre_{\alpha}, eff_{\alpha} \rangle$  where:

- $head_{\alpha}$  is the name of the action and a set of variables  $\mathcal{V}' \subseteq \mathcal{V}$  referenced in  $pre_{\alpha}$  and  $eff_{\alpha}$ ,
- $pre_{\alpha}$  is a well-formed formula with a conjunction of literals that represent the action preconditions,
- $eff_{\alpha}$  is the list of effects which consists of literals that are updated either to true (add/positive effect) or false (delete/negative effect). Often,  $eff_{\alpha}$  is splitted into  $add_{\alpha}$  and  $del_{\alpha}$  lists.

#### **Online Action Recognition**

The problem of Action Recognition is a trivial task under certain assumptions such as fully observable transitions, finite set of relations and objects that are known to be true (closed-world assumption), and a complete knowledge library of STRIPS action models to represent the possible interactions between an agent and the world. From an observer perspective, the action applied by an agent would be easily identified with a naive linear algorithm that loops over the set of grounded actions. Nevertheless, the problem becomes more complex when the knowledge library only contains action signatures (set of action symbols with their arity) and transitions are partially observable. Then, the action models must be learned (Amir and Chang 2008; Mourao et al. 2012; Aineto, Jiménez, and Onaindia 2018). The problem is even harder when the knowledge library starts empty and the interpretation of the each relation between objects is unknown beforehand (open-world assumption) (Minker 1982). We focus on the online version of the latter paradigm.

#### **Input: set of observations**

Inputs consist of observations over a set of transitions. Implicitly, each transition is a (s,a,s') triplet, where that s is the previous state, a is the action applied by an agent, and s' is the successor state. However, in OAR, the action applied

by the agent is never observed, so each transition observation is o=(s,s'). When the input consists of a set of observations  $\mathcal{O}$ , each observation  $o\in\mathcal{O}$  is sequentially processed following the order in which they were observed.

We use a factored state representation where each state s is defined by a set of predicates  $p_n(d_1, \ldots, d_n)$  (or their negation), such that  $p_n$  is a predicate symbol of arity n, and every  $d_i \in \mathcal{D}$  for  $1 \le i \le n$  is an object in the universe.

In an open world, a grounded predicate  $p_n(d_1,\ldots,d_n)$  is known to be true if each  $d_i\in\mathcal{C}$  and it is explicitly true in a given state s (resp.  $\neg p_n(d_1,\ldots,d_n)$ ). Therefore, a state s is fully observable if for every predicate and constants, there is an interpretation  $\mathcal{I}_s(p_n(d_1,\ldots,d_n))$  which maps the grounded predicate either to true or false. Thus, a state s is partially observable if there is no such interpretation  $\mathcal{I}_s$  for all known predicates and constants.

Next, we introduce some definitions over  $\varphi = p_n(d_1, \ldots, d_n)$  that will become handy:

- A function type( $\varphi$ ) :=  $p_n$ .
- A function  $\arg_i(\varphi) := d_i$ .
- A function  $\iota_s(\varphi) := \text{true iff } \mathcal{I}_s(\varphi) \text{ is defined, else false.}$

#### **Output: first-order STRIPS action model**

The output for each observation  $o \in \mathcal{O}$  is a grounded action a composed of the action model  $\alpha \in \mathcal{A}$  and its substitution  $\sigma \in \Sigma$ . We denote as  $\mathcal{D}_{\alpha}$  the set of objects (either constants or variables) referenced within  $\alpha$ 's effects and preconditions. An action a is said to be grounded or instantiated when the variables in  $head_{\alpha}$  are substituted by a tuple of constants from  $\mathcal{D}$  of equal size. We denote this by  $\alpha\sigma$ , where  $\sigma \in \Sigma$  is an object substitution (and thus  $\Sigma$  is the set of all possible substitutions), as those introduced earlier. This nomenclature extends naturally to the substitution of arbitrary objects, not just variables. We adopt the convention that if  $\sigma$  does not define an explicit substitution for an object  $d \in \mathcal{D}_{\alpha}$ , then its application leaves all references to d unchanged.

A **solution** to the problem is then a *grounded action* a, such that  $a=\alpha\sigma$  where  $\alpha\in\mathcal{A}$  and  $\sigma\in\Sigma$ , which completes the observation (s,s') into (s,a,s') and the following conditions hold:

**Condition 1** (Valid Preconditions). The interpretation evaluates the preconditions to true, i.e.  $\mathcal{I}_s(pre_a) = \top$ , in other words, the action preconditions hold in the previous state, i.e.  $pre_a \subseteq s$ .

**Condition 2** (Valid Transition). The successor state s' is a direct consequence from applying a in s, such that  $s' = (s \setminus del_a) \cup add_a$ .

#### **Problem statement**

The OAR problem is tabular in that background theories, such as schemata  $\mathcal{A}$ , are unknown. Indeed, the problem is twofold, where the action is recognized if exists in the knowledge library, otherwise learns a new action model (Arora et al. 2018) and unifies it with previous knowledge.

**Definition 3** (Online Action Recognition). *The OAR problem consists in finding a function*  $\mathcal{P}: \mathcal{O} \to \mathcal{A} \times \Sigma$  *that* 

sequentially maps each observation  $o \in \mathcal{O}$  into an action  $\alpha \in \mathcal{A}$ , and a substitution  $\sigma \in \Sigma$  s.t. Conditions 1 and 2 hold. If  $\alpha \notin \mathcal{A}$  initially, a new  $\alpha$  must be learned.

### Methodology

The main algorithm is OARU. It starts with the set of predicate symbols but neither universe nor action library knowledge are known (i.e.  $\mathcal{D} = \emptyset$  and  $\mathcal{A} = \emptyset$ ). Both are greedily learned with every new observation o = (s, s') as follows:

- 1. For each pair of consecutive states s and s', it generates a Trivial Grounded Action (TGA) a that explains this transition, but does not generalize.
- 2. OARU merges a with the *closest* action in  $\alpha \in \mathcal{A}$  via Action Unification (AU) using a weighted partial MaxSAT, which results in a new action  $\alpha'$  and a grounding  $\sigma$ .
- 3. OARU recognizes grounded action  $a' = \alpha' \sigma$  as the underlying action that produced observation o.

#### **Construction of Trivial Grounded Actions**

A TGA is a grounded STRIPS action a that explains just one transition from s to s'. Hence, s' can be reached if a is applied to s, but does not generalized to other states. This may be also denoted by  $s \xrightarrow{a} s'$ . To construct a in a full observability setting, we set the conjunction of the predicates in s as the precondition, i.e.  $pre_a = s$ . The effects are defined as an add list  $add_a = (s' \setminus s)$ , and a delete list  $del_a = (s \setminus s')$ . To extend this basic construction to support partial observability, we consider uncertain predicates in s and s' as potentially true and false, and mark their occurrences within  $pre_a$ ,  $add_a$  and  $del_a$  and effect as uncertain s.

The preconditions and effects of any action can be joined into a single set  $L_a$  of *labeled predicates*, each defined by a triplet  $\chi = (\varphi, l, k)$ , where:

- $\varphi = p_n(d_1, \dots, d_n)$  is a predicate as defined earlier,
- $l \in \{\text{PRE}, \text{ADD}, \text{DEL}\}$  is the label to denote either the precondition, add or delete list,
- k is either true if the predicate is known to belong to l with total certainty, or false if it is unknown.

Notice that  $L_a$  is sufficient to represent a, since the parameters of a can be constructed as the union of all the variables appearing in  $L_a$  (i.e. all variables in  $\mathcal{D}_a$ ).

The k flag of a labeled predicate is set depending on whether  $\varphi$  is known in s and s'.

**Example 4.** Consider sokoban, and a predicate  $\varphi = \text{at}(\text{agent}, \text{pos-1-1})$ . Suppose  $\mathcal{I}_s(\varphi) = \text{true}$ , but  $\iota_{s'}(\varphi) = \text{false}$  (i.e. the location of the agent is known to be (1,1) in s, but this is not known with certainty in s'). Then, the created TGA contains a labeled predicate  $\chi = (\varphi, \text{DEL}, \text{false})$  because it is unknown whether  $\varphi$  is removed in the transition.

<sup>&</sup>lt;sup>1</sup>The full details are given in the extended version of this article in arXiv.org

### **Action Unification with Weighted Partial MaxSAT**

Let us focus on Action Unification (AU), OARU's mechanism to merge a TGA into its action library.

AU's goal is to find an action  $\alpha_u$  that generalizes  $\alpha_1$  and  $\alpha_2$ , if it exists. We say that  $\alpha_u$  generalizes  $\alpha_1$  and  $\alpha_2$  if there are two substitutions  $\sigma_1, \sigma_2$  such that  $\alpha_i' = \alpha_u \sigma_i$ ,  $eff_{\alpha_i'} = eff_{\alpha_i}$  and  $pre_{\alpha_i'} \models pre_{\alpha_i}$  for  $i \in \{1, 2\}$ . Intuitively,  $\alpha_u$  preserves the effects of  $\alpha_1$  and  $\alpha_2$ , while its preconditions are relaxations of  $pre_{\alpha_1}$  and  $pre_{\alpha_2}$ , lifting some objects in the process. First, we seek to preserve as many predicates in the precondition as possible. Second, among all the  $\alpha_u$  actions with maximal preconditions, we want one with the least number of new parameters. This relaxation/lifting mechanism makes  $\alpha_u$  applicable in a wider range of situations. So far, we have described AU as a dual-objective optimization problem.

To perform AU given  $\alpha_1$  and  $\alpha_2$ , we encode as a WPMS the problem of finding an injective partial function au:  $\mathcal{D}_{\alpha_1} \to \mathcal{D}_{\alpha_2}$  such that  $\tau(o_1) = o_2$  implies that  $o_1$  and  $o_2$ will map to the same object in  $\alpha_u$ . If  $o_1 = o_2 = o \in \mathcal{C}$ , the reference to o is maintained within  $\alpha_u$  as a constant. Otherwise,  $o_1$  and  $o_2$  will be lifted. We say that two labeled predicates  $\chi_1=(\varphi_1,l_1,k_1)\in \mathcal{L}_{\alpha_1}$  and  $\chi_2=(\varphi_2,l_2,k_2)\in \mathcal{L}_{\alpha_2}$ match iff  $l_1 = l_2$ ,  $type(\varphi_1) = type(\varphi_2) = p_n$ , and  $\tau(\arg_i(\varphi_1)) = \arg_i(\varphi_2)$  for  $1 \leq i \leq n$ . We denote as  $M_{\alpha_1,\alpha_2}$  the set of tuples  $(\chi_1,\chi_2)$  of potential matches. We say that a labeled predicate  $\chi \in L_{\alpha_i}$  is *preserved* if it has a match in the other action. A labeled predicate is preserved as certain if it is matched with a certain one. In order of priorities, our conditions for  $\tau$  are: (1) all certain effect predicate are preserved; (2) as many precondition and uncertain predicates as possible are preserved; and (3) as fewer parameters as possible are introduced.

In the WPMS encoding, denoted as  $\Phi_{\alpha_1,\alpha_2}$ , we use the following decision variables:

- $x_{o_1,o_2}$  for  $o_i \in \mathcal{D}_{\alpha_i}$ , means that  $\tau$  maps  $o_1$  to  $o_2$ ,
- $y_{\chi_1,\chi_2}$  for  $(\chi_1,\chi_2) \in M_{\alpha_1,\alpha_2}$ , means that  $\chi_1$  matches  $\chi_2$ ,
- $z_{i,\chi_i}$  s.t.  $i \in \{1,2\}, \chi_i \in \mathcal{L}_{\alpha_i}$ , means that  $\chi_i$  is preserved.

We define four constraints that must be satisfied (Hard), and two constraints to be optimized (Soft):

**(H1)**  $\tau$  is an injective partial function, so  $\forall o_1, o_2$ :

At-Most-1(
$$\{x_{o_1,o'_2} \mid \forall o'_2\}$$
),  
At-Most-1( $\{x_{o'_1,o_2} \mid \forall o'_1\}$ ). (3)

(**H2**) Two potential matches  $(\varphi_1, \ldots)$  and  $(\varphi_2, \ldots)$ , from  $\alpha_1$  and  $\alpha_2$  respectively, match iff  $\tau$  maps every ith argument of  $\varphi_1$  to the corresponding ith argument of  $\varphi_2$ :

$$y_{\chi_1,\chi_2} \Leftrightarrow \bigwedge_i x_{\arg_i(\varphi_1),\arg_i(\varphi_2)}.$$
 (4)

(H3) A labeled predicate  $\chi_i \in L_{\alpha_i}$  is preserved iff it has at least one match in the other action:

$$z_{1,\chi_1} \Leftrightarrow \bigvee_{\chi'_2} y_{\chi_1,\chi'_2},$$

$$z_{2,\chi_2} \Leftrightarrow \bigvee_{\chi'_1} y_{\chi'_1,\chi_2}.$$
(5)

(**H4**) Preserve non uncertain effects (i.e.  $\chi_i = (\varphi_i, l_i, \text{true}) \in L_{\alpha_i}$ , with  $l_i \neq \text{PRE}$ ):

$$z_{i,\chi_i}$$
. (6)

**(S1)** (Weight=1) Avoid lifting objects, i.e.  $\forall o_1, o_2 \in \mathcal{C}$  s.t.  $o_1 \neq o_2$ :

$$\neg x_{o_1,o_2}. \tag{7}$$

**(S2)** (Weight= $W_{big}$ ) Preserve preconditions and uncertain effects (i.e.  $\chi_i = (\varphi_i, l_i, k_i)$  with  $l_i = \text{PRE} \vee \neg k_i$ ):

$$z_{i,\gamma_i}$$
. (8)

For compactness, **(H2)** and **(H3)** have not been expressed in *Clausal Normal Form* (CNF), but transforming them to CNF is trivial through the Tseitin transformation (Tseitin 1983). In **(H1)**, At-Most-1 forbids more than one of the literals in the given set to become true. We use the quadratic encoding. Different weights are given to **(S1)** and **(S2)**.  $W_{big}$  must be large enough so that preserving predicates has priority over avoiding the introduction of parameters.  $W_{big} \geq \min(|\mathcal{D}_{\alpha_1}|, |\mathcal{D}_{\alpha_2}|) + 1$  accomplishes this. Let us highlight that  $\mathcal{I}^*(\Phi_{\alpha_1,\alpha_2})$  may be interpreted as a scaled distance between  $\alpha_1$  and  $\alpha_2$ :

$$\mathcal{I}^*(\Phi_{\alpha_1,\alpha_2}) = W_{big} \cdot N_{np} + N_{param},$$
  
$$dist_{\alpha_1,\alpha_2} = \mathcal{I}^*(\Phi)/W_{big},$$
 (9)

where  $N_{np}$  is the number of non-preserved predicates, and  $N_{param}$  is the number of introduced parameters.  $dist_{\alpha_1,\alpha_2}$  has an intuitive meaning: its integer part represents the number of eliminated predicates  $(N_{np})$  while its fractional part is proportional to the number of introduced parameters  $(N_{param}/W_{big})$ . If  $\alpha_1$  and  $\alpha_2$  cannot be unified, we define  $dist_{\alpha_1,\alpha_2} = \infty$ .

**Complexity of Action Unification.** We have proposed an approach for AU that requires a reduction to a WPMS problem. However, WPMS is known to be *NP-Hard*, so it is reasonable to wonder if AU is in a more tractable complexity class. We claim that this is not the case.

**Theorem 1.** Action Unification's problem is NP-Hard.<sup>2</sup>

Despite the worst-case complexity of AU, we show that, in practice, real-time performance is possible.

#### **Online Action Recognition through Unification**

OARU shows similarities to *Hierarchical Clustering* (HC). Actions act as data points, and AU computes distances and builds clusters. Unlike in standard HC, we may cluster only actions whose effects can be preserved.

OARU's recognition subroutine is outlined in Algorithm 1. This subroutine updates  $|\mathcal{A}|$  on the basis of a new observation o=(s,s'), and explains  $s\to s'$  with a grounded action. It starts building a TGA a from o (line 1). After initializing some bookkeeping variables (line 2), it obtains, if possible, the closest action to a from  $|\mathcal{A}|$ , and the result of AU (lines 3-8). If a could be unified to at least some  $\alpha \in \mathcal{A}$ ,  $\alpha$  is replaced by the unified action  $\alpha'$ , and a' (the

<sup>&</sup>lt;sup>2</sup>Proof in extended article in arXiv.org

## **Algorithm 1** OARU

```
Input: Observation o = (s, s'), action library A
   Output: Grounded action a' s.t. s \xrightarrow{a'} s'
  1: a \leftarrow \text{BuildTGA}(s, s')
 2: \alpha \leftarrow \emptyset, \alpha' \leftarrow \emptyset, d_{min} \leftarrow \infty
 3: for all \beta \in \mathcal{A} do
           (\alpha_u, dist_{\beta,a}) \leftarrow ActionUnification(\beta, a)
 4:
          if dist_{\beta,a} < d_{min} then \alpha \leftarrow \beta, \alpha' \leftarrow \alpha_u, d_{min} \leftarrow dist_{\beta,a}
 5:
 6:
          end if
 7:
 8: end for
 9: if \alpha' \neq \emptyset then
10:
          Remove \alpha from A
           a' \leftarrow \alpha' \sigma, with \sigma s.t. eff_{\alpha' \sigma} = eff_a
11:
12:
           Add \alpha' to \mathcal{A}
13: else
14:
           a' \leftarrow a
15:
           Add a to A
16: end if
17: return a'
```

return value) is set to the grounding of  $\alpha'$  that fills the gap in o (lines 9-12). Otherwise, the TGA is assigned to the return value and added as is to  $|\mathcal{A}|$  (lines 13-16).

In practice, we also have a top-level procedure that initializes  $\mathcal{A}$  to an empty set and runs Algorithm 1 for each observation it encounters. OARU fills  $\mathcal{A}$  progressively, and outputs the grounded action that explains each transition.

## **Evaluation**

We have implemented<sup>3</sup> and evaluated<sup>4</sup> OARU in a benchmark of 9 domains (Muise 2016; Silver and Chitnis 2020). We have conducted two sets of experiments: with full and with partial observability. We use goal-oriented traces, i.e. list of pairs of consecutive states where a goal condition holds in the last state, so that all relevant actions are produced.

For each domain, OARU observes the transitions that arise while solving 8 problems in succession. Each problem depicts a different number of objects and, thus, has a different impact in the number of variables and clauses that AU needs to encode<sup>5</sup>. For each domain, we report:

- $|\mathcal{A}|$ : final size of OARU's library.
- $|\mathcal{O}|$ : total number of transitions across the 8 problems.
- T: Average CPU time taken by Algorithm 1.
- M: Peak memory usage for solving AU.
- Prec.: Precision of a recognized action a<sub>g</sub>, compared to the grounded action a<sub>ref</sub> that was used to perform the tran-

Table 1: Results with (a) full and (b) partial observability. **T** is in milliseconds, **M** in MB, and **Prec.** and **Rec.** in %.

	,		,			
Domain	$ \mathcal{A} $	$ \mathcal{O} $	T	M	Prec.	Rec.
blocks	4	96	$14 \pm 5$	1.0	$100 \pm 1$	$100 \pm 0$
depot	5	300	$70 \pm 57$	1.7	$92\pm12$	$96 \pm 5$
elevator	3	147	$31\pm32$	1.6	$87\pm11$	$73 \pm 10$
gripper	3	262	$46\pm30$	1.7	$100\pm3$	$100\pm0$
minecraft	4	23	$7\pm4$	1.5	$97 \pm 6$	$100 \pm 0$
onearmedgripper	3	284	$38\pm21$	1.7	$100 \pm 3$	$100 \pm 0$
rearrangement	4	42	$19\pm11$	1.7	$93 \pm 9$	$97 \pm 5$
sokoban	4	598	$49\pm24$	1.7	$90 \pm 3$	$91 \pm 2$
travel	5	48	$22\pm17$	1.9	$84\pm15$	$89\pm11$
			(a)			
Domain	$ \mathcal{A} $	0	Т	M	Prec.	Rec.
blocks	4	96	$16 \pm 7$	1.3	$90 \pm 15$	$99 \pm 6$
depot	5	300	$120\pm66$	3.8	$88\pm15$	$95 \pm 7$
elevator	3	147	$48\pm26$	5.3	$83 \pm 22$	$66 \pm 19$

gripper 262  $45 \pm 25$ 5.2  $96 \pm 10$  $100 \pm 2$ minecraft 4 23  $30 \pm 27$ 5.4  $65 \pm 23$  $99 \pm 6$  $32 \pm 14$ onearmedgripper 284 5.4  $95 \pm 10$  $100 \pm 3$ rearrangement 4 42  $28 \pm 15$ 5.5  $80 \pm 20$  $96 \pm 7$ 4 598  $158\pm146$ 13  $89 \pm 12$  $86 \pm 5$ sokoban 5 48  $91 \pm 114$ 16  $68 \pm 27$  $85 \pm 10$ travel (b)

sition. It is the ratio of correct labeled predicates in  $a_q$ :

$$Prec_{a_{ref}}(a_g) = 100 \frac{|\mathcal{L}_{a_g} \cap \mathcal{L}_{a_{ref}}|}{|\mathcal{L}_{a_g}|}.$$
 (10)

• **Rec.**: Average recall (%) of  $a_g$  compared to  $a_{ref}$ . It is the ratio of predicates in  $a_{ref}$  that have been captured by  $a_g$ :

$$Rec_{a_{ref}}(a_g) = 100 \frac{|\mathcal{L}_{a_g} \cap \mathcal{L}_{a_{ref}}|}{|\mathcal{L}_{a_{ref}}|}.$$
 (11)

**T** and **M** are performance-related (quantitative evaluation). **Prec.** and **Rec.** measure the correctness and completeness of  $a_g$  (qualitative evaluation). We report both evaluations for full and partial observability in Table 1.

Let us focus first on the full-observability results (Table 1a). Times are in general well below 1 second. Therefore, our claim about the real-time potential of OARU holds, as long as the throughput of observations is reasonably limited (e.g. a few seconds between observations). Memory requirements are also within a reasonable bound, requiring in the order of a few MBs to solve AU. However, we have found that some domains are challenging to OARU when it has just started with an empty  $\mathcal{A}$ . For instance, OARU cannot cope with *gripper* in a timely fashion if it starts with a problem with 20 objects or more. This is understandable, because the size of the WPMS encoding grows quadratically with the matches of predicates and objects. We believe this is not a

<sup>&</sup>lt;sup>3</sup>https://github.com/sprkrd/sat\_strips\_learn

<sup>&</sup>lt;sup>4</sup>Machine specifications: AMD Ryzen 7 3700X @ 3.6GHz, 32GB of DDR4 RAM CL15 @ 3200MHz.

<sup>&</sup>lt;sup>5</sup>See extended article for domain and problem characteristics

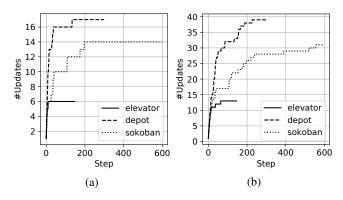


Figure 2: Accumulated updates for three domains in (a) full and (b) partial observability.

huge setback, since it is very natural to ramp up the number of objects progressively. Additional performance is achieved through some optimizations not described here, like a *broad phase* stage in Algorithm 1 to avoid a full call to AU when it is obvious that actions cannot be unified.

We see in general that the recall is very large. The reason lies in how OARU works. In deterministic settings, effects are almost always identified with high accuracy, while preconditions are relaxed as little as possible. In 5 cases the recall is lower than 100%. In 3 of those, moreover, |A| is not equal to the number of actions in the expert's domain or Ground Truth Model (GTM). In elevator (GTM with 4 actions), there are two very similar actions for going up and down that only differ in one predicate of the precondition. OARU recognizes them as a generic move up or down action and discards that predicate. On the other hand, sokoban (GTM with 3 actions) contains an action for moving stones to non-goal positions that deletes an at-goal $(\cdot)$  predicate which is not necessarily in the state. Thus, its deletion is not always observed, and OARU fills |A| with two different actions that address different contexts. Travel (GTM with 4 actions) presents a similar situation, but for adding an already present predicate. The 2 remaining cases (rearrangement and depot) have lower than 100% recalls for similar reasons, but their |A| is equal to their GTM's.

The precision is not as high as the recall because OARU holds onto precondition predicates that are not present in the GTM, but are confirmed by all the observed transitions. For instance, grid-based worlds with adjacency predicates, like *sokoban*, show symmetries. To move from pos-5-6 to pos-6-6, a predicate such as move-dir(pos-5-6, pos-6-6, dir-right) must hold true. However, OARU observes that, whenever that happens, the predicate move-dir(pos-6-6, pos-5-6, dir-left) is also true. However, the GTM lists only one adjacency precondition. In most cases, OARU is not incorrect from a semantic standpoint, but the precision metric counts it as an error.

Table 1b shows the results for partial observability. These have been obtained making parts of the state ignored by OARU at random. Namely, the interpretation of from 0 to 5 predicates has been removed. Results are averaged over 5 executions of the 8 problems for all the domains. We

see penalties in the performance. This is most evident in *sokoban*, because all problems present more than 50 objects. The introduction of uncertain predicates drives up the number of potential matches significantly. Other domains show smaller increases in recognition times and memory requirements. Precision and recall have suffered, but are not significantly worse than the full observability results. We have also experimented with greater proportions of unknown predicates (0 to 10). OARU starts requiring large computational times for AU in *elevator* and *sokoban* (in the order of several minutes). As we would expect, partial observability has a much larger computational burden. A way to address all these problems is to start with smaller problems and build |A| progressively.

Figure 2 shows how often A is updated (via action addition or replacement) in three domains. The X axis shows the number of steps taken, while the Y axis shows the accumulated number of updates. We consider that an update is made only when a TGA is not entirely subsumed by an action already present in A (i.e. either a parameter is added or a labeled predicate is removed). Notice that A stabilizes early on for elevator. For sokoban, it plateaus half-way towards the final number of steps, and depot at approximately two thirds. The drop in the rise rate of the curves shows that, early on, A is empty and is updated rather frequently with new observations. However, as A is filled, OARU sees more actions and comes up with good general schemata that generalize all the transitions that could happen. The shapes of the full and partial observability curves are very similar. The most notable differences are in the scale and that the partial observability curves do not entirely plateau, but still exhibit some small jumps towards the final steps. This is intuitive: partial observations cause an increased number of updates, and that A does not stabilize until later.

#### **Discussion and Conclusions**

In this paper, we have proposed OARU, an algorithm for recognizing STRIPS action models from partially observable state transitions. It uses AU to merge observations into its action library, constructing an action hierarchy and improving its action models through generalization. OARU shows a high computing performance and the recognized actions have strong similarities to expert handcrafted models. Thus, OARU ability to learn without action signatures makes it a promising contender to other model learning approaches.

As future work we consider to learn more expressive actions with generalized planning (Jiménez, Segovia-Aguas, and Jonsson 2019), and refine actions with negative examples (Aguas, Jiménez, and Jonsson 2020). Allow noisy observations extracted from sensor sampling (Yang 2009) which would make it suitable for more realistic applications on the wild. Also recognizing hidden variables and intermediate states in action sequences with Bayesian inference (Aineto, Jimenez, and Onaindia 2020). Finally, we think there is a great potential for an application-ready methodology adopting OARU's philosophy to learn from low-level data, i.e. robot motions (Konidaris, Kaelbling, and Lozano-Perez 2018), images (Asai and Fukunaga 2018) or graphs (Bonet and Geffner 2020).

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