Passivation blocks for fault tolerant control of nonlinear systems[☆]

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ABSTRACT

This paper describes a novel passivity/dissipativity based approach for fault tolerant control (FTC) of nonlinear systems by means of fault hiding, i.e., by inserting reconfiguration blocks (RBs) between the plant and controller to mitigate the fault effects. The proposed approach is used to design a new kind of RB, called passivation block (PB), which is generically employed for sensor and actuator faults and achieves simultaneously series, feedback and feedfoward passivation of the controller during a fault occurrence. Based on the dissipativity theory, new conditions are obtained to design a dynamic PB (DPB) which requires minimum information on the system model. In particular, the proposed DPB can be systematically obtained by combining the LMI-based conditions based on the knowledge about the passivity indices. Numerical simulations are carried out and indicate that the PBs are able to stabilize an example of a faulty nonlinear system.

1. Introduction

During the last decades, fault tolerant control (FTC) for safety-critical and industrial processes has been extensively studied to improve the reliability and availability of such systems which are subject to sensor and actuator faults, such as offsets and stucks, that degrade the system performance and may lead to instability (Argha, Su, & Celler, 2019; Yang, Jiang, & Zhang, 2012). Therefore, FTC aims to ensure the stability and desirable performance even during the fault occurrence.

In general, FTC techniques can be divided into two groups: passive and active techniques. Passive FTC (PFTC) (Stefanovski, 2018) considers the fault occurrence as a perturbation or uncertainty that must be rejected by the designed controller. In this case, it is not necessary further information on the severity and localization of faults, since the FTC is designed to be robust with respect to them at the expense of performance loss. Otherwise, active FTC (AFTC) (Lan & Patton, 2016) uses the information of a fault detection and isolation (FDI) system to modify the control law aiming to mitigate the fault effects avoiding the unnecessary performance loss in fault-free conditions.

Among the AFTC techniques, the fault hiding stands out due to its ability to maintain the same controller designed for the nominal (fault-free) system during the fault occurrence. Indeed, fault hiding consists in inserting a reconfiguration block (RB) between the faulty plant and the controller that corrects the sensor measurements and translates the control signals by control allocation provided by a controller that does not receive the information about the fault occurrence. Most of fault hiding applications deal with linear systems (Lunze & Steffen, 2006), although there are also applications for linear parameter varying (Quadros, Bessa, Leite, & Palhares, 2020; Rotondo, Cristofaro, & Johansen, 2018), Hammerstein–Wiener (Richter, 2011), piecewise affine (Richter, Heemels, Wouw, & Lunze, 2011), and Takagi–Sugeno fuzzy (Bessa, Puig, & Palhares, 2020; Filasová, Krokavec, & Liščinský, 2016) systems.

Dissipativity and passivity theory is an important paradigm of nonlinear system analysis due to its relation with input–output stability (Khalil, 2000; Kottenstette, McCourt, Xia, Gupta, & Antsaklis, 2014). In particular, the passivity indices (McCourt & Antsaklis, 2010) and passivation techniques (Xia, Antsaklis, Gupta, & Zhu, 2017; Zhu, Xia, & Antsaklis, 2017) are studied to obtain closed-loop systems with desired passivity properties. The dissipativity/passivity framework has been already used to design

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controllers for different classes of systems, e.g., switched (McCourt & Antsaklis, 2010) and networked systems (Zhu et al., 2017), but there are few results for FTC. In Yang, Cocquempot, and Jiang (2008), the concepts of global dissipativity and passivity are proposed to quantify the fault tolerance and to decide if it is necessary to design an FTC law for each fault mode. Dissipativity and passivity theory are also used to obtain PFTC systems in some recent works (Sakthivel, Saravanakumar, Kaviarasan, & Lim, 2017; Selvaraj, Kaviarasan, Sakthivel, & Karimi, 2017).

In this work, a novel RB called passivation block (PB) is presented. It can be generically used for actuator and sensor faults and to perform simultaneously series, feedback and feedfoward passivation of the controller for mitigation of fault effects. In addition, LMI-based conditions are obtained for designing the PB to ensure asymptotic stability of the reconfigured system based on the dissipativity theory. Unlike previous approaches, the proposed PB is suitable for different classes of nonlinear systems since their passivity indices can be determined. In summary, the main contributions of this work are: *(i)* the concept of PB for FTC of nonlinear systems is introduced and compared to other passivation strategy for fault mitigation presented in Xia, Rahnama, Wang, and Antsaklis (2018); *(ii)* it is shown that the proposed PB structure generalizes Virtual Actuators and Sensors (respectively, VAs and VSs) for linear systems, although it can also be used for nonlinear systems; *(iii)* new LMI-based stability recovery conditions after sensor or actuator fault occurrence are established.

The rest of this paper is organized as follows: Section 2 presents the problem of stability recovery by fault hiding based on passivity indices; Section 3 presents the concept of PB; Section 4 provides the condition for asymptotic stability recovery with PBs; Section 5 presents numerical simulation results; and Section 6 draws the conclusions.

Notation For a matrix X, $X \succ (\prec)\ 0$ means that X is a positive (negative) definite matrix; $X^\top$ is its transpose; I_n and $0_{n\times m}$ denote, respectively, the nth order identity matrix and the null matrix of order $n \times m$. $\mathrm{He}\{X\}$ denotes $\mathrm{He}\{X\} = X + X^\top$. In a symmetric block matrix, '$\star$' is the term deduced by symmetry and $\mathrm{diag}\{d_1, \dots, d_n\}$ is a block-diagonal matrix with blocks $d_1, \dots, d_n$ in the main diagonal.

2. Preliminaries

2.1. Dissipativity and passivity indices

Consider the nonlinear system Σ described as follows

$$\Sigma : \begin{cases} \dot{x}(t) = f_x\left(x(t)\right) + f_u\left(x(t)\right)u(t) \\ y(t) = h_x\left(x(t)\right) + h_u\left(x(t)\right)u(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^m$ are, respectively, the state, input and output vectors, and the maps f_x, f_u, h_x, and h_u satisfy Assumption 1. Assume also that Σ is zero-state detectable (ZSD) (Hill & Moylan, 1976).

Assumption 1 (*Hill & Moylan, 1976*)**.** The maps f_x, f_u, h_x, and h_u are sufficiently smooth to ensure that (1) is well defined, i.e., for any $x(t_0)$ and admissible $u(t)$, there exists a unique solution for $t \geq t_0$ such that $y(t)$ is locally integrable. Furthermore, $f_x(0) = 0$ and $h_x(0) = 0$.

Remark 1. For the concept of passivity indices that is discussed in the following, it is required that the number of inputs is equal to the number of outputs. If it is not possible to have the same number of inputs and outputs, it is still possible to extend $u(t)$ or $y(t)$ by adding virtual inputs/output that are negligible in the system dynamics.

The system Σ is dissipative if its stored energy, represented by the time derivative of a non-negative continuously differentiable function $V(x(t))$, is always less than or equal to the supplied energy, represented by a supply rate function $S(u(t), y(t))$. A special case of dissipativity is the (Q, S, R)-dissipativity introduced in Definition 1.

Definition 1 (*Hill & Moylan, 1976*)**.** The system Σ is (Q, S, R)-dissipative if it is dissipative with respect to the following supply rate

$$S(u(t), y(t)) = y(t)^\top Q y(t) + 2u(t)^\top S y(t) + u(t)^\top R u(t). \tag{2}$$

In the literature, the relation between dissipativity theory and stability of nonlinear systems is well established (Kottenstette et al., 2014). In particular, Lemma 1 states on stability of (Q, S, R)-dissipative systems.

Lemma 1 (*Hill & Moylan, 1976*)**.** *Let the system Σ be (Q, S, R)-dissipative and zero-state detectable. Then, the unforced origin of Σ is stable if $Q \preceq 0$ and asymptotically stable if $Q \prec 0$.*

Passivity is a special case of dissipativity, i.e., a system is said to be passive if it is passive with respect to the supply rate $S(u(t), y(t)) = u(t)^\top y(t)$. In particular, the framework of passivity indices is useful to obtain an indication of the passivity degree (Kottenstette et al., 2014) as shown in Definition 2 and to develop passivation strategies (Xia et al., 2018).

Definition 2 (*Xia et al., 2018*)**.** The system Σ described in (1) is input feedforward output feedback passive (IF-OFP) if for some $\nu, \rho \in \mathbb{R}$ it is dissipative with respect to

$$S\left(u(t), y(t)\right) = u(t)^\top y(t) - \nu u(t)^\top u(t) - \rho y(t)^\top y(t). \tag{3}$$

In this case, Σ is denominated IF-OFP(ν, ρ), where ν and ρ are called input feedforward and output feedback passivity indices, respectively, which correspond to the excess of passivity of Σ. If Σ is IF-OFP(ν, ρ) with $\nu > 0$ and $\rho > 0$, then it is said very strictly passive (VSP).

The passivity indices provide intuitiveness to the dissipativity framework because they are related to the concept of lack (or excess) of passivity of a system. This means that ν indicates how much feedfoward gain can be inserted preserving Σ stability given the excess of input passivity $\nu u(t)^\top u(t)$ and, similarly, ρ indicates how much feedback gain can be inserted to preserve Σ stability given the excess of output passivity $\rho y(t)^\top y(t)$. In this sense, every system is IF-OFP(ν, ρ) for some ν and ρ, but the maximum value that ν and ρ can assume are indicators of the stability margins of that system. For instance, if the maximum ρ that can be assigned for a system Σ is negative, then it is possible to ensure that Σ is open-loop unstable and requires a negative feedback action to compensate for such lack of passivity that results in the instability. Based on that, the following fact is considered regarding the framework of passivity indices.

Lemma 2. *If Σ is IF-OFP(ν, ρ) for any scalars ν and ρ, then Σ is also IF-OFP($\nu^\star$, $\rho^\star$) for any $\nu^\star \leq \nu$ and $\rho^\star \leq \rho$.*

Proof. Defining

$$S(u(t), y(t)) = u(t)^\top y(t) - \nu u(t)^\top u(t) - \rho y(t)^\top y(t),$$
$$\bar{S}(u(t), y(t)) = u(t)^\top y(t) - \bar{\nu} u(t)^\top u(t) - \bar{\rho} y(t)^\top y(t),$$

it follows that $\bar{S}(u(t), y(t)) \geq S(u(t), y(t))$ for any $\bar{\nu} \leq \nu$ and $\bar{\rho} \leq \rho$. Thus, given that Σ is dissipative with respect to $S(u(t), y(t))$ it is also dissipative with respect to $\bar{S}(u(t), y(t))$, and therefore Σ is IF-OFP($\bar{\nu}$, $\bar{\rho}$). □

The concepts of passivity indices and (Q, S, R)-dissipativity are closely related. Indeed, it is observed that the supply function (3) is a special case of (2) by choosing $Q = -\rho I_m$, $S = -\frac{1}{2}I_m$, and $R = -\nu I_m$. Furthermore, the feedback interconnection of IF-OFP systems is (Q, S, R)-dissipative as stated in Lemma 3.

Lemma 3 (*Madeira & Adamy, 2015*)**.** *Consider the systems Σ_1 and Σ_2 described as in (1) and interconnected by negative feedback. If Σ_1 and Σ_2 are IF-OFP(ν_1, ρ_1) and IF-OFP(ν_2, ρ_2), respectively, then the interconnected system is $(\bar{Q}, \bar{S}, \bar{R})$-dissipative with matrices*

$$\bar{Q} = \begin{bmatrix} -(\rho_1 + \nu_2)I_m & 0_{m\times m} \\ 0_{m\times m} & -(\rho_2 + \nu_1)I_m \end{bmatrix} \tag{4}$$

$$\bar{S} = \begin{bmatrix} \frac{1}{2}I_m & \nu_1 I_m \\ -\nu_2 I_m & \frac{1}{2}I_m \end{bmatrix} \tag{5}$$

$$\bar{R} = \begin{bmatrix} -\nu_1 I_m & 0 \\ 0 & -\nu_2 I_m \end{bmatrix} \tag{6}$$

with input $w(t) \triangleq \left[w_1(t)^\top \; w_2(t)^\top\right]^\top$ and output $y \triangleq [y_1(t)^\top \; y_2(t)^\top]^\top$. In addition, the origin of (Σ_1, Σ_2) is asymptotically stable if $\rho_1 + \nu_2 > 0$ and $\rho_2 + \nu_1 > 0$.

2.2. Problem statement

Consider the nominal system Σ_P subject to faults whose faulty model is Σ_{P_f} described as follows

$$\Sigma_P : \begin{cases} \dot{x}(t) = f_x\left(x(t)\right) + f_u\left(x(t)\right) u_p(t) \\ y_p(t) = h_x\left(x(t)\right) + h_u\left(x(t)\right) u_p(t) \end{cases} \tag{7}$$

$$\Sigma_{P_f} : \begin{cases} \dot{x}(t) = f_{x,f}\left(x(t)\right) + f_{u,f}\left(x(t)\right) u_p(t) \\ y_p(t) = h_{x,f}\left(x(t)\right) + h_{u,f}\left(x(t)\right) u_p(t) \end{cases} \tag{8}$$

and interconnected with an output feedback controller Σ_C described as follows

$$\Sigma_C : \begin{cases} \dot{x}_c(t) = f_{c,x}\left(x_c(t)\right) + f_{c,y}\left(x_c(t)\right) y_c(t) \\ u_c(t) = h_{c,x}\left(x_c(t)\right) + h_{c,y}\left(x(t)\right) y_c(t) \end{cases} \tag{9}$$

where $x_c(t) \in \mathbb{R}^{n_c}$, $u_c(t), u_p(t) \in \mathbb{R}^m$, and $y_c(t), y_p(t) \in \mathbb{R}^m$ are, respectively, the state, input and output of Σ_C.

As shown in Fig. 1, an RB denoted by Σ_R is inserted between the faulty plant and the controller to recover the fault-free performance or stability. In this work, Σ_R is used as a PB, i.e., Σ_R performs series, feedback, and feedfoward passivation on Σ_C to ensure that the closed-loop system presents desired dissipativity properties. The concept of passivation for fault mitigation proposed by Xia et al. (2018) can be extended to a dynamic structure called dynamic PB (DPB) whose model is described as follows

$$\Sigma_R : \begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_{r,y} y(t) + B_{r,u} u_c(t) \\ y_r(t) = C_{r,y} x_r(t) + R_1 y(t) + R_2 u_c(t) \\ u_r(t) = C_{r,u} x_r(t) + R_3 y(t) + R_4 u_c(t) \end{cases} \tag{10}$$

with $x_r(t) \in \mathbb{R}^n$, $y(t), u_c(t), y_r(t), u_r(t) \in \mathbb{R}^m$ and gain matrices A_r, $B_{r,y}$, $B_{r,u}$, $C_{r,y}$, $C_{r,u}$, R_1, R_2, R_3, and R_4 with proper dimensions.

During the fault-free operation, the nominal plant Σ_P is connected by feedback to the controller Σ_C such that $u \leftarrow u_c$ and $y_c \leftarrow y$ (see fault-free system in Fig. 1). However, if a fault occurs, the system dynamics is changed and a proper FDI system can indicate the fault occurrence and obtain the passivity indices of Σ_{P_f}. Then, Σ_R is activated between Σ_{P_f} and Σ_C such that $u_p \leftarrow u_r$ and $y_c \leftarrow y_r$ (see reconfigured system in Fig. 1).

In this paper, the exact models of Σ_P, Σ_{P_f} and Σ_C do not need to be exactly known but only the passivity indices of Σ_{P_f} and Σ_C are.

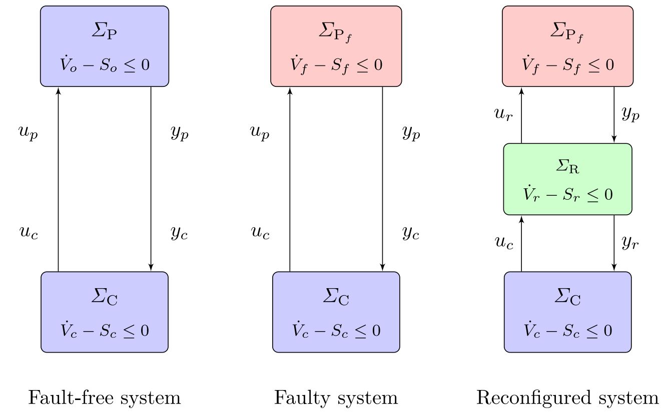


Fig. 1. Control reconfiguration by fault hiding.

Assumption 2. Assume that Σ_C is IF-OFP(ν_c, ρ_c) with given ν_c and ρ_c. In addition, there is no further knowledge about Σ_P and Σ_C beyond the passivity indices of Σ_C.

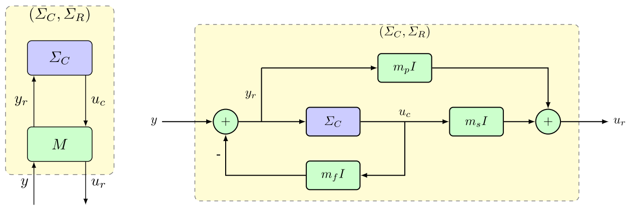
Assumption 3. It is assumed that Σ_{P_f} is IF-OFP(ν_f, ρ_f) and there exists an FDI system that is able to correctly detect the fault occurrence as soon as it occurs. Furthermore, the FDI system provides the values of ν_f and ρ_f.

Remark 2. The results presented in this paper are based on the knowledge of the dissipative properties of the controller and the estimation of dissipativity properties of the faulty system as stated in Assumptions 2 and 3. The estimation of dissipativity properties of the faulty system is achieved by means of any efficient FDI strategy (notice that the FDI design is not addressed in this paper and it is supposed to be available beforehand). The study of fault detection and control reconfiguration is usually separated in the literature on AFTC (Argha et al., 2019; Richter, 2011; Richter et al., 2011), although both should be implemented together. There are few results on the integrated design AFTC and FDI systems (Lan & Patton, 2016). Although it is still an open issue, the integration between FTC, FDI, and passivity indices estimation is not addressed here.

Remark 3. The estimation of dissipativity properties is a challenging problem. It is still lacking general procedures for estimating it for different classes of nonlinear systems. There are some available methodologies for estimating dissipativity properties from data (Romer, Berberich, Köhler, & Allgöwer, 2019; Romer, Montenbruck, & Allgöwer, 2018; Tanemura & Azuma, 2019; Zakeri & Antsaklis, 2019), although most of them are only applicable for linear time invariant systems (Romer et al., 2019, 2018; Tanemura & Azuma, 2019). There are few results for local dissipativity of nonlinear systems under operational constraints based on polynomial approximations (Zakeri & Antsaklis, 2019) and for dissipativity with respect to a periodic orbit (Berberich, Köhler, Allgöwer, & Müller, 2020).

In the context of dissipativity theory, the following fault hiding problem can be stated.

Problem 1 (*Asymptotic Stability Recovery by Fault Hiding*)**.** Let Σ_P be a nominal system with fault model Σ_{P_f} and connected to a controller Σ_C. Considering Assumptions 2 and 3, find an RB Σ_R such that the origin of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is asymptotically stable.



(a) Passivation of Σ_C. (b) Equivalent interconnection with feedback, series, and feedfoward passivation gains.

Fig. 2. Interconnection between Σ and M.

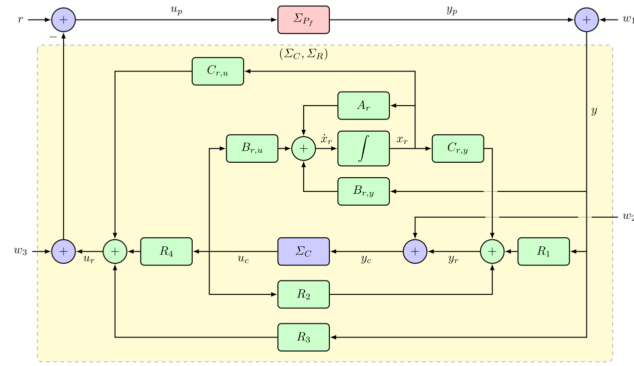


Fig. 3. Equivalent interconnection of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ expressed as controller passivation with DPB.

3. Passivation blocks for fault hiding

3.1. Passivation blocks

Passivity- and dissipativity-based control are important paradigms in control theory that have been employed for different kind of problems, since they also related to the concept of input–output stability. However, eventually, the dissipativity and passivity theory cannot be directly applied to non-passive systems. Then, passivation techniques enable the use of these techniques for a wider class of systems. In the literature, feedback, feedfoward and series passivation techniques have been successfully employed to stabilize dynamical systems. In Xia et al. (2018), a general passivation technique expressed by a matrix transformation that comprises the feedback, feedforward, and series passivation actions is presented.

Consider the system Σ_C and an input–output transformation matrix M described as follows

$$\begin{bmatrix} u_r \\ y_r \end{bmatrix} = M \begin{bmatrix} y \\ u_c \end{bmatrix} \tag{11}$$

such that their interconnection is depicted in Fig. 2a.

As shown in Xia et al. (2018), the matrix transformation M can be expressed as the combination of passivation gains in series, feedfoward, and feedback. In particular, Xia et al. (2018) define the matrix M as follows

$$M \triangleq \begin{bmatrix} m_p I & (m_s - m_p m_f)I \\ I & -m_f I \end{bmatrix}, \quad m_s \neq m_p m_f \tag{12}$$

where m_p, m_f, and m_s are feedfoward, feedback, and series gains, such that the (Σ_C, Σ_R) is equivalent to the system illustrated in Fig. 2b.

Lemma 4, originally presented in Xia et al. (2018), allows to design the M to ensure desired passivity indices for (Σ_C, Σ_R).

Lemma 4 (*Xia et al., 2018*). *Let Σ_C be finite-gain $\mathcal{L}_2$-stable with gain γ with the passivation matrix M given in (12). The equivalent system (Σ_C, Σ_R) is*

- *passive if M is chosen such that*

$$m_p > 0, \quad m_s = -m_f m_p, \quad 0 < |m_f|\gamma < 1 \tag{13}$$

- *OFP(ρ_r) with $\rho_r > 0$, such that $\rho_r = \frac{1}{2}(\frac{1}{m_p} + \frac{m_f}{m_s})$, if M is chosen such that*

$$m_p \geq m_s \gamma > 0, \quad m_s > m_f m_p > 0 \tag{14}$$

- *IFP(ν_r) with $\nu_r > 0$, such that $\nu_r = \frac{1}{2}(m_p + \frac{m_s}{m_f})$, if M is chosen such that*

$$1 > m_f \gamma > 0, \quad m_f m_p > m_s > 0 \tag{15}$$

- *VSP with indices $\nu_r = \frac{a}{2} m_p$ and $\rho_r = \frac{a}{2}\frac{1}{m_p}$, if M is such that*

$$m_p >> 0, \quad m_s = -m_f m_p, \quad m_f^2 \gamma^2 \leq \frac{1-a}{1+a} \tag{16}$$

for an arbitrary $0 < a < 1$.

3.2. DPB and its relation with RBs

This paper extends the concept of passivation for fault mitigation proposed by Xia et al. (2018) for a DPB described as (10) used for solving the FTC Problem 1.

The equivalent reconfigured loop obtained by means of the interconnection $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ with Σ_R described as (10) is depicted in Fig. 3, where w_1, w_2, and w_3 are external disturbances and r is the reference signal.

Note that the RB proposed in Bessa et al. (2020) described as

$$\Sigma_R : \begin{bmatrix} y_r \\ u_r \end{bmatrix} = \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix} \begin{bmatrix} y \\ u_c \end{bmatrix} \tag{17}$$

is a particular DPB case obtained from (10) if one defines $A_r \triangleq 0_{n_r \times n_r}$, $B_{r,y} \triangleq 0_{n_r \times p}$, $B_{r,u} \triangleq 0_{n_r \times m}$, $C_{r,y} \triangleq 0_{p \times n_r}$, $C_{r,u} \triangleq 0_{m \times n_r}$, and $x_r(0) \triangleq 0_{n_r \times 1}$.

Comparing (12) and (17), it is possible to note that M is a particular matrix for Σ_R, where $R_1 = I$, $R_2 = -m_f I$, $R_3 = m_p I$, and $R_4 = (m_s - m_p m_f)I$. Thus, the RB for passivation purpose is defined as a PB in this work.

Indeed, the PB proposed by Xia et al. (2018) is employed in an adaptive control strategy for fault mitigation (Zakeri & Antsaklis, 2019) that is similar to fault hiding. However, the conditions for tuning that kind of PB are too strict and nonconvex, making their design difficult. The PB presented in this paper can be easily obtained by solving a semi-definite programming problem, such that LMI-based conditions are provided. In our experimental evaluation, the PB proposed in this paper is compared to that proposed in Xia et al. (2018).

The DPB described as (10) also generalizes the existing linear dynamic RBs found in the literature (Richter, 2011). However, these RBs, namely VSs and VAs, are usually applicable only for linear and polytopic systems and are based on the internal model principle. To show that, consider for the plant the nominal model, given by Σ_P, and fault models, given by Σ_{P_f}, described as follows

$$\Sigma_P : \begin{cases} \dot{x}(t) = Ax(t) + Bu_p(t) \\ y_p(t) = Cx(t) \end{cases} \tag{18}$$

$$\Sigma_{P_f} : \begin{cases} \dot{x}(t) = Ax(t) + B_f u_p(t) \\ y_p(t) = C_f x(t) \end{cases} \tag{19}$$

A VS is usually described as follows (Richter, 2011):

$$\Sigma_R : \begin{cases} \dot{x}_r(t) = (A + LC)x_r(t) - Ly_p(t) + B_f u_c(t) \\ y_r(t) = (C - C_f)x_r(t) + y_p(t) \\ u_r(t) = u_c(t) \end{cases} \tag{20}$$

and it is designed to compensate sensor faults. Note that the VS described in (20) is a particular case of (10) taking $A_r \triangleq A + LC$, $B_{r,y} \triangleq -L$, $B_{r,u} \triangleq B_f$, $C_{r,y} \triangleq C - C_f$, $C_{r,u} \triangleq 0_{m\times n_r}$, $R_1 \triangleq I_m$, $R_2 \triangleq 0_{p\times m}$, $R_3 \triangleq 0_{m\times p}$, $R_4 \triangleq I_m$ and $x_r(0) \triangleq 0_{n_r\times 1}$. On other hand, a VA is described as follows (Richter, 2011):

$$\Sigma_R : \begin{cases} \dot{x}_r(t) = (A - B_f M)x_r(t) + (B - B_f M)u_c(t) \\ y_r(t) = Cx_r(t) + y_p(t) \\ u_r(t) = Mx_r(t) + Nu_c(t) \end{cases} \tag{21}$$

and is employed to compensate for actuator faults. The VA in (21) is also a particular case of the DPB in (10) if one defines $A_r \triangleq A - B_f M$, $B_{r,y} \triangleq 0_{n_r\times p}$, $B_{r,u} \triangleq B - B_f$, $C_{r,y} \triangleq C$, $C_{r,u} \triangleq M$, $R_1 \triangleq I_m$, $R_2 \triangleq 0_{p\times m}$, $R_3 \triangleq 0_{m\times p}$, $R_4 \triangleq N$ and $x_r(0) \triangleq 0_{n_r\times 1}$.

4. Stability recovery with DPB

In this section, LMI-based conditions for computing the DPB are presented to ensure the asymptotic stability recovery by fault hiding. For this purpose, it is worth to obtain conditions for dissipativity analysis of DPB.

All the results described in this section deal with the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ interconnected as Fig. 3, under Assumptions 2 and 3 where Σ_{P_f}, Σ_C and Σ_R are, respectively, described in (1), (9) and (10).

In particular, Lemma 5 provides conditions for Σ_R in (10) to be IF-OFP(ν_r, ρ_r).

Lemma 5. *Σ_R is IF-OFP(ν_r, ρ_r) if there exist a matrix $P = P^\top$, and scalars ν_r and ρ_r such that the following inequalities are satisfied:*

$$P \succ 0 \qquad \Theta \preceq 0 \tag{22}$$

with

$$\Theta \triangleq \begin{bmatrix} \mathrm{He}\{PA_r\} + W_{11} & PB_{r,y} - \frac{1}{2}C_{r,y}^\top + W_{12} & PB_{r,u} - \frac{1}{2}C_{r,u}^\top + W_{13} \\ \star & \nu_r I_m - \frac{1}{2}\mathrm{He}\{R_1\} + W_{22} & -\frac{1}{2}R_2 - \frac{1}{2}R_3^\top + W_{23} \\ \star & \star & \nu_r I_m - \frac{1}{2}\mathrm{He}\{R_4\} + W_{33} \end{bmatrix}$$

$$W_{11} \triangleq \rho_r C_{r,y}^\top C_{r,y} + \rho_r C_{r,u}^\top C_{r,u}, \tag{23}$$

$$W_{12} \triangleq \rho_r C_{r,y}^\top R_1 + \rho_r C_{r,u}^\top R_3, \tag{24}$$

$$W_{13} \triangleq \rho_r C_{r,y}^\top R_2 + \rho_r C_{r,u}^\top R_4, \tag{25}$$

$$W_{22} \triangleq \rho_r R_1^\top R_1 + \rho_r R_3^\top R_3, \tag{26}$$

$$W_{23} \triangleq \rho_r R_1^\top R_2 + \rho_r R_3^\top R_4, \tag{27}$$

$$W_{33} \triangleq \rho_r R_2^\top R_2 + \rho_r R_4^\top R_4. \tag{28}$$

Proof. The DPB depicted in Fig. 3 and described in (10) is represented by using, respectively, augmented input and output vectors: $\bar{u}_r(t) \triangleq \begin{bmatrix} y(t)^\top & u_c(t)^\top \end{bmatrix}^\top$ and $\bar{y}_r(t) \triangleq \begin{bmatrix} y_r(t)^\top & u_r(t)^\top \end{bmatrix}^\top$. Defining $V_r(x_r(t)) = x_r(t)^\top P x_r(t)$ as a storage function, its time derivative is computed as

$$\dot{V}(x_r(t)) = \mathrm{He}\left\{x_r^\top PA_r x_r + x_r^\top PB_{r,y} y + x_r^\top PB_{r,u} u_c\right\} \tag{29}$$

Consider the following supply function for Σ_R

$$S(\bar{u}_r(t), \bar{y}_r(t)) = \bar{u}_r^\top \bar{y}_r - \nu_r \bar{u}_r^\top \bar{u}_r - \rho_r \bar{y}_r^\top \bar{y}_r. \tag{30}$$

Taking into account that $y_r(t) = C_{r,y}x_r(t) + R_1 y(t) + R_2 u_c(t)$ and $u_r(t) = C_{r,u}x_r(t) + R_3 y(t) + R_4 u_c(t)$, (30) results in

$$\begin{aligned} S(\bar{u}_r, \bar{y}_r) = &- x_r^\top W_{11} x_r - y^\top(-R_1 + W_{22})y - \nu_r y^\top y \\ &- u_c^\top(-R_4 + W_{33})u_c - \nu_r u_c^\top u_c + y^\top C_{r,y} x_r \\ &- \mathrm{He}\left\{x_r^\top W_{12} y + x_r^\top W_{13} u_c + y^\top W_{23} u_c\right\} \\ &+ u_c^\top C_{r,u} x_r + y^\top R_2 u_c + u_c^\top R_3 y. \end{aligned} \tag{31}$$

where W_{ij} are defined by (23)–(28). Based on the identity $x^\top W y = \frac{1}{2}\mathrm{He}\left\{x^\top W y\right\}$, (31) is equivalent to

$$\begin{aligned} S(\bar{u}_r, \bar{y}_r) = &- x_r^\top W_{11} x_r - y^\top\left(\nu_r I_m - \tfrac{1}{2}\mathrm{He}\{R_1\} \right.\\ &\left.+ W_{22}\right)y - u_c^\top\left(\nu_r I_m - \tfrac{1}{2}\mathrm{He}\{R_4\}\right.\\ &\left.+ W_{33}\right)u_c - \mathrm{He}\left\{x_r^\top\left(-\tfrac{1}{2}C_{r,y}^\top + W_{12}\right)y\right\} \\ &+ \mathrm{He}\left\{x_r^\top\left(-\tfrac{1}{2}C_{r,u}^\top + W_{13}\right)u_c\right\} \\ &+ \mathrm{He}\left\{y^\top\left(R_2 + R_3^\top + W_{23}\right)u_c\right\}. \end{aligned} \tag{32}$$

Σ_R is dissipative with respect to (32) if $\dot{V}(x_r(t)) - S(\bar{u}_r, \bar{y}_r) \leq 0$ or, equivalently, subtracting (32) from (29) one obtains:

$$\bar{x}_r^\top \Theta \bar{x}_r \leq 0 \tag{33}$$

with $\bar{x}_r = \begin{bmatrix} x_r(t)^\top & y(t)^\top & u_c(t)^\top \end{bmatrix}^\top$. If (22) is satisfied for some ν_r and ρ_r, then (33) is satisfied and Σ_R is IF-OFP(ν_r, ρ_r) according to Definition 2. □

The next Lemma presents sufficient conditions for ensuring that the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is a (Q, S, R)-dissipative system.

Lemma 6. *If Σ_{P_f}, Σ_C, and Σ_R are, respectively, IF-OFP(ν_f, ρ_f), IF-OFP(ν_c, ρ_c), and IF-OFP(ν_r, ρ_r), then the reconfigured system $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ is (Q, S, R)-dissipative with:*

$$Q = \begin{bmatrix} -(\rho_f + \nu_r)I_m & 0_{m\times m} & \frac{1}{2}I_m & 0_{m\times m} \\ \star & -(\rho_c + \nu_r)I_m & \frac{1}{2}I_m & \frac{1}{2}I_m \\ \star & \star & -(\nu_c + \rho_r)I_m & 0_{m\times m} \\ \star & \star & \star & -(\nu_f + \rho_r)I_m \end{bmatrix} \tag{34}$$

$$S = \begin{bmatrix} \frac{1}{2}I_m & 0_{m\times m} & 0_{m\times m} & \nu_f I_m \\ -\nu_r I_m & 0_{m\times m} & \frac{1}{2}I_m & 0_{m\times m} \\ 0_{m\times m} & \frac{1}{2}I_m & -\nu_c I_m & 0_{m\times m} \\ -I_m & 0_{m\times m} & 0_{m\times m} & -\nu_f I_m \end{bmatrix} \tag{35}$$

$$R = \begin{bmatrix} -\nu_f I_m & 0_{m\times m} & 0_{m\times m} & \nu_f I_m \\ 0_{m\times m} & -\nu_r I_m & 0_{m\times m} & 0_{m\times m} \\ 0_{m\times m} & 0_{m\times m} & -\nu_c I_m & 0_{m\times m} \\ \nu_f I_m & 0_{m\times m} & 0_{m\times m} & -\nu_f I_m \end{bmatrix} \tag{36}$$

Proof. Define

$$X_1 \triangleq \begin{bmatrix} I_m & 0_{m\times m} & 0_{m\times m} & -I_m \end{bmatrix}, X_2 \triangleq \begin{bmatrix} 0_{m\times m} & 0_{m\times m} & 0_{m\times m} & I_m \end{bmatrix},$$

$$X_3 \triangleq \begin{bmatrix} I_m & 0_{m\times m} & 0_{m\times m} & 0_{m\times m} \end{bmatrix}, X_4 \triangleq \begin{bmatrix} 0_{m\times m} & I_m & 0_{m\times m} & 0_{m\times m} \end{bmatrix},$$

$$X_5 \triangleq \begin{bmatrix} 0_{m\times m} & 0_{m\times m} & I_m & 0_{m\times m} \end{bmatrix}, \bar{w}^\top \triangleq \begin{bmatrix} r(t)^\top & w_1(t)^\top & w_2(t)^\top & w_3(t)^\top \end{bmatrix},$$

$$\bar{y} \triangleq \begin{bmatrix} y_p(t)^\top & u_c(t)^\top & y_r(t)^\top & u_r(t)^\top \end{bmatrix}^\top.$$

Considering the equivalent loop in Fig. 3, the inputs and outputs of Σ_{P_f}, Σ_C, and Σ_R can be expressed as follows

$$u(t) = r(t) - w_3(t) - u_r(t) = X_1\bar{w} - X_2\bar{y}, \tag{37}$$

$$y_p(t) = X_3\bar{y}, \quad u_r(t) = X_2\bar{y}, \quad y_r(t) = X_5\bar{y}, \tag{38}$$

$$y(t) = y_p(t) + w_1(t) = X_4\bar{w} + X_3\bar{y}, \tag{39}$$

$$u_c(t) = X_4\bar{y}, y_c(t) = y_r(t) + w_2(t) = X_5(\bar{w} + \bar{y}). \tag{40}$$

Assuming that Σ_{P_f} is IF-OFP(ν_f, ρ_f), the following inequality holds according to Definition 2:

$$\dot{V}_f(x) \leq u_p^\top y_p - \nu_f u_p^\top u_p - \rho_f y_p^\top y_p \tag{41}$$

where V_f is a valid storage function for Σ_{P_f}. Substituting (37) and (38) into (41), it follows that:

$$\dot{V}_f(x) \leq \bar{y}^\top \bar{Q}_f \bar{y} + 2\bar{w}^\top \bar{S}_f \bar{y} + \bar{w}^\top \bar{R}_f \bar{w}, \tag{42}$$

$$\bar{Q}_f = -\nu_f X_2^\top X_2 - \rho_f X_3^\top X_3, \tag{43}$$

$$S_f = \frac{1}{2} X_1^\top X_3 - \frac{1}{2} X_2^\top X_3 + \nu_f X_1^\top X_2, \tag{44}$$

$$\bar{R}_f = -\nu_f X_1^\top X_1. \tag{45}$$

Moreover, given that Σ_C is IF-OFP(ν_c, ρ_c), then

$$\dot{V}_c(x_c) \leq u_c^\top y_c - \nu_c y_c^\top y_c - \rho_c u_c^\top u_c \tag{46}$$

where V_c is a valid storage function for Σ_C. Substituting (38) and (40) into (46), it follows that:

$$\dot{V}_c(x_c) \leq \bar{y}^\top \bar{Q}_c \bar{y} + 2\bar{w}^\top \bar{S}_c \bar{y} + \bar{w}^\top \bar{R}_c \bar{w}, \tag{47}$$

$$\bar{Q}_c = X_5^\top X_4 - \nu_c X_5^\top X_5 - \rho_c X_4^\top X_4, \tag{48}$$

$$\bar{Q}_c = X_5^\top X_4 - \nu_c X_5^\top X_5 - \rho_c X_4^\top X_4, \tag{49}$$

$$\bar{R}_c = -\nu_c X_5^\top X_5. \tag{50}$$

Similarly, assuming that Σ_R is IF-OFP(ν_r, ρ_r) and

$$\bar{w}_r \triangleq \begin{bmatrix} y \\ u_c \end{bmatrix} = \begin{bmatrix} X_4 \\ 0_{m\times 4m} \end{bmatrix} \bar{w} + \begin{bmatrix} X_3 \\ X_4 \end{bmatrix} \bar{y}, \tag{51}$$

$$\bar{y}_r \triangleq \begin{bmatrix} y_r \\ u_r \end{bmatrix} = \begin{bmatrix} X_5 \\ X_2 \end{bmatrix} \bar{y}, \tag{52}$$

it follows that

$$\dot{V}_r(x_r) \leq \bar{w}_r^\top \bar{y}_r - \nu_r \bar{u}_r^\top \bar{u}_r - \rho_r \bar{y}_r^\top \bar{y}_r \tag{53}$$

where V_r is a valid storage function for Σ_R. Substituting (51) and (52) into (53), it is equivalent to

$$\dot{V}_r(x_r) \leq \bar{y}^\top \bar{Q}_r \bar{y} + 2\bar{w}^\top \bar{S}_r \bar{y} + \bar{w}^\top \bar{R}_r \bar{w}, \tag{54}$$

$$\begin{aligned} \bar{Q}_r =& X_3^\top X_5 + X_4^\top X_2 - \nu_r \left(X_3^\top X_3 + X_4^\top X_4\right) \\ & - \rho_r \left(X_5^\top X_5 + X_2^\top X_2\right), \end{aligned} \tag{55}$$

$$S_r = \frac{1}{2} X_4^\top X_5 - \nu_r X_4^\top X_3, \tag{56}$$

$$\bar{R}_f = -\nu_r X_4^\top X_4. \tag{57}$$

Thus, adopting the storage function $\bar{V}(x, x_r, x_c) \triangleq \dot{V}_f(x) + \dot{V}_r(x_r) + \dot{V}_c(x_c)$ for the reconfigured system (Σ_{P_f}, Σ_R, Σ_C), the following inequality can be obtained by summing the inequalities (42), (47) and (54)

$$\begin{aligned} \bar{V}(x, x_r, x_c) \leq& \bar{y}^\top \left(\bar{Q}_f + \bar{Q}_r + \bar{Q}_c\right) \bar{y} + 2\bar{w}^\top \left(\bar{S}_f + \bar{S}_r + \bar{S}_c\right) \bar{y} \\ & + \bar{w}^\top \left(\bar{R}_f + \bar{R}_r + \bar{R}_c\right) \bar{w}. \end{aligned}$$

It is straightforwardly obtained that $S = \bar{S}_f + \bar{S}_r + \bar{S}_c$ and $R = \bar{R}_f + \bar{R}_r + \bar{R}_c$ are, respectively, equivalent to (35) and (36). In addition, $\bar{Q}_f + \bar{Q}_r + \bar{Q}_c$ is

$$X^\top \Gamma X \tag{58}$$

where $X \triangleq \begin{bmatrix} X_3^\top & X_4^\top & X_5^\top & X_2^\top \end{bmatrix}^\top$ and

$$\Gamma \triangleq \begin{bmatrix} -\left(\rho_f + \nu_r\right) I_m & 0_{m\times m} & \frac{1}{2} I_m & 0_{m\times m} \\ \star & -\left(\rho_c + \nu_r\right) I_m & \frac{1}{2} I_m & \frac{1}{2} I_m \\ \star & \star & -\left(\nu_c + \rho_r\right) I_m & 0_{m\times m} \\ \star & \star & \star & -\left(\nu_f + \rho_r\right) I_m \end{bmatrix}$$

Note that since X is an identity matrix then (58) is equivalent to $Q = \bar{Q}_f + \bar{Q}_r + \bar{Q}_c$ which results in (34). Therefore, (Σ_{P_f}, Σ_R, Σ_C) is (Q, S, R)-dissipative with Q, S and R defined by (34)–(36), respectively. □

Based on Lemma 6, the next Theorem provides sufficient conditions to ensure the stabilization of faulty systems by means of DPB and based on passivity indices.

Theorem 1. *Assume that Σ_{P_f} is IF-OFP(ν_f, ρ_f) and Σ_C is IF-OFP(ν_c, ρ_c) for given ν_f, ρ_f, ν_c, and ρ_c. The origin of reconfigured system (Σ_{P_f}, Σ_R, Σ_C) is asymptotically stable if there exist scalars γ_1, γ_2, γ_3, γ_4, ν_r and μ_r, and matrices $P = P^\top$, Z_1, Z_2, Z_3, $C_{r,y}$, $C_{r,u}$, R_1, R_2, R_3 and R_4 that satisfy the following inequalities:*

$$P \succ 0,$$

$$\gamma_1^{-1} \leq \nu_f^{-1}, \quad \gamma_2^{-1} \leq \nu_c^{-1}, \quad \gamma_3 \leq \rho_f, \quad \gamma_4 \leq \rho_c, \tag{59}$$

$$\begin{bmatrix} -\mu_r I_m & 0_{m\times m} & C_{r,y} & R_1 & R_2 \\ \star & -\mu_r I_m & C_{r,u} & R_3 & R_4 \\ \star & \star & \text{He}\{P + Z_1\} & Z_2 - \frac{1}{2} C_{r,y}^\top & Z_3 - \frac{1}{2} C_{r,u}^\top \\ \star & \star & \star & \nu_r I_m - \frac{1}{2}\text{He}\{R_1\} & -\frac{1}{2} R_2 - \frac{1}{2} R_3^\top \\ \star & \star & \star & \star & \nu_r I_m - \frac{1}{2}\text{He}\{R_4\} \end{bmatrix} \prec 0, \tag{60}$$

$$\begin{bmatrix} -(\gamma_3 + \nu_r) & 0 & \frac{1}{2}\mu_r & 0 & 0 & 0 \\ \star & -(\gamma_4 + \nu_r) & \frac{1}{2}\mu_r & \frac{1}{2}\mu_r & 0 & 0 \\ \star & \star & -\mu_r & 0 & -\mu_r & 0 \\ \star & \star & \star & -\mu_r & 0 & \mu_r \\ \star & \star & \star & \star & \gamma_1 & 0 \\ \star & \star & \star & \star & \star & \gamma_2 \end{bmatrix} \prec 0. \tag{61}$$

In this case, the gains A_r, $B_{r,y}$, and $B_{r,u}$ of (10) are given by $A_r = I_n + P^{-1} Z_1$, $B_{r,y} = P^{-1} Z_2$, and $B_{r,u} = P^{-1} Z_3$.

Proof. Using a Schur's complement argument, for $\mu_r > 0$ given, if (60) is satisfied then

$$\begin{bmatrix} \text{He}\{P + Z_1\} & Z_2 - \frac{1}{2} C_{r,y}^\top & Z_3 - \frac{1}{2} C_{r,u}^\top \\ \star & \nu_r I_m - \frac{1}{2}\text{He}\{R_1\} & -\frac{1}{2} R_2 - \frac{1}{2} R_3^\top \\ \star & \star & \nu_r I_m - \frac{1}{2}\text{He}\{R_4\} \end{bmatrix} + \mu_r^{-1} T_1^\top T_1 \prec 0, \tag{62}$$

where

$$T_1 \triangleq \begin{bmatrix} C_{r,y} & R_1 & R_2 \\ C_{r,u} & R_3 & R_4 \end{bmatrix}.$$

Further, define

$$W \triangleq \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ \star & W_{22} & W_{23} \\ \star & \star & W_{33} \end{bmatrix},$$

where W_{ij} are defined as in (23)–(28), $i, j = 1, 2, 3$. Defining $\rho_r \triangleq \mu_r^{-1}$, it follows that

$$W = \mu_r^{-1} T_1^\top T_1.$$

Defining $Z_1 \triangleq PA_r - P$, $Z_2 \triangleq PB_{r,y}$, and $Z_3 \triangleq PB_{r,u}$, (62) is equivalent to (22). Thus, according with Lemma 5, if (60) is satisfied for some $\mu_r > 0$, then (22) is also satisfied and Σ_R is IF-OFP(ν_r, ρ_r). Lemma 6 ensures that if Σ_{P_f}, Σ_C and Σ_R are, respectively, IF-OFP(ν_f, ρ_f), IF-OFP(ν_r, ρ_r) and IF-OFP(ν_c, ρ_c), then (Σ_{P_f}, Σ_R, Σ_C) is (Q, S, R)-dissipative with Q, S and R defined, respectively, by (34), (35), and (36). Using the Schur's complement Lemma, if (61) is satisfied then

$$\begin{bmatrix} -(\gamma_3 + \nu_r) & 0 & \frac{1}{2}\mu_r & 0 \\ \star & -(\gamma_4 + \nu_r) & \frac{1}{2}\mu_r & \frac{1}{2}\mu_r \\ \star & \star & -\mu_r & 0 \\ \star & \star & \star & -\mu_r \end{bmatrix} - T_2^\top Y_1 T_2 \prec 0. \tag{63}$$

is also satisfied for $T_2 \triangleq [0_{2\times 2}\ \mu_r I_2]$ and $Y_1^{-1} \triangleq \text{diag}\{\gamma_2, \gamma_1\}$. Thus (63) is equivalent to

$$Y_2 \triangleq \begin{bmatrix} -(\gamma_3+\nu_r) & 0 & \frac{1}{2}\mu_r & 0 \\ \star & -(\gamma_4+\nu_r) & \frac{1}{2}\mu_r & \frac{1}{2}\mu_r \\ \star & \star & -\mu_r-\mu_r^2\gamma_2 & 0 \\ \star & \star & \star & -\mu_r-\mu_r^2\gamma_1 \end{bmatrix} \prec 0. \quad (64)$$

Considering (59) and Lemma 2, it is clear that Σ_{P_f} and Σ_C are, respectively, IF-OFP(γ_1^{-1}, γ_3) and IF-OFP(γ_2^{-1}, γ_4). Adopting the congruence transformation $T_3^\top Y_2 T_3$ with $T_3 \triangleq \text{diag}\{1, 1, \mu_r^{-1}, \mu_r^{-1}\}$ and $\mu_r^{-1} = \rho_r$, then notice that $T_3^\top Y_2 T_3 \succeq Q$ with Q defined by (34). Thus, it demonstrates that (61) implies that $Q \prec 0$, which it is sufficient to ensure the asymptotic stability of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ according to Lemma 1. Finally, any Σ_R described as (10) with matrix gains that satisfy (59)–(61) with $P \succ 0$ ensures the asymptotic stability of $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$ and this concludes the proof. □

5. Numerical example

Consider the following nonlinear faulty system Σ_{P_f}:

$$\Sigma_{P_f} : \begin{cases} \dot{x}_1 = -0.5x_1^3 + 0.5x_2 \\ \dot{x}_2 = -0.5x_1 - 1.25x_2 - 4f_a u_p \\ y_p = -f_s x_2 - 0.5 f_s u_p \end{cases} \quad (65)$$

where the signals f_a and f_s denote, respectively, actuator and sensor multiplicative faults such that $f_a = 1$ and $f_s = 1$ represent the fault-free operation.

Let $V(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2$ be a storage function for Σ_{P_f}. Taking its time-derivative, it follows that:

$$\dot{V}(x_1, x_2) = -0.5x_1^4 - 1.25x_2^2 - 4f_a u_p x_2. \quad (66)$$

Furthermore, given that Σ_{P_f} is IF-OFP(ν_f, ρ_f), $S(u, y)$ is the supply rate function in (3) for (65):

$$S(u_p, y_p) = -\left(\frac{1}{2}f_s f_a + f_a^2\nu_f + \frac{1}{4}f_s\rho_f\right)u_p^2 - \left(f_s f_a + f_s\rho_f\right)u_p x_2 - f_s\rho_f x_2^2. \quad (67)$$

Disregarding the negative term $-0.5x_1^4$ in (66), the following condition, obtained from (66) and (67), is sufficient to ensure that the passivation indices satisfy $\dot{V}(x_1, x_2) - S(u_p, y_p) \le 0$

$$\begin{bmatrix} x_2 \\ u_p \end{bmatrix}^\top \begin{bmatrix} -1.25 + f_s\rho_f & \frac{1}{2}\left(f_s f_a + f_s\rho_f - 4f_a\right) \\ \star & \frac{1}{2}f_s f_a + f_a^2\nu_f + \frac{1}{4}f_s\rho_f \end{bmatrix} \begin{bmatrix} x_2 \\ u_p \end{bmatrix} \le 0 \quad (68)$$

Thus, the above condition allows computing ρ_f and ν_f by means of semi-definite programming (SDP) for both the fault-free and faulty operation modes since the fault estimates f_s and f_a are known (cf. Assumption 3). To test different fault scenarios (fault-free; actuator, sensor or simultaneous faults), f_s and f_a are chosen as follows:

$$f_s = \begin{cases} 0.75, & \text{if } 10 < t \le 85, \\ 1, & \text{otherwise} \end{cases}, \qquad f_a = \begin{cases} 1, & \text{if } t \le 55, \\ -0.2, & \text{if } t > 55. \end{cases}$$

Notice that the fault-free operation happens until $t = 10$ s, when a sensor attenuation fault starts. While the sensor fault still is present, an actuator fault with $f_a = -0.2$ starts at $t = 10$ s. For $t > 85$ s, the sensor fault is no longer occurring but only the actuator fault affects the system. The passivity indices are computed solving the maximization of the cost $J = \rho_f + \nu_f$ with constraint (68). Then, the passivity indices are:

$$\nu_f = \begin{cases} -2.563, & \text{if } t \le 10, \\ -0.249, & \text{if } 10 < t \le 55, \\ -2.843, & \text{if } 55 < t \le 85, \\ 0.686, & \text{if } t > 85, \end{cases} \qquad \rho_f = \begin{cases} 0.375, & \text{if } t \le 10, \\ 0.512, & \text{if } 10 < t \le 55, \\ 0.001, & \text{if } t > 55. \end{cases}$$

For the simulations, a controller Σ_C, that is IF-OFP($-0.25, 2.75$) is used to stabilization and disturbance rejection for the fault-free system. It is given by:

$$\Sigma_C : \begin{cases} \dot{x}_c = -3.517x_c - 4.5y_c \\ u_c = x_c + 0.2045y_c \end{cases} \quad (69)$$

The proposed DPB described in (10) is designed based on Theorem 1 by using the LMILAB. The computed gains of the DPBs Σ_R^i for the scenarios $i = 1, 2, 3$ are presented in the sequel. Notice that Scenario 1 denotes the case when only the sensor fault occurs ($10 < t \le 55$ s). Scenario 2 corresponds to the case when only the actuator fault occurs ($t \ge 85$ s). And Scenario 3 is the case when both sensor and actuator faults occur simultaneously ($55 < t \le 85$ s). The superscript indices in the matrices indicate the scenario

$$A_r^1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad A_r^2 = A_r^3 = \begin{bmatrix} -1.4142 & 0 \\ 0 & -1.4142 \end{bmatrix},$$

$$B_{r,u}^1 = B_{r,u}^2 = B_{r,u}^3 = B_{r,y}^1 = B_{r,y}^2 = B_{r,y}^3 = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top,$$

$$C_{r,u}^1 = C_{r,u}^2 = C_{r,u}^3 = C_{r,y}^1 = C_{r,y}^2 = C_{r,y}^3 = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$R_1^1 = R_4^1 = 0.126, \quad R_1^2 = R_4^2 = 0.0181,$$

$$R_1^3 = R_4^3 = 0.0207, \quad R_2^1 = R_3^1 = R_2^2 = R_3^2 = R_2^3 = R_3^3 = 0,$$

The procedure proposed in Zakeri and Antsaklis (2019) and the PB in (11) proposed by Xia et al. (2018) can also be employed to recover the stability of the reconfigured system. For this purpose, Lemma 3 indicates that the passivation controller must be OFP(ρ_r) with $\rho_r > -\nu_f$ for Scenarios 1 and 3. In the Scenario 2, it is sufficient (Σ_C, Σ_R) to be passive since $\nu_f > 0$ and $\rho_f > 0$. According to Lemma 4, given that Σ_C is finite gain $\mathcal{L}_2$-stable with gain $\gamma = 0.9688$, the passivation gains may be chosen as

$$m_p^1 = 0.25, \quad m_f^1 = 0.2, \quad m_s^1 = 0.1,$$

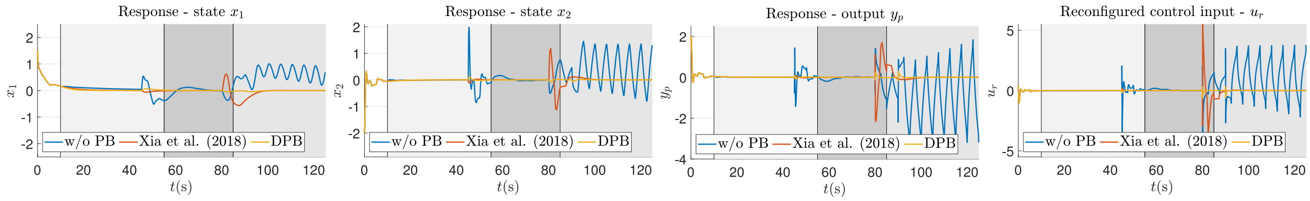
$$m_p^2 = 2, \quad m_f^2 = 0.5, \quad m_s^2 = -1,$$

$$m_p^3 = 0.3, \quad m_f^3 = 0.25, \quad m_s^3 = 0.05.$$

The numerical simulation results are depicted in Fig. 4 and compare the faulty system response with the reconfigured system responses with the PB proposed by Xia et al. (2018) and with the proposed DPB designed based on Theorem 1. Disturbances $w_3 = 10$ are added in the system output at $t = 45$ s, $t = 80$ s and $t = 90$ s with duration of 0.1 s. The initial states are $x_1(0) = 1.5$ and $x_2(0) = -2$. The results illustrate that the proposed DPB is able to recover the stability in all the fault scenarios and presents good disturbance rejection action without requiring much control effort. Otherwise, the system without PB became oscillatory in Scenario 2 and unstable in Scenario 3. The PB proposed by Xia et al. (2018) also recovers the stability, but it became too sensitive to the disturbance when both faults are occurring (Scenario 2).

6. Conclusion

In this work, a novel fault hiding strategy by means of PBs for asymptotic stability recovery of nonlinear systems based on dissipativity and passivity theory is presented. Differently from other fault hiding applications for nonlinear systems, the proposed approach can be applied to different classes of nonlinear system since it is possible to obtain the passivity indices of faulty system and controller. LMI-based conditions for asymptotic stability recovery by means of the proposed PBs from passivity indices are provided. Numerical examples indicate that the proposed approach is able to recover the stability after sensor and actuator fault occurrence with better disturbance rejection ability than recent approaches in the literature as Xia et al. (2018). Further work should include investigation on the estimation of dissipativity properties from data to allow the FTC of nonlinear systems with unknown models and on the integration of PB-based FTC with inaccurate and delayed FDI systems.



(a) State x_1. (b) State x_2. (c) Output. (d) Reconfigured input.

Fig. 4. Comparison between the proposed DPB and the PB in Xia et al. (2018) for the nonlinear system (65).

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