Technical Note

Hydrocavitation piezoelectric ocean wave energy harvesting

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The possibility to convert the ocean wave energy into electrical energy by piezoelectric layers has excited the imagination of ocean wave energy conversion designers for decades owing to its relative robustness (no mechanical parts are needed), the capability to cover large areas and its relative low cost. Unfortunately, the very poor efficiency featured by piezoelectric layers in application of ocean waves has prevented its application even as energy harvester. Here, the possibility to induce hydrocavitation and then working with more higher local pressures for substantial efficiency enhancement is discussed. Utilizing a simplified geometrical and physical model and the linear and potential theory, a first theoretical estimation for the energy enhancement driven by hydrocavitation was calculated. It was found that the power could be enhanced several orders of magnitude which, although still rather low, however, the enhanced electric outputs can be used now as energy harvesters. Additional R&D is encouraged in order to explore the possibilities to harness hydrocavitation to enhance piezoelectric converters.

Keywords. *Ocean wave energy conversion; Piezoelectricity; Hydocavitation; Renewable energy,*

I. INTRODUCTION

Whereas piezoelectric energy harvesting has attracted wide attention from researchers particulary in the last decade owing to its advantages such as high power density, architectural simplicity, and scalability, [1]-[3], however its application to ocean wave energy conversion has been limited. In fact, the idea of piezoelectric conversion for ocean wave energy conversion seems an almost unavoidable idea owing to the oscillatory a nature and then pressure fluctuations of the waves. Back to the 1970s Burfoot and Taylor presented a complete study of the theory and application of piezoelectric, [4], and Taylor (1970), gave a preliminary approach for piezoelectric power generation from ocean waves,[5]. Unfortunately, in that early work it was found that piezoelectric materials were not attractive for wave energy conversion due to its very poor efficiency -as poor as $\simeq 10^{-13}$ for typical ocean waves [6]. Since then, extensive research in the field have been carried out in order to improve the efficiency. Several strategies to tackle the problem have been investigated but they may be classified as those strategies addressed to improve the properties of the piezoelectric materials (transverse or compressional materials) including the use of new innovative materials, [7], and those strategies tackling the problem by mechanical design, as for example, [8]-[10]. Nevertheless, even with the substantial improvements in the field the efficiency is still very poor and its use even as energy harvester for very small amount of power for, say, low-energy electronics is unattractive.

The object of this work was to analyze a novel hydrodynamic approach which may potentially boost the piezoelectric efficiency in a considerable manner. Because piezoelectric energy is related with the square of the pressure, and because hydrocavitation larger than the surrounding pressure, then if instead of using the direct fluctuations of pressure by the ocean wave it is induced hydrocavitation near the piezoelectric layer a very important amplification of piezoelectric energy seems possible

II. MATERIALS AND METHODS

In this section a first estimation of the potential for piezoelectric ocean wave harvesting by inducing hydrocavitation will be inferred. To begin with, the most simple physical model will be analyzed which although does not correspond to the most effective optimized design, nonetheless will provide an upper limit and guidance for further future research.

With such a goal, let us assume a single symmetrical duct immersed horizontally at a certain depth from the surface of water. The duct has a throat in the middle as schematically depicted in Fig. 1 which by continuity will accelerate the fluid and then inducing cavitation when pressure is reduced below the local vapor pressure. Now, if the length of the duct has a length which is equal to the ocean traveling wavelength, then for a duct with a small cross section area it is permissible to assume that the horizontal velocity at the inlet of the duct will be equal to the velocity at the outlet at any time and also their pressures. Let us call the velocity,

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pressure and cross section area at the inlet/outlet as v_e; p_e, and A_e, respectively. With this configuration, it is easy to see that the function of the central throat will be the acceleration of the fluid which is coming from the inlet and then reducing its pressure in the center where cavitation is desired, and after passing the throat, the fluid will recover the pressure and the velocity and will be exhausted at the outlet which because is just at a wavelength distance from the inlet will be equal velocity or very similar and then causing the minor distortion. The objective is to calculate the reduction of pressure in the throat as function of time and then when cavitation takes place and during which fraction of a wave period. It is clear that the configuration presented is an ideal model which is unavoidable if analytical expressions are pursued, and therefore the results are not intended to typify estimates but rather to give an upper limit. Effects of the duct on the flow can translates into a departure from a linear-potential theory (nor irrotational neither axysymmetric flow), performance should be assessed over a range of wave frequencies, etc. A complete analysis will require studies coupled with computational simulations in which, for example, performance over a range of wave frequencies, etc, which is out of the scope of the present note, which is a first assessment.

Referring to Fig. 1, let us z be the vertical coordinate with its origin on the Still Water Level SWL and negative in the downward direction; and x as the horizontal direction. The water depth being h.

A. The pressure profile at the throat

Let us call the pressure at the central throat as p_c and its velocity as v_c.

The fluid is incompressible and not stationary and within the linear-theory approximation is also nonrotational. For a first upper limit estimation we will neglect primary and secondary losses in the duct. Thus, within the conventional linear approximation the pressure in the throat can be calculated as function of the inlet velocity and pressure by applying the Bernoulli *energy equation*

$$p_e + \frac{\rho v_e^2}{2} + \rho\frac{\partial \varphi_e}{\partial t} = p_c + \frac{\rho v_c^2}{2} + \rho\frac{\partial \varphi_c}{\partial t} \qquad (1)$$

where p_e and v_e are the *pressure* and *horizontal velocity* at the inlet and outlet section at a given *time* t, respectively; and p_c and v_c the pressure and velocity in the throat section at the same time, respectively; ρ is the fluid density; and φ_e and φ_c are the *velocity potentials* for the inlet and the throat section, respectively. From the equation of *continuity*

$$A_e v_e = A_c v_c \qquad (2)$$

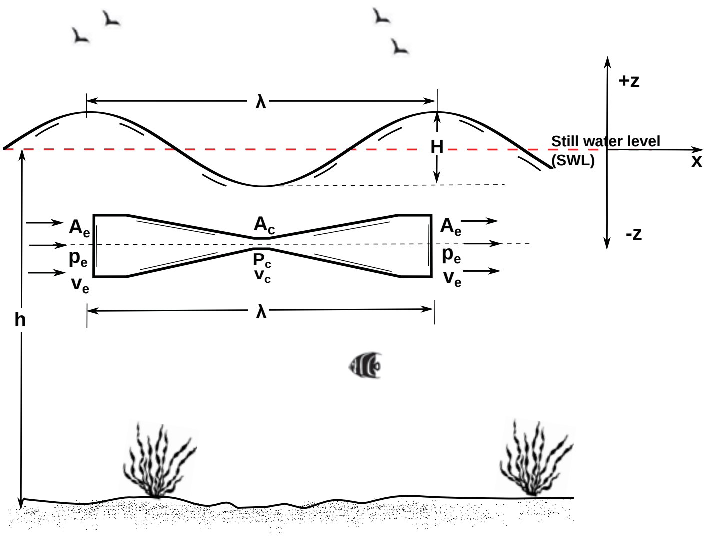

FIG. 1: Physical model used for energy conversion driven by hydrocavitation.

where A_e and A_c are the cross section flow areas of the inlet and the throat, respectively. In addition, if the duct doesn't affect the pressure and velocity conditions at the inlet which is allowable if its cross section is comparatively small, then the pressure and velocity at the inlet (or outlet which is located at a wavelenght of distance) of the duct are the same than the surrounding. Therefore, the *pressure* p_e; *velocity* v_e and the *velocity potential* φ_e at the inlet/outlet will be approximately the same than the wave at that location and time, let us called just as p, v and φ. Therefore, Eq.(1) becomes

$$p_c = p - \frac{\rho v^2}{2}\left[\frac{A_e^2}{A_c^2} - 1\right] + \frac{\rho\partial\varphi}{\partial t} - \frac{\rho\partial\varphi_c}{\partial t} \qquad (3)$$

Likewise the *velocity potential* $\varphi_c(t)$ are approximated as

$$\varphi_c \simeq \left(\frac{A_e}{A_c}\right)\varphi_e$$

$$\simeq \left(\frac{A_e}{A_c}\right)\varphi \qquad (4)$$

Inserting Eq.(4) into Eq.(3) one obtains

$$p_c = p - \frac{\rho v^2}{2}\left[\frac{A_e^2}{A_c^2} - 1\right] - \left[\frac{A_e}{A_c} - 1\right]\frac{\rho\partial\varphi}{\partial t} \qquad (5)$$

On the other hand, also from *Bernoulli´s equation* we know, Streeter, [11]

$$-\frac{\rho\partial\varphi}{\partial t} = p + \rho g z + \frac{\rho v^2}{2} \tag{6}$$

where g is gravity and z is the depth coordinate being negative beneath the *still water level* (SWL), (see Fig. 1).

Taking into account Eq.(6) into Eq.(5) and after simple arrangement of terms we obtain

$$p_c = p\frac{A_e}{A_c} - \frac{\rho v^2}{2}\left[\frac{A_e^2}{A_c^2} - \frac{A_e}{A_c}\right] + \rho g z\left[\frac{A_e}{A_c} - 1\right] \tag{7}$$

Finally, the pressure at any point beneath the ocean wave is given by [6]

$$p = p_o \cos(\kappa x - \omega t) - \rho g z \tag{8}$$

where $\kappa = \frac{2\pi}{\lambda}$ is the *wave number* being λ the *wave-length*, $\omega = \frac{2\pi}{T}$ the *circular wave frequency* being T the *wave period*; g is *gravity*, and

$$p_o = \frac{\rho g H}{2}\frac{\cosh(\kappa z + \kappa h)}{\cosh(\kappa h)} \tag{9}$$

where H and h are the *wave height* and water depth, respectively. Likewise, the *horizontal velocity component* at any point beneath the wave is given by [6]

$$v = v_o \cos(\kappa x - \omega t) \tag{10}$$

where

$$v_o = \frac{\pi H}{T}\frac{\cosh(\kappa z + \kappa h)}{\sinh(\kappa h)} \tag{11}$$

for the sake of compactness, if one define the origin or the *x-coordinate* at the inlet, i.e., $x = 0$, then Eq.(11) is rewritten as

$$p_c \simeq a\cos(\omega t) - b\cos^2(\omega t) - \rho g z \tag{12}$$

where

$$a = \left[\frac{A_e}{A_c}\right] p_o \tag{13}$$

and

$$b = \left[\frac{A_e^2}{A_c^2} - 1\right]\frac{\rho v_o^2}{2} \tag{14}$$

• **Discussion**

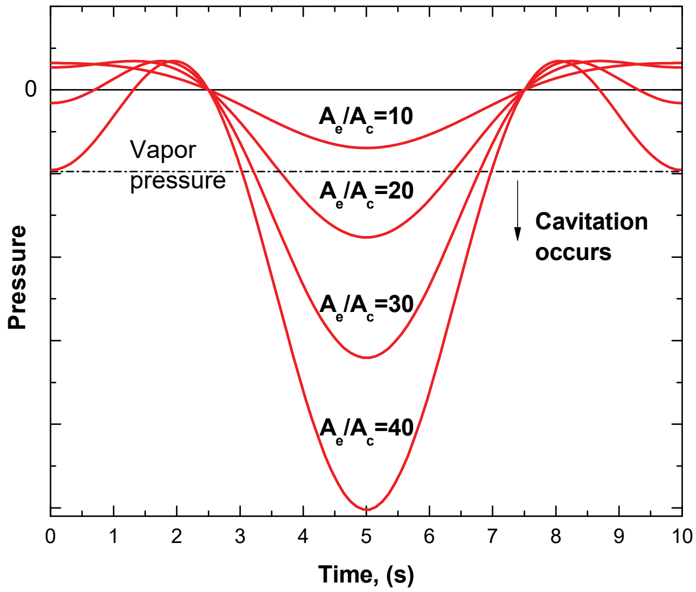

FIG. 2: Time during which cavitation occurs in the throat according with Eq.(12)

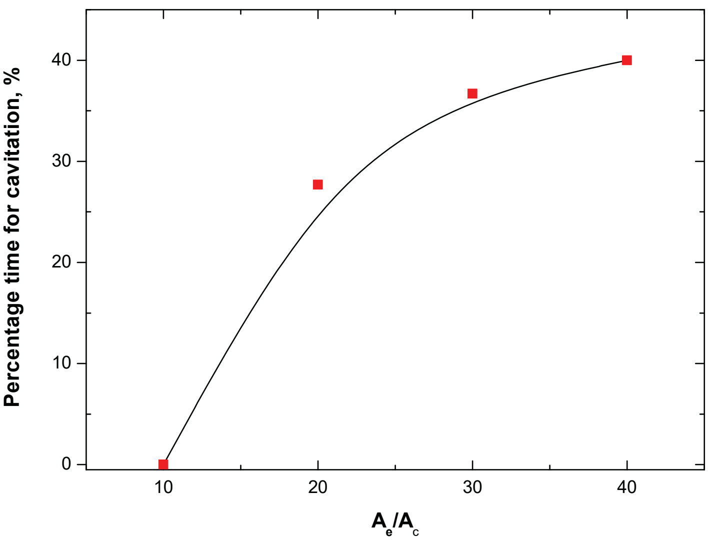

FIG. 3: Fraction of time during which cavitation occurs as a function of the ratio $\frac{A_e}{A_c}$

Eq.(12) allows us to know the pressure in the throat at a given time, where, of course, cavitation only will occur in the interval of time when the throat pressure p_c drops below the local vapor pressure e_v, i.e, when $p_c < e_v$. Then, Eq.(12) will allow us to calculate the fraction of time during a period T in which cavitation actually occurs and to define the *average power* converted by the piezoelectric converter.

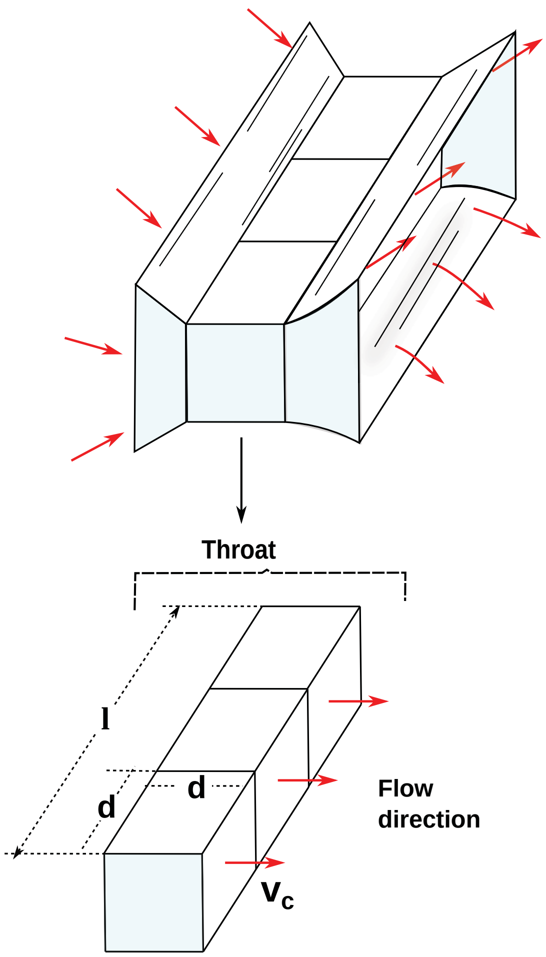

FIG. 4: Physical model of the throat.

In order to obtain some idea of interval of time during which cavitation occurs in the throat as predicted by Eq.(12), we assume some typical values of the parameters considering an average ocean swell with an amplitude $H = 1$ m height, a depth of water $h = 9.5$ m, at a depth $z = 2$ m; a wave period $T = 10$ s; and a wavelength $\lambda = 95$ m; $\rho \simeq 10^3$ kg/m^3; $g = 9.8$ m/s^2. The resulting curves are shown in Fig. 2 for ratios $\frac{A_e}{A_c} = 10$ 10, 20, 30, and 40. From Fig. 2 it is easy to see, that cavitation only occurs in certain fraction of the period of the wave and is not possible for $\frac{A_e}{A_c} < 15$ or thereabouts. Fig. 3 is the fraction of time in which cavitation occurs in Fig. 2.

B. The energy conversion

The energy conversion of a piezoelectric layer with surface area A_{pz} is calculated as, [5]

$$E_{pc} = c_{pz}\delta_{pz}p_\infty^2 A_{pz} \tag{15}$$

where c_{pz} is a parameter of the specific piezoelectric material properties (transverse or compressional material); δ_{pz} is the thickness of the piezoelectric material; and p_∞ is the surrounding pressure.

Focusing our attention in the throat which is now pictorially shown in Fig. 4. In the left side the flow is being converged toward the throat and after is exhausted at the right side where a sudden expansion occurs. Now, if the width of the throat is similar than the diameter of the bubble cavity d then every time that a volume of fluid $\simeq d^3$ is passing through the throat the cavitation of a single bubble occurs. Therefore, the flux of bubbles, let us call Φ_c, in the throat is given by

$$\Phi_c \simeq \frac{v_c l}{d^2} \tag{16}$$

Because each bubble -immediately after collapse, will produce a pressure blast p_{pz} covering an area of the piezoelectric layer approximately $A_{pz} = d^2$ then from Eq.(15) the energy per bubble-cavitation assuming a double piezoelectric stack (top and bottom) is given by

$$E_{pz} \simeq 2c_{pz}\delta_{pz}p_\infty^2 d^2 \tag{17}$$

and the total power w_l per unit length of the piezoelectric stack given by

$$w_l \simeq E_{pz}\Phi_c \tag{18}$$

taking into account Eq.(16) and Eq.(17), Eq.(18) yields

$$w_l \simeq 2c_{p_\infty}\delta_{pz}p_\infty^2 v_c \tag{19}$$

- **Hydrocavitation**

Hydrocavitation is a phenomenon in which the static pressure of the liquid reduces to below the liquid's vapour pressure, leading to the formation of small vapor-filled cavities or bubble in the liquid. Then, when the small bubbles are subjected to higher pressure, these bubbles collapse and can generate shock waves with large pressure peaks strong enough to cause significant damage to parts, and for this reason cavitation is typically an undesirable phenomenon in engineering,[12],[13]. The peak pressure p_b is strong when they are very close to the imploded bubble of radius, say, R_m, but rapidly weaken as they propagate away from the implosion, Fujikawa and Akamatsu (1980), [14] found the following relationship for the pressure as function of the radial distance from the bubble immediately after implosion

$$p_b = 100 \times \frac{R_c}{r} p_\infty \tag{20}$$

where $R_c = \frac{d}{2}$ is the radius of the bubble (immediately before to start the collapse), r is the radial distance from the origin of the bubble and p_∞ is the surrounding pressure in which the bubble implodes. It can be seen that the effect which we want to be harnessed is precisely the amplification of the surrounding pressure by the shock wave. From Eq.(20) it is seen that at a distance near to the size of the bubble $r = R_c$ the pressure from the blast wave can be 100 times higher than the surrounding pressure. The surrounding pressure in which the implosion occurs can be assumed as the local vapor pressure, e_v, i.e., $p^2_\infty = e^2_v$. Then, assuming a throat with a gap just with the dimensions of the bubble and then $r = R_m$ considering the above and the Eq.(20), Eq.(19) yields

$$w_l \simeq 2 \times 10^4 c_{pz} \delta_{pz} e^2_v v_c \tag{21}$$

which inserting Eq.(10) with $x = 0$, becomes

$$w_l \simeq 2 \times 10^4 c_{pz} \delta_{pz} v_o \left(\frac{A_e}{A_c}\right) e^2_v \cos(\omega t) \tag{22}$$

Finally, the *average power per unit of length* over one wave period is given by

$$\tilde{w}_l = \frac{1}{T}\int_0^T w_l dt \tag{23}$$

However, because the piezoelectricity power obtained when there is not cavitation will be negligible in comparison with the obtained during cavitation, then it can be omitted in the integration and if the interval of time in which cavitation occurs is between t_1 and t_2, Eq,(23) is approximated as

$$\tilde{w}_l \simeq \frac{1}{T}\int_{t_1}^{t_2} w_l dt \tag{24}$$

which after integration yields

$$\tilde{w}_l \simeq \frac{10^4 c_{pz} \delta_{pz} v_o e^2_v}{\pi} \left(\frac{A_e}{A_c}\right) [\sin(\omega t_2) - \sin(\omega t_1)] \tag{25}$$

From Fig. 2, it is easy to see that the time of cavitation during a period of wave is centered at $t = \frac{T}{2}$, and then if Δt_c is the total time of cavitation, $t_1 = \frac{T-\Delta t_c}{2}$ and $t_2 = \frac{T+\Delta t_c}{2}$ and after applying the product-to-sum identity $\sin(\alpha+\beta) - \sin(\alpha-\beta) = 2\cos(\alpha)\sin(\beta)$, Eq.(25) becomes

$$\tilde{w}_l \simeq \frac{2 \times 10^4 c_{pz} \delta_{pz} v_o e^2_v}{\pi} \left(\frac{A_e}{A_c}\right) \sin\left(\frac{\Delta t_c}{2}\right) \tag{26}$$

- **Discussion**

In order to obtain an idea of the power per unit of length predicted by Eq.(26) we will analyze the case for the wave already studied for $\frac{A_e}{A_c} = 40$. The only additional factor which is needed in Eq.(26) is the piezoelectric parameter c_{pz} which takes into account the specific properties of the piezoelectric material (transverse or compressional). Unfortunately, since the early works by Taylor (1979), [5], on piezoelectric power generation from ocean waves, not much substantial technological improvements have been reported, and the values in any case are in the same order. For example, piezoelectric transverse materials (for example a honeywell composite) $c_{pz} \simeq 2 \times 10^{-10}$ m^2/N and $c_{pz} \simeq 4.6 \times 10^{-12}$ m^2/N for a transverse material (Kureha piezofilm). Thus, taking a best optimistic value $c_{pz} \simeq 2 \times 10^{-10}$ m^2/N and with a typical thickness of the piezoelectric material $\delta_{pz} = 50$ μm. The resulting curve is shown in Fig. 5. It is seen that the *average power per unit of length* is on 7.5 mW/m. Although this power is rather small, however, it must be put into the context. The *average power per unit of length* from an ocean wave $\tilde{w}_w$, which for the case of study corresponds to a shallow water, is given by [6]

$$\tilde{w}_w = \frac{\rho g^{\frac{3}{2}} h^{\frac{1}{2}} H^2}{8} \tag{27}$$

For our case of study this results into a linear power $\tilde{w}_w \simeq= 11.8$ kW/m and then our *efficiency* ϵ is around 6.3×10^{-7}. Nevertheless, this small value represents, actually, a considerable quantum leap if compared with the traditional piezoelectric ocean wave conversion, if one considers that for the same wave using the convectional piezoelectric approach (without hydrocavitation) the efficiency is around 3.24×10^{-13}, [6] i.e., the enhancement in efficiency is in the order of 2×10^6.

We can say, that although the use of piezoelectric is still no feasible for large wave energy generation, however, hydrocavitation open its potential use as energy harvester for capture and store small, wireless autonomous devices, like those used in wearable electronics and wireless sensor networks or to light buoys.

III. CONCLUSIONS

The use of hydrocavitation to boost piezoelectric conversion in ocean wave energy was proposed and discussed. Some important conclusions are raised by the preliminary work as follows.

a) Hydrocavitation could increase the efficiency for piezoelectric conversion several orders of magnitude in comparison with traditional studies

b) Despite that the efficiency is still small, however, hydrocavitation allows the use of hydrocavitation as energy harvester for capture and store small, wireless autonomous devices.

c) Linear average power was enhanced up to $\simeq 10$ mW/m.

c) Additional R&D is encouraged in order to explore the possibilities to harness hydrocavitation to enhance piezoelectric converters.

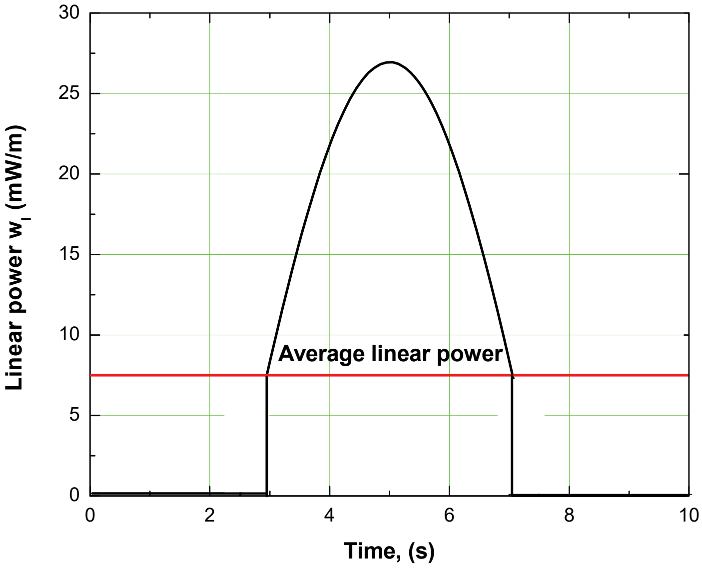

FIG. 5: Piezoelectric linear power driven by hydrocavitation during over one wave period and with ratio $\frac{A_e}{A_c} = 40$

NOMENCLATURE

a = parameter defined by Eq.(13)
A = area
b = parameter defined by Eq.(14)
d = width of the throat, diameter bubble
e_v = vapor pressure
g = gravity
h = water depth
H = wave height
k = secondary loss pressure factor
l = length throat
p = pressure
R_c = radius of the bubble
v_o = amplitude pressure defined by Eq.(9)
t = time
v = velocity
v_o = amplitude velocity defined by Eq.(11)
w = linear average power
x = longitudinal co-ordinate
z = vertical co-ordinate

Greek symbols

φ = velocity potential
κ = wave number
λ = wavelength
ρ = density
ω = wave frequency

subscripts

c = throat zone
d = downstream
e = inlet
l = linear
pz = piezoelectric
∞ = surrounding

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