Permutation entropy analysis of the output of a laser diode under stimulated Brillouin scattering optical feedback

LEIDY JOHANA QUINTERO-RODRÍGUEZ,[1] IGNACIO ENRIQUE ZALDÍVAR-HUERTA,[1] YANHUA HONG,[2] CRISTINA MASOLLER,[3] AND MIN WON LEE[4],*

[1] *Dep. de Electrónica, Instituto Nacional de Astrofísica, Óptica y Electrónica, Puebla 72840, Mexico*
[2] *School of Computer Science and Electronic Engineering, Bangor University, LL57 1UT Wales, UK*
[3] *Departament de Física, Universitat Politècnica de Catalunya, 08222, Terrassa, Barcelona, Spain*
[4] *Laboratoire de Physique des Lasers CNRS - UMR7538, Université Sorbonne Paris Nord, 93430 Villetaneuse, France*
*min.lee@univ-paris13.fr

Abstract: The chaotic output emitted by a diode laser with optical feedback has fascinated the community for decades. The external cavity delay time imparts a weak level of periodicity to the laser output (the so-called "time delay signature", TDS) that is a drawback for applications that require random optical signals. A lot of efforts have focused in suppressing the TDS either by post-processing the signal or by using alternative ways to generate random optical signals. Here, we compare the signals generated by two optical feedback setups: in the first one, the stimulated Brillouin backscattered light from a standard optical fibre is re-injected into the laser (stimulated Brillouin scattering optical feedback, SBSOF); in the second one, the light transmitted through the fibre is re-injected into the laser (conventional optical feedback, COF). We analyse the permutation entropy, a well-known measure of complexity that captures order relations between values of a time series. We find that, on average, the signal generated by the SBSOF setup has slightly lower PE than the one generated by the COF setup, except when the sampling time of the intensity signal is an exact multiple of the delay; in that case, due to TDS, the entropy of the COF signal is lower than that of the SBSOF signal. We interpret the lower entropy value of the SBSOF signal as due to oscillations at the Brillouin frequency shift. Taken together, our results show that TDS suppression can have an undesirable side effect: a decrease of the entropy of the signal.

1. Introduction

Chaotic dynamics in laser diodes subject to optical feedback has been widely studied for decades [1]. The complexity of chaotic dynamics has been of great interest in many applications [2]. Complex dynamics have been exploited in reflectometry applications [3,4], reservoir computing [5,6], rogue wave studies [7,8] and random number generation [9–11].

Chaotic dynamics in laser diodes subject to optical feedback exhibit a weak level of periodicity which arises from the round-trip delay time in the feedback cavity. This periodicity has been referred to as "time delay signature" (TDS) [12] and is a drawback in applications that require fully random or chaotic optical signals. In Ref. [13] high speed random bit generation (640 Gbit/s) was demonstrated by choosing only 4-least significant bits of the 8-bit digitization of a chaotic signal generated by a monolithic integrated dual-mode amplified feedback laser; however, post-processing was needed to suppress TDS.

A lot of efforts have focused on suppressing TDS, not only by using signal post-processing, but also, hardware alternatives: different feedback configurations have been proposed that have different difficulties, limitations and challenges. Self-interference of chaotic signals has

been shown to suppress TDS [14], but it requires two high-speed photodetectors. Brillouin backscattering of optical fibre was examined in Ref. [15], where the chaotic light generated by an external cavity DFB laser was injected into an Er-doped fibre amplifier (EDFA). While significant reduction (but not complete suppression) of TDS was obtained, the setup has the drawbacks of using a rather high injection power (in the range 400–1500 mW) and rather expensive optical components (an EDFA and an optical isolator). TDS has also been shown to reduce significantly when using fibre Bragg grating (FBG) feedback, but it is not suppressed completely [16–19].

Stimulated Brillouin scattering (SBS) is a nonlinear optical process that is initiated by quantum noise [20], and thus, it has potential for generating, all-optically, highly random signals. In Ref. [21] it was shown that broadband optical chaos generated from a single-mode semiconductor laser with optical injection increases the SBS threshold (as compared to that of the cw output from the free-running laser). In the setup in Ref. [21], SBS was not used to generate a chaotic signal; in contrast, we have recently proposed a new setup [22] in which SBS backscattered light in the fibre is re-injected into the laser and generates a chaotic output in which TDS is completely absent. By using spectral and autocorrelation analysis, we showed that TDS is present in the chaotic signal generated by conventional optical feedback (COF), while it is removed in the signal generated by the SBSOF setup, without any need of signal post-processing.

An open question is whether the SBSOF signal is genuinely more random than the COF signal. To address this issue, here we analyse the permutation entropy (PE) of the signals generated by the two setups, COF and SBSOF. The PE is a well-known nonlinear measure of complexity that captures order relations between values of a time series [23–27]. We show that TDS suppression in the SBSOF setup can have an unwanted side effect: it can decrease the entropy of the signal.

2. Experimental setup and linear analysis

Figure 1 illustrates the experimental setups (they were described in detail in Ref. [22]). A high-power multimode laser diode emitting at 1450 nm with a mode-spacing of 0.1 nm is used together with a standard 4 km-long optical fibre spool. The spectrum of the free-running laser diode is displayed in Fig. 1(c). The maximum free-running power of the laser diode is $P_0 = 270$ mW for a pump current of 1000 mA, but the power injected into the fibre is around 140 mW (21.5 dBm) because of the components losses. The light backscattered by SBS from the fibre is injected back into the laser diode as optical feedback forming a closed-loop in Fig. 1(b). In the COF setup, as seen in Fig. 1(a), only the light transmitted through the fibre is injected back into the laser diode. In both setups the time delay is around 21 μs. We note that the SBS threshold power for chaotic laser light in a fibre of 4 km is higher than 30 dBm [21]; therefore, with an injected power of 21.5 dBm, SBS is not present in the COF setup. In contrast, in the SBSOF setup, the SBS threshold power is 16 dBm; therefore, the injected power, 21.5 dBm, is higher than the SBS threshold. Rayleigh scattering also exists in the setup, but SBS is almost 12 dB greater than Rayleigh scattering [22] and thus dominant in dynamics.
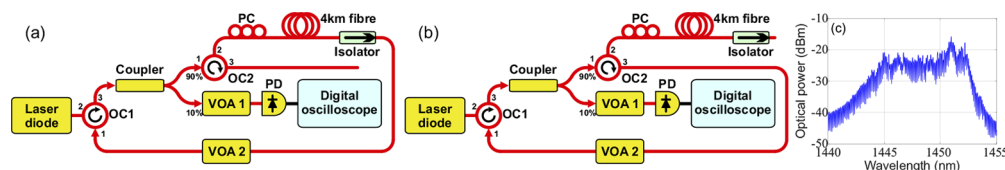

Fig. 1. Schematics of the (a) COF and (b) SBSOF setups. OC: optical circulator, PC: polarisation controller, VOA: variable optical attenuators, PD: photo-detector. (c) Spectrum of the free-running laser.

The dynamical behaviours in the time domain for both the setups appear complex, but very different as shown in Figs. 2(a) and 2(b). The time traces are recorded with 8M sample points

with a sampling rate of 40 GS/ over a time window of 200 µs. But only a window of 10 µs is shown. Their mean amplitudes are measured as 0.84 V (P = 295 mW and 0.14 V_{RMS}) for COF and 0.78 V (P = 287 mW and 0.12 V_{RMS}) for SBS. For the COF setup, the dynamics look like noise. For the SBSOF setup, the signal manifests complex with a burst form which appears irregularly. Their fast dynamics are also shown in Figs. 2(c) and 2(d). In the SBSOF case, a fast oscillation can be observed. This is due to the Brillouin frequency shift (BFS) of 11.46 GHz as seen clearly in Fig. 2(f). This frequency matches to the theory at 1450 nm [22]. The power spectrum for COF in Fig. 2(e) does not show such a peak for the BFS.
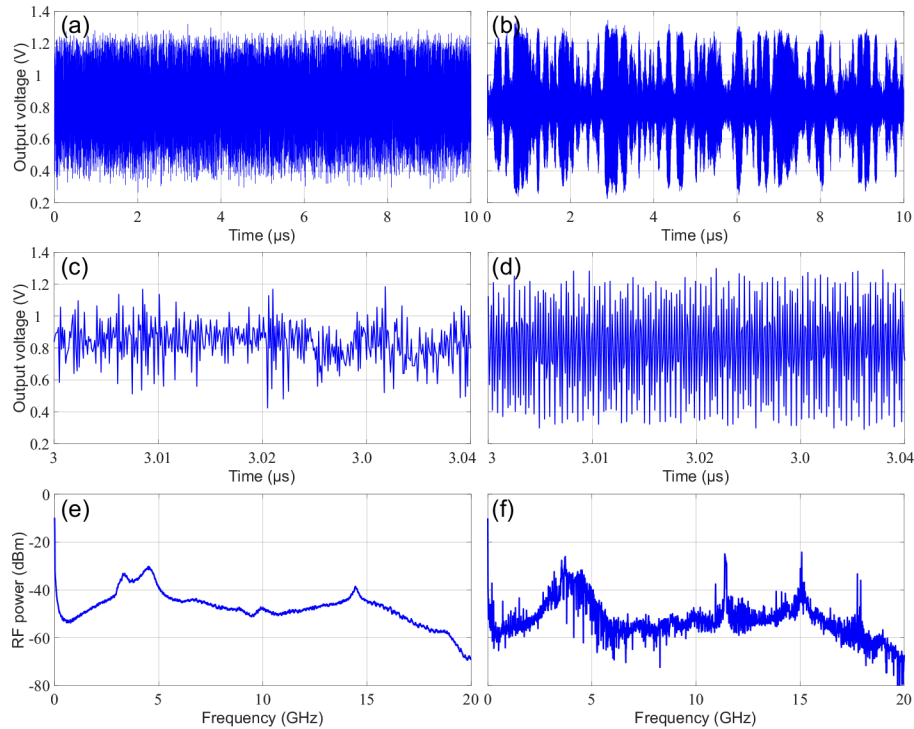

Fig. 2. Left column for COF and right column for SBSOF at 1000 mA and 0.2 %. (a), (b) Time-traces of the intensity dynamics over 10 μs. (c), (d) Time-traces showing the fast dynamics over 40 ns. (e), (f) Corresponding RF power spectra.

The auto-correlation function (ACF) of the laser intensity in both the setups is presented in Fig. 3. In the COF case the ACF displays a distinct peak at 21.09 µs in Fig. 3(a). The peak corresponds to the time delay (τ_{ext} = 21.09 µs) from the 4 km-long fibre. Another peak is also present at 42.18 µs ($2\tau_{ext}$). On the other hand, the TDS vanishes in the SBSOF case as seen in Fig. 2(b). In the SBS setup, the round-trip time is 42.18 µs, but no peak is seen at this time-shift. Therefore, the TDS is suppressed in the SBSOF dynamics. However, the ACF for COF decays much faster than that for SBSOF as shown in the insets of the figure. The large oscillation seen in the inset of Fig. 3(b) is due to the peak at around 4 GHz. Such oscillation with the similar period as the SBSOF one can also be observed in the COF case, but decays much faster than in the SBSOF case. The background is noisy, but oscillation can be noiticed in the SBSOF case and its period corresponds to the BFS at 11.46 GHz. This is because of the strong presence of the BFS in the RF power spectrum shown in Fig. 2(f). Despite of the background oscillation, the TDS is completely suppressed. The TDS suppression is also confirmed by the external-cavity frequency measurement with RF power spectra available in Supplement 1.
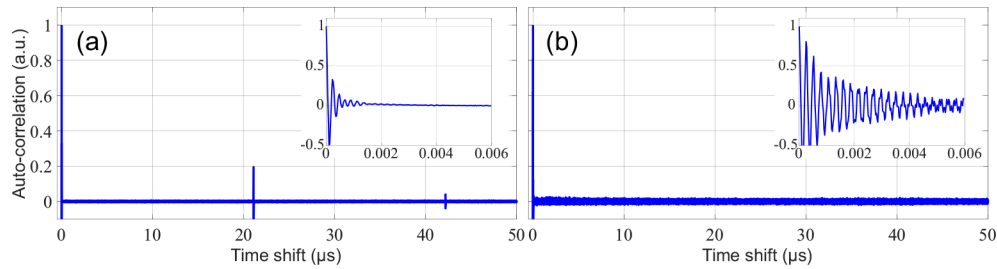
Fig. 3. Auto-correlation functions obtained at 1000 mA and 0.2 % for (a) COF and (b) SBSOF setups, respectively. The ACFs are zoomed in a range of 0.006 μs in the insets.

As a form of distributed feedback, SBSOF can be considered similar to FBG feedback, but in the SBSOF setup, feedback comes from an infinitely large number of reflectors, while in the FBG setup, it comes from a finite number of reflectors. This may be the reason why SBSOF fully suppresses TDS, while in a FBG setup, TDS does not vanish completely [16–19].

3. Permutation entropy nonlinear analysis

The permutation entropy (PE) [23] is a complexity measure that is calculated from the probabilities of symbols, known as ordinal patterns, that are defined by the temporal order relations of datapoints in the time series (see Ref. [24] for details). The plot of the PE as a function of a time shift τ used for defining the ordinal patterns can show well-defined dips that allow unveiling periodicities in the dynamics [25,26]. Here we compute the normalised PE with patterns of length $D = 5$.

Figure 4 shows PE with time shifts τ in the range 0–50 μs, for the times series shown in Fig. 2. We note that τ represents an effective sampling time, as the ordinal patterns are defined in terms of the relative ordering of the intensity values $I(t)$, $I(t + \tau)$, $I(t + 2\tau) \ldots I(t + 4\tau)$. For fully random signals, PE ≈ 1, and lower values reveal the presence of temporal ordering in the sequence of data points. The signals analysed in this paper are highly random, therefore, PE values close to 1 are expected. Because the data acquisition system and the characteristics of the datasets (length and resolution) are the same for the SBSOF and COF setups, we can make a fair comparison, even if the differences found are very small.
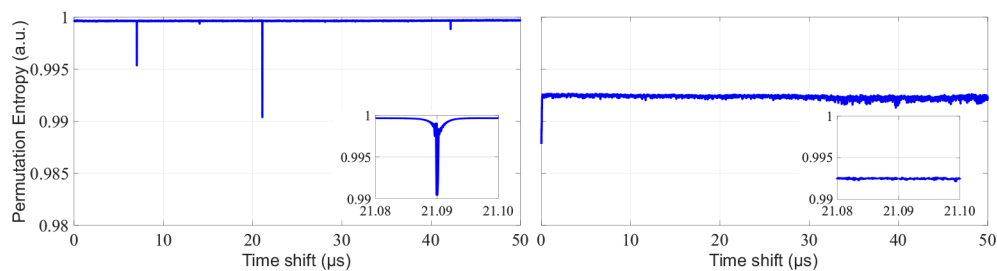

Fig. 4. Permutation entropies obtained from the time traces recorded at 1000 mA and 0.2 % for (a) COF and (b) SBSOF setups in the ordinal pattern length $D = 5$. The insets show PE from 21.08 μs to 21.10 μs for zoom-in purpose.

For the COF setup in Fig. 4(a), a dip at $\tau_{ext} = 21.09$ μs (see the inset of the figure) confirms the presence of TDS. Additional dips are seen at 7.03 μs, 14.06 μs, 42.18 μs, which correspond to $\tau_{ext}/3$, $2\tau_{ext}/3$ and $2\tau_{ext}$, respectively. The baseline value (for $\tau \neq n\tau_{ext}$ with n an integer number) PE ≈ 0.9996 reveals a high degree of randomness, when the intensity sampling time is not an exact multiple of the delay time. For the signal generated with the SBSOF setup, Fig. 4(b), dips

occurring at $\tau = n\tau_{ext}$ are not observed, confirming the total suppression of TDS. However, the baseline PE value is ≈ 0.9925, slightly lower than that of the COF signal. We speculate that the slight decrease of the entropy is due to the fact that the SBSOF dynamics has oscillations at the BFS, which are not present in the dynamics induced by COF.

We have mapped the PE as a function of the laser current and feedback strength in the COF and SBSOF setups. The maps, depicted in Fig. 5, are established by scanning the laser current from 100 mA to 1000 mA at every 2 mA and the feedback ratio from 0.005 % to 0.2 % at every 0.005 %. For each feedback configuration, a total of 18048 intensity time-traces of 2M samples have been recorded. The entropy values obtained with time shift $\tau = \tau_{ext} = 21.09$ µs are coded in colour scale. In both maps the laser threshold reveals clearly: the PE values are represented with light green colour. The reason for which the entropy, when the laser is below the threshold, is not higher is due to the resolution of the data acquisition system: all readings were done using the same amplitude scale on the oscilloscope (8 bits). This results in very small amplitude fluctuations below the threshold, that lower the PE (this is in contrast with the procedure followed in Ref. [25], where the PE maps do not unveil the threshold).
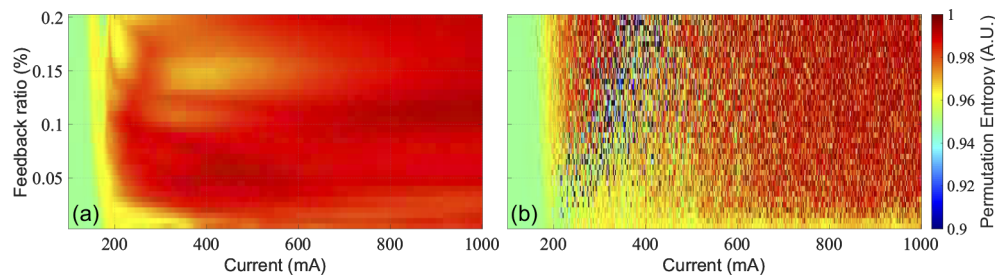

Fig. 5. Permutation entropy maps for (a) COF and (b) SBSOF setups.

The map patterns are very different. Particularly the SBSOF map pattern in Fig. 5(b) appears irregular unlike the one for the COF setup in Fig. 5(a). PE drops are seen in the SBSOF map (blue dots). It seems that the PE values decreases in the early process of SBS, but increase rapidly with the laser current, reaching values close to 1. In the COF map it can also be noticed that the PE values decrease slightly after the threshold with a feedback stronger than 0.15 %, showing a green zone surrounded by the yellow zone. In fact, the well-known dynamical regime of low frequency fluctuations [1] is observed in this parameter region.

4.   Conclusions

We have compared the permutation entropies of the output of a high power laser diode with optical feedback from a standard optical fibre in two configurations: when the light transmitted through the fibre is re-injected into the laser (conventional optical feedback, COF) and when the light backscattered from the fibre is re-injected into the laser (stimulated Brillouin scattering optical feedback, SBSOF). While it was expected that the SBSOF setup would produce a more random output than the COF one (because in the SBSOF setup TDS is fully suppressed), we have found that the SBSOF setup produces signals that are, on average, less random than those produced by the COF setup. Therefore, our results indicate that COF and SBSOF generate chaotic signals with different characteristics, that might have advantages and drawbacks for different applications of optical chaos [2,6,11].

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

Supplemental document. See Supplement 1 for supporting content.

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