Coupled solid-fluid response of deep tunnels excavated in saturated rock masses with a time-dependent plastic behaviour

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A B S T R A C T

This article provides a general numerical approach for modelling the response of deep tunnels excavated in saturated time-dependent plastic rock masses, considering a coupled solid-fluid interaction and time-dependent plastic behaviour. In order to do that, a Burgers-viscoplastic strain-softening model has been developed and implemented into the finite element method software CODE_BRIGHT, and a coupled solid-fluid model is used to simulate the interaction between solid deformations and fluid flow. Parametric analyses are then performed to analyse the influence on the tunnelling response of different time-dependent models, different standstill times and different excavation rates. It has been observed that the time-dependent model selection is crucial to simulate the response of underground excavations. Additionally, the coupled solid-fluid results are significantly different from the purely mechanical ones. The liquid pressure build-up in the vicinity of the tunnel face and the overpressure dissipation with time due consolidation can be accounted for. Moreover, the higher the excavation rate, the larger build-up of liquid pressures occurs.

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1. Introduction

The multi-physics multi-phase coupling problems concerning mass transport, fluid flow and solid deformations are of significance in engineering applications, such as nuclear waste repositories, geological carbon storage, geothermal energy or underground excavation [1–5,8,7,88]. Multi-stage underground excavation in saturated ground is a characteristic process in many geotechnical applications. In some cases, the surrounding rock masses can deform gradually, showing large delayed deformations that may lead to creep-induced failure [6–9]. A better understanding of time-dependency, damage evolution, post-failure behaviour and creep-induced failure response is a topic of considerable interest in rock mechanics [9–13]. Furthermore, the effect of the coupled solid-fluid response cannot be ignored when an underground excavation is performed below the water table [15–17]. Therefore, a comprehensive analysis of the entire process of multi-stage excavation incor-

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porating time-dependent plastic behaviour of geomaterials and the coupled solid-fluid interaction is crucial to perform a proper numerical simulation of underground excavation problems.

Theoretical approaches, the convergence-confinement method (CCM) and numerical simulations have been widely used in tunnelling design [7,8,11,16,17,19–36,87,88]. Among all these methods, the CCM provides an efficient way to determine support forces by considering the ground-structure interaction [10,38]. The CCM consists of three basic components in the form of graphs: the ground reaction curve (GRC), the longitudinal deformation profile (LDP) and the support characteristic curve (SCC) [9,13,38,39]. The main focus of this article is to analyze solid deformations (i.e. LDP) and fluid flow in longitudinal multi-stage excavation problems, considering the coupled solid-fluid interaction, and time-dependent plastic behaviour of geomaterials.

Some researchers have studied the LDP based on field measurements or numerical simulations, but most of them are based on elastic [25,40–42] or elastic-plastic [10,26,43] constitutive models. However, when using these constitutive models, the properties of the geomaterial are time-independent. This condition can be quite different from that in real projects in which the rock masses exhibit strong time-dependent creep properties, such as in soft or squeezing rock masses [9,12,20,44]. Based on the Burgers-creep viscous (CVISC) model introduced by Itasca [45], Paraskevopoulos [12] and Paraskevopoulos and Diederichs [20] presented numerical simulations of LDP for time-dependent viscoelastic rock masses, although no plastic yield was considered. Moreover, in the CVISC model, the plastic slider is not coupled with the viscous dashpot, which means that the CVISC model cannot simulate creep-induced failure behaviour [9,12]. Recently, Song et al. [9] proposed a coupled Maxwell-viscoelastic strain-softening (MVSS) model and programmed it in a finite element method software CODE_BRIGHT [46]; this MVSS model considers the coupled behaviour of viscous and strain-softening models, which can simulate creep-induced failure response. Then, Song et al. [9] presented analysis and modelling of the LDP for tunnelling in time-dependent plastic rock masses, but the primary creep stage behaviour cannot be simulated by their model. Thus, the proposed MVSS model in Song et al. [9] cannot adequately represent the response of geomaterials with high strength or subject to low stresses, due to neglecting the primary creep response.

Furthermore, the previously mentioned research about the LDP was carried out without considering the coupled solid-fluid interaction, although the majority of deep tunnels are excavated under the water table. When tunnelling in saturated ground, the coupled solid-fluid response of geomaterials is important for estimating the tunnelling behaviour [20]. Nam and Bobet [15] proposed empirical best-fit solutions for the LDP for saturated ground with steady-state flow condition. Furthermore, considering coupled interactions between solid deformations and fluid flow, Prassetyo and Gutierrez [16] proposed a LDP for predicting time-dependent radial displacements along the tunnel axis. However, the surrounding rock masses were assumed as linear elastic materials for the coupled solid-fluid problems [16]. Recently, Song et al. [1] presented empirical best-fit solutions of the LDP for hydro-mechanical plastic problems under steady-state fluid flow condition, but without considering time-dependent behaviour, strain-softening post-failure behaviour and the coupled solid-fluid interaction. However, the LDP heavily depends on both the behaviour model selected [9,10,23,24] and the coupled solid-fluid interaction [16,47,48].

In summary, research on the underground excavation problems has been mostly concentrated on purely mechanical problems, or the coupled solid-fluid problems but with simpler constitutive models. Instead, in this article, we present the coupled solid-fluid modelling of multi-stage underground excavation in time-dependent plastic geomaterials, taking into consideration time-dependent plastic properties of geomaterials, post-failure behaviour models, the coupling between creep response and damage evolution, and a coupled interaction between solid deformations and fluid flow.

2. Statement of the problem and theoretical background

2.1. Statement of the problem

This study considers the coupled time-dependent solid-fluid responses of deep tunnels excavated in saturated soft rock masses below the groundwater table. The following assumptions are made:

1) The tunnel is excavated deep enough so that variation of stresses with depth can be neglected and, hence, the far-field initial stresses $p_0$ can be considered isotropic. For the same reason, the variation of the water table can be neglected and, hence, the far-field liquid pressure $p_c$ can be considered isotropic. Note that the water table does not change significantly in deep tunnels, as described in the work of other researchers [9,15,16,47,48].

2) The surrounding rock mass exhibits homogeneous, isotropic and time-dependent plastic behaviour. A Burgers-viscoplastic strain-softening (BVSS) model has been proposed to simulate the primary, secondary and creep-failure responses. In section 2.2, the mechanical responses of soft or squeezing rock masses are described, and the corresponding constitutive models are introduced to represent this type of rock mass behaviour.

3) The couplings between solid deformations and fluid flow in both directions are taken into account, as described in section 2.3. In this study, saturated conditions are considered. From a modelling perspective, discontinuities of the rock mass can be represented as an equivalent porous medium [1,46,49,50] and, hence, with an 'equivalent permeability'. This approximation may be appropriate in low-quality rock masses with many joints, such as the considered soft or squeezing rock masses in this article, in which permeability can be considered isotropic. However, may be not
appropriate in cases where permeability cannot be assumed isotropic, e.g. high-quality rock masses with a few joints per cubic meter.

On the other hand, the numerical model in this article is a 2D axisymmetric model with a circular tunnel excavated in a time-dependent plastic saturated rock mass. Two-dimensional (2D) axisymmetric numerical models can reasonably represent the rock mass behaviour during the excavation of deep tunnels considering isotropic initial stresses and constant liquid pressures at the far-field boundaries. Under these conditions, 2D axisymmetric models can save significant computational time during calculations, compared with three-dimensional models. In fact, they have been widely used for mechanical problems.

2.2. Theoretical background for solid deformations

2.2.1. Rock mechanics

Experimental creep tests show that the time-dependent deformation of soft (or squeezing) rocks or rock masses accounts for a large amount of the total displacements [9,10,12,20,23,24,51,52]. Time dependency (or creep behaviour) results in delayed deformation, which must be taken into account to design underground projects more accurately and safely [12,20].

Moreover, the creep-induced failure behaviour of geomaterials may be relevant, as some underground structures do not fail during the excavation process, but ultimately fail after a long-term [8,9,53]. Thus, time-dependent plastic behaviour, especially the creep-induced failure response of geomaterials, is a topic of considerable interest in rock mechanics.

As shown in Figure 1, the typical creep response of rock samples may usually be characterized by four stages under a constant applied load [9,12,20,54]:

1) Instantaneous elastic response: the reversible elastic strain occurs instantaneously upon loading.
2) The primary creep stage: the strain rate decreases with time.
3) The secondary creep stage: a constant strain rate with time occurs.
4) The accelerated (or named tertiary) creep stage: the strain rate starts to accelerate once the material reaches the yield surface.

Furthermore, the post-failure behaviour of geomaterials may play a significant role in the performance of underground excavations [5–11,38,57]. For instance, different quality rock masses may exhibit different post-failure behaviour, as shown in Figure 2(a) [10,58], and the response of rock masses will differ depending on the selected model [10,38,59]. The strain-softening model is adopted in this study, which can be easily simplified to purely-brittle behaviour (strain-softening with an infinite drop modulus, M, as shown in Figure 2a) or perfectly-plastic behaviour (strain-softening with a drop modulus, M, equal to zero, as shown in Figure 2a) [9,10], if desired.

In a strain-softening model, as shown in Figure 2(b), the yield surfaces F(σ, η) depend on both the stress tensor σ and the softening parameter η [10,11]. The softening parameter η describes the gradual transition from the peak yield surface to the residual failure surface, being η∗ the value of the softening parameter at which the softening phase ends and the residual phase begins [9,10].

Song et al. [9] proposed a viscoelastic-viscoplastic strain-softening model (named Maxwell-viscoplastic strain-softening or MVSS model in the following), combining the Maxwell model and the rate-dependent viscoplastic model in series. Due to the coupling between the creep deformation and the strain-softening model, the MVSS model can represent creep-induced
failure [9]. However, the primary creep stage was neglected in the MVSS model [9]. However, the primary creep-induced deformations may be relevant for rock masses with high strength or subjected to low stresses [24,44]. Hence, in this article, by introducing the Kelvin model to the MVSS model, a new Burgers-viscoplastic strain-softening (BVSS) model has been developed. Section 2.2.2 shows the detailed description of the proposed BVSS model. Section 3 presents the numerical implementation method of the BVSS model in a finite element method (FEM) software, CODE_BRIGHT [46].

2.2.2. The Burgers-viscoplastic strain-softening constitutive model

In rock mechanics, the Hookean elastic springs and the Newtonian viscous dashpot can be coupled in series or in parallel to model the time-dependent behaviour [60,61]. Moreover, a plastic slider or a viscoplastic model can be used to simulate the plastic response [9,12].

In the current research, a coupled Burgers-viscoplastic strain-softening (BVSS) model is proposed. It consists of the Burgers model in series with the Perzyna viscoplastic model, with the aim of representing rock behaviour more realistically. The Mohr-Coulomb strain-softening and the Hoek-Brown strain-softening models [9,10,13] have been selected for this study. Note that the proposed BVSS model should be considered as an approximation of the actual rock mass behaviour that is adequate for a continuous approach, within the framework of wider methodological approaches, as recommend by Starfield and Cundall [62].

The BVSS model is developed to capture the critical aspects of geomaterials, which include:

1) Time-dependent properties, including the primary, secondary and accelerated creep responses.
2) Post-failure behaviour, including perfectly-plastic, strain-softening and purely-brittle models. Note that the strain-softening model can incorporate perfectly-plastic (strain-softening with a drop modulus, M, equal to zero) and purely-brittle (strain-softening with an infinite drop modulus M) models, as shown in Figure 2.
3) Coupled behaviour between creep response and damage evolution, so that creep-induced (or delayed) failure can be simulated.

As shown in Figure 3, some simpler models can be considered as particular cases of the proposed BVSS model. For example, the Burgers model can be simplified to the Maxwell model, the Kelvin model and the Generalized Kelvin model. Similarly, the rate-dependent viscoplastic model can be simplified to the purely plastic slider. Thus, the Maxwell-viscoplastic strain-softening (MVSS) model proposed in Song et al. [9] is just a particular case of the proposed BVSS model. Figure 4 presents the creep response corresponding to three different mechanical models.

The total strain rate tensor of the proposed BVSS model \( \frac{ds}{dt} \) can be decomposed into components describing viscoelasticity \( \frac{d\varepsilon_{el}}{dt} \) and viscoplasticity \( \frac{d\varepsilon_{pl}}{dt} \), as shown in Eq. (1). The viscoelasticity consists of two components: Maxwell \( \frac{d\varepsilon_{M}}{dt} \) and Kelvin \( \frac{d\varepsilon_{K}}{dt} \) parts, and the Maxwell part can be decomposed into elastic spring \( \frac{d\varepsilon_{M}^{e}}{dt} \) and viscous \( \frac{d\varepsilon_{M}^{v}}{dt} \) parts.

\[
\frac{d\varepsilon}{dt} = \frac{d\varepsilon_{el}}{dt} + \frac{d\varepsilon_{pl}}{dt} = \frac{d\varepsilon_{M}}{dt} + \frac{d\varepsilon_{K}}{dt} + \frac{d\varepsilon_{vp}}{dt} \]
\[
= \frac{d\varepsilon_{M}^{e}}{dt} + \frac{d\varepsilon_{M}^{v}}{dt} + \frac{d\varepsilon_{K}}{dt} + \frac{d\varepsilon_{vp}}{dt} \]  
(1)
Figure 3. The proposed Burgers-viscoplastic strain-softening (BVSS) model for geomaterials.

The strain rates corresponding to Maxwell, Kelvin and viscoplastic models can be expressed as in Eqs. (2)-(4), respectively.

\[
\frac{d\varepsilon_M}{dt} = \frac{d\varepsilon^e_M}{dt} + \frac{d\varepsilon^\nu}{dt} = C_M \frac{d\sigma’}{dt} + \frac{\sigma’}{\eta_M} = (D^{\mu}_M)^{-1} \frac{d\sigma’}{dt} + \frac{\sigma’}{\eta_M}
\]

\[
\frac{d\varepsilon_K}{dt} = C'_K (\sigma’ - \varepsilon_K) = (D'_K)^{-1} (\sigma’ - \varepsilon_K)
\]

\[
\frac{d\varepsilon_{vp}}{dt} = \frac{1}{\eta_{vp}} (\Phi(F)) \frac{\partial G}{\partial \sigma’}
\]

where \(D^e_i\) (or \(C^e_i\)) and \(D^\nu_i\) (or \(C^\nu_i\)) represent the tangent stiffness matrices (or the compliance matrices) of the elastic spring and of the viscous dashpot in the \(i\) model (\(i = M\) or \(K\) represents the Maxwell model or the Kelvin model); \(\eta_M\) (or \(\eta_K\)) is the viscosity of the viscous dashpot in the Maxwell (or the Kelvin) model; \(\eta_{vp}\) is the viscosity of the viscoplastic model; \(F\) and \(G\) represent the overstress function and the viscoplastic potential, respectively, of the viscoplastic model. The symbol \(\langle \Phi(F) \rangle\) can be expressed as in Eq. (5) and \(\Phi(F) = F^m\) (\(m \geq 1\)) [9,63,64] is adopted in this article.

\[
\langle \Phi(F) \rangle = \begin{cases} 
\Phi(F) & \text{if } \Phi(F) \geq 0 \\
0 & \text{if } \Phi(F) < 0
\end{cases}
\]

The expression of the Mohr-Coulomb strain-softening model is shown in Eq. (6), where \(c(\eta)\) and \(\varphi(\eta)\) are cohesion and friction angle, respectively. \(p’, J_2\) and \(\theta\) represent the mean effective stress, the second invariant of the deviatoric stress tensor, and the Lode angle, respectively.

\[
F = p' \sin \varphi(\eta) + \sqrt{J_2} \left[ \cos \theta - \frac{1}{\sqrt{3}} \cdot \sin \varphi(\eta) \sin \theta \right] - c(\eta) \cos \varphi(\eta)
\]

The viscoplastic potential \(G\) is expressed as in Eq. (7), where \(\alpha\) is a parameter for the viscoplastic potential (\(0 \leq \alpha \leq 1\)) and \(\psi(\eta)\) is the dilatancy angle.

\[
G = \alpha p' \sin \psi(\eta) + \sqrt{J_2} \left[ \cos \theta - \frac{1}{\sqrt{3}} \cdot \sin \psi(\eta) \sin \theta \right]
\]
the tunnel deformation of the Hoek-Brown model. Hence, if a Hoek-Brown strain-softening model is used, an equivalent cohesion and friction angle for each rock mass and stress range can be determined, which can be used in the numerical calculation. This equivalent cohesion and friction angle can be obtained based on the approach proposed by Hoek et al. [66] – see Eqs. (8) and (9).

\[
\phi = \sin^{-1} \left[ \frac{6a_{\text{HB}}m_{\text{HB}}(s_{\text{HB}} + m_{\text{HB}}\sigma_{\text{ci}}^{\text{max}})}{2(1 + a_{\text{HB}})(2 + a_{\text{HB}}) + 6a_{\text{HB}}m_{\text{HB}}(s_{\text{HB}} + m_{\text{HB}}\sigma_{\text{ci}}^{\text{max}})a_{\text{HB}}^{-1}} \right] \tag{8}
\]

\[
c = \frac{\sigma_{\text{ci}}(1 + 2a_{\text{HB}})s_{\text{HB}} + (1 - a_{\text{HB}})m_{\text{HB}}\sigma_{\text{ci}}^{\text{max}}(s_{\text{HB}} + m_{\text{HB}}\sigma_{\text{ci}}^{\text{max}})a_{\text{HB}}^{-1}}{(1 + a_{\text{HB}})(2 + a_{\text{HB}})\left[ 1 + \frac{6a_{\text{HB}}m_{\text{HB}}(s_{\text{HB}} + m_{\text{HB}}\sigma_{\text{ci}}^{\text{max}})a_{\text{HB}}^{-1}}{(1 + a_{\text{HB}})(2 + a_{\text{HB}})} \right]} \tag{9}
\]

where \(\sigma_{\text{ci}}^{\text{max}}\) is the upper limit of the confining stress over which the relationship between the Hoek-Brown and the Mohr-Coulomb criteria is considered. The reader is referred to the original source [66] for a detailed description of the equivalent method between the Hoek-Brown and Mohr-Coulomb failure criteria.

A piecewise linear plastic parameter function of the plastic parameters \(k(\eta)\) is adopted [9,10,57,65,67–69], as shown in Eq. (10), where \(k\) can represent the cohesion \(c\), the friction angle \(\phi\) and the dilatancy angle \(\psi\). \(k_{\text{peak}}\) and \(k_{\text{res}}\) are the peak and residual values of \(k\), respectively. The softening parameter \(\eta\) is defined in Eq. (11), where \(\varepsilon^{P}\) (or \(\psi^{P}\)) represents the accumulated plastic strain and \(\varepsilon^{P}_{\text{m}} = \frac{1}{2}(\varepsilon^{P}_{1} + \varepsilon^{P}_{2} + \varepsilon^{P}_{3})\) [9,63]. Note that, we have selected a linear decrease in the these strength parameters to represent softening, following the work of other researchers that used a linearly decreasing function for the cohesion, friction angle and dilatancy angle [9,10,57,65,67–69]. It is a simplification of a more complex problem. However, as a first approach, it is important to have a rather basic understanding of the tunnelling response in different time-dependent models selection and different excavation methods using simple post-failure behaviour models.

\[
k(\eta) = \begin{cases} 
k_{\text{peak}}, & \text{for } \eta < 0 \\
k_{\text{peak}} + \left( \frac{k_{\text{res}} - k_{\text{peak}}}{\eta^{*}} \right)\eta, & \text{for } 0 \leq \eta < \eta^{*} \\
k_{\text{res}}, & \text{for } \eta \geq \eta^{*} \end{cases} \tag{10}
\]

\[
\eta = \sqrt{\frac{3}{2} \left[ (\varepsilon^{P}_{1} - \varepsilon^{P}_{\text{m}})^{2} + (\varepsilon^{P}_{2} - \varepsilon^{P}_{\text{m}})^{2} + (\varepsilon^{P}_{3} - \varepsilon^{P}_{\text{m}})^{2} + \left( \frac{1}{2} \gamma^{P}_{12} \right)^{2} + \left( \frac{1}{2} \gamma^{P}_{23} \right)^{2} + \left( \frac{1}{2} \gamma^{P}_{31} \right)^{2} \right]} \tag{11}
\]
The strains developed in the viscous dashpot of the Maxwell model ($\varepsilon_M^v$) and in the viscoplastic model ($\varepsilon_{vp}$) are accumulated to account for the softening evolution, as shown in Figure 5. In that way, although the rock mass behaviour may be inside the yield surface initially, eventually it may reach failure. This is due to the yield surface shrinkage caused by the increase of the accumulated plastic strain ($\varepsilon_M^p$) [9]. Thus, creep-induced failure behaviour can be simulated using the proposed BVSS model.

Moreover, regarding creep-induced failure behaviour, some experimental results show that the creep response differs according to the applied load level (see Figure 1b). In addition, for stress values below the so-called 'limited stress level', i.e. stress points inside the residual failure surface domain (see Figure 5 and line 1 in Figure 6), no failure occurs even in the long term [9,12,70], because the yield surface would never be reached.

As shown in Figure 6, after some stress increment $\Delta\sigma$ is applied up to points A, C and E, the applied stress is kept constant for a long period of time. It shows that the stress state will result in a failure response for stress states above the so-called 'limited stress level' (lines 2 and 3). This failure will happen at different accumulated plastic strain values: for lines 2 and 3, failure initiates at points D and F, respectively. Based on Eq. (2), higher applied stresses would result in bigger values of $\frac{d\varepsilon_M^p}{dt}$ and thus, earlier failure times will occur due to the faster softening evolution. Hence, different stress levels will result in a different creep response in the proposed BVSS model.

2.3. The multi-physics multi-phases theory

2.3.1. The coupled solid-fluid interaction

In this study, the problem is formulated in a multi-physics (hydro-mechanical) multi-phase (solid-fluid) approach. From a modelling perspective, saturated rock masses are treated as an equivalent porous medium [1,46,49,50]. The adopted gen-
eralized effective stress expression is that in Eq. (12).
\[ \sigma' = \sigma - \rho_l I \] (12)
where \( \sigma \) is the total stress tensor, \( \sigma' \) is the effective stress tensor, \( \rho_l \) is the liquid pressure, \( B \) is Biot’s coefficient and \( I \) is the identity tensor.

As shown in Figure 7, two aspects (hydraulic and mechanical) and two phases (solid and fluid), are considered, including the coupling between them in both directions. Changes in hydraulic conditions influence the stress/strain field through changes in pore water pressure. Moreover, changes in the volume of the pores (or joints) caused by volumetric strains influence pore (or joint) liquid pressure distributions.

In this case, saturated rock masses are composed of two phases –rock and water–. The water component is always in the liquid phase \((l)\), and the rock component is always in the solid phase \((s)\). Conceptually, this means that the water is considered to be in the joints of the rock mass, which are the main vector of water seepage.

The mass balance of the solid phase is expressed in Eq. (13). \( \rho_s \) represents the solid phase density, i.e. the density of the rock; \( \phi \) represents the porosity, which can be understood as the volume of the joints per unit volume of rock mass; \( j_s \) represents the mass flux of the rock concerning a fixed reference system.

\[ \frac{\partial}{\partial t}(\rho_s(1 - \phi)) + \nabla \cdot (j_s) = 0 \] (13)

The mass balance of the liquid phase is expressed in Eq. (14). \( q_l \), \( \rho_l \) represent the liquid flux and liquid density, respectively; \( f^w \) is a source/sink term.

\[ \frac{\partial (\rho_l \phi)}{\partial t} + \nabla \cdot \left( \rho_l q_l + \rho_l \phi \frac{du}{dt} \right) = f^w \] (14)

The stress equilibrium equation is expressed in Eq. (15). \( \sigma \) represents the stress tensor and \( b \) represents the vector of body forces.

\[ \nabla \cdot \sigma + b = 0 \] (15)

Using the material derivative with respect to the solid –Eq. (16)–, the mass balance of solid phase becomes that in Eq. (17).

\[ \frac{D_s(\bullet)}{Dt} = \frac{\partial (\bullet)}{\partial t} + \frac{du}{dt} \cdot \nabla (\bullet) \] (16)

\[ \frac{D_s \phi}{Dt} = \frac{(1 - \phi) D_s \rho_s}{\rho_s} + (1 - \phi) \nabla \cdot \frac{du}{dt} \] (17)

Combining Eq. (17) with the mass balance of the liquid phase and then making use of the material derivative –Eq. (16)– again, Eq. (18) can be obtained.

\[ \phi \frac{D_l(\rho_l)}{Dt} + \rho_l \frac{(1 - \phi) D_s \rho_s}{\rho_s} + \rho_l \nabla \cdot \frac{du}{dt} + \nabla \cdot (\rho_l q_l) = f^w \] (18)

The theory of the coupled multi-physics multi-phases formulation used herein is a particular case of the general formulation presented in Olivella [49] and Olivella et al. [50]. More details on the finite-element formulation employed in CODE_BRIGHT, including the weak forms of the governing equations and explicit definitions of the resulting matrices and vectors, can be found in [46,49,50].
2.3.2. Fluid flow

The liquid advective flux related to phase motion is governed by the generalized Darcy’s law, as shown in Eq. (19).

\[
\mathbf{q}_l = - \frac{k_{ij}}{\mu_i} (\nabla p_l - \mathbf{\rho}_l \mathbf{g}) \tag{19}
\]

where \(p_l\) represents the liquid pressure; \(k\) represents the intrinsic permeability tensor, which depends on the joint structure of the rock mass; \(k_{ij}\) and \(\mu_i\) represent the relative permeability and dynamic viscosity of the fluid, respectively; \(\mathbf{g}\) represents the gravity forces vector.

3. Numerical implementation

The Burgers-viscoplastic strain-softening (BVSS) model is programmed in a finite element method software CODE_BRIGHT [46]. If another code is adopted, the same constitutive model presented in this article can be used. The programming of the proposed BVSS model should consider the following aspects:

1) The existence of corners in the yield and potential surfaces at which the gradients are not uniquely defined.
2) The strain localization and the development of strain-softening behaviour.
3) The coupled behaviour between creep response and softening evolution.
4) The fact that the development of the Kelvin model depends not only on the current state of stress and strain, but also on the full history of their variation.

The first three aspects are already addressed in Song et al. [9], from which the same approaches of the smoothing method for the yield and potential surfaces, strength parameters update for the strain-softening model and the viscoplastic model for strain localization have been assumed for this study. In addition to that, the implementation of the Kelvin model in CODE_BRIGHT is introduced in this article. However, for the sake of clarity, a brief description of the first three aspects is presented in this section, and more detailed information can be found in Song et al. [9].

On the other hand, multi-physics modelling of multi-stage excavation problems usually encounters numerical inefficiency, especially for non-linear geomaterials [1] such as the adopted BVSS model in this study. Thus, the smoothed excavation method [1] is used in the numerical simulations, which can improve the numerical reliability and numerical efficiency for multi-stage excavation problems. More detailed information of the smoothed excavation method can be found in Song et al. [1].

3.1. Numerical implementation of the BVSS model

The strain rate of the BVSS model is composed into the Maxwell, Kelvin and viscoplastic parts, as shown in Eq. (1). The strain rate of the Maxwell model only depends on the current stress state and the material properties, as shown in Eq. (20).

\[
\frac{d\mathbf{e}_M}{dt}{|_{t=t^k}} = \mathbf{C}_M^e \frac{d\mathbf{\sigma}}{dt}{|_{t=t^k}} + \mathbf{C}_M^s \mathbf{\sigma}^k{|_{t=t^k}} \tag{20}
\]

\(\mathbf{C}_M^e\) and \(\mathbf{C}_M^s\) are shown in Eq. (21).

\[
\mathbf{C}_M^e = \begin{bmatrix}
M_{Me_{12}} & 0_{3 \times 3} \\
0_{3 \times 3} & N_{Me_{23}}
\end{bmatrix}, \quad \mathbf{C}_M^s = \begin{bmatrix}
M_{Mv_{12}} & 0_{3 \times 3} \\
0_{3 \times 3} & N_{Mv_{23}}
\end{bmatrix}
\tag{21}
\]

where \(M_{Me} = \begin{bmatrix}
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} \\
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} \\
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M}
\end{bmatrix}, \quad N_{Me} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix};
\]

\(M_{Mv} = \begin{bmatrix}
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} \\
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} \\
\frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M} & \frac{1}{3c_M} + \frac{1}{\eta_M}
\end{bmatrix}, \quad N_{Mv} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.\)

Moreover, the elastic spring in the Maxwell model is characterized by an elastic modulus \(E_M\), a Poisson’s ratio \(v_M\), a shear modulus \(G_M = \frac{E_M}{2(1+v_M)}\), and a volumetric modulus \(K_M = \frac{E_M}{3(1-2v_M)}\). \(\eta_M^v\) (or \(\eta_M^d\)) represents the volumetric (or deviatoric) viscosity of the viscous dashpot in the Maxwell model.

The strain rate of the Kelvin model is shown in Eqs. (22) and (23). Note that, the numerical calculation and numerical implementation of the Kelvin model involve some theoretical difficulties, since this model depends on not only the current stress state, but also the full history of its development [71] and thus, history variables [46] are required in the numerical implementation.

\[
\frac{d\mathbf{e}_K}{dt}{|_{t=t^k}} = \mathbf{C}_K^v \mathbf{\sigma}{|_{t=t^k}} - \mathbf{\varepsilon}_K{|_{t=t^k}} \tag{22}
\]
\[ \varepsilon_K(t_{k+1}) = \left[ \varepsilon_K + \frac{d\varepsilon_K}{dt} \right]_{t=t_k} \]  

where \( \varepsilon_K(t_{k+1}) \) and \( \varepsilon_K(t_{k+1}) \) are the accumulated strains of the Kelvin model at time \( t = t_k \) and \( t = t_{k+1} \), respectively. \( C_k \) is shown in Eq. (24). History variables [46] are applied to store the accumulated strain of the Kelvin model.

\[ C_k = \begin{bmatrix} M^{Kv}_{3 \times 3} & 0_{3 \times 3} & N^{Kv}_{3 \times 3} \end{bmatrix} \]  

Furthermore, the strain rate of the viscoplastic model is shown in Eq. (4), and the derivative of Eq. (4) with respect to the stress tensor can be expressed as in Eq. (25).

\[ \frac{d}{d\sigma} \left( \frac{d\varepsilon_{\text{vp}}}{dt} \right) = \frac{1}{\eta_{\text{vp}}} \frac{d}{\sigma} \left( F^m \frac{\partial G}{\partial \sigma} \right) = \frac{1}{\eta_{\text{vp}}} \left[ m^{F_{m-1}} \frac{\partial F}{\partial \sigma} \frac{\partial G}{\partial \sigma} + F^m \frac{\partial^2 G}{\partial \sigma^2} \right] \]  

However, due to the gradient discontinuities of the yield surfaces at both edges and corners, computational inefficiency or non-convergence issues may happen in the stress integration scheme during numerical calculations. To eliminate these gradient discontinuities of the yield surface, a smoothed Mohr-Coulomb strain-softening yield surface [9,72] has been adopted in the numerical implementation, as shown in Eqs. (26) and (27). \( \theta_T \) is a specified transition angle [72]. Note that, if not specified, \( a \) and \( \theta_T \) are adopted as 0.25 and 25 deg in this article.

\[ F = \sqrt{J_2 K_{MC}^2(\theta) + a^2 \sin^2 \varphi(\eta) + p' \sin \varphi(\eta) - c(\eta) \cos \varphi(\eta)} \]  

\[ K_{MC}(\theta, \eta) = \begin{cases} A_{MC} + B_{MC} \sin 3\theta, & \text{for} |\theta| > \theta_T \\ \cos \theta - \frac{1}{\sqrt{3}} \sin \varphi(\eta) \sin \theta, & \text{for} |\theta| \leq \theta_T \end{cases} \]  

where:

\[ A_{MC} = \frac{1}{3} \cos \theta \left[ 3 + \tan \theta_T \tan 3\theta_T + \frac{1}{\sqrt{3}} \langle \theta \rangle (\tan 3\theta_T - 3 \tan \theta_T) \sin \varphi(\eta) \right], \]

\[ B_{MC} = -\frac{1}{3} \cos 3\theta_T \left[ \langle \theta \rangle \sin \theta_T + \frac{1}{\sqrt{3}} \sin \varphi(\eta) \cos \theta_T \right], \]

\[ \langle \theta \rangle = \begin{cases} +1, & \theta \geq 0 \\ -1, & \theta < 0 \end{cases} \]

Since the second derivative of the viscoplastic potential should also be continuous, the C2 method [9,73] has been used to smooth the potential function. The adopted smoothed viscoplastic potential can be expressed in Eq. (28). The alternative form of \( K_G(\theta) \) in the vicinity of the singularities can be expressed as shown in Eq. (29). Note again that, \( \theta_T \) is adopted as 25 deg in this study.

\[ G = \alpha p \sin \psi + J_2 K_G^2(\theta) \]  

\[ K_G(\theta) = \begin{cases} D_C + E_C \sin 3\theta + F_C \sin^2 3\theta, & \text{for} |\theta| > \theta_T \\ \cos \theta - \frac{1}{\sqrt{3}} \sin \psi \sin \theta, & \text{for} |\theta| \leq \theta_T \end{cases} \]  

where:

\[ F_C = -\cos 3\theta_T \left( \cos \theta_T - \frac{1}{\sqrt{3}} \sin \psi(\theta) \right) - 3 \langle \theta \rangle \sin 3\theta_T \left( \langle \theta \rangle \sin \theta_T + \frac{1}{\sqrt{3}} \sin \varphi \cos \theta_T \right) \]  

\[ E_C = \frac{\langle \theta \rangle \sin \theta_T \left( \cos \theta_T - \frac{1}{\sqrt{3}} \sin \psi(\theta) \sin \theta_T \right) - 6 \cos \theta_T \left( \langle \theta \rangle \sin \theta_T + \frac{1}{\sqrt{3}} \sin \varphi \cos \theta_T \right)}{18 \cos^3 3\theta_T} \]  

\[ D_C = -\frac{1}{\sqrt{3}} \sin \psi(\theta) \sin \theta_T - E \langle \theta \rangle \sin 3\theta_T - F \sin^2 3\theta_T + \cos \theta_T \]
These equations, Eqs. (26)-(29), have been adapted from Abbo et al. [72] and Abbo and Sloan [73]. The reader is referred to the original source for a detailed description of the smoothed approximation (C1 and C2) to the smoothed Mohr-Coulomb yield surfaces.

Regarding the plastic evolutions, the accumulated plastic (unrecoverable) strain of the proposed BVSS model is the sum of creep strain from the Maxwell model ($\epsilon_{\text{cr}}^f$) and the viscoplastic strain ($\epsilon_{\text{vp}}$). In the code, a history variable [46] has been introduced to calculate the accumulated unrecoverable strain. In the numerical calculations, the accumulated plastic strain is updated in the elements, and then, the softening parameter $\eta$ is updated following the Eq. (11). After that, the strength parameters (cohesion, friction angle, and dilatancy angle for the Mohr-Coulomb model) are updated based on Eq. (10). Finally, the failure surface ($F$) and the viscoplastic potential ($G$) are updated. Note that softening behaviour may introduce bifurcation and localization of the plastic solutions. However, the viscoplastic approach [9,63,64] and the overstress theory [9,63,74] are adopted in this study to eliminate the numerical difficulties of softening problems. In the next section (section 3.2), an analysis of the effect of mesh on the resulting tunnelling responses (solid deformations and fluid flow) is carried out.

### 3.2. Analyses of mesh independency

Numerical analysis involving softening problems may exhibit a marked dependency on the employed finite element mesh. In some cases, instability occurs since softening may concentrate at isolated elements, while other elements in their vicinity experience stress relaxation [9]. In this study, the viscoplastic approach is adopted for the proposed BVSS model, which can be capable of homogenizing the spatial distribution of softening strain, and thus, benefits the control of the size of the localized zone and allows avoidance of the dependency on the employed mesh [9,63,74,76]. On the other hand, from an engineering point of view, strain localization effects are residual when considering an appropriately large spatial scale [9,38], as would be the case of the tunnelling problems in this article.

In this section, analyses of the effect of meshes on the resulting tunnelling deformations and liquid pressures are carried out. Several 2D axisymmetric models were developed to assess the performance of the viscoplastic approach in the simulation of softening problems. In the CODE_BRIGHT numerical model, the tunnel is represented as a rectangle of length 50 m, excavated in 40 steps 1.25 m long. The excavation rate selected is 5 m/day, as recommended by Prassetyo and Gutierrez [16], and, therefore, the whole excavation process takes 10 days. The dimensions and boundary conditions of the CODE_BRIGHT numerical model are shown in Figure 8. The tunnels analysed in this section have a diameter of 5 m. The selected initial stress ($p_0 = 4.5$ MPa), initial liquid pressure ($p_l = 2.25$ MPa), intrinsic permeability ($k = 5.12 \times 10^{-17}$ m²) and initial poros-
ity (0.39) values are based on the former research [16,47,48]. After excavation, atmospheric pressure (0.1 MPa) is prescribed on the tunnel surface. Two rock masses (set#1 and set#2) with different input parameters are considered (Table 1).

As shown in Figure 9, four meshes with different mesh sizes have been adopted to analyze the mesh sensitivity on the resulting tunnelling deformations and liquid pressures. The meshes are composed of the following number of quadratic triangular elements (i.e. triangles with 6 nodes): (a) mesh_01: 2845 elements, (b) mesh_02: 6232 elements, (c) mesh_03: 8913 elements, and (d) mesh_04: 14205 elements. Figure 10 shows the radial displacements and liquid pressures versus the normalized distance to the tunnel face \((x_d/R_1)\) for the above mentioned four mesh sizes (mesh_01, mesh_02, mesh_03, mesh_04) and for two different rock masses with different input parameters (set#1 and set#2). Note for reference that radial deformation is taken at a radius of 2.5 m, while liquid pressure is taken at a radius of 4.5 m. A good agreement can be observed among the results obtained with different meshes, which verifies the mesh-independency of the proposed softening model in tunnelling applications.

### 4. Numerical verification and comparison with other solutions

#### 4.1. Creep tests

Uniaxial creep numerical tests are carried out to analyze the response of different mechanical models. Firstly, CODE_BRIGHT viscoelastic results are compared with analytical solutions. After that, creep tests are carried out to analyze the creep-induced failure behaviour of the proposed Burgers-viscoplastic strain-softening (BVSS) model. Five different models are adopted in the comparison: three different viscoelastic models, including Maxwell, Kelvin and Generalized Kelvin models; two different Burgers-viscoplastic models, including Burgers-viscoplastic perfectly-plastic (BVPP) model and Burgers-viscoplastic strain-softening (BVSS) model.
Figure 10. Radial displacements versus the normalized distance to the tunnel face \((x_d/R_1)\) for (a) set #1, and (b) set #2. Liquid pressures versus the normalized distance to the tunnel face \((x_d/R_1)\) for (c) set #1, and (d) set #2. Four different meshes (mesh_01: 2845 elements; mesh_02: 6232 elements; mesh_03: 8913 elements; mesh_04: 14205 elements) are adopted for each case. \(t_{	ext{standstill}} = 0\).

Table 2

<table>
<thead>
<tr>
<th>Viscoelastic model</th>
<th>Maxwell model</th>
<th>(E_m)</th>
<th>6000 MPa</th>
<th>(\eta_m)</th>
<th>(7.77 \times 10^6) MPa.s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelvin model</td>
<td>(E_K)</td>
<td>3000 MPa</td>
<td>(\eta_K)</td>
<td>1.296 ( \times 10^9) MPa.s</td>
<td></td>
</tr>
<tr>
<td>Viscoplastic model</td>
<td>Cohesion</td>
<td>(\zeta_{\text{peak}})</td>
<td>10 MPa</td>
<td>(\zeta_{\text{res}})</td>
<td>5 MPa</td>
</tr>
<tr>
<td>Friction angle</td>
<td>(\psi_{\text{peak}})</td>
<td>50 deg</td>
<td>(\psi_{\text{res}})</td>
<td>20 deg</td>
<td></td>
</tr>
<tr>
<td>Dilatancy</td>
<td>(\alpha)</td>
<td>1</td>
<td>(\psi_{\text{peak}} = \psi_{\text{res}})</td>
<td>5 deg</td>
<td></td>
</tr>
<tr>
<td>Perzyna model</td>
<td>(m)</td>
<td>5</td>
<td>(\eta_{\text{vp}})</td>
<td>(10^{10}) MPa.s</td>
<td></td>
</tr>
</tbody>
</table>

\(a = 0.25\) and \(\theta_1 = 25\) deg in Eqs. (26) and (28).

The numerical model used here is 2D axisymmetric with dimensions of 0.01 m \(\times\) 0.05 m. The normal displacements along the bottom and left boundaries have been restrained, as shown in Figure 11(a). A mesh with 2250 quadratic triangle elements has been considered for the analysis, as shown in Figure 11(b). In the numerical simulations, a constant stress \(p_y\) is applied on the top boundary at time \(t = 0\). The calculation is stopped at 150 days.

The analytical solutions of creep strains are shown in Eq. (30) for three different viscoelastic models. The rock mass is considered incompressible and the input material parameters are shown in Table 2. No dilatancy is considered in the perfectly-plastic behaviour model. Note that the analyses do not represent any particular experiment.

\[
\varepsilon(t) = \begin{cases} 
\frac{p_y}{E_m} + \frac{p_y}{\eta_m} t & \text{for the Maxwell model} \\
\frac{p_y}{E_m} + \frac{p_y}{\zeta_{\text{peak}}} \left(1 - e^{-\frac{t}{\zeta_{\text{peak}}/\eta_m}}\right) & \text{for the Generalized Kelvin model} \\
\frac{p_y}{E_m} + \frac{p_y}{\zeta_{\text{res}}} + \frac{p_y}{E} \left(1 - e^{-\frac{t}{\zeta_{\text{res}}/\eta_m}}\right) & \text{for the Burgers model}
\end{cases}
\]  

(30)

Figure 12(a) shows that the responses of different viscoelastic models (at Point A; see Figure 11a) differ and, hence, the selection of time-dependent models might be relevant. Moreover, a good agreement is observed between analytical solutions and CODE_BRIGHT results, which verifies the correctness of the implementation of the CODE_BRIGHT viscoelastic models.
Figure 11. Creep numerical test: (a) basic features and boundary conditions; (b) mesh (2250 quadratic triangle elements).

Figure 12. Axial strains of point A versus time: (a) comparison between analytical solutions and CODE_BRIGHT results for three different viscoelastic models; (b) comparison between Burgers, BVPP and BVSS models. Note that C_B represents the CODE_BRIGHT results.

For further verification, another group of creep tests is performed to analyze creep-induced failure behaviour. The numerical geometry, mesh and input parameters are the same as in the former example (see Figure 11). In Figure 12(b), the resulting strains of the Burgers model are the same as those of the BVPP model, which means that the viscoplastic part of the BVPP model is not engaged in this case.

In the case of the BVSS model, the resulting strain is the same as that of Burgers at the start of the simulation –i.e. when there is only viscoelastic response–. However, when the stress reaches the yield surface at point A, the geomaterial starts to fail, given that the yield surface shrinks due to creep-induced softening (see Figure 5), and hence the strain rate accelerates (see Figure 12b) [9]. Therefore, the proposed BVSS model can simulate creep-induced failure behaviour.

4.2. Comparison with tunnelling results based on the viscoelastic model

The proposed BVSS model can be simplified to a viscoelastic model (see Figure 3). In order to verify the BVSS model for tunnelling in viscoelastic rock masses, examples of circular tunnels excavated in rock masses using the Generalized Kelvin model and the Burgers model are carried out using CODE_BRIGHT. The numerical results are compared with the analytical
The expressions for tunnels excavated in the Generalized Kelvin and Burgers viscoelastic geomaterials under isotropic initial stresses are shown in Eqs. (31) and (32), respectively. The complex variable method and the Laplace transform technique have been used in the process of obtaining the analytical solutions [60].

\[
\begin{aligned}
    u_r(t) &= -\frac{p_0}{2\tau} \int_0^\tau \left[ \frac{1}{\eta_K^d} \exp \left( -\frac{G_K}{\eta_K^d} (t - \tau) \right) + \frac{1}{G_M} \delta(t - \tau) \right] R^2(\tau) d\tau \\
    u_r(t) &= -\frac{p_0}{2\tau} \int_0^\tau \left[ \frac{1}{\eta_M} + \frac{1}{\eta_K^d} \exp \left( -\frac{G_K}{\eta_K^d} (t - \tau) \right) + \frac{1}{G_M} \delta(t - \tau) \right] R^2(\tau) d\tau
\end{aligned}
\]

where \( R(t) \) represents the time-dependent radius of the tunnel cross-section; \( t \) represents time; \( p_0 \) represents the isotropic initial stress; \( u_r \) represents the incremental radial displacements occurring during the excavation; \( r \) represents the radial location in polar coordinates \((r, \theta)\), where \( r = 0 \) represents the location of the tunnel centre.

The numerical model developed using CODE_BRIGHT is consistent with the hypothesis made in the analytical solutions, and both of them are calculated under plane-strain conditions with small deformations. Only a quarter of the domain is analyzed in the numerical model (see Figure 13) because of the double symmetry of the geometry and the boundary conditions on both the \( x \) and \( y \) axes. In addition, the normal displacements along the bottom \((y = 0)\) and the left \((x = 0)\) boundaries are restrained. Figure 13(b) shows the mesh of the numerical model. A mesh of 1560 quadratic triangular elements is adopted, with smaller elements near the excavation. The isotropic initial stress in the model is \( p_0 = 20 \text{ MPa} \).

First, the model is run for a sufficiently long time (100 days in this example) before excavation, to reach the equilibrium stress-strain conditions before starting the excavation, ensuring that we only consider the incremental deformations induced by excavation. Subsequently, the first section of the tunnel is instantaneously excavated. That time will be considered zero \((t = 0 \text{ day})\) in this study. The following sections of the tunnel are excavated step by step, and the radii of the tunnel cross-section can be expressed by Eq. (33). After the completion of all excavation steps, the calculation is stopped at \( t = 60 \text{ days} \).

\[
R(t) = \begin{cases} 
2\text{m, for} 0 \leq t < 5\text{days} \\
4\text{m, for} 5\text{days} \leq t < 10\text{days} \\
6\text{m, for} 10\text{days} \leq t < 15\text{days} \\
8\text{m, for} t \geq 15\text{days}
\end{cases}
\]

In the comparison, for both the Generalized Kelvin and Burgers models, three different groups of material properties are considered for each model, as shown in Table 3. The rock masses are assumed incompressible, i.e. Poisson’s ratio is close to 0.5. A comparison of the time-dependent incremental radial displacements that occurred during the excavation along the tunnel wall \((r = 8 \text{ m}, \theta = 0 \text{ deg})\) predicted by the analytical solutions and the numerical simulations is shown in Figure 14. A good agreement between the numerical and analytical results is observed, hence verifying the viscoelastic models implemented in CODE_BRIGHT.

4.3. Comparison with the elastic solutions of the LDP for hydro-mechanical problems

Numerical simulations of the longitudinal deformation profiles (LDP) for the coupled solid-fluid problems with elastic materials are carried out in this section. Thus, the CODE_BRIGHT results are compared with the solutions for steady-state problems found in Nam and Bobet [15] and the solutions for coupled solid-fluid problems found in Prassetyo and Gutierrez.
Table 3
Input parameters of the viscoelastic models.

<table>
<thead>
<tr>
<th></th>
<th>$E_M$ (MPa)</th>
<th>$E_K$ (MPa)</th>
<th>$\eta_d^M$ (MPa.s)</th>
<th>$\eta_d^K$ (MPa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized Kelvin model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>2000</td>
<td>2000</td>
<td>-</td>
<td>10000</td>
</tr>
<tr>
<td>Case 2</td>
<td>2000</td>
<td>5000</td>
<td>-</td>
<td>20000</td>
</tr>
<tr>
<td>Case 3</td>
<td>5000</td>
<td>100</td>
<td>4.32 $\times 10^9$</td>
<td>8.64 $\times 10^8$</td>
</tr>
<tr>
<td>Burgers model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>5000</td>
<td>2000</td>
<td>$8.64 \times 10^8$</td>
<td>1.728 $\times 10^9$</td>
</tr>
<tr>
<td>Case 2</td>
<td>2000</td>
<td>500</td>
<td>$1.728 \times 10^9$</td>
<td>2.529 $\times 10^9$</td>
</tr>
</tbody>
</table>

$\eta_d^M \rightarrow \infty$ and $\eta_d^K \rightarrow \infty$.

Figure 14. Comparison between analytical solutions [60] and CODE_BRIGHT results for the incremental radial displacements $u_r$ along the tunnel wall ($r = 8$ m, $\theta = 0$ deg): (a) Generalized Kelvin model, and (b) Burgers model. Note that C_B represents CODE_BRIGHT results.

Figure 15. Comparison of the longitudinal deformation profiles for steady-state saturated problems, according to CODE_BRIGHT results and Nam and Bobet’s analytical solutions [15].

Figure 16. Comparison of the longitudinal deformation profiles for two different standstill times ($t_{stand} = 5$ days, 80 days), according to CODE_BRIGHT and Prassetyo and Gutierrez’s solutions (P&G in the figure) [16]: (a) $t_{stand} = 5$ days, (b) $t_{stand} = 80$ days.
Figure 17. Conceptual solid-fluid coupled model for longitudinal deformation profiles (LDP).

Figure 18. Radial displacements of the tunnel wall along the normalized distance to the tunnel face for three (MVSS-M, BVSS-M and BVSS-HM) models and two different standstill times (0 and 1 years): (a) set #1 and $t_{\text{stand}} = 0$, (b) set #1 and $t_{\text{stand}} = 1$ year, (c) set #2 and $t_{\text{stand}} = 0$, (d) set #2 and $t_{\text{stand}} = 1$ year.

[16]. A more detailed description of the steady-state and the time-dependent coupled solutions of the LDP for tunnels excavated in saturated rock masses can be found in the aforementioned references [15,16].

Considering tunnels below the water table with drainage towards the tunnel under steady-state conditions, Nam and Bobet [15] proposed the steady-state empirical solutions for normalized radial displacements ahead of ($x_d < 0$) and behind
Figure 19. Contours of radial displacements for two different rock masses (set #1 and set #2), three different models (MVSS-M, BVSS-M and BVSS-HM) and two different standstill times ($t_{\text{stand}} = 0$ and 1 year): (a) set #1 at $t_{\text{stand}} = 0$, (b) set #1 at $t_{\text{stand}} = 1$ year, (c) set #2 at $t_{\text{stand}} = 0$, and (d) set #2 at $t_{\text{stand}} = 1$ year.

\[ \frac{u_r}{u_{r,\text{max}}} = \begin{cases} 0.28 \exp\left(1.05 \frac{x_d}{R_1}\right) + 0.2 \exp\left(\frac{x_d}{10R_1}\right) \left[1 - \exp\left(-\frac{p_l}{p'}\right)\right], & x_d < 0 \\ \frac{u_{r,0}}{u_{r,\text{max}}} + \left(1 - \frac{u_{r,0}}{u_{r,\text{max}}}\right) \left[1 - \left(\frac{0.75}{0.75 + x_d/R_1}\right)^2\right], & x_d \geq 0 \end{cases} \] (34)

\[
\frac{u_{r,0}}{u_{r,\text{max}}} = 0.28 + 0.19 \left[1 - \exp\left(-\frac{p_l}{p'}\right)\right] \]

(35)

where $p_l$, $p'$ represent the far-field liquid pressure and effective stress at the centre of the tunnel, respectively; $u_{r,0}$ represents the radial displacement of the tunnel wall at the tunnel face; and $u_{r,\text{max}}$ represents the maximum radial displacement of the tunnel wall.
Considering the effect of the coupled solid-fluid interaction, Prasetyo and Gutierrez [16] proposed the transient coupled solutions of LDP shown in Eqs. (36) - (40).

\[
\frac{u_t}{u_{r,max}} = \begin{cases} 
\frac{u_{i0}}{u_{r,max}} \exp \left( \frac{2.11 x_d}{2 R_1} \right) + 0.2 \exp \left( \frac{x_d}{10 R_1} \right) \left[ 1 - \exp (-A) \right], x_d < 0 \\
\frac{u_{i0}}{u_{r,max}} + \left( 1 - \frac{u_{i0}}{u_{r,max}} \right) \left[ 1 - \left( \frac{B}{B + 0.5 x_d/R_1} \right)^2 \right], x_d \geq 0
\end{cases}
\]  

(36)

\[
\frac{u_{i0}}{u_{r,max}} = 0.28 + 0.028 \left[ 1 - \exp (-0.56 \log t^*) \right]
\]

(37)

\[
A = - \frac{1}{2.4} \ln \left( - \frac{u_{i0}}{u_{r,max}} - 0.305 \right) / 0.028
\]

(38)

\[
B = 0.85 - 2.10^5 \exp \left( -47 \frac{u_{i0}}{u_{r,max}} \right)
\]

(39)

\[
\frac{u_{r,max}}{R_1} = 2.0 - 0.3 \exp \left[ -0.005 t^* \right]
\]

(40)
where $t^*$ represents the normalised consolidation time [16,81,82].

In the CODE_BRIGHT numerical model, the numerical model geometry, boundary conditions, excavation process and mesh, are the same as those described in section 3.2 (see Figure 8). Mesh_02 in Figure 9(b) is adopted in the following numerical analysis. Table 4 contains the input parameters – rock mass properties and tunnel geometry. For the steady-state case, the intrinsic permeability chosen is $10^{-10}$ m$^2$, which is a big enough value to ensure drained conditions [1]; while for the coupled solid-fluid case, the intrinsic permeability adopted is $5.12 \times 10^{-17}$ m$^2$, the same used by Prassetyo and Gutierrez [16]. Note that, in this section, no pressure is prescribed on the tunnel surface after excavation to be consistent with the numerical models found in Prassetyo and Gutierrez [16].

Figure 15 presents the comparison between the CODE_BRIGHT results and the solutions of Nam and Bobet [15] of the LDP for tunnels excavated in saturated elastic rock masses under steady-state conditions. Similarly, Figure 16 shows the comparison of LDP considering the coupled solid-fluid interaction between the CODE_BRIGHT results and the solutions of Prassetyo and Gutierrez [16]. A good agreement is observed for all the cases analyzed. Therefore, the use of CODE_BRIGHT for representing the LDP considering the solid-fluid interaction and elastic materials has been verified.

### 5. Application in the design of tunnels

In this section, several analyses are carried out to examine the coupled solid-fluid response of tunnels excavated in time-dependent plastic rock masses (Figure 17). The time-dependent deformation of a tunnel depends on the creep response of the geomaterial and on the tunnel advancement rate [12,20,23,24]. In addition, the coupled solid-fluid interaction also affects the deformation of the tunnel [16,47,48]. In this respect, the effects of selecting different time-dependent models
are analysed in section 5.1 and the influence of the standstill time and of the excavation rate on the tunnelling response is analysed in sections 5.2 and 5.3.

The tunnels analysed in this section have a diameter of 5 m. The selected initial stress \( p_0 = 4.5 \text{ MPa} \), initial liquid pressure \( p_l = 2.25 \text{ MPa} \), intrinsic permeability \( k = 5.12 \times 10^{-17} \text{ m}^2 \) and initial porosity \( 0.39 \) values are based on the former research \[16,47,48\]. The numerical model geometry, boundary conditions, excavation process and mesh, are the same as those described in section 3.2 (see Figure 8). After excavation, atmospheric pressure (0.1 MPa) is prescribed on the tunnel surface. Mesh_02 is adopted, as shown in Figure 9(b). Note that no hydraulic boundary conditions are applied for the purely mechanical problems. If not specified, the implied excavation rate is 5 m/day \[16,47\]. Two rock masses (set#1 and set#2) with different input parameters are considered (Table 1). Set#1 mainly consists of sandstones and schists under extreme squeezing conditions \[77\], while set#2 is mostly composed of grey to black shale, Marl and calcareous Shale \[78\].

5.1. Influence of the selected behaviour model

This sub-section focuses on analysing the effect of the selection of different rock mass behaviour models on the resulting radial displacements of tunnels. Three different cases are investigated:

1) MVSS-M: a mechanical problem (M) using the Maxwell-viscoplastic strain-softening (MVSS) model.
2) BVSS-M: a mechanical problem (M) using the Burgers-viscoplastic strain-softening (BVSS) model.
3) BVSS-HM: a hydro-mechanical (HM) coupled problem using the BVSS model.

Note that the MVSS model is the same as the proposed model by Song et al. \[9\], which is a particular case of the BVSS model. Moreover, to investigate the long-term response of tunnelling, two different standstill times \( t_{\text{stand}} = 0 \text{ and 1 year} \) are considered. The reader can find more information about the concept of standstill time in previous work \[16,47,48\].

Unlike the MVSS-M model, the BVSS-M model incorporates the Kelvin model, which can simulate the primary creep behaviour of geomaterials. Furthermore, the BVSS-HM model considers the coupled solid-fluid interaction. However, the

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Figure 21. Liquid pressure profile (LPP) evolution for two different rock masses: (a) 2D LPP for set#1, (b) 2D LPP for set#2, (c) 3D LPP for set#1, (d) 3D LPP for set#2.
MVSS-M and BVSS-M models can only analyze mechanical problems. Regarding the Kelvin model, the displacements increase over time under constant applied stress, while the strain rate progressively decreases to zero. Therefore, after a long enough standstill period, the Kelvin model will have no contribution to the induced deformations. Note that $T_K = \eta_M^d / G_K$ denotes the retardation time, which is the delayed response to applied stress and can be described as ‘delay of the elasticity’. As the value of $T_K$ decreases, less time is needed for the Kelvin model to achieve a very low strain rate, i.e. the response of the rock mass to applied stress will be faster.

The first rock mass (set#1 in Table 1) is expected to exhibit a stronger primary and secondary creep response, because the Maxwell ($\eta_M^d$) and Kelvin ($\eta_K^d$) viscosities are lower. At time $t_{\text{stand}} = 0$, the resulting deformations using the BVSS-M model are significantly higher than those using the MVSS-M (Figures 18 and 19), due to the additional contribution of the Kelvin model. However, with increasing standstill time, the response of the Kelvin model may achieve a stable condition. Hence, after that, the difference between MVSS-M and BVSS-M results may be caused only by the creep response of the Maxwell model. This may be the reason why the difference between MVSS-M and BVSS-M results is smaller, although still significant, at $t_{\text{stand}} = 1$ year. On the other hand, at both $t_{\text{stand}} = 0$ and $t_{\text{stand}} = 1$ year, the BVSS-HM and the BVSS-M results are significantly different. This is caused by the additional contribution of the coupled solid-fluid response.

Regarding the second rock mass (set#2 in Table 1), its secondary creep response is expected to be residual, given the large value of $\eta_K^d$ considered. Moreover, the retardation time of set#2 has a higher value ($T_K = 1.58 \times 10^7$ s), which means that the primary creep develops during a longer time than in the case of set#1. For set #2, in the short-term ($t_{\text{stand}} = 0$), both MVSS-M and BVSS-M results are similar (Figures 18 and 19), probably because there is not enough time to develop a significant primary creep (or creep of the Kelvin model), given the aforementioned high value of $T_K$. On the contrary, at $t_{\text{stand}} = 1$ year, there is a considerable difference between MVSS-M and BVSS-M results, probably due to the additional contribution of the Kelvin model, which now has had enough time to induce deformations. Furthermore, it can be observed that BVSS-HM and BVSS-M results are significantly different, due to the coupled solid-fluid effect, both for short-term ($t_{\text{stand}} = 0$) and long-term ($t_{\text{stand}} = 1$ year) cases.

Figure 22. Two-dimensional (2D) and three-dimensional (3D) longitudinal deformation profile (LDP) evolution for two different rock masses (set#1 and set#2): (a) 2D LDP for set#1, (b) 2D LDP for set#2, (c) 3D LDP for set#1, (d) 3D LDP for set#2.
Hence, the MVSS model may represent the response of some tunnels in which large deformations occur due to secondary and accelerated creep and in which the primary creep-induced deformation is marginal. For example, this may be the case of tunnels excavated in weak or altered rock masses, or the case of hard rock masses excavated very deep, or the case of rock masses with significant values of $\eta_g$ [9,54,83,84].

However, in some other cases, where the primary creep-induced deformations may be relevant, the proposed BVSS model should be able to represent more accurately the time-dependent behaviour. Hence, it may contribute to a safer and more efficient tunnel design than the MVSS model. This is the case of tunnels excavated in rock masses exhibiting high strength or subjected to low stresses [24,44]. In addition, note that the MVSS model is actually a particular case of the BVSS model (as described in section 2).

Additionally, hydro-mechanical (HM) results are significantly different from the purely mechanical results. Therefore, both the selected rock mass behaviour model and the coupled solid-fluid interaction may be relevant in the preliminary design of tunnels.

5.2. Time-dependent responses of tunnels during the standstill period

Some tunnels do not fail during excavation but ultimately fail after a long period of operation [8,53]. This phenomenon might be caused by the creep-induced failure behaviour and/or by the pore pressure time-dependent dissipation process due to the coupled solid-fluid interaction. In this section, the tunnelling response for four different standstill times ($t_{\text{stand}} = 0$, 0.1, 1 year and 2 years) is presented. Two different rock masses (set#1 and set#2) are considered. The BVSS-HM model has been used for this analysis.

Concerning the pore pressure evolution, it can be observed that the liquid pressure builds up just ahead of the tunnel face (Figure 20) in the short term after the excavation ($t_{\text{stand}} = 0$), increasing around 5% and 7% for set#1 and set#2, respectively. This phenomenon may be due to the fact that the liquid pressure dissipation rate is lower than the excavation rate [16,47,48]. Then, with the increase of the standstill time, this overpressure dissipates and, eventually, the liquid pressure achieves a steady-state condition, after a long enough period [16,47,48], as it can be observed in Figure 20.

Moreover, although the hydro-mechanical boundary conditions are the same for both cases, it can be observed that the fluid responses are different for set#1 and set#2 (Figure 20). This difference is due to the coupled solid-fluid interaction (see Figure 7), which may cause different solid deformations for different geomaterials, in turn affecting fluid flow differently.

Figure 23. Contours of the accumulated unrecoverable deviatoric strain (EDP [46]) for two different rock masses (set#1 and set#2) and four different standstill times ($t_{\text{stand}} = 0$, 0.1, 1 year and 2 years) for: (a) set#1, and (b) set#2.
**Figure 24.** Contours of liquid pressure for three different excavation rates (5 m/day, 10 m/day, 20 m/day) and two different rock masses: (a) set#1, and (b) set#2. A standstill time $t_{\text{stand}} = 0$ is assumed.

**Figure 25.** Profiles of liquid pressure for two different rock masses: (a) set#1 and (b) set#2. Three different excavation rates (5 m/day, 10 m/day, 20 m/day) are considered.

**Figure 21** shows the liquid pressure profile along the normalized distance to the tunnel face ($x_d/R_1$). Note for reference that the liquid pressure is taken at a radius of 4.5 m. For both sets#1 and set#2, it can be observed that the liquid pressure profile is initially higher all along the tunnel, and then it experiences a certain dissipation as the standstill time increases. In addition, **Figure 21** shows that there is some overpressure just ahead of the tunnel face, as it has been commented on above. This overpressure seems to dissipate relatively fast. Certainly, the liquid pressure profiles are similar after a standstill time of 1 year, which means that most of this dissipation process has been completed. Moreover, note that the dissipation rate decreases with the standstill time, according to the results. Finally, it can be observed that this dissipation is much slower.
as we move away from the tunnel face in the tunnel advance direction (e.g. $x_d/R_1 = -15$). These effects may be caused by the coupled solid-fluid interaction.

Finally, Figure 22 shows the longitudinal deformation profile (LDP) evolution. For set#1, the induced displacements increase linearly even after a long standstill time, which indicates that the displacements may be fundamentally induced by secondary creep (Maxwell part of the Burgers model). Furthermore, Figure 23 presents the contours of the evolution of accumulated unrecoverable deviatoric strain (EDP [46]). It can be observed in Figure 23(a) that, for set#1, the accumulated unrecoverable strain and the plastic zone obviously increase with time, even after a long standstill time, which may be caused by the contribution of the unrecoverable strain from the Maxwell model. This phenomenon may be consistent with some reported engineering cases [9,54,83,84].

In contrast, for set#2, it can be observed that the incremental rates of induced displacements decrease along the standstill time (Figure 22), which indicates that the induced displacements by secondary creep may not be significant. This phenomenon may be caused by (1) the decreasing liquid dissipation rates with increased standstill time, (2) the decreasing contribution of the viscous response of the viscoplastic model with standstill time, (3) and the response of primary creep induced displacements (Kelvin part of the Burgers model), which are more relevant at short-term. Furthermore, Figure 23(b) shows that the incremental accumulated unrecoverable strain rates are more relevant at a shorter term and less significant afterwards. This phenomenon of decreasing strain rates with standstill time may represent the behaviour of rock masses exhibiting high strength or subjected to low stresses [24,44].

### 5.3. Influence of the excavation rate

During the excavation process, solid deformations and fluid flow are coupled, and thus, tunnelling responses become more complex and difficult to anticipate. The excavation rate may heavily affect the creep response of tunnels [9] and the pressure dissipation of the fluid phase. In this section, three different excavation rates (5 m/day, 10 m/day and 20 m/day) are adopted to analyze the influence of the excavation rate on the tunnelling response. The liquid pressure is taken at a radius of 4.5 m in these examples. All presented results in this section assume a standstill time $t_{stand} = 0$, i.e. just after the excavation of the tunnel face.

During excavation, the high undrained loads experienced by the rock mass near the tunnel face, causes the liquid pressure to increase from its initial value (2.25 MPa). This phenomenon can be called overpressure. Figure 24 shows contours of liquid pressure for two different rock masses (set#1 and set#2) and three different excavation rates (5 m/day, 10 m/day and 20 m/day). It can be observed that different excavation rates result in different overpressure values. Table 5 presents the maximum overpressure reached during the different simulations, both in absolute value and in percentage increment from the initial liquid pressure value. It can be observed that overpressure increases with the excavation rate, being slightly higher for set#2.

### Table 5

<table>
<thead>
<tr>
<th>Excavation rate [m/day]</th>
<th>Maximum liquid pressure [MPa]</th>
<th>Increment of $p_l$ [MPa]</th>
<th>Percentage increment of $p_l$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set#1</td>
<td>Set#2</td>
<td>Set#1</td>
</tr>
<tr>
<td>5 m/day</td>
<td>2.36</td>
<td>2.41</td>
<td>0.11</td>
</tr>
<tr>
<td>10 m/day</td>
<td>2.45</td>
<td>2.5</td>
<td>0.2</td>
</tr>
<tr>
<td>20 m/day</td>
<td>2.59</td>
<td>2.58</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Furthermore, Figure 25 plots the liquid pressure profiles along the normalized distance to the tunnel face ($x_d/R_1$). Behind the tunnel face ($x_d > 0$), faster excavation rates result in larger liquid pressures. Moreover, ahead of the tunnel face ($x_d < 0$), the faster the excavation rate, the more significant build-up values of liquid pressures occur in the vicinity of the tunnel face, due to the shorter time available for dissipation. In addition, note that the maximum values of liquid pressure are attained at points closer to the tunnel face, as the excavation rate increases. However, there is no significant effect of the excavation rate on the liquid pressure as we move away from the tunnel face in the tunnel advance direction.

Similarly, Figure 26 presents the radial displacements of the tunnel wall ($r = R_1$) along the normalized distance to the tunnel face ($x_d/R_1$), i.e. the longitudinal deformation profiles, for three different excavation rates and two different rock masses. It can be observed that lower excavation rates may lead to larger deformations, since there is more time for creep deformations to develop [9] and for pore pressures to dissipate [16,47]. Moreover, the radial displacements ahead of the tunnel face do not seem to be affected by the excavation rate. However, behind the tunnel face, the radial displacement differences corresponding to different excavation rates grow as we move away from the tunnel face.

In summary, the different excavation rates can result in different fluid flow for two reasons: (1) the faster the excavation rate, the lower dissipation of liquid pressure ahead of the tunnel face and thus, the higher liquid pressure build-up; (2) different excavation rates result in different periods of time for the development of creep deformations, in turn affecting the fluid flow.

6. Conclusions

This article provides an alternative approach for modelling underground excavation in saturated ground, considering the coupled solid-fluid interaction and time-dependent plastic behaviour of geomaterials. To do that, a Burgers-viscoplastic strain-softening (BVSS) model has been proposed and implemented into the finite element method software CODE_BRIGHT to be able to simulate the time-dependent behaviour and the plastic behaviour (stress-induced or/and creep-induced) of geomaterials. In addition, a coupled solid-fluid model is used to simulate the interaction between solid deformations and fluid flows.

Firstly, the Burgers viscoelastic model and the viscoplastic model are combined in series, to simulate the mechanical response of geomaterials. The Mohr-Coulomb strain-softening and Hoek-Brown strain-softening models, and a non-associated plastic flow rule have been used. The creep deformations are coupled with the strain-softening model to simulate the damage evolution, and hence, the creep-induced failure behaviour can be simulated. The generalized Darcy’s law has been adopted in the simulation of fluid flow.

The numerical results are verified by comparing the CODE_BRIGHT results with other analytical or numerical results. Finally, parametric analyses are performed to analyze the effects of the selection of different constitutive model, the standstill time and the excavation rate on tunnelling response. Some conclusions can be drawn from this study:

1. The proposed Burgers-viscoplastic strain-softening (BVSS) model provides the possibility of modelling rock masses in which both plastic and time-dependent responses take place.
2. The results obtained considering the coupled solid-fluid interaction are significantly different from the purely mechanical results. Liquid pressure builds up just ahead of the tunnel face in the short term after the excavation. Afterwards, this overpressure dissipates. The dissipation rate decreases with the increasing standstill time. Moreover, overpressure dissipation is much slower as we move away from the tunnel face in the tunnel advance direction.
3. Excavation rate is relevant to tunnelling response. Behind the tunnel face, a higher excavation rate results in a higher liquid pressure. In addition, ahead of the tunnel face, the higher the excavation rate, the more significant build-up values of liquid pressure occur in the vicinity of the tunnel face, due to the shorter time available for dissipation. In addition, as the excavation rate increases, the maximum values of liquid pressure are attained at points closer to the tunnel face.

Note that, even if the proposed method in this article can simulate different cases of coupled solid-fluid response for tunnelling in saturated rock masses with time-dependent plastic behaviour, it still has some limitations, since e.g. the effect of anisotropy and heterogeneity have not been considered. Moreover, the delayed installation of the lining and the non-linear damage evolution during the softening process have not been considered.

Codes availability

The proposed Burgers-viscoplastic model has been implemented in the finite element method computer code CODE_BRIGHT (version: Code_bright_2020_15), which can be found in the following link: https://deca.upc.edu/en/projects/code_bright.

Conflict of interest

None
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