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Introduction to the Numerical Solution of the Navier-Stokes equations.

Module 10: Hands on session 4

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Introduction to the Numerical Solution of the Navier-Stokes equations

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Module 10. Hands on session 4

Make sure you understand the following before you begin

- Structure of the Navier-Stokes equations, physical meaning of the convective and diffusive terms (M1).
- Ladyzhenskaya theorem and Poisson equation for the pseudo-pressure (M9).
- Compatibility conditions for the Poisson equation (M9).

Class questions:

1. Write the *idiv* function (M9.5), that returns the integral of the divergence of a vector field.

```
function [ d ] = idiv( u,v,L)
```

To check it, generate a small vector field zero everywhere except at one or two positions where it is set to 1. Using *print_field* function, check the results.

Recall that u and v must be halo-updated before calling *idiv* !

To check this, set to 1 positions next to the inner domain limits.

2. Write the *ilaplacianMatrix* function (M9.5), that returns the coefficients of a matrix expressing the integral of the Laplacian of a scalar field. Recall that the matrix has to be perturbed to avoid the singularity.

```
function [ A ] = ilaplacianMatrix( N)
```

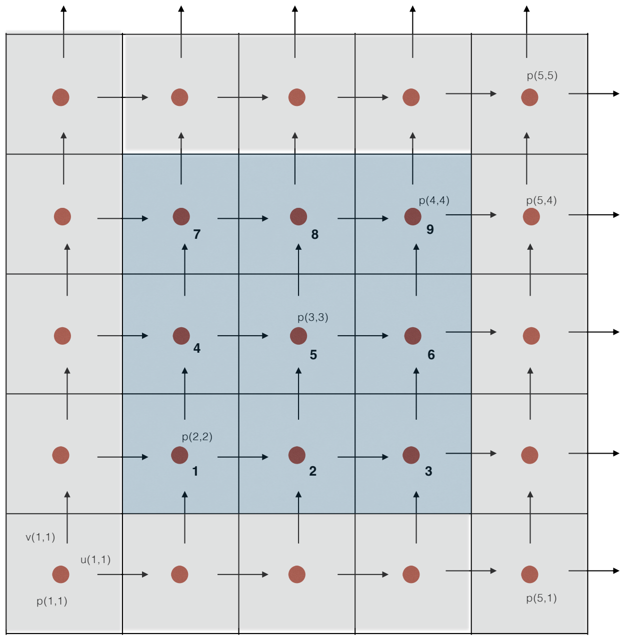
Hint: Be careful with the periodic structure of the domain.

To build the matrix, the best is to implement a double loop with horizontal and vertical indices (i,j) that sweep the domain from bottom-left to top-right (only inner region).

The horizontal loop has to be the inner loop. An additional counter (p) will incremented for each node visited. It will provide the numeration of the following figure.

When the last node in horizontal direction is reached (e.g., *i==4, p==6*) then the east coefficient will be assigned to the first node in the row (4, in this example).

To check it, examine the structure of the matrix. Take your time, check several values of N. Check that before perturbing it, the matrix is singular and non-singular after.



3. Write the *gradient* function (M9.5), that returns the gradient of a vector field (and NOT the integral of the gradient).

```
function [ gx,gy ] = gradient(s)
```

To check it, proceed as with the *idiv* function.

4. Write the function *field2vector* (M9.5), that converts a field (two dimensional, with halos) to an algebraic vector (one-dimensional, without halos).

5. Write the function *vector2field* (M9.5) that does the opposite.

```
function [ v ] = field2vector(f)
  f(N+2,N+2)
  v(N^2)

function [ f ] = vector2field(v)
  v(N^2)
  f(N+2,N+2)
```

To check them, try several simple fields (print them with *print_field*) and check the vectors obtained.

6.Implement the pressure-velocity coupling algorithm (M9.6)

```
function [u2 , v2, pp] = pressureVelocity( u , v , L)
```

```
u(N+2,N+2), v(N+2,N+2): arbitrary vector field (will be predictor velocity)
L: domain size
u2(N+2,N+2), v2(N+2,N+2): divergence-free vector field (will be next step velocity)
pp(N+2,N+2):  pseudo-pressure field
```

To check it, generate a small vector field zero everywhere except at one or two positions where it is set to 1.
Check that the sum of the right-hand-side of the Poisson equation is zero.
Check that the divergence of (u2,v2) is zero at all the points (the maximum of its absolute value has to be very small, typically less than 10^{-10}).

7. If there are errors, do not despair. Try to check all the steps individually and the errors will be found.

8. Once you have finished, you have the second block of your Navier-Stokes solver ready.

Congratulations, it is an important accomplishment and by no means easy !!