Operation of transit corridors served by two routes. Physical design, synchronization and control strategies

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Abstract

Many transit network layouts imply the operation of multiple routes along a common transit segment in the busiest area of the city. At some points, these routes branch out to provide spatial coverage to the city periphery. These schemes allow the more efficient deployment of resources at the expenses of introducing more complexity into the system operation.
The paper aims to estimate the effect of the branched layout of the corridor, the demand distribution and the traffic lights on the total cost of the system as well as its service regularity. The transit corridor is operated by buses, although it can be generalized for other transit modes. A kinematic model to estimate the natural motion of buses of different routes along the corridor is presented. The model considers the stochastic effect of the passenger arrivals at stops and vehicle acceleration-deceleration rates. An optimization procedure is presented to determine the optimal headway and the relative synchronization of routes that would minimize the total cost incurred by transit agencies and users or the headways variations in the common route segment. The performance of bus control strategies based on a combination of holding points and green extensions at traffic signals is also addressed in the H10 cross-town corridor of Barcelona's new bus network.

Keywords: transit route design, bus bunching, control strategies, branched lines

1 Introduction

The efficiency of urban transit networks is highly influenced by the spatial configuration of the different transit routes over the city. Other tactical and operating decisions in the transit planning problem are based on the network design, previously defined in the logical sequence of planning steps (Desaulniers and Hickman, 2007; Ceder, 2007). The transit route layout, together with the frequency setting, present a trade-off between the performance experienced by users (accessibility, temporal coverage and speed) and the operating costs (mileage, resources) incurred by transit agencies (Ibarra –Rojas et al., 2015). Therefore, urban decision-makers are required to find the equilibrium point between route performance and cost, considering economic, social and political constraints.

The most suitable fixed transit route structure for users and transit agencies has been widely analysed in the literature under the forms of total cost minimization problems. Continuous approximation and parsimonious models were developed to estimate both user and agency costs, in idealized route pattern designs. The best element of each network concept could be chosen by continuous optimization. Perfect grid networks (Holroyd, 1967) and radial or hub and spoke route schemes (Newell, 1979; Byrne, 1975) were assessed. The radial designs benefit transit agencies with low infrastructure investment, while grid structures provide competitive door-to-door travel time for users (Nourbakhsh and Ouyang, 2012). Recently, hybrid configurations, combining a mesh of routes in the city centre and branching lines in the city periphery (Daganzo, 2010 and Estrada et al., 2011), were developed to merge the potentialities of both networks in the same design concept. In all these idealized network concepts, the demand is assumed to be uniformly distributed on the area of operation, and any trip can be made transferring at the corridors' intersections. These designs, complementary to the street network, shape the physical distribution of trips over the city. Recently, Badia et al. (2016) compared the performance of the previous models based on transfer, with the corresponding level of service provided by ubiquitous designs. The latter schemes connect the most demanded trip origin and destination zones by a direct route, without transfers. They concluded that the transfer-based hybrid design generally outperforms the total cost of the transit service, mostly when demand is consolidated in the city centre. Other discrete-based heuristic models (Ceder and Wilson, 1986; Baaj and Mahmassani, 1990 and 1995; Zhao, 2006) builds the transit network with the aim of serving the highest number of trips by direct services without transfers, given the O-D trip matrix in the city. In these contributions, the demand distribution over the region configures the deployment of the transit infrastructure, creating bus network layouts that result in neither readable nor comprehensive bus design. These schemes differ from the designs proposed by continuous approximation models, where the route layout tend to be simple and easy to understand.

In both discrete-based or continuous-based models, a single transit corridor may be served by multiple lines. Generally, these lines run along with a shared sketch of the corridor, and at particular points, they branch out to provide service to the outskirts. When doing this, accessibility in the whole area of the city is guaranteed. However, the waiting time in the branched parts of the corridor is worsened in the economic balance between user and agency cost. The design of branched corridors has been widely used in rail services, due to the high investment cost (Metro of Amsterdam, S-Bahn in Berlin, Tram, FGC Regional Rail and Metro in Barcelona, L. Underground, L. Overground and DLR in London, Metro and Tram in Milan and Rome, and Metro and RER in Paris, among others). Branching routes have begun to be deployed in large, medium and even small bus systems around the world (Barcelona, Dublin, Lorient, Metz, Nantes, etc).

The branching scheme reduces the length of the whole corridor and the temporal coverage in the branches, in comparison to the situation in which we have two parallel lines with their own infrastructure. Nevertheless, the flow of vehicles belonging to multiple lines along the same corridor increases the operation's complexity and contributes to the transit vehicle bunching phenomena, firstly described by Newell and Potts (1964). The demand at stops, and consequently, the stop boarding time in the central corridor depends on the real-time headway between consecutive buses. In transit corridors with high passenger flows, primarily operated by buses, the natural motion of vehicles causes irregular vehicle arrivals at stops, so that any potential exogenous disturbance is propagated to the whole route. Therefore, these multiple routes should be jointly planned. Control strategies are required to maintain constant headways among buses and alleviate the unstable motion of buses.

A lot of research regarding control strategies in bus systems have been developed. Some researchers proposed the introduction of slack times into bus schedules at holding points to control the system regularity (Barnett, 1974; Turnquist and Blume, 1980; Rossetti and Turitto, 1998). Holding times allow recovering the target bus headway when vehicles are delayed, at the expenses of increasing the target roundtrip travel time. However, schedule-based holding points are significantly inefficient since each bus is controlled without any input of the rest of the vehicles. A dynamic slack introduction was proposed in several works, to tackle this problem, predicting the evolution of the system in a rolling horizon (Eberlein et al., 2001, Delgado et al., 2009; Liu et al., 2013). This control strategy was also combined along the route with the possibility of skipping some stops (Delgado et al., 2012, Sáez et al., 2012; Cortés et al., 2010) and vehicle overtaking (Fonzone et al., 2015) to recover the desired time-headway. Muñoz et al. (2013), pointed out the capacity problems arisen by holding point strategies due to the scalable reduction of bus speeds.

Other contributions proposed adaptive strategies to the actual performance that do not consider system predictions. Daganzo (2009), presented a new control principle, based on dynamic holding points. Later, Daganzo and Pilachowski (2011) proposed a control strategy based on variable cruising speeds, to improve the regularity of transit systems. This procedure, similar to the holding point strategy, reduces the cruising velocity of buses based on the current headway with the vehicle ahead.

However, the first analysis of the bus bunching effect and joint operation of multiple routes in the same corridor was addressed in Hernández et al. (2015). A control system based on dynamic holding points at stops previously proposed by Delgado et al. (2012) was implemented. It compared the transit system performance under two scenarios: i) all routes are jointly managed by a central operator that considers the multiple routes as a whole system and, ii) each route is operated independently to maximize its profit. The objective function to be minimized was the total travel time of passengers. The join control strategy demonstrated to reduce the waiting time of passengers by 55% in comparison to the independent operation of lines. Later, Schmöcker et al. (2016) extended the former Newell and Potts (1964) propagation model to the motion of buses along corridors operated by two lines when overtaking is allowed. Under this situation, a new passenger queue distribution for buses was presented. The overtaking operation always produces a positive effect on the system performance in terms of total waiting time of passengers and the time-headway variance metrics. Finally, Argote-Cabanero et al. (2015) extended the adaptive control strategy proposed in Daganzo (2009) and Daganzo and Pilachowski (2011) to complex transit systems, conformed by many lines operating along the same corridor. The slack time at each stop for a given bus was based on the current deviation of the vehicle under study and previous vehicles (belonging to different lines) from the target headway. These deviations were multiplied by a dimensionless parameter responsible for propagating the bunching effect, which accounted for the expected increase in the dwell time due to boarding when the one-time unit increases the headway.

Unfortunately, all these contributions for multiple services assumed that the unstable motion of buses is caused by exogenous disruption. Their modelling approaches did not consider those key route design aspects that would generate irregular arrivals of buses in a corridor with multiple bus services, such as traffic lights, demand distribution and lengths of the branched routes. Besides, a small fraction of these works addressed the impacts of control strategies on the operating cost incurred by transit agencies, apart from the combined effect on the user side.

This paper addresses the bus corridor design problem served by two-branched lines. An optimization model is presented, aimed at minimizing the total cost incurred by both transit agency and users. The effects of the length uniformity of the branched segments, the fraction of demand captured by the branched sections concerning the central sketch, and traffic lights settings on the efficiency and regularity of the bus corridor are analyzed. The analysis considers a base case scenario when the bus motion is not controlled and overtaking is not allowed. Results obtained under other plans for control strategies based on holding points, speed modification and traffic light priority measures are also provided. The objective of this work, as Schmöcker et al. (2016) suggested as further research, is to provide network design recommendations and the implementation of the best control strategy to alleviate the lack of regularity in a corridor with multiple lines at the minimum operating cost. To the best of our knowledge, this is the first time that the effect of traffic light settings and the operational cost variation caused by the deployment of control strategies are considered in corridors operated by bus branched lines. In Section 2, the formulation for the bus corridor design problem with two bus services is presented, with the associated constraints, significant assumptions and boundary conditions. A bus motion model is then developed, with analytical formulations to estimate the time spent in segments between stops, dwell time at stops and delays intersections due to traffic lights. The model considers a variable acceleration in the kinematic equations that follows a truncated normal distribution. This fact outperforms previous simplified models (Estrada et al. 2016) that supposed instantaneous speed profile changes at stops or intersections. The different metric analysis measuring the effects in stakeholders, as well as the optimization models, are presented in Section 3. Section 4 gathers the optimization problem and the formulas needed to consider the control strategies in the bus modelling approach. Later, the numerical results of the optimization process in a set of problem instances are introduced in Section 5. Finally, in Section 6, several physical network design, control strategies recommendations and general conclusions are drawn.

2 Notation and modelling framework

We consider a ground transit corridor in a given city that runs along the East-West direction. This corridor presents two routes, route *A* and *B*, that operate a central segment between points *PE* and *PW* (Figure 1). At the two edges *PE* and *PW*, the corridor branches out into two segments, to provide a wider accessibility in the peripheral regions. Therefore, four independent branches are identified: the West and East branches operated by route A and referred by AW and AE respectively, and the corresponding West and East branches only served by route *B*, denoted by BW and BE. The central segment is denoted by both *AC* and *BC*, depending on the route under analysis. Let $R = \{AW, BW, AC, BC, AE, BE\}$ be the set of the 6 route segments, while $R_A = \{AW, AC, AW\}$, $R_B = \{BW, BC, BE\}$ capture the set of route segments operated by route *A* and *B* respectively ($R_A \cup R_B = R$), and $R_C = \{AC, BC\}$ the route central segments. The term $r(i)$ returns the bus route *A* or *B* that is serving the segment $i \in R$. Each route segment $i \in R$ is operated in two direction of service: direction *East* or *West*. From this point on, we will use the subscript *z* to represent the two available directions of service (*z*=*E* towards *East* or *z*=*W* towards *West*). Therefore, a single roundtrip of route *A* consists of the sequence of route segments *AW-AC-AE* (run in the East and West directions), while in route *B* is defined by segments *BW-BC-BE* (two way of service). Moreover, in order to represent the passenger transfer movements, we create the subsets $R_W = \{AW, BW\}$, $R_E = \{AE, BE\}$ that contain the branched route segments in the West and East respectively, and $R_{T1} = \{AW, BE\}$, $R_{T2} = \{BW, AE\}$ that contain the branched route segments operated by different routes and located in opposite edges of the central segment.

Stops. Each route segment $i \in R$ presents $n_{i,z}$ stops in direction *z*. In each route segment, the distance between stop *k* and *k*+1 is labelled by $s_{i,z}(k)$ $\left(s_{i,z}(k) \geq 0\right)$ in direction *z* (*k*=1, …, $n_{i,z} - 1$). Note that stop *k*=1 is equivalent to the starting point of the route segment and stop $k = n_{i,z}$ to the ending point. Since this variable $s_{i,z}(k)$ is dependent to every stop and route segment, this model can reproduce variable stop densities in the central and branching segments of the route. Stops are considered to present one on-line loading area, consequently only one bus can simultaneously perform boarding/alighting operation. Moreover, all buses must stop over all stops scattered along the branched and central segments of the route (skip -stop pattern is not allowed). Therefore, the length of each route segment *i*$\in R$ in direction *z* is computed by $l_{i,z} = \sum_{k=1}^{n_{i,z}-1} s_{i,z}(k)$. The characteristics of stops in the route segments *AC* and *BC* referring the central part of the corridor are the same. All vehicles of route *A* and *B* will follow the same stop pattern in *AC* and *BC route segments*.

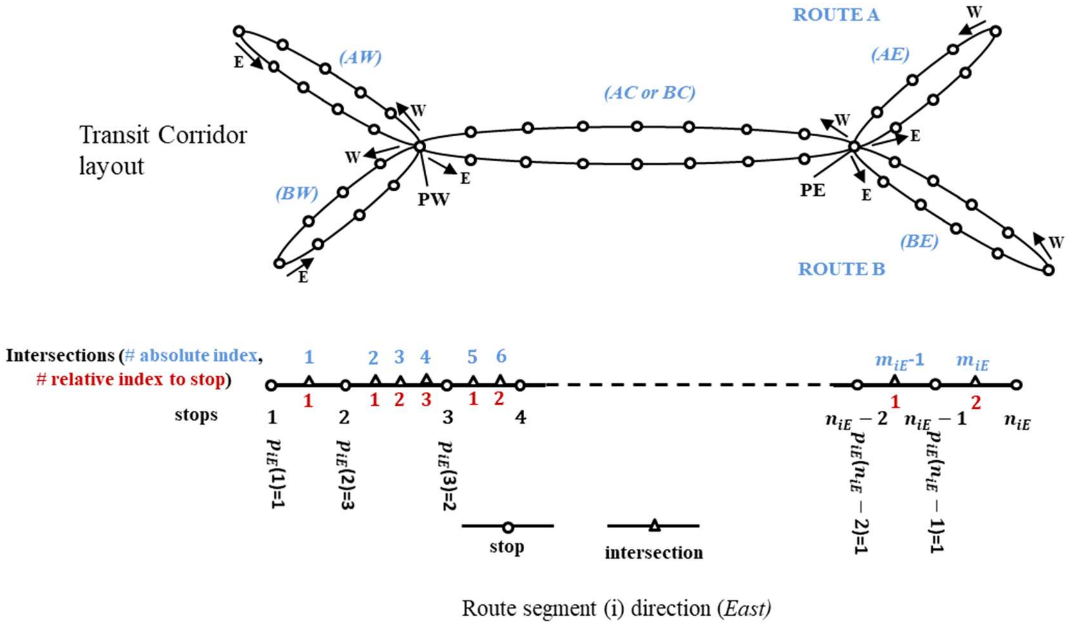

Figure 1. Scheme of the transit system composed by a branched corridor with two routes (route A and B).

Intersections. We assume that there are $m_{i,z}$ signalized intersections along the route segment $i \in R$, direction z, and $I_{i,z}$ represents the set of these intersections. Moreover, the variable $p_{i,z}(k)$ (k=1, ..,$n_{i,z}-1$) captures the number of intersections located in the sketch between two consecutive stops k and k+1 in route segment i, direction z, so that $\sum_{k=1}^{n_{i,z}-1} p_{i,z}(k) = m_{i,z}$. The variable $x_{i,z}(m)$ denotes the distance between the location of the *m-th* intersection to the initial stop along the route segment $i \in R$ in direction z $\left(m = 1, \dots, m_{i,z}\right)$. We assume that all intersections of the problem present the same traffic signal cycle time C_p. However, each intersection m $\left(m = 1, \dots, m_{i,z}\right)$ in direction z of the route segment i is defined by its green phase time $g_{i,z}(m)$ and traffic light offset $\Delta_{i,z}(m)$. The signals attributes in the intersections along route segments *AC* and *BC* are identical.

Vehicles. The number of transit vehicles required to provide the service in route *A* and *B* are, respectively, M_A and M_B. We assume that these vehicles are homogeneous and their vehicle capacity is *C*. Each vehicle will run along the length of the segments to be covered in each route, stopping at each stop and obeying the traffic signal regulation.

Demand. The total number of boarding passengers in one hour of service along the whole route segment i (direction z), whose destination is located at any stop of route segment j ($i, j \in R$) is defined by $B_{i,j,z}$. Moreover, $\beta_{i,j,z}(k)$=$B_{i,j,z}$ $F^{\beta}_{i,j,z}(k)$ denotes the cumulative number of boarding passengers per hour at stop k ($1 < k \leq n_{i,z}$) in direction z of route segment i, with regard to the initial stop, that will alight in route segment j. Indeed, the variable $0 \leq F^{\beta}_{i,j,z}(k) \leq 1$ is the cumulative distribution function at stop $k$of the boarding passengers traveling from route segment i to j, direction z. Similarly, the term $A_{i,j,z}$ captures the total number of alighting passengers per hour along the whole route segment j (direction z), whose origin was located in a stop of route segment i ($i, j \in R$). In this case, the number of cumulative alighting passengers from stop 1 to stop k' ($1 \leq k' \leq n_{i,z}$) in direction z of route segment j is estimated by $\alpha_{i,j,z}(k')$=$A_{i,j,z}$ $F^{\alpha}_{i,j,z}(k')$, where $F^{\alpha}_{i,j,z}(k')$ is the cumulative distribution function of alighting passengers from stop k=1 to stop k' ($0 \leq F^{\alpha}_{i,j,z}(k') \leq 1$).

Along Section 2, we present an analytical model to reproduce the bus motion in a transit system with two branched lines. This model will be able to estimate arrival and departure times of buses along the route segments and, eventually, several bus route metrics required to assess user performance and operating cost. To represent feasible transit demand states, the definition of the previous variables must satisfy some properties. Equation (1) states that the boarding passenger rate in between segments *i* and *j* on both directions of service (*E* and *W*) must equal the alighting rate between the same route segments. Equation (2) states the same conservation condition when the boarding and alighting operations are made in the same route segment *i* in direction *z*. Finally, Equation (3) obliges that the cumulative number of boarding counts of passengers whose origin and destination are in the same route segment must be higher or equal to the corresponding number of alighting counts, at a given stop k.

$$B_{i,j,E} + B_{i,j,W} = A_{i,j,E} + A_{i,j,W} \quad i \neq j,\ i, j \in R \tag{1}$$

$$B_{i,i,z} = A_{i,i,z} \quad i \in R,\ \ z \in \{E, W\} \tag{2}$$

$$\beta_{i,i,z}(k) \geq \alpha_{i,i,z}(k) \quad i \in R, z \in \{E, W\},\ k=1,\ldots, n_{i,z} \tag{3}$$

2.1 Operational assumptions

The operation of transit services in the two branching lines is supposed to fulfil the following operational hypotheses:

i Routes *A* and *B* may be operated at different constant headways H_A (H_A >0) and H_B (H_B >0) respectively, according to the potential demand in the branched route segments.

ii For sake of simplicity, the entrance times of the first vehicle trip at the first stop, direction *z=E* in route segment AW and BW are referred by $t_{0,A}$ and t_{0B} respectively. Indeed, the entrance times vary in the following domains $0 < t_{0,A} < H_A$, and $0 < t_{0,B} < H_B$. Moreover, these variables also determine the temporal separation of vehicles in the trunk route segment *AC,BC* where two routes provide a joint service.

iii The transit operator controls the cost and service performance by means of the headways H_A, H_B and the entrance times $t_{0,A}$, $t_{0,B}$ of the first vehicle at the first stop in each route *A* and *B*. These four terms are considered as the decision variables of the model.

iv For the sake of generality, we assume that buses operate both transit routes *A* and *B*, where the dwell time at stops is highly affected by the number of boarding and alighting passengers. However, the model can be also used in other transit technologies if the dwell times are properly formulated with regard to the particular boarding/alighting process in each system. We suppose that each passenger needs τ^{β} units of time to get on the bus, and τ^{α} units of time to alight, depending on the validation technology and other bus features. In Sun et al (2014) there is a wide review of available modelling approaches to calculate dwell times. We assume that boarding and alighting operations have different vehicle doors assigned, since this is the situation that better represents our implementation site. Therefore, since these operations can be done independently, the dwell time can be calculated as the maximal time spent at each operation (model II in Sun et al, 2014).

v The arrival of passengers at any stop *k* in route segment *i*, direction *x*, is assumed to follow a Poisson distribution. Therefore, the number of passengers that get in each vehicle is a random variable. Therefore, the time interval between two passengers arriving at stop k of route segment $i \in R$ direction z presents an exponential distribution of term $\lambda_{i,z}(k) = \sum_{j \in R} \left(\beta_{i,j,z}(k) - \beta_{i,j,z}(k-1)\right)$, where $\beta_{i,j,z}(0) = 0$.

vi Passengers choose the route path that minimizes the number of transfers to reach their destination. In the specific case that both the trip origin and destination are included in the trunk segment (*AC or BC*), this passenger will get on the next bus of route *A* or *B* arriving at the stop.

vii In trips connecting branched segments *AE-BW, AW-BE* and vice versa, the passengers transfer at any potential stop along the central route segment.

viii Each vehicle modifies the instantaneous speed v_f due to presence of stops and intersections at an acceleration rate a_f (positive or negative). This acceleration is supposed to follow a Normal distribution (μ_a, σ_a), of mean μ_a and standard deviation σ_a, to represent the different driver sensitivities to the traffic conditions. Nevertheless, the Normal distribution has been truncated to only accept acceleration and deceleration rates in the domains (0, a_{max}) and (-a_{min},0), respectively. The maximal allowable instantaneous speed of buses is v_{max} ($v_f \leq v_{max}$) along the whole route segments.

ix The capacity of all vehicles operating on both routes *A* and *B*, referred by *C* is homogeneous.

x In the general version of the analysis, overtaking between buses operating the same bus route is not allowed. However, overtaking is only permitted between vehicles of different routes *A* and *B* in the central route segment.

xi Transit vehicles are supposed to be benefitted by right of way measures (segregation of lanes) and the effect of the rest of the traffic is negligible.

2.2 Bus motion

Let *M* be a large number, representing the maximal number of vehicle departures from the initial stop of the segment *AE* of route *A* (or segment *BE* in route *B*) that we would like to analyze. This term *M* may be higher than the fleet size in route *A* or *B*, assuming that a single vehicle would complete a roundtrip and depart from the first stop several times. Let $a_{i,z}(k,f)$ be the arrival time at stop *k* of bus trip *f* (*f*=1,..., *M*), and $d_{i,z}(k,f)$ be the dwell time at stop *k* of bus trip *f* on the bus route segment $i \in R$ in the direction *z* (*z*=*East* or *West*). The entrance time $a_{i,z}(1,f)$ at the first stop of route segments *AW* and *BW* in direction *East* of the *f*-th bus trip are calculated by $a_{AW,E}(1,f) = (f-1)H_A + t_{0A}$ and $a_{BW,E}(1,f) = (f-1)H_B + t_{0B}$, once the entrance times of first vehicles in route *A* and *B* have been defined, based on the assumptions i)-iii). In other segments and directions, these entrance times should be defined in accordance to the exit times of the vehicle trip at the last stop of the previous segment of the route, ensuring travel time compatibility between segments. Hence, variables $a_{i,z}(1,f)$, except for $a_{AW,E}(1,f)$, $a_{BW,E}(1,f)$are defined imposing compatibility of travel times between route segment boundaries. In Appendix 1, the estimation of these variables is provided.

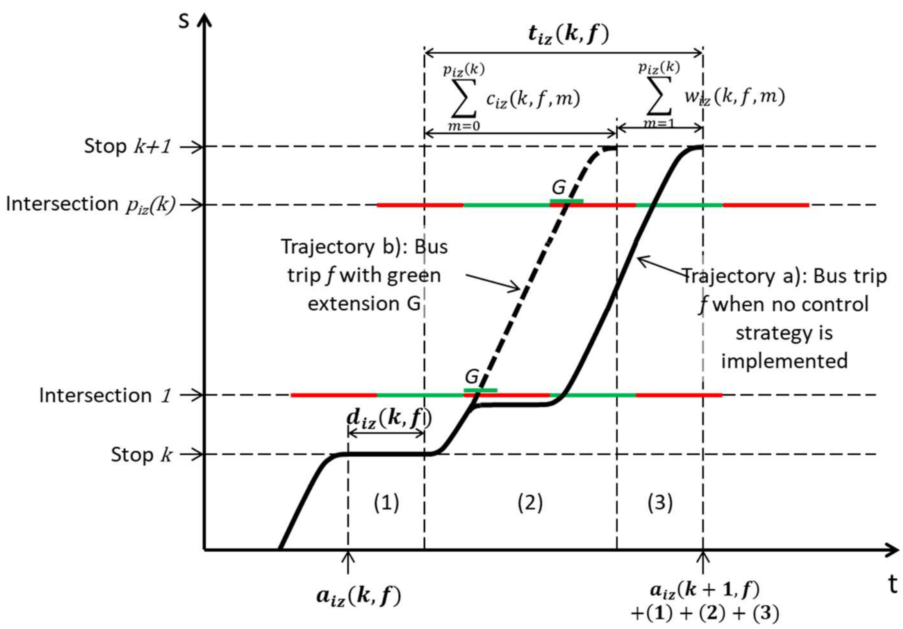

Figure 2. Motion of vehicle trip *f* between consecutive stops *k*,*k*+1 in route segment $i \in R; z \in \{E, W\}$

The calculation of travel time of bus trip *f* between two consecutive stops *k*, *k*+1 ($k < n_{iz}$) along route segment $i \in R$ in direction $z \in \{E, W\}$ can be made by means of Equation (4). The arrival time of bus trip *f* at the next stop *k+1*, $a_{i,z}(k+1,f)$, in the route segment $i \in R$ direction *z* (*z*=*E* or *W*) is the sum of the arrival time $a_{i,z}(k,f)$ at the stop *k*, the dwell time $d_{i,z}(k,f)$ at the stop *k*, and the time needed to overcome the distance $s_{i,z}(k)$ between stops k, k+1, $t_{i,z}(k,f)$. The sum of the two first terms determines the departure time from stop k, while the third term is affected by the delays caused by traffic lights. We calculate consecutively the travel time, starting from the first stop (where we have already defined $a_{i,z}(1,f)$) and ending at the last stop $k = n_{i,z}$ of the route segment. Hence, the trajectory of bus trip f between stops k,k+1 is depicted in a space-time diagram in Figure 2 (trajectory a), with reference to the variables used in the modeling approach.

$$a_{i,z}(k+1,f) = a_{i,z}(k,f) + d_{i,z}(k,f) + t_{i,z}(k,f) \quad k = 1,\ldots,n_{i,z}-1; \forall f, i \in R; z \in \{E,W\} \qquad (4)$$

The evaluation of term $a_{i,z}(k,f)$ is known from the analysis in the previous iteration at stop k or by setting the initial condition when k=1. The evaluation of the dwell time is addressed separately in Section 2.2.1. Eventually, the estimation of the travel time $t_{i,z}(k,f)$ to overcome the distance $s_{iz}(k)$ is made by Equation (5). We assume that there are $p_{i,z}(k)$ intersections along the sketch between stops (k, k+1) in route segment i, direction z. The term $c_{i,z}(k,f,0)$ captures the travel time to overcome the distance between stop k and the first intersection m=1 (if $p_{i,z}(k)$>0); $c_{i,z}(k,f,m)$ the travel time to run the distance between intersections m, m+1 (m=1,.., $p_{i,z}(k)-1$); $c_{i,z}(k,f,p_{i,z}(k))$ the travel time to cover the distance between the last intersection m= $p_{i,z}(k)$ and stop k+1, and finally, $w_{i,z}(k,f,m)$ the potential traffic delay caused by the complete detention of vehicle at intersection m ($m = 1,\ldots,p_{i,z}(k)$). The estimation of the first three components of Equation (5) can be developed by means of the vehicle motion laws under uniform acceleration.

$$t_{i,z}(k,f) = c_{i,z}(k,f,0) + \sum_{m=1}^{p_{i,z}(k)-1} c_{i,z}(k,f,m) + c_{i,z}(k,f,p_{i,z}(k)) + \sum_{m=1}^{p_{i,z}(k)} w_{i,z}(k,f,m) \qquad (5)$$

We introduce a random noise in the three first terms, when vehicle is adapting the speed due to the presence of traffic lights and stops. In fact, the acceleration a_f of vehicle trip f can be assumed to be a random variable with a known probabilistic distribution. This would lead to different acceleration/deceleration profiles and different cruising speed values among vehicles arriving at the same traffic light or stop. Based on modeling assumption viii), the acceleration a_f is assumed to follow a Normal distribution, whose mean and standard deviation are, respectively, (μ_a, σ_a). Nevertheless, we truncated the previous distribution to only accept acceleration modules in the domain (0, a_{max}).

Equation (5) allows calculating the arrival time of the bus at intersections in an increasing order in route segment $i \in R$ along direction $z \in \{E,W\}$. This arrival time requires to check whether the traffic light at intersections will be in green or red phase. In the latter condition, we may expect a delay of a bus. Therefore, the term $e_{iz}(k,f,m)$ denotes the arrival time at intersection m (m=1,…, $p_{i,z}(k)$) in route i in direction z (Equation 6), where the departure time from the last stop k and from the (m-1) previous intersections are known.

$$e_{i,z}(k,f,m) = a_{i,z}(k,f) + d_{i,z}(k,f) + \sum_{h=1}^{m-1} c_{i,z}(k,f,h) + \sum_{h=1}^{m-1} w_{i,z}(k,f,h) \qquad (6)$$

Let $q_{i,z}(k,f,m) = \left[\frac{e_{i,z}(k,f,m) - \Delta_{i,z}(m+\sum_{h=1}^{k-1} p_{i,z}(h))}{C_p}\right]^{-}$ be the number of traffic light cycles that have been completed from t=0 when the bus trip f under study arrives at the m-th intersection among stops k,$k+1$. The term $\Delta_{i,z}(m_T)$ is the global traffic light offset of this intersection m (m=1,..., $p_{i,z}(k)$) with regard to the segment origin, $m_T = m + \sum_{h=1}^{k-1} p_{i,z}(h)$. Therefore, the delay at intersection m (m=1,..., $p_{i,z}(k)$) is estimated by Equation (7), considering the amount of time that the vehicle is completely stopped at an intersection. Finally, the terms $g_{iz}(m_T)$ and $\Delta g_{i,z}(k,f,m)$ are respectively the green time at intersection m (m=1,…, $p_{i,z}(k)$) and the green extension parameter to be defined when the control strategy S2 is activated (see Section 4). For the base case implementation of the bus motion modeling, this parameter is set $\Delta g_{i,z}(k,f,m)=0$ seconds.

$$w_{i,z}(k,f,m) = \begin{cases} 0 & \text{if } \; e_{i,z}(k,f,m) \le C_p q_{i,z}(k,f,m) + \Delta_{i,z}(m_T) + g_{i,z}(m_T) + \Delta g_{i,z}(k,f,m) \\ \left(C_p\left(q_{i,z}(k,f,m)+1\right) + \Delta_{i,z}(m_T) - e_{i,z}(k,f,m)\right) & \text{otherwise} \end{cases} \quad (7)$$

2.2.1 Dwell time at stops

For any stop k ($1\leq k\leq n_{i,z}$) in the route segment $i\in R$, the dwell time is calculated in a general form by Equation (8), assuming that buses have independent doors for boarding and alighting (assumption iv). The first and second parentheses capture, respectively, the total number of boarding and alighting passengers in/from the bus trip f at stop k in route segment $i\in R$, direction z; while τ^{β} , τ^{α} represent the unit boarding and alighting time per passenger, respectively.

$$d_{i,z}(k,f) = \max\left\{ \begin{array}{l} \tau^{\beta}\left(\sum_{j\in R} N^{\beta}_{i,j,z}(k,f) + \sum_{\substack{j\in R_W\cup R_E \\ j\neq i}} \eta^{\beta}_{j,i,z}(k,f) + \sum_{\substack{h\in R_{T1} \\ h\in R_{T2} \\ h\neq j;\, r(h)\neq r(i)}} \zeta_{h,i,j,z}(k,f)\right); \\ \tau^{\alpha}\left(\sum_{j\in R} N^{\alpha}_{j,i,z}(k,f) + \sum_{\substack{j\in R_W\cup R_E \\ j\neq i}} \eta^{\alpha}_{i,j,z}(k,f) + \sum_{\substack{h\in R_{T1} \\ h\in R_{T2} \\ h\neq j;\, r(h)=r(i)}} \zeta_{h,i,j,z}(k,f)\right) \end{array} \right\} \quad (8)$$

The term $N^{\beta}_{i,j,z}(k,f)$ represents the number of boarding passengers on the bus trip f at stop k of the route segment $i\in R$, direction z that will get off the bus on route segment $j\in R$. Similarly, $N^{\alpha}_{j,i,z}(k,f)$ captures the alighting passengers at the bus stop k of route segment $i\in R$, who previously got on the bus in route segment $j\in R$. Both terms $N^{\beta}_{i,j,z}(k,f)$ and $N^{\alpha}_{j,i,z}(k,f)$ only capture the initial boarding and last alighting operations of passengers, without considering transfers between origin and destination. Nevertheless, any trip between route segments i,j when $i\in R_A, j\in R_B$ (or vice versa) implies a transfer movement between routes A and B. Therefore, the additional boarding and alighting movements due to transfer operations are captured in the second and third term within the parentheses of Equation (8). There are two possible transfer movements:

i) The first transfer movement typology is made when the route segment origin (i) and destination (j) of the passenger are served by different routes but are located in the same corridor edge, $i,j\in R_E$ or $i,j\in R_W$, $i\neq j$, $r(i)\neq r(j)$. In that case, the transfer passengers never travel along the trunk segment *AC or BC*, since the transfer operation is only performed at points PE or PW of Figure 3. Let $\eta^{\alpha}_{i,j,z}(k,f)$ be the number of transferring passengers that alight from bus trip f at stop k of route segment i (direction z). This transfer can only be performed at the last stop of segment i ($\eta^{\alpha}_{i,j,z}(k,f)=0$ when $k<n_{i,z}$). Similarly, $\eta^{\beta}_{j,i,z}(k,f)$ denotes the number of boarding passengers at transfer stop k of bus route segment i, that are transferring from route segment j. Due to transfer compatibility between route segments i and j at points *PW* or *PE*, this term is $\eta^{\beta}_{j,i,z}(k,f)=0$ when $k>1$. This term is also 0 in all stops of the central segment *AC or BC*.

ii) The second transfer typology happens when the passenger's origin and destination are located at opposite branches (i,j) separated by the central segment and served by different routes, i.e. $i,j\in R_{T1}$ *or* $i,j\in R_{T2}$, $i\neq j$, $r(i)\neq r(j)$. In these cases, the transfer movements are considered to be made at any stop along the whole length of the central route segment. Hence, the term $\zeta^{\alpha}_{i,j,z}(k,f)$ captures the number of transferring passengers alighting from vehicle trip f at stop k of the central route segment operated by the same route that runs along segment i. Similarly, $\zeta^{\beta}_{i,j,z}(k,f')$ refers the transferring passengers from segment i that will board for second time on the complementary central segment to finally alight at route segment j, in bus trip f'.

The estimation of the first term of Equation (8) is made assuming that $N^{\beta}_{i,j,z}(k,f)$ follows a Poisson distribution of parameter $\Delta\beta_{i,j,z}(k)$ considering assumption v), where $\Delta\beta_{i,j,z}(k) = \beta_{i,j,z}(k) - \beta_{i,j,z}(k-1)$. Let $h(f,f-1)$ be the time period between the consecutive arrivals of bus trip f-1 and f at a given stop k. Hence, passenger arrivals at a stop within this headway $h(f,f-1)$ may be considered independent. Under this circumstance, the interarrival time between two passenger arrivals follows an exponential distribution of parameter $\Delta\beta_{i,j,z}(k)$. Thus, if we want to generate variates of this exponential distribution of parameter $\Delta\beta_{i,j,z}(k)$, the resulting interarrival time can be calculated by $T_{i,j,z}(k) = \frac{-\ln(U)}{\Delta\beta_{i,j,z}(k)}$, where U is a random variable with uniform distribution $U(0,1)$. Therefore, the number of boarding passengers $N^{\beta}_{i,j,z}(k,f)$is defined as the integer value x that fulfills the condition of the right hand of Equation (6), where U_m are uniform distributed variates in (0,1).

$$P\left(x = N^{\beta}_{i,j,z}(k,f)\right) = P\left\{\sum_{m=1}^{x} \frac{-\ln(U_m)}{\Delta\beta_{i,j,z}(k)} \leq h(f,f-1) < \sum_{m=1}^{x+1} \frac{-\ln(U_m)}{\Delta\beta_{i,j,z}(k)}\right\} \tag{9}$$

The previous equation is only valid if the vehicle has enough capacity to allow the service to all waiting passengers at stops. In the case of both trip origin and destination are located in the central route segments $i,j \in R_C$, the headway has to be estimated between consecutive vehicle trips, even if they belong to different routes.

The number of alighting and boarding passengers due to a transfer operation between route segments contained in R_W or R_E (typology 1) are estimated by Equations (10) and (11) respectively, once $N^{\beta}_{i,j,z}(k,f)$ terms are known. Note in Equation (8) that the boarding passengers $\eta^{\beta}_{j,i,z}(1,f')$ in vehicle trip f' at the first stop of route segment i would have arrived from different vehicle trips f operating segment j. It forces that we have to sum up the potential contribution of all bus trips operating route segment j in Equation (11). For this reason, we define the operator $\Phi_{i,j,z}(k,f)$ to denote the correspondence of transferring passenger flows between bus trips of different routes. Let's consider that the vehicle trip f operating route segment i in direction z is transferring passengers to route segment j at stop k. Hence, the operator $\Phi_{i,j,z}(k,f) = f'$ returns the vehicle trip f' operating route segment j that has arrived at transfer stop k just before vehicle trip f. An example of this transfer operation is depicted in Figure 3.

$$\eta^{\alpha}_{i,j,z}(n_{i,z},f) = \begin{cases} \sum_{k=1}^{n_{i,z}} N^{\beta}_{i,j,z}(k,f), & i,j \in R_W \; or \; i,j \in R_E, i \neq j,;\forall f \\ 0 & otherwise \end{cases} \tag{10}$$

$$\eta^{\beta}_{j,i,z'}(1,f') = \begin{cases} \sum_{g=1}^{M} \eta^{\alpha}_{j,i,z}(n_{j,z},g)\kappa_1(g,f') = \sum_{g=1}^{M}\sum_{k=1}^{n_{j,z}} N^{\beta}_{j,i,z}(k,g)\kappa_1(g,f'), i,j \in R_W \; or \; i,j \in R_E, i \neq j,;\forall f' \\ 0 & otherwise \end{cases} \tag{11}$$

Where $\kappa_1(g,f') = \begin{cases} 1 \; if \; \Phi_{j,i,z}(n_{j,z},g) = f'+1 \\ 0 \qquad otherwise \end{cases}$ and
$\Phi_{j,i,z}(n_{j,z},g) = \max f' \left|\left(a_{j,z}(n_{j,z},g) - a_{i,z'}(1,f')\right)\right. > 0$ when $j,i \notin R_C$.

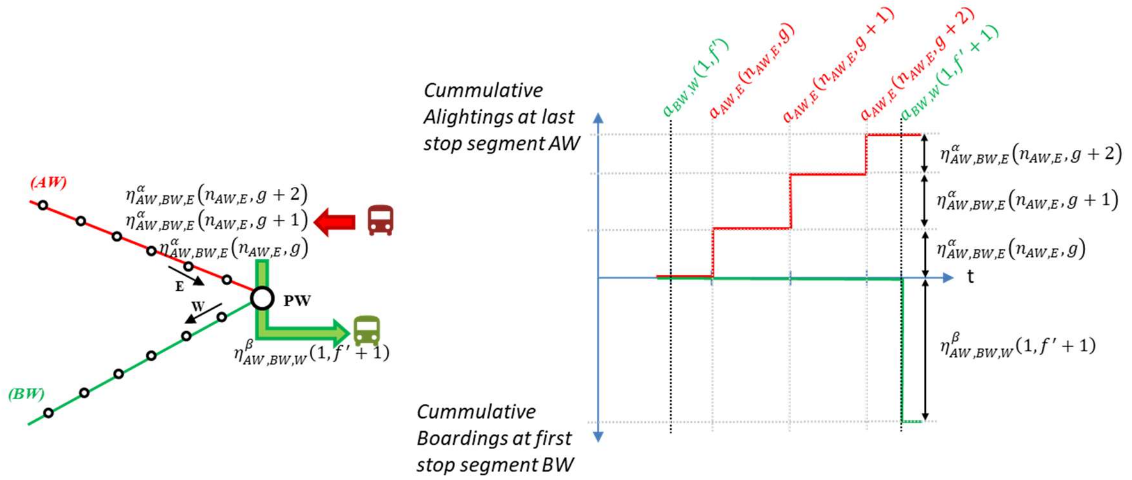

Figure 3. Example of transferring passengers at station *PW* from vehicle trips in segment *AW* to vehicle trip (f'+1) in segment *BW*.

Concerning the second transfer typology, Equation (12a) estimates the alighting passengers $\zeta^{\alpha}_{i,j,z}(k,f)$ from vehicle trip f at transfer stop k of the central segment h (h =*AC* if $i \in R_A$ or h=*BC* if $i \in R_B$) direction z. These passengers will board on bus trip $f' = \Phi_{h,h',z}(k,f)+1$ operating the complementary route h' in the central route segment. Nevertheless, this bus trip f' may find at stop k transferring passengers that have alighted from different bus trips f operating route segment i. Therefore, the term $\zeta^{\beta}_{i,j,z}(k,f')$ is estimated in Equation (12b) as all boarding passengers at transfer stop k of the central route segment, direction z, that go to the branched segment j. Figure 4 shows an example of the transshipment operation in the central route segment. In order to consider assumption vii), we define $F^T_z(k)$ as the cumulative distribution of all passengers transferring from the first stop to stop k of central route segment, where $\Delta F^T_z(k) = F^T_z(k) - F^T_z(k-1)$ and $F^T_z(0) = 0$. If $F^T_z(1) = 1$, it means that all passengers will transfer at the first stop of the central route segment. Oppositely, if we set $F^T_z(k) = 0$ (k=1,.., $n_{AC,z}$-1) and $F^T_z(n_{AC,z}) = 1$, it represents that all passengers will transfer at the last stop of this route segment *AC*.

$$\zeta^{\alpha}_{i,j,z}(k,f) = \Delta F^T_z(k) \sum_{r=1}^{n_{i,z}} N^{\beta}_{i,j,z}(r,f); \quad \forall f,k;\ i,j \in R_{T1}\ or\ i,j \in R_{T2} \tag{12a}$$

$$\zeta^{\beta}_{i,j,z}(k,f') = \Delta F^T_z(k) \sum_{g=1}^{M} \sum_{r=1}^{n_{i,z}} N^{\beta}_{i,j,z}(r,g) \cdot \kappa_2(g,f')\ \forall f,k;\ i,j \in R_{T1}\ or\ i,j \in R_{T2} \tag{12b}$$

Where $h = \begin{cases} AC\ \ if\ i \in R_A \\ BC\ if\ i \in R_B \end{cases}, h' = \begin{cases} BC\ \ if\ i \in R_A \\ AC\ if\ i \in R_B \end{cases}, \quad \kappa_2(g,f') = \begin{cases} 1\ \ if\ \Phi_{h,h',z}(k,g) = f'+1 \\ \quad 0\ otherwise \end{cases}$ and

$\Phi_{h,h',z}(k,g) = \max f' \left| \left(a_{h,z}(k,g) - a_{h',z}(k,f') \right) \right. > 0.$

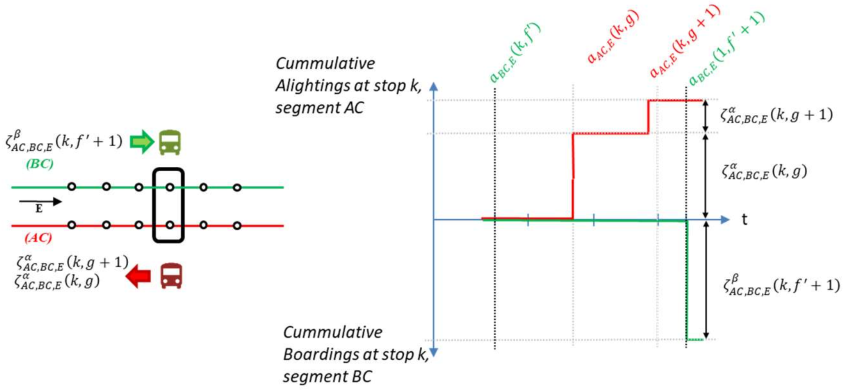

Figure 4. Example of transferring passengers at intermediate stop k from vehicle trips in segment *AC* to vehicle trip (*f'*+1) in segment *BC*.

Eventually, the term $N^{\alpha}_{i,j,z}(k,f)$ accounts for the number of alighting passengers from bus *f* at stop *k* of route segment *i* in the direction of service *z* (Equation 13). This term does not consider the transfer movements. The estimation of this term must be consistent to the vehicle occupancy at this stop, and therefore depends on the stochastic number of boarding passengers that this bus trip *f* has found along the previous service. Therefore, the number of alighting passengers is based on the vehicle occupancy when the vehicle arrives at the stop. The vehicle occupancy is calculated by $O_{i,z}(k,f) = P_{i,z}(k,f) + Q_{i,z}(k,f)$, considering Equations (14-15). The term $P_{i,z}(k,f)$ represents the onboard passengers of bus trip *f* between stops *k*, *k+1* whose origin and destination are both located along the same route segment $i \in R$, in direction *z*. Oppositely, the term $Q_{i,z}(k,f)$ represents the onboard passengers between stops *k*,*k+1* whose origin or destination are not located along the route segment *i* in direction *z*. The sum of both terms accounts for the real occupancy of the vehicle. Equation (14) overpredicts the estimation of the alighting of passengers at stop *k* when the vehicle trip presents a significant higher time headway than the target one. If trip length distributions of passengers and an OD matrix is considered, the service irregularity effects would be even further pronounced.

$$N^{\alpha}_{i,j,z}(k,f) = P_{i,z}(k-1,f)\cdot\frac{A_{i,i,z}(k)}{B_{i,i,z}(k)} + \sum_{\substack{j\in R\\ j\neq i}} \Delta F^{\alpha}_{j,i,z}(k)\sum_{r=1}^{n_{j,z}} N^{\beta}_{j,i,z}(r,f) \quad i,j\in R;\ k,f>0 \tag{13}$$

Hence, the number of passengers alighting at stop *k* that previously boarded along the same route segment *i* is calculated by the first term of Equation (10), as the product between the vehicle occupancy and the quotient $\frac{A_{i,i,z}(k)}{B_{i,i,z}(k)}$. Note that this quotient is equal to 1 at the last stop $k = n_{i,z}$. Nevertheless, the estimation of alighting passengers that have boarded in another route segment is calculated by the second term of Equation (10), considering the boarding passengers in other route segments *j*, $j \neq i$.

The estimation of vehicle occupancies terms $P_{i,z}(k,f)$ and $Q_{i,z}(k,f)$ are provided through Equations (14) and (15) respectively.

$$P_{i,z}(k,f) = P_{i,z}(k-1,f) + N^{\beta}_{i,i,z}(k,f) - P_{i,z}(k-1,f)\frac{A_{i,i,z}(k)}{B_{i,i,z}(k)};\ i\in R; z\in \{W,E\}; k,f>0 \tag{14}$$

$$\begin{aligned} Q_{i,z}(k,f) = {} & Q_{i,z}(k-1,f) + \sum_{\substack{j\in R\\ j\neq i}} N^{\beta}_{i,j,z}(k,f) + \sum_{\substack{j\in R_W\cup R_E\\ j\neq i}} \eta^{\beta}_{j,i,z}(k,f) + \sum_{h\in R_{T1}\cup R_{T2}} \sum_{\substack{h\neq j\\ r(j)=r(i)\\ r(j)\neq r(h)}} \zeta^{\beta}_{h,j,z}(k,f) \\ & - \sum_{\substack{j\in R\\ j\neq i}} \Delta F^{\alpha}_{j,i,z}(k) \sum_{r=1}^{n_s^{jz}} N^{\beta}_{j,i,z}(r,f) - \sum_{\substack{j\in R_W\cup R_E\\ j\neq i}} \eta^{\alpha}_{i,j,z}(k,f) - \sum_{\substack{h\in R_{T1}\cup R_{T2}\\ r(h)=r(i)}} \sum_{\substack{h\neq j\\ r(j)\neq r(h)}} \zeta^{\alpha}_{h,j,z}(k,f) \\ & i\in R; z\in\{W,E\};\ k,f>0 \end{aligned} \tag{15}$$

The passenger flow conservation principle has to be imposed between consecutive route segments. In our model, this principle is formulated at stop k=0 of each route segment i, imposing $Q_{i,z}(0,f) = Q_0$, where Q_0 is different for each route segment i, direction z. For instance, $Q_{AC,E}(0,f) = \sum_{r=1}^{n_{AW,E}} \left(N^{\beta}_{AW,AC,E}(r,f) + N^{\beta}_{AW,AE,E}(r,f) + N^{\beta}_{AW,BE,E}(r,f)\right)$. Therefore, Appendix 1 shows the boundary conditions that must prevail.

3 Optimization

The key performance indicators to analyze the proper operational design of the corridor as well as the effects on the user side are presented in Section 3. Compact estimations for the operating cost incurred by transit agency and the temporal costs experienced by users are provided, based on the modelling framework presented in Section 2. An optimization procedure of the vehicle synchronization and headway is developed, aimed at minimizing the total cost of the system.

3.1 Agency metrics

The operating cost incurred by the bus operator will depend on the fleet size and the distance run by the whole fleet in the period of analysis. The estimation of the fleet size in route A and B (denoted by M_A and M_B respectively) can be obtained through Equation (16). It is computed as the maximal difference in the number of vehicles trips between a vehicle trip departure ($f_A$in route A, f_B in route B) from the initial stop of the first route segment, and the previous vehicle trip arriving at the last stop of the last segment of the route ($f_{n,A}(f_A)$in route A, $f_{n,B}(f_B)$ in route B, where $f_{n,A}(f_A) < f_A$; $f_{n,B}(f_B) < f_B$).

$$M_A = \max_{f_A}\{(f_A - f_{n,A}(f_A)) | a_{AW,E}(1,f_A) > a_{AW,W}(n_{AW,W}, f_{n,A}(f_A)) \geq a_{AW,E}(1,f_A - 1)\}\ f_A = 1,..,M \quad (16)$$

$$M_B = \max_{f_B}\{(f_B - f_{n,B}(f_B)) | a_{BW,E}(1,f_B) > a_{BW,W}(n_{BW,W}, f_{n,B}(f_B)) \geq a_{BW,E}(1,f_B - 1)\}\ f_B = 1,..,M$$

On the other hand, the distance run in one hour of service by the vehicles of each bus route (V_A and V_B respectively) can be estimated by Equation (19), as the total length of each route divided by the target headway. The estimation of the fleet size of Equation (16) guarantees that vehicles are dispatched regularly from the beginning of each route at constant headways.

$$V_A = \frac{\sum_{i \in R_A}(l_{i,E} + l_{i,W})}{H_A} \quad ; \quad V_B = \frac{\sum_{i \in R_B}(l_{i,E} + l_{i,W})}{H_B} \quad (17)$$

3.2 User performance

The effects of the bus performance on the user's side will be mainly evaluated by the total travel time of users and the coefficient of variation of headways. The former metric is the sum of the time spent by users in the waiting (W_i), in-vehicle ($IVTT_i$) and transferring ($W_{T,i}$) components of all user trips in all route segments $i \in R$ in one hour of operation. The latter, denoted by c_{vH}, is a commonly used metric to assess the quality of the service, in terms of reliability and time-headway adherence. These metrics are consistent to Estrada et al. (2016).

On one hand, the in-vehicle travel time $IVTT_i$ in route segment $i \in R$ is estimated by Equation (21) during one hour of service. The $IVTT_i$ is calculated multiplying the passenger occupancy in each segment between two stops, times the travel time along this segment. The time $t^*$is defined by Equation (22) and represents the maximal time when the first vehicle trip of route A or B has completed a full round trip. The second departure (or trip) from the initial stop of the first vehicle is referred by f=$M_A + 1$ in route A or f=$M_B + 1$ in route B. The time period $(0; t^*)$ is needed to preposition all vehicles along the routes A and B. Therefore, Equation (21) only takes into account the travel time of vehicles during the next hour after t^*, i.e. in the time period $(t^*; t^*+1)$ measured in hours. To do this, we introduce the Heaviside function *H(x)* in Equation (21), that returns $H(x \geq 0) = 1$, and $H(x < 0) = 0$.

$$IVTT_i = \sum_{z\in\{E,W\}} \sum_{f=1}^{M} \sum_{k=1}^{n_{i,z}-1} \left\{O_{i,z}(k,f) \cdot \left(a_{i,z}(k+1,f) - a_{i,z}(k,f)\right)\right\} \cdot H\left(a_{i,z}(k,f) - t^*\right) \cdot H\left(t^* + 1 - a_{i,z}(k+1,f)\right) \quad i \in R \tag{18}$$

$$t^* = max\{a_{AW,E}(1, M_A + 1); a_{BW,E}(1, M_B + 1)\} \tag{19}$$

Moreover, the waiting time of passengers at stops scattered along segment $i \in R$ is denoted by W_i and calculated through Equation (23). To do this, we consider the arrival time $\tau_{i,j,z}(k,f,p)$ of each passenger p=1,.., $N^{\beta}_{i,j,z}(k,f)$ boarding on vehicle trip f at stop k. In Section 2.3.1, we have justified that the p-th boarding passenger in vehicle trip f at stop k of route segment i (direction z) traveling to route segment j is assumed to arrive at this stop at time $\tau_{i,j,z}(k,f,p) = \tau_{i,j,z}(k,f,p-1) - \frac{\ln(U_m)}{\Delta\beta_{i,j,z}(k,f)}$; where $\tau_{i,j,z}(k,f,0) = a_{i,z}(k,f)$, and U_m is uniform distributed variates in (0,1). Again, we only consider those passengers that have arrived at stops in the one-hour period after the first vehicles of route A and B have completed one roundtrip. The transfer time of passengers in a one-hour of service is evaluated in Equation (24) in a similar way to the estimation of waiting time in the previous equation. Due to the two different transfer typologies, two formulas are proposed.

$$W_i = \sum_{z\in\{E;W\}} \left\{ \sum_{f=1}^{M} \sum_{k=1}^{n_{i,z}} \sum_{p=1}^{N^{\beta}_{i,j,z}(k,f)} \left(\min[a_{i,z}(k,f); t^* + 1] - \tau_{i,j,z}(k,f,p)\right) \cdot H\left(\tau_{i,j,z}(k,f,p) - t^*\right) \cdot H\left(t^* + 1 - \tau_{i,j,z}(k,f,p)\right) \right\} \quad i \in R \tag{20}$$

$$W_{T,i} = \sum_{z\in\{E,W\}} \left\{ \sum_{f=1}^{M} \sum_{\substack{j\in R \\ j\neq AC;BC}} \eta^{\alpha}_{i,j,z}(k,f) \cdot \left(\min[a_{j,u}(1, \Phi_{i,j}(f) + 1); t^* + 1] - a_{i,z}(n_{i,z}, f)\right) \cdot H\left(a_{i,z}(n_{i,z}, f) - t^*\right) \cdot H\left(t^* + 1 - a_{i,z}(n_{i,z}, f)\right) \right\} \quad i \in R - R_C; \quad u = \begin{cases} E & if\ z = W \\ W & if\ z = E \end{cases} \tag{21}$$

$$W_{T,i} = \Sigma_{z\in\{W,E\}} \left\{ \Sigma_{f=1}^{M} \Sigma_{\substack{j\in R_{T1} \\ j\in R_{T2}}} \Sigma_{\substack{h\in R_{T1} \\ h\in R_{T2} \\ h\neq j; \\ r(h)=r(i)}} \Sigma_{k=1}^{n_{iz}} \zeta_{h,i,j,z}(k,f) \left(\min[a_{i',z}(k, \Phi_{i,i',z}(f) + 1); t^* + 1] - a_{i,z}(k,f)\right) H\left(a_{i,z}(k,f) - t^*\right) \cdot H\left(t^* + 1 - a_{i,z}(k,f)\right) \right\} \quad i \in R_C; i' \in R_C - \{i\}$$

Eventually, the coefficient of variation of headways is calculated for all segments $i \in R_A$ operated by route A by $c_{v,h,A} = \frac{s_H}{H_A}$, and segments $i \in R_B$ of route B by $c_{v,h,B} = \frac{s_H}{H_B}$. The term s_H is the standard deviation of the time headways between consecutive buses from the target headway in all stops along segment. According to the value of this coefficient of variation, TRB (2013) proposes 6 different levels of service domains to assess the regularity of service. This classification will be used in the numerical instances to rank the bus performance in terms of time headway adherence.

3.3 Optimization procedure

The problem optimization aimed at minimizing the user and agency costs is defined by means of Equations (22)-(24). The agency cost component Z_A, expressed in EUR/h, considers the number of resources and distance travelled by the whole fleet, multiplied by the unit temporal and unit distance cost parameters, c_t (€/veh-h) and c_d (€/veh-km). User costs Z_U (EUR/h) are computed as the product of waiting and in-vehicle travel times of all passengers by the value of the time parameter μ_N (€/pax − h). This parameter accounts for the monetary value of one hour spent by an average user in the system.

$$\min_{H_A, H_B, t_{0A}, t_{0B}} \quad Z = Z_A + Z_U = (M_A + M_B)c_t + (V_A + V_B)c_d + \mu_N\left\{\sum_{i\in R}\left(W_i + IVTT_i + W_{T,i}\right)\right\} \tag{22}$$

$$s.t.$$

$$0 \le t_{0A} \le H_A; 0 \le t_{0B} \le H_B \tag{23}$$

$$O_{i,z}(k,f) \le C; \forall k, f, i, z \tag{24}$$

Equation (26) states the non-negative nature of the decision variables of the problem: time-headways (H_A and H_B) and the entrance time of buses at the beginning of segments *AW* and *BW* in direction *E* (variables t_{0A} and t_{0B}). This problem must also verify the capacity constraint stated in Equation (27).

Nevertheless, the optimal set of $H_A^*, H_B^*, (t_{0A})^*, (t_{0B})^*$ minimizing the objective function may cause a poor regularity in the central segment. In fact, this situation may occur when the transfer time component W_T of the objective function is quite important due to the significant demand between branched segments. Hence, we also propose to estimate those optimal values of the decision variables that provide the minimal coefficient of variation of headways.

The optimization procedure followed was based on a grid search of the objective function Z (H_A', H_B', t_{0A}', t_{0B}'), enumerating Z for constant intervals in the domain of the decision variables. The headways H_A', H_B', are enumerate every minute. For other decision variables, we define an enumeration interval of Δ=5-20 seconds, depending on the problem size. The entrance times of vehicles in segment *AW*=1 (route *A*) and segment *BW* (route *B*) are then calculated respectively as $t_{0A}' = k_1 \cdot \Delta$ and $t_{0B}' = k_2 \cdot \Delta$; k_1, k_2 integers.

4 Control strategies

As it is explained in section 2, there are different sources of instability in the definition of the problem that will cause a low level of service in route segments, especially in the central segment, in terms of regularity. To tackle this problem, we consider three potential bus control strategies to alleviate the lack of time headway adherence, in addition to the "do nothing" or no control strategy defined as Strategy S0. They are enumerated in the following lines.

Strategy S0. This strategy resembles the assumptions listed in Section 2.1 where there is no key constraint to the bus motion. It considers that buses do not have holding points along the route and overtaking between buses of the same route is not allowed.

Strategy S1. We consider the provision of slacks at holding points (stops) in the route *A* and *B*. This strategy is aimed at maintaining constant the target headway of H_A and H_B in all segments of the route, at the expenses of enlarging bus travel times. We assume that the maximal slack (i.e. available synchronization time at any stop) is θ_{max}. Therefore, we allow that a vehicle trip *f* at stop *k* of route segment *i,* running at rear of a delayed vehicle, may wait $\theta_{i,z}(k,f)$ units of time ($\theta_{i,z}(k,f)<\theta_{\max}$) to recover the target headway. The modeling formulation modification to represent this strategy is presented in the following lines.

Consider that the arrival time of vehicle trip *f* at stop k *of* route segment *i* is $a_{i,z}(k,f)$. The vehicle trip (*f*-1) of the same route (A or B) running ahead would have arrived at time $a_{i,z}(k,f-1)$. Hence, the real time headway between these vehicles is $a_{i,z}(k,f)-a_{i,z}(k,f-1)$. The control strategy forces vehicle trip *f* of this route segment *i* to wait $\theta_{i,z}(k,f)$ at stop *k* when the previous real headway is lower than the target H_A or H_B value. The determination of these slacks is estimated by Equation (28) depending on the difference between real and target headways.

Let $\varepsilon_{f-1,f}=a_{i,z}(k,f)-a_{i,z}(k,f-1)-H_x$ be the forward time headway adjustment between vehicle trip *f* and the vehicle ahead (*f*-1) at this stop k ($x=A$ if $i\in R_A$ and $x=B$ if $i\in R_B$). We can also define the corresponding time headway adjustment between vehicles *f* and *f*+1 by $\varepsilon_{f,f+1}=a_{i,z}(k^p,f+1)-a_{i,z}(k^p,f)-H_x$. Nevertheless, the latter headway adherence metric must be evaluated at the last stop $k^p<k$ already visited by vehicle trip *f*+1.The first and second conditions of Equation (28) hold the vehicle trip *f* an amount of time $\theta_{i,z}(k,f)$ at stop *k* of route *i* in direction *z*. Since the deviation of the headway with regard the vehicle at rear is higher than the corresponding value with the vehicle ahead, this slack helps reducing the headway with the vehicle at rear. The slack to be introduced depends on the relative value of $(\varepsilon_{f,f+1}-\varepsilon_{f-1,f})$ and θ_{max}. If the maximal slack time is not sufficient to correct the real time headway $(\varepsilon_{f,f+1}-\varepsilon_{f-1,f})\geq\theta_{max}$, we hold vehicle trip *f* an amount of θ_{max}. On the contrary, when the headway deviation is lower than maximal slack $(\varepsilon_{f,f+1}-\varepsilon_{f-1,f})\geq\theta_{max}$, we only hold vehicles this deviation. Under other situations (third condition), the slack to be introduced is 0. If we would hold the vehicle at this stop, we would worsen the system regularity.

$$\theta_{i,z}(k,f)=\begin{cases}\theta_{max} & if\,\varepsilon_{f,f+1}>\varepsilon_{f-1,f}\ and\,\varepsilon_{f,f+1}>0\ and\,(\varepsilon_{f,f+1}-\varepsilon_{f-1,f})\geq\theta_{max}\\(\varepsilon_{f,f+1}-\varepsilon_{f-1,f}) & if\,\varepsilon_{f,f+1}>\varepsilon_{f-1,f}\ and\,\varepsilon_{f,f+1}>0\ and\,(\varepsilon_{f,f+1}-\varepsilon_{f-1,f})<\theta_{max}\\0 & otherwise\end{cases}\qquad(25)$$

These slacks must be added into the formulation of vehicle dwell times at stop *k* of route segment $i\in R$ defined by Equation (5). It will suppose an extra dwell time of the vehicle trajectory a) depicted in Figure 2.

Strategy S2. This strategy extends the duration of the green phase at traffic lights to avoid delayed buses to stop at intersections. Therefore, we assume that the trajectories of vehicles are monitored to activate traffic light priority to buses in real time. Let *G* be the maximal extension time of the green phase allowed by traffic conditions. In fact, this green extension should be defined by a maximal percentage ψ of the total traffic light cycle, $\frac{G}{C_p}<\psi$. However, this green extension time is only provided to those buses whose time headway with the regard to the vehicle ahead is higher than the value corresponding to the vehicle at rear. Therefore, the extension parameter $\Delta g_{i,z}(k,f,m)$ of Equation (18) for the vehicle trip *f* at intersection *m* (*m*=1,…, $p_{i,z}(k)$) between stops *k,k*+1 of the route segment *i*, direction *z,* is replaced by the value given in Equation (29).The resulting bus motion under this strategy is depicted by trajectory b) in Figure 2

$$\Delta g_{i,z}(k,f,m)=\begin{cases}G & if\,\varepsilon_{f,f+1}\leq\varepsilon_{f-1,f}\\0 & if\,\varepsilon_{f,f+1}>\varepsilon_{f-1,f}\end{cases}\qquad(26)$$

Strategy S3. This strategy is a combination of Strategy S1 and S2. It extends the duration of the green phase at traffic lights (G>0) and introduces some slack at each stop ($\theta_{\max}$>0). As suggested in Estrada et al (2016), this was the most efficient control strategy for minimizing the total cost of the system, in corridors operated by a single line with exogenous bus disruptions.

The optimization procedure is able to obtain the best solution of the problem by a grid search considering only one control strategy from the previous list at each execution. Therefore, the sequences of travel times of buses along the route are particular to the strategy chosen in each execution.

5 Numerical analysis

Formulas (1)-(29) reproduce the transit vehicle motion and were coded in Visual Basic programming language, compatible to the input files developed in Microsoft Excel. The model was implemented in the corridor layout of the H10 route of the Barcelona bus network, to analyse how the total cost and the selected metrics regarding the performance of the branched transit routes are influenced by transit layout, vehicle entrance synchronization and control strategies.

Section lengths (km)

Section	Length
AW	1.450
AC/BC	8.750
AE	2.975
BW	0.645
BE	4.600

Demand matrix (pax/h)

Section	AW	AC/BC	AE	BW	BE
AW	100	150	50	10	20
AC/BC	150	1,805	100	100	100
AE	50	100	30	20	50
BW	10	100	20	25	25
BE	20	100	50	25	200

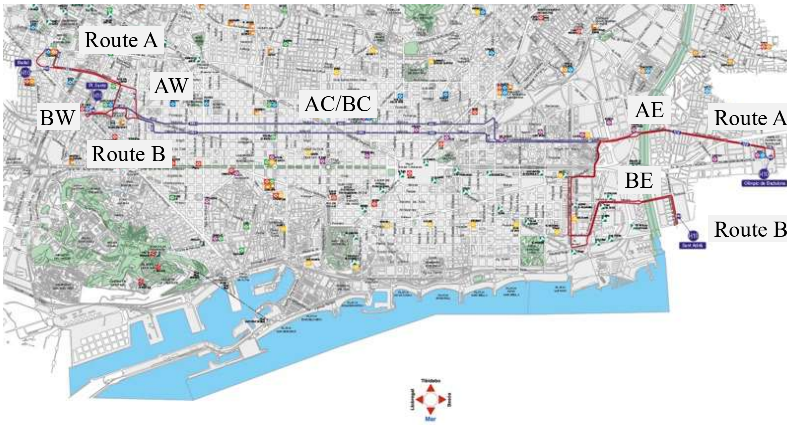

Figure 5. Barcelona's H10 bus corridor layout, with the section lengths and the matrix showing the demand distribution through its five sections

The current layout of this corridor does not show any branched segment and it is operated at H= 6 minutes. The occupancy at the edges of the route is significantly low at this headway. Therefore, a potential modification of the current bus corridor H10 into two branched routes in the edges (H10-A and H10-B) is proposed to correct this malfunction. In Figure 5, the layouts of the two the routes are depicted. Route H10-A is 13.175 km long, from Badal St. to Olimpic de Badalona Station (roundtrip distance 26.3km), while the length of route H10-B trip, from Pl. de Sants St. to Sant Adrià St., is 14.00 km. The branches of route H10-B run closer to the sea front. The corridor layout, the length of each branch as well as demand matrix are depicted in Figure 5. In this physical problem, we considered a demand matrix that has been obtained from the actual ridership in the central corridor of H10 and in other lines currently operating the branched segments (TMB, 2018). From the available data, the total demand estimation along the corridor was supposed to be 3410 pax/h. Moreover, the cycle time in the Barcelona neighbourhood where the corridor run along is C_p=90 s and the green phase time $g_{iz}(m)$ of intersection m in route segment i (direction z) ranges between 36-60 seconds. The kinematic and economic input parameters are the following: the maximal cruising speed, $v_{\max}$=50 km/h, mean acceleration rate μ_a=1.1 m/s^2, standard deviation of the acceleration rate σ_a=0.3m/s^2, vehicle capacity C=120 pax/veh, unit distance cost c_d=5 EUR/veh-km, unit temporal cost c_t=60 EUR/veh-h, unit boarding time τ^{β}=2.73 sec/pax, unit alighting time τ^{α}=1.73 sec/pax and value of time μ_N=10 EUR/pax-h.

5.1 Effect of the stochastic processes on the cost and bus performance

We have repeated the optimization process aimed at minimizing Equation (22)-(24) in 16 different multirandom runs, considering different variants of the demand and acceleration random variables of the H10 problem. These variables are the number of boarding passengers on bus at stops and acceleration rates. Considering the implementation of Strategy S0, we have obtained that the optimal headways are H_A^*=7 min and H_B^*=6 minutes for the situation represented by the input parameters.

In Figure 6a, we plot the best ten combinations of entrance times (t_{0A}) and (t_{0B}) that provide the lowest total cost of the system in the 16 problem instances under Strategy (S0). The points labelled by numbers p=1,..,16 represent the pair of optimal values $(t_{0A})^*$,$(t_{0B})^*$ corresponding to the p-*th* set of random instance. The results show that there is no a well-defined subdomain of t_{0A} and t_{0B} that recurrently minimizes the total cost for all random variable sets. Nevertheless, the pair of (t_{0A}) and (t_{0B}) in the same random execution that produce the lowest cost of the system are usually located along a line of unitary slope in the diagram. It means that the optimal domain of time entrances is identified for a constant offset between these entrance times, i.e. $(t_{0B}) - (t_{0A})$ =ct. This constant offset is particular for each random execution, hence, it depends on the random variable demand boarding at stops. For example, the vast majority of best combinations of entrance times in Random Execution #5 (blue crosses) are found when $(t_{0B}) - (t_{0A})$ =0 seconds (blue line). However, in Random Execution #11, the best pairs of entrance times maintain the following offsets $(t_{0B}) - (t_{0A}) = -120; 80; 140$ (three red lines). On the other hand, Figure 6b depicts the main cost and performance metrics for the optimal solution in each of the 16 random executions.

The maximal difference between the best solution among random instances is less than 10%. A fact that deserves comment is the fleet size required to operate the bus route. There is a maximal variation of 4 buses among test instances. In real implementations, depending on the stochastic boarding and acceleration process, some instances would theoretically require additional buses to operate at the given headway. If these buses are not provided (we only deploy the fleet size requried in the best solutions), it would mean that some buses would present higher roundtrip travel times than expected. Therefore, the buses experiencing higher roundtrip travel times will initiate the next roundtrip with delays regarding the timetable, not satisfying the target headways. The maximal variation of coefficient of headway variation is 0.26 and 0.33 in route A and B respectively. Therefore, the synchronization of the entrances in the corridor highly influences the time headway adherence of buses along the corridors. Nevertheless, it does not imply a large increment of operating cost in the bus route.

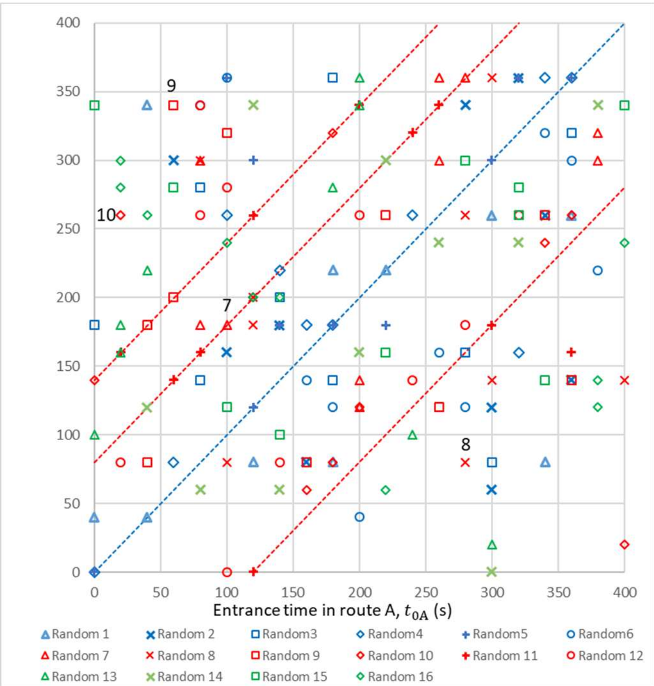

Figure 6a. (t_{0A}, t_{0B}) pairs of the 10 lowest solutions in terms of total cost for the 16 random instances.

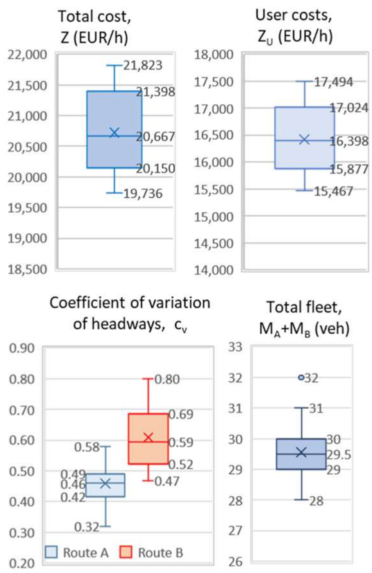

Figure 6b. Box plot of the optimal cost and performance metrics in the 16 random instances.

5.2 Effect of traffic signals and control strategies

We compared the route performance under the same control strategies defined in Section 4, although we considered more combinations of parameters in Strategy S3 (θ_{max}=5 sec, G=5sec in option a; θ_{max} =5 sec, G=5sec in option b; θ_{max} =5 sec, G=20sec in option c; and θ_{max} =10 sec, G=10sec in option d).

The results obtained are summarized in Figures 7a and 7b for the random set of parameters #1 and the optimal values of the decision variables ($t_{0A} = 0\ s, t_{0B} = 40\ s, H_A = 420\ \mathrm{s}, H_B = 360\ s$). The operation under the no-control strategy (S0) requires 28 vehicles (13 buses in route A and 15 in route B) and presents a total cost of 20,297 EUR/h. The headway adherence is not satisfactory. According to the assessment criterion of TRB (2013) and the values of the coefficient of variation of headways in routes A ($c_{vH,A} = 0.44$) and B ($c_{vH,B} = 0.60$), the level of service can be cathegorized as D and E respectively. Strategy S1 based on slacks is able to marginally reduce the variation of headways (level of service) at the expenses of increasing the roundtrip travel time of vehicles. Therefore, the operational cost of transit agency is increased (two additional vehicles with regard to Strategy S0) as well as the in-vehicle travel time of users. It resulted to be the most expensive strategy among the set of strategies under analysis. Strategy S2 based on traffic signal extension is able to reduce the total cost of the system by 5.5% with regard to Strategy S0. However, this strategy presents a very poor service regularity, with a c_{vh} metric even 15% higher than the strategy with no control (S0). The provision of the discrete green extension strategy to delayed buses increases the instability of the system (poorer c_{vh}), but allows minimizing travel times. To fix this effect, hybrid strategy S3 (combining short slack times at holding points and the green extension measure) is able to provide similar results in terms of total cost to the best strategy S2, while improving the variation of headways. Strategy S3c is the most cost-efficient control strategy, being able to reduce the total cost by 3.99% with regard to S0 counterpart, maintaining the coefficient of variation of headways below 0.38 (level of service C).

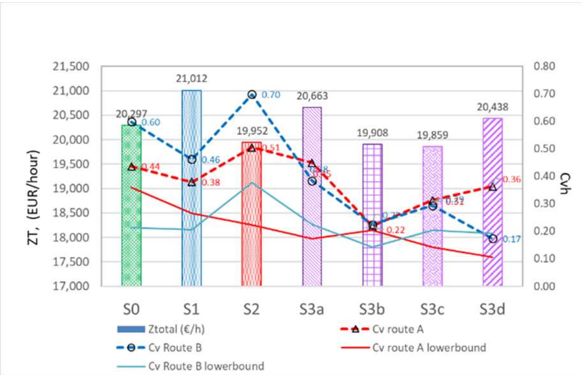

Figure 7a. Total cost and coefficient of variation of headways corresponding to each control strategy

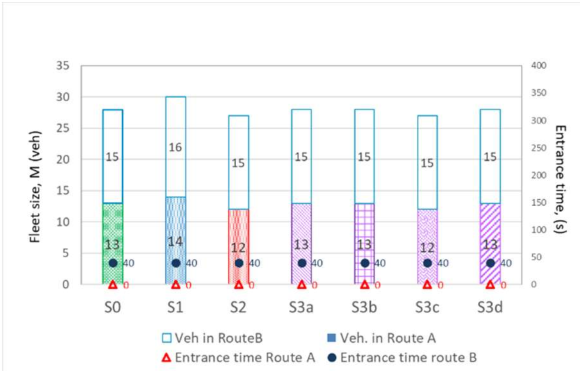

Figure 7b. Fleet size and route entrance times corresponding to each control strategy

The second best solution in terms of total cost is Strategy S3b, only 1.5% higher than S3c. Nevertheless, this strategy is able to maintain the level of service within the A domain (c_{vh} <0.2). Figure 7a also exhibits the lowerbound of the coefficient of variation of headways corresponding to route A and B. These curves are obtained by the enumeration of the coefficient of variation of headways for different entrance times t_{0A}, t_{0B}. Generally, the pair (t_{0A}, t_{0B}) that outperforms the service regularity in route A and B is not the same that minimizes the objective function defined in Equation (25). As it was suggested in Estrada et al (2016), the most recommendable control strategy is the hybrid one, with small holding points distributed along the line and the possibility of the green extension of traffic lights. Nevertheless, the branched layout of routes and the inclusion of the stochastic effects reduce the potential savings (around 30% of cost reduction) reported in Estrada et al (2016).

5.3 Effect of unit boarding times and total demand

The results presented in section 5.1 are calculated with a given unit boarding time and average hourly demand rates, based on the real data gathered by the bus agency in Barcelona. In this subsection we carry out a sensitivity analysis of the cost and bus performance with regard to these input parameters, since they are the ones that present more uncertainty. The cost and coefficient of variation of headways have been evaluated when the current unit boarding time (τ^{β}=2.73 s/pax) is scaled by factors 0; 0.5; 1.0; 1.25 and 1.5 while the current hourly demand rates between route segment $i,j \in R$, direction z ($B_{i,j,z}$) are magnified by the scaling factors ρ=1.0; 1.25 and 1.50. In fact, the assumption that τ^{β}=0 represents the situation when the source of instability is due to the different acceleration of vehicles. The arrival rate of boarding passengers does not have any effect on the disturbance propagation and the headway adherence.

Figure 8a depicts the evolution of the total cost of users and transit agency in the domain of variation of the aforementioned parameters when no control strategy is considered (S0). As it is expected, the higher the unit boarding time and demand rates are, the more expensive the service becomes. Nevertheless, the monotonically increasing function of total cost with regard to unit boarding time is not linear. In the most crowded route scenario, the total cost increment rate is significantly larger for high values of unit boarding times. In fact, this domain embraces the situations when the system is more irregular. Indeed, any stochastic effect may vary the relative time spacing among two consecutive vehicles with regard to the target headway. This variation is propagated and increased by the term $\Delta\beta_{i,j,z}(k) \cdot \tau^{\beta}$, where $\Delta\beta_{i,j,z}(k)$ is the arrival passenger rate at stop k of route segment i traveling to route segment j. In this figure, we have also provided the curve of the minimal total cost of the system when Strategy S3 is implemented only in the most crowded scenario when ρ=1.50. The estimation has been made considering the optimal values of slacks and green extension time in the domain θ_{max} , $G \in \{0,30\}$ seconds. Strategy S3 is able to reduce the total cost of the system by 9-19%, when demand is scaled up by ρ=1.50.

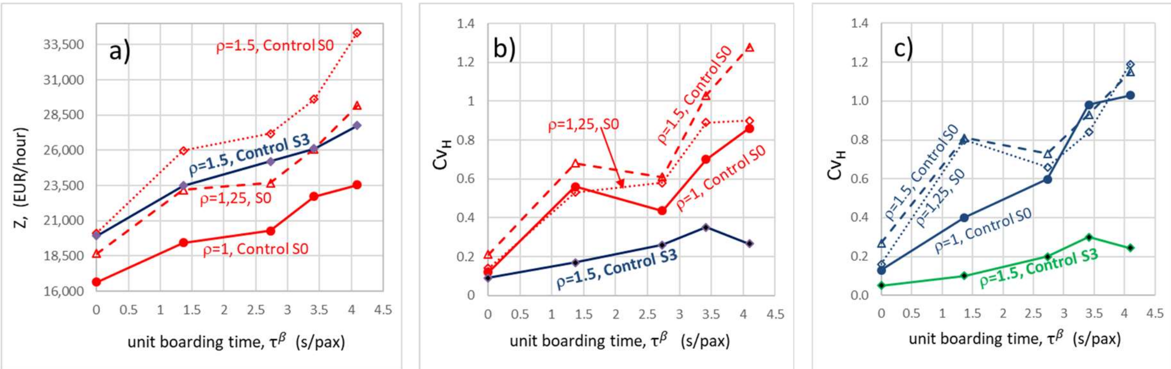

Figure 8. Total cost of the system (a), Coefficient of variation of headways in route A (b), and in route B (c) with regard to unit boarding time for different demand scaling factors (ρ). Input parameters: v_{max}= 50 km/h, μ_a=1.1 m/s^2, σ_a=0.3m/s^2, C= 120 pax/veh, c_d=5 EUR/veh-km, c_t=60 EUR/veh-h, τ^{α}=1.73 sec/pax, μ_N=10 EUR/pax

Moreover, Figure 8b and 8c plot the coefficient of variation of headways as a function of the unit boarding time. Although the tendency is similar to the total cost behavior, the curves estimating the headway adherence do not present a monotonically non-decreasing increment with regard to the unit boarding times. In these figures, we have also plotted the best coefficient of variation of headways obtained by the implementation of Strategy S3. The optimal green extension time and slack is not the same as the calculated in Figure 6a to obtain the minimal total cost. The control strategy S3 depicted in Figure 6a is aimed at minimizing total cost and it is able to fulfill $c_{v,H} \leq$0.5 in the whole domain of analysis when G=10s and θ_{max}=30s. If the goal is modified and we will maximize the headway adherence, strategy S3 is able to maintain the level of service within region A ($c_{v,H} \leq 0.32$), incurring a similar total cost to the Strategy S0. These situations are achieved when the Strategy S3 is implemented with the highest green extension times and slacks (G=30s and θ_{max}=30s). The idealistic situation that τ_b=0 sec will imply that the coefficient of headway variation will be maintained below $c_{v,H}$=0.9. In that case, the total cost of the system is practically the same in Strategy S3 and Strategy S0 (no control).

5.4 Effect of demand distribution between route segments

In this subsection, we analyse the effect that a different spatial demand distribution between the trunk and branched route segments would have on the bus service performance. Hence, the passenger flows between route segments ($B_{i,j,z}$) have been modified to represent relative passenger flows between the central trunk segment and the rest of the branched segment, and the flow captured by the two routes A and B under study. Each demand scenario is controlled by two scaling factors. The central scaling factor ϕ_C generates a new demand rate of passengers whose trip origin and destination are located in route segment i=AC or BC, i.e. $B_{AC,AC,z}{}' = \phi_C \cdot B_{AC,AC,z}{}^0$, assuming that $B_{AC,AC,z}{}^0$ is the demand rate provided by the transit operator in the current situation (base case). This term increases the dependency of the coupled service between the two routes, since these passengers may board on both routes. The second scaling factor, ϕ_A, is aimed at varying the demand to be captured by route A, so that the new demand rates are calculated by $B_{i,j,z}{}' = \phi_A \cdot B_{i,j,z}{}^0$, where $B_{i,j,z}{}^0$ are the demand rates in the base case scenario. This formula is only applied for the following relationship of origin and destination route segments: $i,j \in R_A$, except when i=j=AC. Figures 9-11 summarize the results obtained by the optimization of Equation (25) in the domain of scaling factors 0.25$\leq \phi_C \leq$ 1.25 and 0.25$\leq \phi_A \leq$ 3. The results are obtained when no control strategy is considered. The terms into brackets in the horizontal axis present the value of the quotient of $\frac{\Gamma_A}{\Gamma_B}$, that estimates the percentage of passengers that can only be served by route A with regard to those only served by route B. Therefore, the term Γ_A is the sum of the boarding terms between the route segments $i,j \in R_A$, except when i=j=AC. On the other hand, Γ_B is the corresponding sum of boarding terms $i,j \in R_B$, except when i=j=BC.

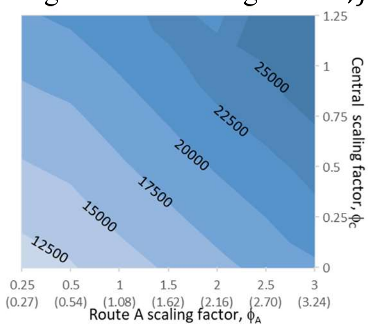

Figure 9. Total cost with regard to the demand scaling factors

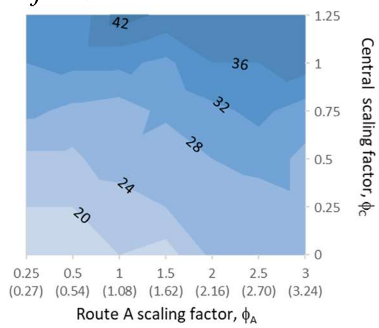

Figure 10. Total fleet size in the system with regard to the demand scaling factors

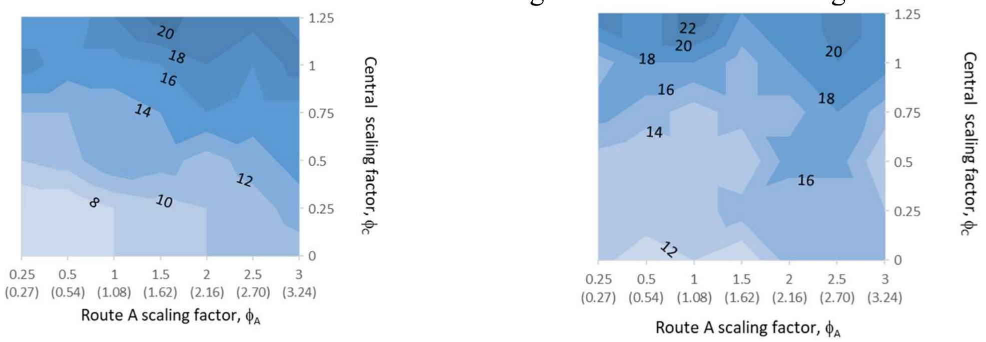

Figure 11. Required fleet size in the coupled routes A (left) and B (right) to minimize the total cost, with regard to the demand scaling factors

Both entrance times in route A and B (decision variables) are determined to be fixed at $t_{0A} = t_{0B}$ =40s to reduce the computational time of the problem. Figure 9 reasonable shows that the total cost of the system is increased when the boarding passenger demand rates are also magnified in the central segment as well as in the segments covered by route A. Despite the stochastic effect of the problem, well-defined iso-cost curves are obtained in the domain of analysis. In addition, Figure 10 depicts the fleet size needed in both coupled bus routes (whole system) to run the service, while Figure 11a)-b) depicts the number of vehicles and headways, respectively, in each route (Route A left, Route B, right). The number of vehicles presents a more unstable behaviour that depends on the vehicle sequence of operation along the central segment. If $\phi_C > 0.60$, the central segment captures the vast majority of trip origins and destinations in the whole system. Under this situation, both route A and B present similar number of vehicles to be allocated and similar headways. It happens because both routes contribute to serve the crowded segments of the system and the travel time to run this segment is practically the same in both routes. Nevertheless, when $\phi_C < 0.60$ the contribution of the branched segments $i \in R_W \cup R_E$ in the performance of the system is more dominant. For very small values of central scaling factor ($\phi_C < 0.25$), the routes A and B are operated decoupled, and both routes present headways according to the expected demand this line would independently capture. In this domain $\phi_C < 0.25$, both lines present headways ranging from 8.5 to 15 minutes. This situation seldom justifies the operation of the corridor with coupled lines, since it would be better operated by independent routes, with just one providing service in the central segment.

Nevertheless, in the domain 0.25<$\phi_C < 0.60$ we clearly identify a coupled effect between lines. When ϕ_A<<1.00, the optimization procedure reasonably allocates the minimal number of vehicles to route A, and therefore, we obtain the highest headways in this route, while in route B the headways are competitive, around 6-7 minutes. However, even if the demand that can exclusively be served by route A is 2.16 times higher than the corresponding to route B (ϕ_A >2.0), the number of vehicles assigned to route B is higher than the fleet size in route A. Therefore, the headway in route B is lower than the route A counterpart. In that case, route B contributes to provide service to the central segment. Indeed, the presence of more frequent service in route B is identified in the whole domain of analysis.

Figure 12. Best combination of headways in coupled routes A (left) and B (right) with regard to the demand scaling factors, in seconds.

6 Conclusions

A transit corridor operated by two routes needs joint scheduling and dispatching of resources in order to guarantee a proper level of service. An operational model such as the presented in this paper is required to find out the optimal headways, vehicle entrance times in the corridor and control strategies that minimize the total cost of the system. The total cost should encompass the total time of users spent in the system as well as the operating cost incurred by transit agencies. The latter depends on the control strategies to tackle bus bunching, since most of these measures improve time headway adherence at the expenses of reducing the commercial speed of buses.

The random effect of the boarding operation and acceleration phase in the motion of a single bus generates variability in the roundtrip travel times. This fact does not enable maintaining a perfect time headway adherence between vehicles. It has been corroborated by the model's results in the base case scenario of bus route H10 of Barcelona, when we suppose that there is no control strategy (S0). The implementations of the model in different random sets of the stochastic variables generate solutions that diverge by 10%, concerning the total cost. Nevertheless, the coefficient of variation of headways in the solution that minimizes the total cost of the system ranges between 0.32 to values higher than 0.8 in the different random tests. It is also impossible to generate an optimal synchronization of between the entrance time of vehicles at the first stop of the branched segment of route *A* and *B*. Nevertheless, if the dwell times are predefined off-line (high enough to allow boarding an alighting operations), there is a well-defined domain of the pair of entrance times in route *A* and *B* that always provide cost-efficient solutions, similar to the best case.

The comparison of the bus performance and total cost among the different control strategies under analysis clearly recommends Strategy S3, based on a combination of short slacks in the timetable and traffic light priority for delayed buses. It is able to maintain an excellent bus regularity (coefficients of headway variation below 0.25) and reduces the total cost by 5%, in comparison with no control. It may allow transit agencies reduce the fleet size by 1-2 units. Strategies based on introducing slacks at holding points increases the fleet size needed and are not competitive in terms of operating cost. On the other hand, strategies only based on the green extension at traffic lights reduce the total cost at the expenses of increasing the lack of regularity of the service.

In this paper we have also corroborated the hypothetical effect that an increase of the route demand rates and the unit boarding time would have on the system metrics. This situation is likely to happen if more passenger car restrictions are implemented by city councils and dwell times are affected by new validation systems or access control (COVID-free protocols). In fact, the product of the boarding demand rates and unit boarding times is the term that propagates and amplifies any potential disturbance in the whole line. Results demonstrate that the unstable motion of buses is essentially affected by the unit boarding time. If we consider instantaneous boarding operations, the performance of the uncontrolled bus motion would present similar results to S3 control strategy. Nevertheless, the increment rate of the cost and the variation of headways with regard the unit boarding time is higher for large values of boarding times. Increments in the demand rates worsen the cost and regularity performance of bus system in the uncontrolled scenario. However, the implementation of hybrid control strategies based on slacks and traffic green extension is able to unbalance the increment on the total cost and maintains an outstanding level of service in terms of headway adherence.

The results also demonstrate that if the central trunk segment concentrates the vast majority of trip origins and destinations, both routes present similar headways and number of resources. Nevertheless, in the hypothetical situation that the central segment would not capture an important number of trips and, there would be one route whose branched segments concentrate more trips than the other coupled route, the assignment of resources is not obvious. The model tends to assign more resources to the less crowded route, to increase the frequency and capture the demand along the central segment. Doing this, the resources of the less crowded route can capture the demand of the central segment and both routes are balanced in terms of travel times and commercial speed. Finally, in the case that the central segment of the route would not practically generate or capture any trip, the coupled effect among lines would not be identified. Each line would be managed independently and, probably, other route schemes can provide a better service in terms of travel time and operating cost.

The model presented in this paper is aimed to reproduce the motion of buses with variable dwell times at stops in branched corridor layouts. Nevertheless, several parts of the modeling framework can be easily replaced by other formulation to better adapt to other vehicle technologies. The forecasting or, at least, the detection of passengers waiting at stops in real time may be crucial to estimate the time lost at stops. Based on this information, transit agencies may determine the best parameters of the control strategies, to tackle the potential deviations caused by the next boarding events.