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Power-Based Equivalent-Modeling Circuit for DC Linear Time-Invariant Resistive One-Port Networks

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ABSTRACT This paper introduces a new equivalent circuit for linear DC networks containing independent voltage and/or current sources and resistors, which represents all internal losses. Since the equivalent-modeling proposal does allow the determination of the efficiency of the original circuit or of all power dissipated in the internal resistors, it can be used for both the power and efficiency analysis of the real linear networks. The proposed equivalent circuit can be considered an extension to and differs from those proposed by Thévenin, and other subsequent proposals with regard to its conservativeness in relation to the actual circuit. Therefore, the paper also demonstrates that there is a direct relation between the model presented here and that proposed by Thévenin.

INDEX TERMS Circuits, circuit theory, equivalent circuit, power conservation, Thévenin's theorem, Thévenin equivalent circuit, efficiency of equivalent circuits.

I. INTRODUCTION

As is well known, electric networks can be studied from a general mathematical point of view. If the specific electrical properties are abstracted, there remains a geometrical circuit, characterized by sets of nodes, branches, and loops. In order to analyze these networks, there is a wide range of laws and theorems that allows obtaining all their variables and predicting the electrical behavior of them. Prominent among them is the Thévenin's theorem [1], [2], [3].

In 1883, Léon Charles Thévenin (1857-1926), a Telegraph Engineer, published his well-known paper which presents the theorem that would later become known as his theorem. This resulted in the Thévenin equivalent circuit for direct current networks [4], [5]. In particular, recently I. Barbi published an interesting article [6] in which the author discusses a new equivalent circuit that is an extension to and differs from that proposed by Thévenin with regard to its conservativeness in relation to the actual circuit. Therefore, as the author says, it serves to calculate the power transferred to a load resistor connected at the terminals, but, in addition, it also helps to model all internal dissipated power and efficiency of the actual circuit.

Although the procedure presented in [6], is quite logic, it have an important weakness: It is only valid when the power dissipated by the circuit in short-circuit conditions is greater than the power dissipated in open-circuit conditions. As a consequence, this paper suggest an enhancement to the analysis carried out in [6], since it serves to model all internal dissipated power and efficiency of the actual circuit in a more proper and accurate way by the addition of a single component. In fact, as will be pointed out below, the modeling proposal presented in this paper, compared with that presented in [6], has only one more current source. Apart from this, the proposed circuit model also allows to calculate the power transferred to a load resistor connected at the output terminals.

It is important to highlight that, as mentioned above, the proposed equivalent circuit can be considered an extension to and differs from those proposed by Thévenin and [6], with regard to its conservativeness in relation to the actual circuit. Therefore, the paper also demonstrates that there is a direct relation between the model presented here and that proposed by Thévenin, as detailed below.

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Notice that this paper introduces a new equivalent circuit for linear DC networks containing independent voltage and/or current sources and resistors, which represents all internal losses. Consequently, since the equivalent-modeling proposal does allow the determination of the efficiency of the original circuit or of all power dissipated in the internal resistors, it can be used in both the power and efficiency analysis of DC networks.

Therefore, the interest for practitioners or engineers can be focus on the analysis of DC microgrids [7]. Notice that some important issues in these microgrids are, among others, stability, protection and losses studies. All these points, for different applications such as those proposed by you (data centers, telecommunication, traction, shipboard power systems, etc.) can be study using equivalent-modeling circuits in which losses and efficiency should be incorporated to them.

The paper is structured as follows: In Section 2, we do a review of the circuit modeling proposal presented by Barbi in [6]; then, in Section 3, the derivation of the proposed circuit model carried out in this work is presented. We move on with a numerical example in Section 4. Finally, the article concludes in Section 6 with the main conclusions and the extension of the theorem presented in [6].

II. THE BARBI'S MODELING CIRCUIT PROPOSAL

Let the circuit in Fig. 1.*a* be a *well-defined*[1] [3] linear time-invariant resistive one-port network, containing independent voltage and current sources and resistors, with two external terminals $a-b$. In addition, as a load, an electrical resistance is connected at these output terminals. As it is well known, and according to [6], we have (1), which represents the basic equation of the 'classical' Thévenin equivalent circuit, shown in Fig. 1.*b*.

$$V_o = V_{TH} - R_{TH}I_o \quad (1)$$

However, notice that the internal dissipated power is not represented by the 'classical' Thévenin equivalent circuit because, according to this circuit, the input power is null when $I_o = 0\,A$. In fact, it serves only to model the power transferred to the aforementioned load resistor connected at the output terminals. As it is discussed in [6], review of publications since 1883 that present different theorems for analyzing circuits and networks reveals that none of them addresses the analysis of input power, internal losses and efficiencies of equivalent circuits for DC networks.

In [6], author discusses an equivalent circuit that is an extension to and differs from that proposed by Thévenin with regard to its conservativeness in relation to the actual circuit. Thus, it serves not only to calculate the power transferred to a load resistor connected at the terminals, but also to model all internal dissipated power and efficiency of the actual circuit. In this regard, the model proposed by author in this paper considers an equivalent circuit given by next equation:

$$P = P_X + R_{TH}I_o^2, \quad (2)$$

where P represents the sum of all internal network losses while P_X represents the internal losses for the disconnected external source, i.e., for $I_o = 0\,A$. Therefore, it is shown that the internal losses of a resistive direct current circuit have two components, according to the proposed model given in Fig. 2.

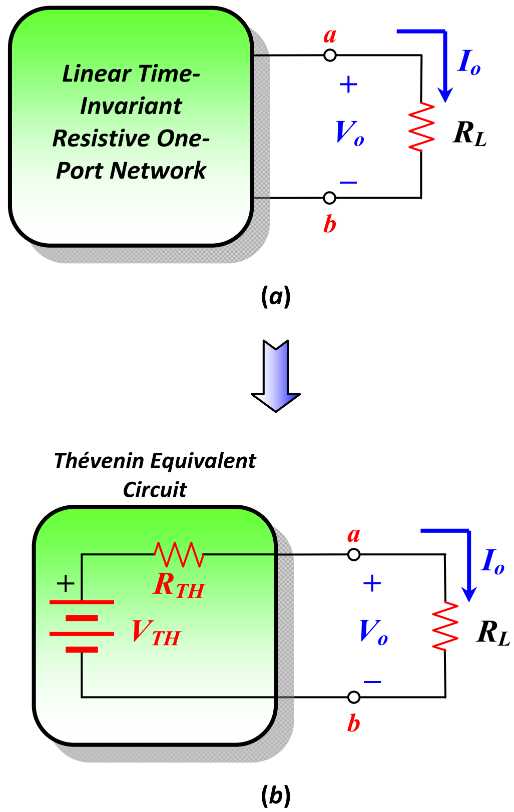

FIGURE 1. (a) Linear resistive network. (b) 'Classical' Thévenin equivalent circuit that consists of an independent voltage source V_{TH} in series with a resistance RTH.

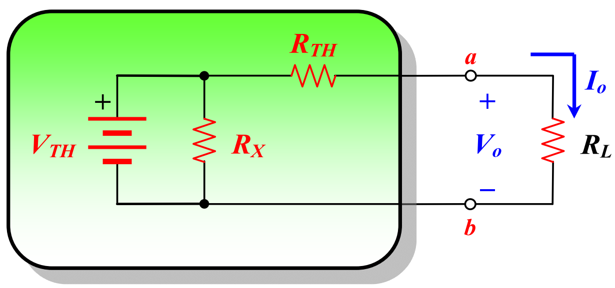

FIGURE 2. Thévenin equivalent model presented in [6].

According to this author, the term $R_{TH}I_o^2$ represents the internal power losses in the equivalent resistance R_T of the Thévenin equivalent circuit. Therefore, the author concludes that the internal losses have these two components. One the one hand, the first one is constant and independent of the

[1] According to [3], a one-port network is said to be well-defined if it does not contain any circuit element which is coupled, electrically or nonelectrically. To some physical variable outside it; e.g., controlled sources depending on a variable external to the network, transformer windings coupled magnetically to an external winding, a photoresistor coupled to an external light source, etc.

power delivered to the external load, which is equal to the internal power dissipated with the terminals $a-b$ open. On the other hand, the second one is variable and dependent on the current at the external terminals $a-b$.

Notice that the constant losses P_X are not represented by the 'classical' Thévenin equivalent circuit. Expression (2) represents the internal power of the equivalent circuit shown in Fig. 3, where V_T and R_T are the parameters of the Thévenin equivalent circuit and R_X is the additional resistance given in Fig. 3. This resistance R_X, which considers the internal losses that are independent of current I_o, is given by (3):

$$R_X = \frac{V_{TH}^2}{P_X} \tag{3}$$

Therefore, author proposed this theorem as an extension of the Thévenin theorem, with the following Corollary: "*In any direct current network with a pair of terminals, the power dissipated internally has two components, one being constant and dependent only on the internal sources and resistances and the other variable and dependent on the internal resistances and the power transferred to the load connected at the terminals*".

However, it starts from an important premise: According to (2) [6], the circuit dissipated power in open-circuited conditions (P_{OC}), that is, when $I_o = 0\,A$, and given by P_X in [6], is always smaller than the power in short-circuited conditions (P_{SC}), given by P; that is, the sum of P_X plus $R_{TH}I_o^2$ term. This can be appreciated in Fig. 3.

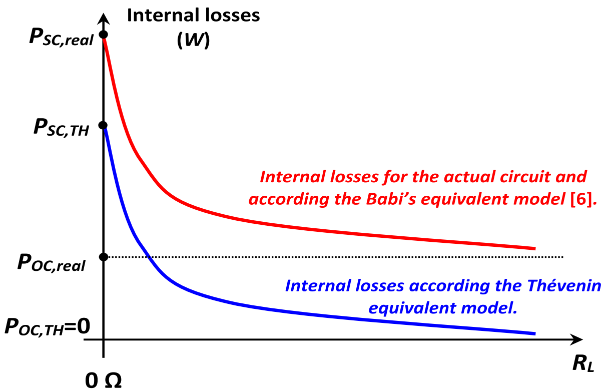

FIGURE 3. According to [6], linear circuit dissipated power in short-circuited conditions (P_{SC}) and, open-circuited conditions (P_{OC}), when Io = 0 *A*.

This supposition or initial premise is accomplished in lots of linear circuits as the example presented in [6]. However, it is important to highlight that this assumption is not always fulfilled for all linear DC networks containing independent voltage and/or current sources and resistors (especially when the circuit included current sources). In this context, notice that for some of these DC networks, the circuit dissipated power in open-circuited conditions (P_{OC}), could be higher than the power in short-circuited conditions (P_{SC}), as it can be presented in Fig. 4.

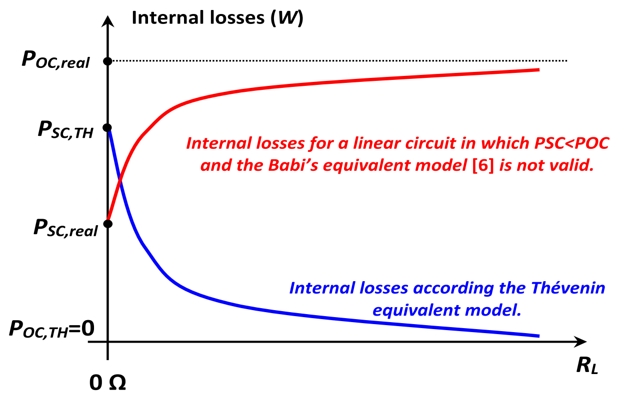

FIGURE 4. Internal losses for a linear circuit in which dissipated power in short-circuited conditions (P_{SC}) is smaller than that in open-circuited conditions (P_{OC}).

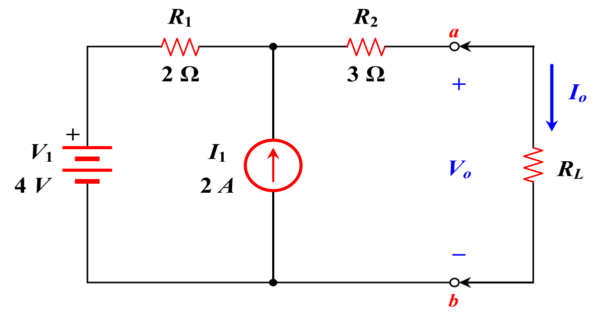

FIGURE 5. Circuit of the numerical example.

Indeed, let us now consider the circuit shown in Fig. 5, with the given values, i.e.: $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, $V_1 = 4\ V$, and $I_1 = 2\,A$. Maybe a more generic circuit would be more useful; however, the circuit analysis could hide the main objective of the paper (i.e., the derivation of the proposed model). In this spirit, the authors consider that the selected linear network, as a proof-of-concept, validates the power-based equivalent modeling circuit proposed by them. In particular, in these DC networks in which the dissipated power in open-circuited conditions (P_{OC}) is higher than the power in short-circuited conditions (P_{SC}), as it can be presented in Fig. 4 of the manuscript, and the Barbi's equivalent model presented in [6] is not valid. In this case, the authors prefer to push the complexity of the circuit and its numerical analysis into the background, and focus on the circuit model and highlight the process to derivate it.

In addition, notice that the example used in the manuscript is quite interesting since, apart from $P_{OC} > P_{SC}$ (and the Barbi's equivalent model presented in [6] is not valid), it has a special peculiarity to corroborate the proposed model: The internal dissipated power by the actual linear network, P_d (and, in fact, the proposed model) has a local minimum when $R_L = 1.66\ \Omega$ with $P_d = 8.8\ W$ (as the reader will see later in Fig. 7).

A quick analysis provides the values of the 'classical' equivalent circuit shown in Fig. 6.*a*, with:

$$V_{TH} = V_1 + I_1 R_1 = 4V + (2A \cdot 2\Omega) = 8V, \tag{4}$$

$$R_X = \frac{V_{TH}^2}{P_X} = \frac{(8V)^2}{16W} = 4\Omega$$

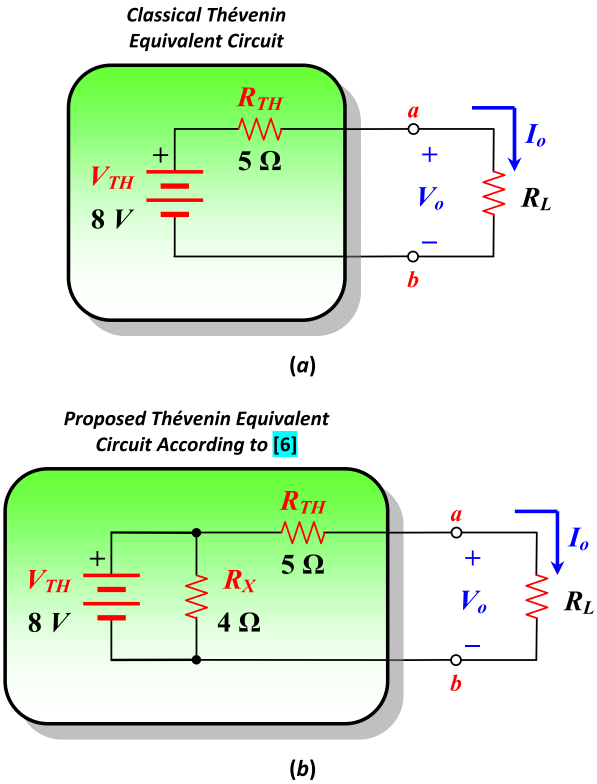

FIGURE 6. (a) 'Classical' Thévenin equivalent circuit, and (b) Proposed Thévenin equivalent model according to [6], for the circuit of the numerical example in Fig. 5.

and:

$$R_{TH} = R_1 + R_2 = 5\Omega \tag{5}$$

According to [6], we can add a third component, i.e., resistance R_X, that meets:

$$R_X = \frac{V_{TH}^2}{P_X}, \tag{6}$$

where P_X represents the internal losses for the disconnected external source, i.e., for $I_o = 0\,A$ (internally dissipated power in open circuit conditions, P_{OC}). Analyzing the circuit, this power is equal to:

$$\begin{aligned} P_X &= P_{OC} = V_1 I_1 + I_1^2 R_1 \\ &= 4V \cdot 2A + (2A)^2 \cdot 2\Omega = 16W \end{aligned} \tag{7}$$

Therefore, we have:

$$R_X = \frac{V_{TH}^2}{P_X} = \frac{(8V)^2}{16W} = 4\Omega \tag{8}$$

The equivalent circuit according to [6] is provided in Fig. 6.*b*. Using PSpice® simulations, a comparative simulation of the relevant involved powers by all three cases (actual linear network, according to the 'classical' Thévenin circuit, and to the model given in [6]), the circuit reveals that the proposed model in [6] fails for this circuit. In this regard, in Fig. 7 we can appreciated the power delivered to the output load and internally dissipated in all three cases, when output load varies between 10 $m\Omega$ (practically a short circuit for the parameters of the circuit), and 1 $k\Omega$ (almost an open circuit).

On the one hand, notice that the power delivered to the output load varies from 0 W when $R_L \to 0$ (i.e., when we have a short circuit), to a maximum when $R_L = R_{TH} = 5\ \Omega$, equal to 3.2 W, according to the maximum power transfer theorem. From this maximum point, if the output load continues increasing its value, the power delivered to it decreases newly to 0 W when $R_L \to \infty$ (i.e., when we have an open circuit).

In addition, the internal dissipated power by the actual linear time-invariant resistive one-port network varies from 9.6 W, which is the internal dissipated power in short-circuited conditions at the output terminals, P_{SC} (obtained thanks to Kirchhoff's first law equation for the circuit's highest node), to a minimum when $R_L = 1.66\ \Omega$, equal to 8.8 W. From this minimum point, and when the load continues increasing its value, the actual circuit dissipated power increases to a maximum value equal to P_{OC} obtained previously; i.e., 16 W when we have an open circuit at the output terminals.

According to the 'classical' Thévenin circuit the progress of the dissipated power in the linear network starts with a maximum values equal to 12.8 W when $R_L \to 0$ (short circuit at the output terminals), to a minimum equal to 0 W when $R_L \to \infty$ (open circuit at the output terminals). As a consequence, the internal dissipated power by the linear network is not represented by this 'classical' Thévenin equivalent circuit since, according to this model, the input power is null when $I_o = 0\,A$. In fact, as it is well known, it serves only to calculate the power *transferred* to a load resistor connected at the linear network's output terminals.

Finally, we also analyze the dissipated power according to the model given in [6]. This plot starts with a maximum values equal to 28.8 W when $R_L \to 0$ (short circuit at the output terminals), to a minimum equal to 16 W when $R_L \to \infty$ (open circuit at the output terminals). As it can be appreciated, the internal dissipated power is neither modeled by this proposal since, according to this model, the input power is maximum when I_o is maximum, $I_{o,max}$, and this model only matches the actual circuit dissipated power when $I_o = 0\,A$.

Thus, the rest of the current article presents and discusses a new proposal for an equivalent circuit model. This model is an extension to and differs from those proposed by Thévenin and [6] with regard to its conservativeness in relation to the actual linear time-invariant resistive one-port network. Thus, it serves not only to calculate the power transferred to a load resistor connected at the terminals, but also to model all internal dissipated power and efficiency of the actual network.

III. DERIVATION OF THE PROPOSED CIRCUIT

The modeling proposal presented in this paper, compared with that presented in [6], has only one more component. In particular, the modeling circuit proposal is presented in

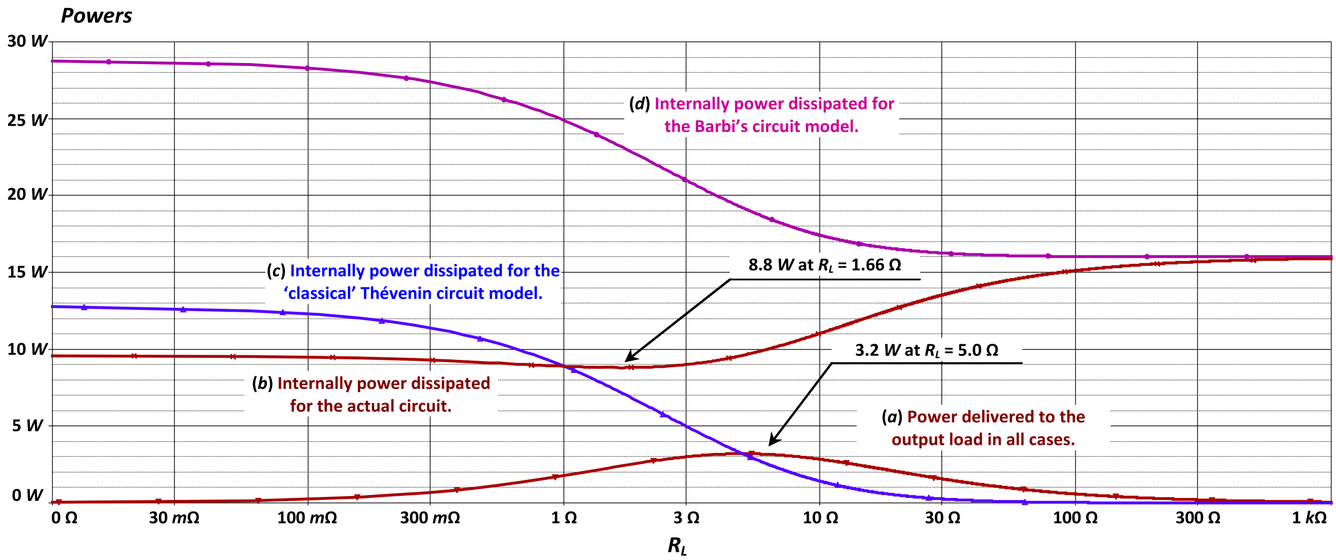

FIGURE 7. (a) Power delivered to the output load, and internal dissipated power (b) by the actual linear network, (c) according to the 'classical' Thévenin circuit, and (d) According to the model given in [6]. The output load varies between 10 mΩ (practically a short circuit for the parameters of the circuit), and 1 kΩ (almost an open circuit).

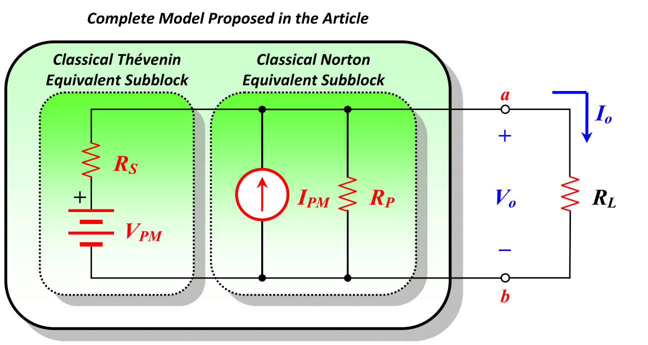

FIGURE 8. The equivalent model proposed in the article for linear time-invariant resistive one-port networks.

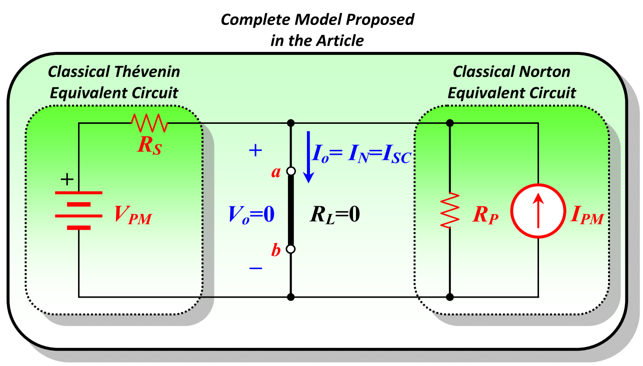

FIGURE 9. The equivalent model proposed in the article for linear time-invariant resistive one-port networks with the terminals a-b short-circuited.

Fig. 8. Notice that the circuit model proposed in this article consists of two subblocks with four components: The first one is a Thévenin network [4] formed by a voltage source V_{PM} plus the resistor R_S; the second one is a Norton linear network [8] formed by the current source I_{PM} plus resistor R_P.

Regarding the proposal, it is important to highlight that:

1) From the point of view of the load resistance R_L connected at the output terminals, the set constitutes an equivalent power model that can be used to calculate the power transferred to a load resistor (in the same way the 'classical' Thévenin or Norton models).
2) It additionally serves to model all internal dissipated power and efficiency of the actual network in a more proper and accurate way.

As a consequence, given a well-defined linear time-invariant resistive one-port network, the objective of the theoretical analysis is to obtain the proper values for the model's components (i.e., voltage source V_{PM}, current source I_{PM}, and resistors R_S and R_P) according to the two previous items. In order to calculate these values, it is necessary to state, from the point of view of the power balancing, the required equations to model the different states of the circuit.

On the one hand, when the output load is replaced by a short circuit (i.e., $R_L = 0\ \Omega$), the power delivered to it is obviously zero, and the actual circuit dissipated power under these conditions, P_{SC}, and according to the model given in Fig. 9, can be obtained thanks to:

$$P_{SC} = \frac{V_{PM}^2}{R_S} \tag{9}$$

Thus, the series resistor R_S can be obtained as:

$$R_S = \frac{V_{PM}^2}{P_{SC}} \tag{10}$$

In addition, also under these conditions, the proposed model circuit's components should also accomplish this other equation:

$$I_{SC} = \frac{V_{PM}}{R_S} + I_{PM}, \tag{11}$$

where I_{SC} is 'classical' Norton current, which coincides with the short circuit current with the output terminals $a - b$ in short circuit conditions (short circuit output current I_{SC}).

On the other hand, when the output load is disconnected (i.e., open circuit with $R_L = \infty$), the power delivered to it obviously is also zero, and the actual circuit dissipated power under these other conditions, P_{OC}, also according to the model given in Fig. 10, can be obtained by:

$$P_{OC} = \frac{(V_{PM} - V_{TH})^2}{R_S} + \frac{V_{TH}^2}{R_P}, \tag{12}$$

where V_{TH} is 'classical' Thévenin voltage, which coincides with the voltage across output terminals without output load (open circuit output voltage V_{OC}). In addition, also under these conditions, the proposed model circuit's components should also accomplish this other equation:

$$I_{PM} + \frac{V_{PM} - V_{TH}}{R_S} = \frac{V_{TH}}{R_P} \tag{13}$$

Finally, notice that both resistances of the proposed model need to conform to this expression:

$$R_{TH} = R_S || R_P = \frac{R_S R_P}{R_S + R_P}, \tag{14}$$

where R_{TH} is equivalent resistance of the 'classical' Thévenin equivalent circuit, which coincides with the electrical resistance measured from the terminals $a - b$ with the circuit passivated; i.e., voltage and current sources are set equal to zero (short- and open-circuited, respectively).

Therefore, after this theoretical analysis, and thanks to equations (9), (12), (13), and (14), it is easy to obtain the proper values of the voltage source V_{PM}, current source I_{PM}, and resistors R_S and R_P in order to complete the circuit model for the DC linear time-invariant resistive one-port network.

The proposed circuit in Fig. 8 does not have mathematical deduction. However, notice that an "intuitive thinking" based on power balance can be considered in order to propose it, in the same way as that proposed by Barbi's in [6]. In fact, the authors propose the model in Fig. 8 starting from what happens with the dissipated power under short-circuited and open-circuited conditions.

In particular, it is necessary to include the operating boundary conditions imposed in the circuit when we have a short circuit at the output terminal (i.e., $R_L = 0\ \Omega$), and when the output load is disconnected (i.e., open circuit with $R_L = \infty$), obtaining the actual circuit dissipated power under these two conditions, P_{SC} (given in (9)), and P_{OC} (expression (12)), respectively. Thus, from these two equations, the model proposed in Fig. 8 can be obtained.

IV. NUMERICAL EXAMPLE

As a proof of concept, the following numerical example illustrates the determination of the parameters of the proposed equivalent-modeling circuit. Let us newly consider the circuit shown in Fig. 5, with the same values considered previously; i.e.: $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, $V_1 = 4\ V$, and $I_1 = 2\ A$.

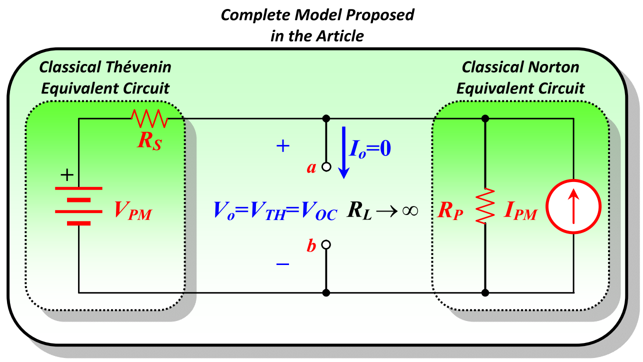

FIGURE 10. The equivalent model proposed in the article for linear time-invariant resistive one-port networks with the terminals $a - b$ open-circuited.

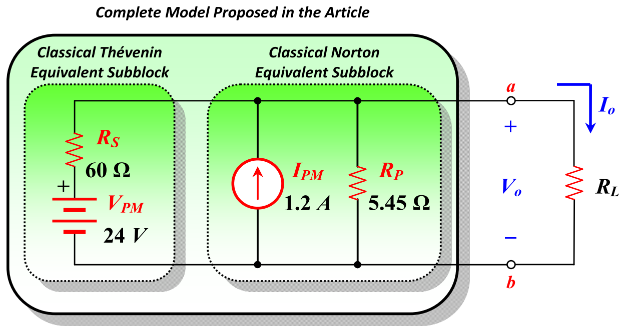

FIGURE 11. The equivalent model proposed for the numerical example discussed in the article.

As mentioned above, according to (9), (12), (13), and (14), we can obtain the proper values for the model's components (i.e., voltage source V_{PM}, current source I_{PM}, and resistors R_S and R_P).

After appropriate numerical substitutions, we obtain:

$$R_S = \frac{4V_{TH}^2 R_{TH}^2 P_{SC}}{\left[V_{TH}^2 + R_{TH}\left(P_{SC} - P_{OC}\right)\right]^2} \tag{15}$$

For our particular case, and considering the values previously obtained for circuit model's components ($V_{TH} = 8\ V, R_{TH} = 5\ \Omega$, actual circuit dissipated power under short-circuited conditions, $P_{SC} = 9.6\ W$, and open-circuited conditions, $P_{OC} = 16\ W$), leads to:

$$R_S = \frac{4(8V)^2(5\Omega)^2 9,6W}{\left[(8V)^2 + 5\Omega\,(9,6W - 16W)\right]^2} = 60\Omega \tag{16}$$

Calculated resistance R_S, and from equation (14), we obtain $R_P = 5.45\ \Omega$, thanks to (9) $V_{PM} = 24\ V$, and, finally, from (13) $I_{PM} = 1.2\ A$. As a result, the equivalent circuit proposed with its appropriate values is presented in Fig. 11.

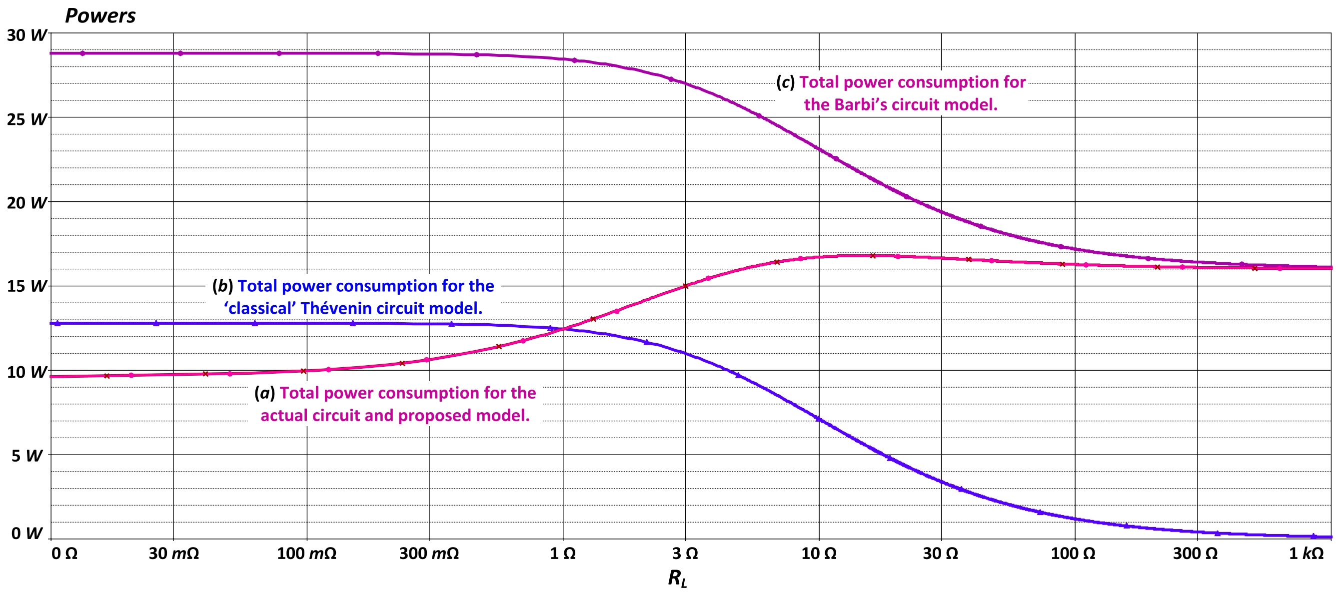

FIGURE 12. Total power consumption: (a) By the actual linear network and proposed model, (b) According to the 'classical' Thévenin circuit, and (c) According to the model given in [6]. The output load varies between 10 mΩ (practically a short circuit for the parameters of the circuit), and 1 kΩ (virtually an open circuit).

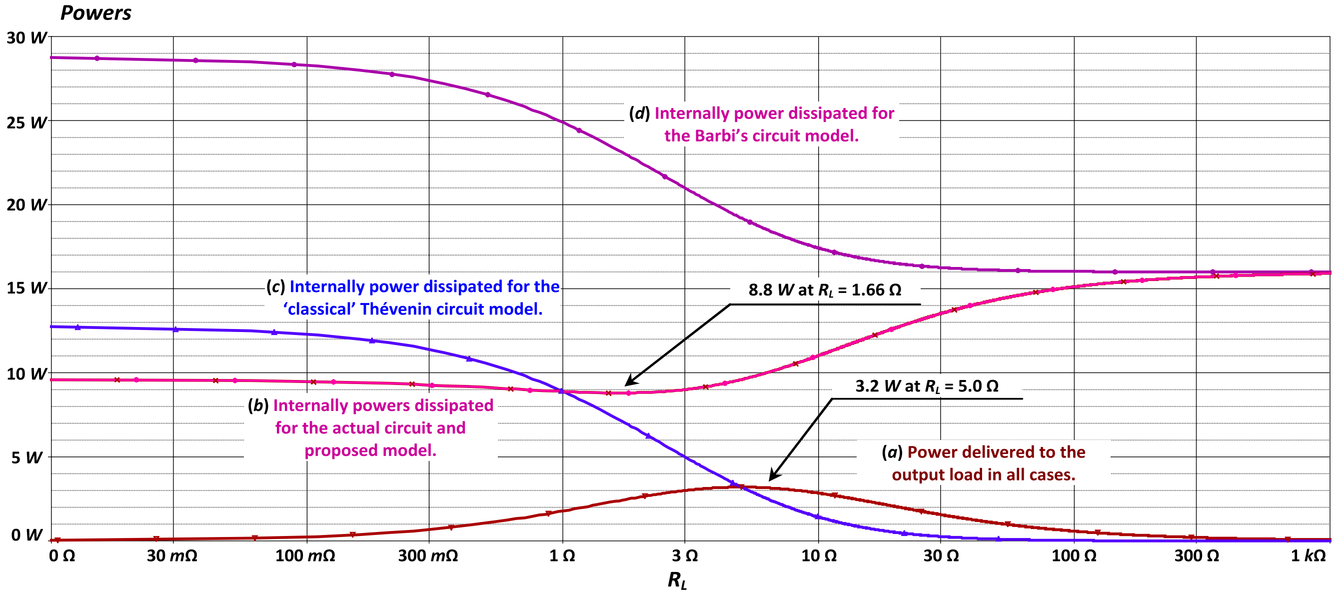

FIGURE 13. (a) Power delivered to the output load, and internal dissipated power (b) By the actual linear network and proposed model, (c) According to the 'classical' Thévenin circuit, and (d) According to the model given in [6]. The output load varies between 10 mΩ (practically a short circuit for the parameters of the circuit), and 1 kΩ (virtually an open circuit).

V. POWER AND EFFICIENCY RESULTS AND DISCUSSION

In order to corroborate the proposed model, PSpice ® simulation have been carried, including total power consumption, power delivered to the output load, and internal dissipated power by all three cases (i.e., the actual linear network, according to the 'classical' Thévenin circuit, to that given in [6] and the model proposed in the article. In this regard, in Fig. 12 we can appreciate the total power consumption in all four cases when output load varies between 10 $m\Omega$ (virtually a short circuit for the parameters of the circuit), and 1 $k\Omega$ (practically an open circuit). In this figure, we can see that the total power consumption for the actual circuit and proposed model match perfectly over the entire R_L value range.

In the same way that the results presented in Fig. 7, in Fig. 13 we can appreciate the power delivered to the output load and dissipated powers when output load also varies between practically a short circuit, and virtually an open circuit. As mentioned in Fig. 7, on the one hand, notice that the power delivered to the output load varies from 0 W when

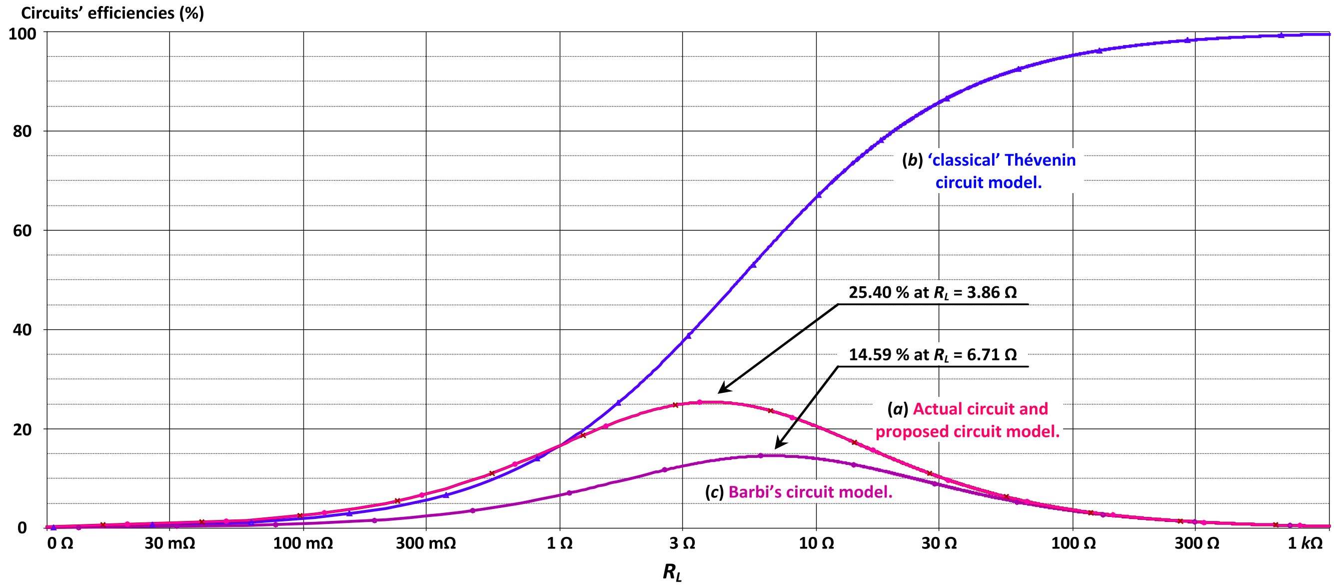

FIGURE 14. Efficiency curves of the original circuit and the different models examined in the article: (a) By the actual linear network and proposed model, (b) According to the 'classical' Thévenin circuit, and (c) According to the model given in [6]. The output load varies between 10 mΩ (practically a short circuit for the parameters of the circuit), and 1 kΩ (virtually an open circuit).

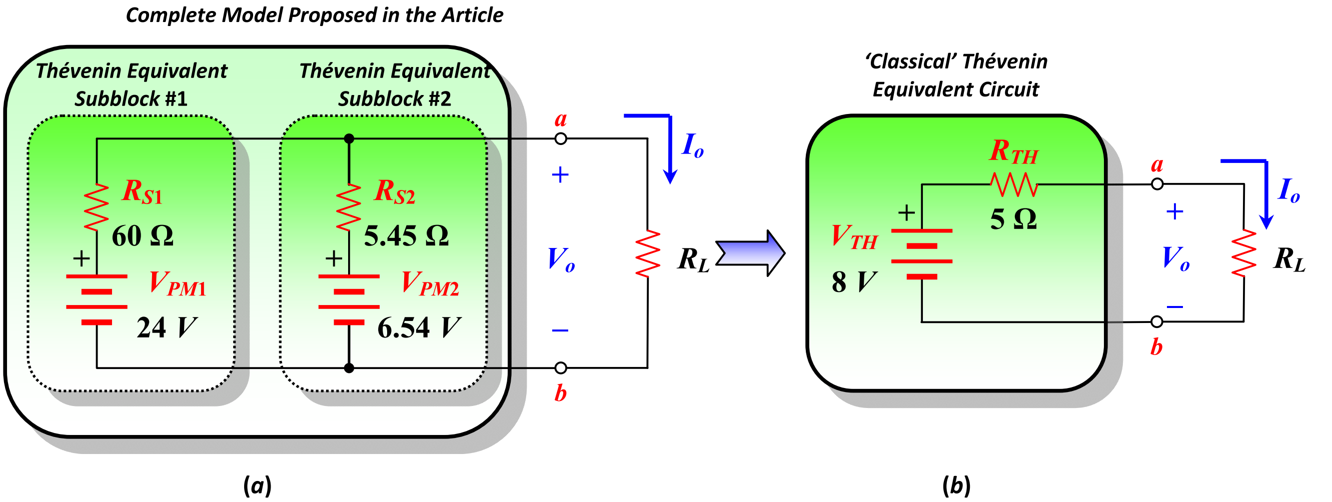

FIGURE 15. (a) Other equivalent model proposed, and (b) 'Classical Thévenin equivalent circuit for the numerical example discussed in the article.

$R_L \rightarrow 0$ (i.e., when we have a short circuit), to a maximum when $R_L = R_{TH} = 5\ \Omega$, equal to 3.2 W, according to the maximum power transfer theorem. From this maximum point, if the output load continues increasing its value, the power delivered to it decreases newly to 0 W when $R_L \rightarrow \infty$ (i.e., when we have an open circuit).

In addition, on the other hand, we can see the internal dissipated power in all four cases. Newly, notice that this power for the proposed model matches perfectly with that for the actual linear time-invariant resistive one-port network. In particular, in both cases, the plots vary in the same form: From 9.6 W when we have a short circuit at the output terminals, to a minimum when $R_L = 1.66\ \Omega$, equal to 8.8 W. From this minimum point, and when the load continues increasing its value, the circuit dissipated power increases to a maximum value equal to 16 W when we have an open circuit at the output terminals.

Finally, the efficiency curves of the original circuit and the different models examined in the article are presented in Fig. 14. Notice that the plot for proposed equivalent model's efficiency is the same than that for the original network. However, they are different from the efficiency curve of the Thevénin equivalent circuit and Barbi's models.

As mentioned above, and in the same way as [6], the proposed equivalent circuit can be considered an extension of the Thévenin equivalent circuit. Furthermore, notice that, from the proposed circuit model, we can obtain an equivalent Thévenin circuit. Indeed, from the proposed model in Fig. 11, and thanks to the Thévenin equivalent circuit for the Norton subblock, an alternative circuit is shown in Fig. 15.*a*. Thus, from the proposed model's components values (Fig. 11), notice that the 'classic' Thévenin circuit's components has the following values (Fig. 15.*b*):

$$R_{TH} = R_{S1}||R_{S2} = R_S||R_{P2} = \frac{R_S R_P}{R_S + R_P} = 5\Omega, \quad (17)$$

and:

$$V_{TH} = \frac{V_{PM1}R_{S2} + V_{PM2}R_{S1}}{R_{S1} + R_{S2}} = \\ = \frac{24V \cdot 5,45\Omega + 6.54V \cdot 60\Omega}{5,45\Omega + 60\Omega} = 8V \quad (18)$$

As a summary, the proof of the theorem formulated below can be presented. This theorem can be considered an extension of the Thévenin's and Barbi's theorems.

Theorem: Any linear DC network consisting of resistors and independent voltage and/or current sources, with two accessible terminals, can be replaced by an equivalent circuit that is power-conservative. This circuit consists of a DC equivalent Thévenin subblock (voltage source and series resistor) plus a DC equivalent Norton subblock (current source and parallel resistor). The first one's components values depends on the actual circuit dissipated power under short-circuited, P_{SC}, and open circuited conditions, P_{OC}, while the second one's components values depends only on the actual circuit dissipated power under these other conditions, P_{OC}.

VI. CONCLUSION

This paper has introduced a new equivalent circuit for linear DC networks containing independent voltage and/or current sources and resistors, which represents all internal losses. Since the equivalent-modeling proposal does allow the determination of the efficiency of the original circuit or of all power dissipated in the internal resistors, it can be used in both the power and efficiency analysis of DC networks.

From the point of view of the output load, the paper has demonstrated that there is a direct relation between the model presented here and that proposed by Thévenin. However, the proposed equivalent circuit can be considered an enhancement to and differs from that proposed by Thévenin, with regard to its energy conservation in relation to the actual circuit. In addition, it completes the model presented in [6].

In conclusion, in the previous section, a proof of the theorem formulated has been presented, considering it an extension of the Thévenin's and Barbi's theorems.

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