Macroscopic Modeling of Connected Autonomous Vehicle Platoons under Mixed Traffic Conditions

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Abstract

Autonomous vehicles (AV) will be present in freeways, and they will have to share current infrastructure with human driven vehicles. This mixed traffic scenario needs to be planed and managed. Some research points that an AV mismanagement could lead to a capacity decrease. This can be solved with connected AV traveling in platoons. Some research exist to that end, but all of them is done using microsimulation tools. These are very powerful and detailed tools but have the shortcoming of strongly rely on an uncertain calibration and give limited insights to the problem. The more robust and simpler to understand macroscopic tools, have almost no platooning models yet. The research presented in this paper fills the gap, by providing a generalized macroscopic model to estimate the average length in vehicle for platoons. This is done by giving a set of rules for AV to form a platoon, including two different platooning schemes representing the best and worst case scenarios. This is of a key importance as greater platoon length is the main factor to drive capacity improvements on highways, which under the appropriate conditions can exceed 10.000 vehicles per hour and lane.

1. Introduction and background

In recent years technology has been evolving at a remarkable pace. In the field of road transportation. This is mainly observed as: introduction of newer and more advanced driver assist systems and the development of new and more efficient powertrains. While the latter is expected to have a great impact in terms of energy consumption and pollution is only the former factor the one having the potential to change the vehicles behavior to tackle congestion and safety problems.

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While some of the early electronic drivers' assist systems had the only purpose to improve safety, such as the Anti Block System (ABS) or the Electronic Stability Control (ESC or ESP). Modern ones aim to partially automate driving as for instance is the ACC (Adaptive Cruise Control) or lane keeping technologies, being the final goal to achieve full automation in the form of Autonomous Vehicles (AV). There is some debate on how long it will take for AVs to be reality outside of pilot tests. But exists some consensus that will eventually happen after solving the great technological challenges AV present (Martinez et al., 2018).

Nonetheless, the aim of this paper is not to discuss the technological challenges, but to set the path of a macroscopic model for platoons of Cooperative Autonomous Vehicles (CAV) at highways. This is important as early studies on partial automation system showed, that congestion, not only will not be solved by AVs but can even be worsened. For instance, greater time gaps and lower acceleration typically in Adaptative Cruise Control (ACC) systems greatly reduce the capacity (Ntousakisa et al. 2014). Thus, in order to increase the capacity the average headway needs to decrease (Taieb-Maimo, and Shinar, 2001; Lin et al., 2009; Shladover et al., 2012; Lioris et al., 2017). Exist different ways of achieving it, as current ACC have reaction time ranging between 0.1 and 0.2 seconds (Ketsing at al. 2007). Thus, the problem is users selecting these possible shorter gaps and still feel safe (Taieb-Maimo, and Shinar, 2001; Rahman et al., 2017, Lin et al., 2009). However, CACC (Coperative ACC) is generally preferred, as it has the potential to achieve even shorter gaps while being still safe. In particular, platooning which is a string of consecutive CACC vehicles traveling at a very short time gaps. This will not only have the possibility to increase capacity, but it also reduces the energy consumption of the vehicles traveling at short gaps (Alam et al., 2010; Tsugawa et al., 2011; Shida et al., 2010).

However, to achieve significant capacity improvements is necessary to have long platoons; as the first vehicle in the platoon will not travel at a reduced headway. Thus, knowing which traffic conditions could potentially result in longer platoons, which would imply greater capacity is of a key importance. The advantage of using macroscopic models is that they rely in simpler models which in turn require fewer inputs. This allows to a better understanding of the phenomena trade-off, at the cost of losing some level of detail. To the authors best knowledge exists just one initial approach to macroscopically model CAV platoons (Chen at al., 2017). This contribution is of great interest, as it quantifies the impact of different parameters and policies on the road capacity. However, it avoids tackling any traffic dynamics nor how different traffic conditions can affect the platoon length. This is precisely the gap this paper fills by providing traffic related platoons, using two different platoon schemes which aim to be representative of the reality. Also, estimates on the capacity for the given methodology are provided. The paper is structured as follows: in section 2 the problem and variables definitions are made. Following, in section 3 the model is presented along with some results on the average platoon length estimation. In section 4 estimates of capacity for different platoon models and penetration rates are presented and finally at section 5 some conclusions are outlined.
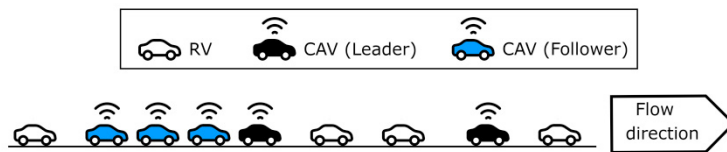
2. Definitions

Figure 1. Platoon components illustration.

The model considers a highway with mixed traffic, which means CAVs and RV (Regular Vehicles) sharing the infrastructure. Platoons are consecutive strings of CAVs traveling at a shorter gaps than RVs. Note that by definition any RV splits one platoon into two different ones. Vehicles in a platoon are classified as: i) platoon leader, which is the first vehicle of the string and ii) platoon follower, which is any vehicle in the platoon behind the leader, see Figure 1 for further clarifications. For sake of briefness, they will be referred as leader and followers. Note that only followers travel at reduced gaps, since leaders have no cooperative information of the vehicle in front. A single CAV is considered to be itself a platoon, which their only component is the leader. The model inputs are defined in Table 1. The endogenous computational variables are defined in Table 2.

Table 1. Input definitions.

Input	Units	Description
q_T	[veh/h]	Total demand.
a	[# lanes]	Total number of lanes.
v	[Km/h]	Lane speed.
β	[pu]	Penetration rate of the CAV over the total demand.
l_d	[Km]	Platooning range.
L_P	[veh/platoon]	Maximum allowed platoon length in vehicles.

Table 2. Variable definitions.

Variable	Units	Description
$k_{i,j}$	[veh]	Platoon length. Subindex i can be c for cooperative or o for opportunistic. Subindex j stands for the max platoon length L_P
m	[platoons]	Number of platoons.
s	[platoons]	Number of times a platoon has to be split to fulfill L_P.
λ	[veh]	Average number of vehicles per lane in the l_d distance.
n	[veh]	Deterministic sample size.
$p^1(\lambda; n)$	[pu]	Zero Truncated Poison Distribution probability for n with a Poisson parameter λ.

3. Platoon length estimation

In order to estimate the platoon length is necessary to make some assumptions on how platoons will actually take place, i.e. the formation dynamics. In this paper is assumed that: i) the process to join a platoon is instantaneous (no time); ii) vehicles only consider to join a platoon or merge with other CAVs within the platoon range (l_d). The first assumption is reasonable as long as the time it takes for one vehicle to join a platoon is negligible in comparison with the time then it travels within the platoon. This time depends on many different factors as: speed differential, l_d, etc. The second one is placed as there will be some communication range limit. Literature points out that currently 5.9-GHz DSRC communication technology provides at least 300 m of communication range (Shladover et al., 2015).

Still, different platoon schemes under the previous hypothesis can be considered. In this paper two possibilities are taken into account and presented: cooperative and opportunistic platooning. Under the cooperative scheme, any CAV within the platoon range will join a single platoon. While on the opportunistic platoon only already consecutive CAVs will form a platoon. This means that platoon length is limited to those CAVs that by chance are already consecutive, and any RV will cut the platoon. It could be argued on how these two schemes are realistic or not, but this will mainly depend on how platooning formation is implemented in a future. Nonetheless, the two presented options, aim to be representative of the best case scenario (cooperative) giving the longest possible platoons at the cost of a much more complex formation, and the worse case scenario (opportunistic), giving the shortest possible platoon, as nothing is done to enlarge them. Of course, shorter than opportunistic platoons are feasible, but to achieve that, some strategy to actively shorten the platoon length would be required. And the only strategy that falls in this category that seems reasonable to be ever implemented, is to enforce a maximum platoon length (L_P), which will be considered further on in this section. Indeed some research points out that a maximum platoon length needs to be enforced, since long platoons could exhibit string instabilities. Shladover et al., (2015) states that a maximum platoon length of 20 vehicles can be considered, and in order to achieve this length, a careful platoon design is required.

The average platoon length will depend on how many vehicles are in a given region. The vehicle distribution is assumed to follow a Poisson distribution (Daganzo, 1997). However, only regions with at least one vehicle are of interest. As the ones with no vehicles, have no impact to the platoon mean length. This is represented by a ZTPD (Zero Truncated Poisson Distribution). The probability mass function of the ZTPD (p^1) is given in equation (1). The formula to compute the Poison parameter λ is given at equation (2). Note that λ represents the average number of vehicles both CAVs and RVs) within the platoon range l_d, and n the actual number of vehicles.

$$p^1(N = n; \lambda) = \frac{e^{-\lambda} \cdot \lambda^n}{n! \cdot (1 - e^{-\lambda})} \; ; \; n \in \mathbb{N}^* \tag{1}$$

$$\lambda = \frac{q_T}{a} \cdot \frac{l_d}{v} \tag{2}$$

3.1. Cooperative average platoon length

When following the cooperative scheme and no platoon length limits is imposed, all CAVs within the platooning region will form platoon a single platoon. Thus, the average platoon length is just the average number of CAVs in this region. Since the amount of CAVs is a fraction β of the total number of vehicles λ, the average platoon length is shown at equation (3) as the ZTPD mean with a parameter $\lambda \cdot \beta$. When a maximum platoon length is enforced, whenever than more than L_P CAVs are in range, these will have to be split into multiple platoons. The minimum number of platoons to be split (s) is shown in equation (4), where $\lceil \ \rceil$ represents the nearest upper integer. Of course, this is 1 when there is no limit. Thus, the average platoon length now is computed as in equation (5).

$$\overline{k}_{c,+\infty} = \frac{\lambda \cdot \beta}{1 - e^{-\lambda \cdot \beta}} \tag{3}$$

$$s = \begin{cases} \left\lceil \dfrac{k}{L_P} \right\rceil ; L_P < +\infty \\ 1 \quad ; L_P = +\infty \end{cases} \tag{4}$$

$$\overline{k}_{c,L_P} = \frac{\sum_{k=1}^{+\infty} p^1(\lambda \cdot \beta, k) \cdot k}{\sum_{k=1}^{+\infty} p^1(\lambda \cdot \beta, k) \cdot s} \tag{5}$$

3.2. Opportunistic platooning

The conceptual problem to solve in this section can be conceptualized as: determine the average number of consecutive successes in a Bernoulli experiment with a constant success probability β in a sample of arbitrary size n. A success is considered to be a CAV while a failure represents a RV. Each vehicle in the sample is considered to have an independent probability of being a CAV, and this is equal to β. Thus, each vehicle can be seen as a Bernoulli experiment with a success probability β. This apparently simple problem involves some complexity. Think of a single platoon of length k, it has a probability to happen as described in equation (6). Note that when the platoon length is smaller than the sample, a failure is required to "cut" the success string (i.e. the platoon). However, this is for a single platoon, and when $k + 1 < n$, other platoons can also happen simultaneously in the same sample. For any n exists a finite (could be very large) number of combinations on which the successes and failures (CAVs and RVs) can be sorted. This problem is solved at Theorem 2.1 form Makri and Psillakis (2011), shown at equation (7), which gives the probability of having exactly m platoons of exactly k vehicles long in a sample of size n, given an independent success probability β. The binomial coefficients are the extended ones as defined in equation (8), with no support for negative numbers.

$$\begin{cases} \beta^k, \ n = k \\ \beta^k \cdot (1 - \beta), \ n > k \end{cases} \tag{6}$$

$$P(E_{n,k} = m) = \sum_{y=0}^{n-k \cdot m} \beta^{n-y} \cdot (1 - \beta)^y \cdot \binom{y+1}{m} \cdot \sum_{j=0}^{\left\lfloor \frac{n-y-k \cdot m}{k} \right\rfloor} (-1)^j \cdot \binom{y+1-m}{j} \cdot \binom{n-(k+1)\cdot(m+j)}{n-y-k\cdot(m+j)} \tag{7}$$

$$k \in [1,n]; \ m \in \left[1, \left\lfloor \frac{n+1}{k+1} \right\rfloor\right]$$

$$\binom{a}{b} = \frac{a \cdot (a-1) \cdots (a-b+1)}{b!} \; ; \; b \in \mathbb{N}^0, a \in \mathbb{R}, a > b \tag{8}$$

Since the binomial coefficient is involved, the problem becomes numerically unstable (overflow) for large samples. A solution to this is provided in Remark 2.3 of Makri and Psillakis (2011) as a normal approximation when $n \gg 1$, see equation (10), with Φ meaning normal N(0,1) cumulative distribution function. In the current paper this is used for $n \geq 100$. Values for μ and σ are defined in equation (9). Note that this approximation can only be used when the conditions at equation (11) are met.

$$\begin{cases} \mu = (1-\beta)^2 \cdot \beta^k \\ \sigma^2 = \mu \cdot \left(1 + \mu \cdot \left(2 \cdot \left(\frac{1-\beta}{\beta} - k \right) - 1 \right) \right) \end{cases} \tag{9}$$

$$P\big(E_{n,k} = m\big) \approx \Phi\left(\frac{m + 0.5 - n * \mu}{\sigma \cdot \sqrt{n}} \right) - \Phi\left(\frac{m - 0.5 - n * \mu}{\sigma \cdot \sqrt{n}} \right) \tag{10}$$

$$\mu - \frac{3 \cdot \sigma}{\sqrt{n}} > 0 \quad and \quad \mu + \frac{3 \cdot \sigma}{\sqrt{n}} < \frac{\left\lfloor \frac{n+1}{k+1} \right\rfloor}{n} \tag{11}$$

So far, only the probabilities of having exactly m platoons of length k in a sample n are known, the mean is still unknown. The definition of the mean implies adding all the probabilities multiplied by the variable value. However to do so, the sum of the probabilities need to add 1, and this is not the case. Still the probabilities are correct, the reason of that is that some combinations can happen at the same time that others happen too. For instance in a sample of size 20, two platoons of 3 vehicles do not exclude that one platoon of 5 vehicles could happen too. Adding all the combinations as shown in equation (12) results not on the average platoon length but on the average total number of CAVs in the sample n, which is $n \cdot \beta$. Equations (7) and (10) can be used to compute the mean number of platoons in the sample ($\bar{m}$) as shown in equation (13). Dividing the average total number of CAVs by the average number of platons in the sample gives the average platoon length ($\bar{k}$) in the sample of size n, see equation (14). The average platoon length in a region with an average number of vehicles equal to λ, on which the vehicles follow a Poisson distribution is given at equation (15). Similar results can be obtained preforming a Montecarlo simulation enough times.

$$n \cdot \beta = \sum_{k=1}^{n} \sum_{m=1}^{\left\lfloor \frac{n+1}{k+1} \right\rfloor} \frac{k}{s} \cdot (m \cdot s) \cdot P\big(E_{n,k} = m\big) \tag{12}$$

$$\bar{m}(n,\beta) = \sum_{o=1}^{n} \sum_{m=1}^{\left\lfloor \frac{n+1}{k+1} \right\rfloor} m \cdot s \cdot P\big(E_{n,k} = m\big) \tag{13}$$

$$\bar{k}(n,\beta) = \frac{n \cdot \beta}{\bar{m}(n,\beta)} \tag{14}$$

$$\bar{k}_{o,LP}(\lambda,\beta) = \sum_{n=1}^{\infty} p^1(\lambda,n) \cdot \bar{k}(n,\beta) \tag{15}$$

3.3. Comparing platoon schemes.

In Figure 2, the average platoon length for both types of platoon is presented for an average number of vehicles within the platooning range up to $\lambda = 100$. Both types of platoons are presented for no maximum platoon length ($L_P = +\infty$) and for a maximum platoon length of 20 vehicles ($L_P = 20$). Additional comparison plots are presented at Figure 3.
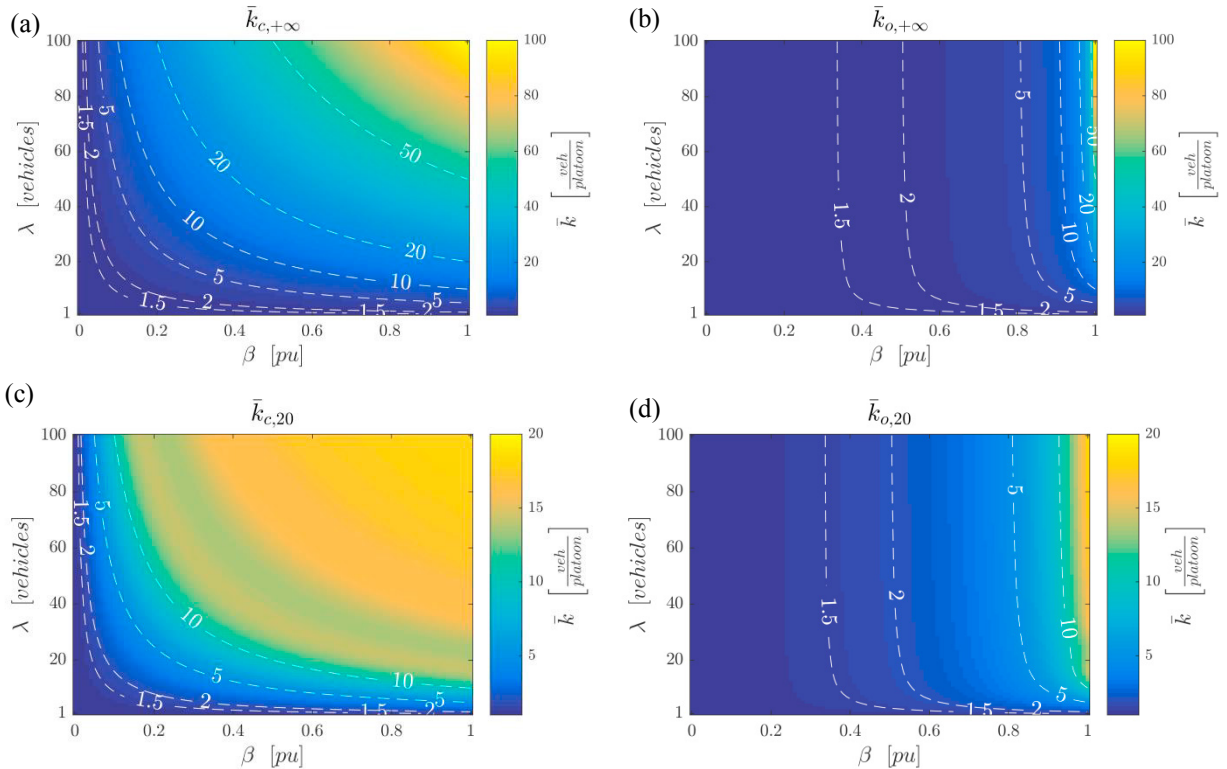

Figure 2. Average platoon lengths under different demands, AV penetration rates and platooning schemes.

Figure 2 (a) shows the cooperative platoon with no length limit. This is the same as showing the ZTPD mean presented in equation (3), which is the average amount of CAVs. However, when a maximum platoon is imposed, the results change, see Figure 2 (c). Apart from all values being equal or smaller than 20, a much shorter average platoon length can be observed in the region nearby $\lambda \cdot \beta = 20$, this is because any platoon that exceeds 20 vehicles is split into two platoons, effectively shortening the average length. When observing the results of opportunistic platoon, Figure 2 (b) and (d), is evident that the average length is much more sensible to the value of β than it is to λ. The latter has only impact when demand is small enough, so the average length of the platoon is limited due to the lack of CAVs to form a long enough platoon. As the penetration rate increases (β), it also increases the likelihood of having longer platoons. Thus, is when β is close to one that average platoon length shows sensitivity to both penetration rate and average number of vehicles.

Figure 3 shows the comparison between the two purposed platooning strategies as the subtraction of average platoon length, for no platoon length limit (Figure 3a) and a platoon limit of 20 vehicles (Figure 3b). In both Figure 3a and 3b, when β is either zero or one, there is no difference between platooning schemes. The explanation is simple, as for $\beta = 0$, there are no CAVs, thus no platoons and both have the same average length of one vehicle. For $\beta = 1$ there are no RV, and since the only thing that makes opportunistic platooning shorter than cooperative are RV cutting the otherwise longer platoons, the platoon length is the same no matter the scheme. The greatest difference between schemes is for mid range penetration rates with big demands. Is in this zone where there is plenty of CAV

to form long platoons but also of RV to cut them if nothing is made to avoid it. The comparison between Figure 3 (a) and (b), shows that differences between platooning schemes are much smaller when a maximum platoon length is enforced. This is, as the cooperative platooning produces much shorter platoons when a maximum length is enforced, see Figure 2 (a) and (c).
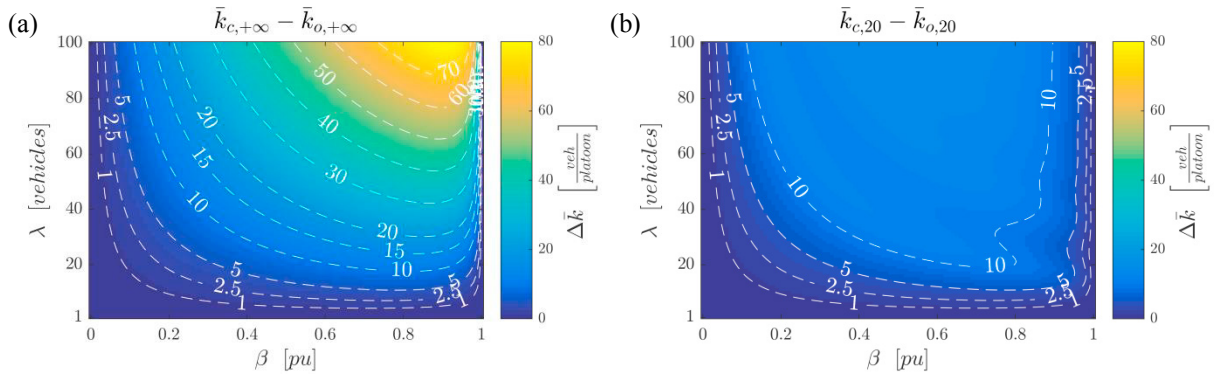

Figure 3. Comparaison between cooperative and opportunistic platooning in terms of average platoon length. a) No platoon length limit. b) Maximum length of 20 vehicles platoon.

4. Mixed lane capacity

In this section, an idea of the capacities that could be achieved using the platoon schemes presented is given. To do so, is necessary to assume some values regarding the vehicles. These are: a speed of 120 Km/h, a vehicular length of 5 meters and a time gap of 1.5 seconds for RV and platoon leaders, and of 0.1 seconds for platoon followers. With these values, is possible to compute the mean headway given the CAV penetration rate. This is done for both platoon schemes and for no platoon limit and a limit of 20 vehicles per platoon. Thus, the lane capacities per each of the four possible combinations through the whole range of CAV penetration is obtained and shown at Figure 4. The capacities that can be achieved are huge. As for instance with a 100% of CAV in any of the possible combinations, the lane capacity exceeds 10.000 veh/h/lane. This is more than 4 times the value with no CAVs at all, which is 2308 veh/h/lane. Capacity values for a 50% penetration rate, are 3407 veh/h/lane under cooperative platooning and 2811 veh/h/lane for opportunistic. Another interesting question that can be observed form Figure 4, is that enforcing a platoon limit, in terms of capacity only has a meaningful effect for great penetration rates, in excess of 90% of CAVs. In lower penetration rates, the differences are indistinguishable.
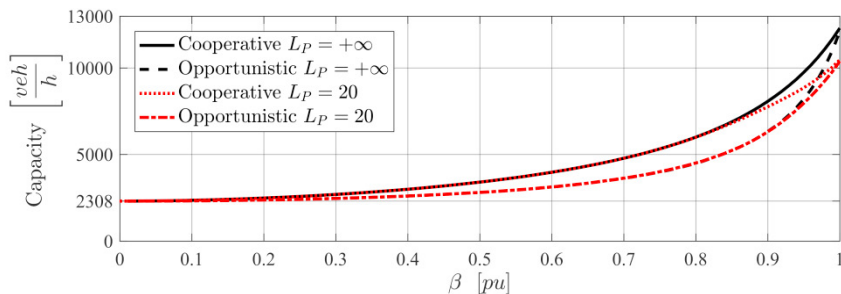

Figure 4. CAV penetration rate vs lane capacity for both platooning schemes (cooperative and opportunistic) and maximum platoon length of $+\infty$ and 20 vehicles per platoon.

5. Conclusions

The presented methodology to estimate platoon length is a key step in order to achieve a macroscopic model that can model traffic dynamics with platoons. As is crucial to be able to model how the different traffic states produce

different platoons. Since only platoon followers travel at a reduced gap (or time gap), they are the only vehicles to actually contribute towards increasing the lane capacity. Thus, the longer the platoons, the more followers in them and greater lane capacities can be achieved. Of course not only traffic will affect the platoon length, but also how they form. To that end two methodologies were presented, which aim to represent the best case scenario (cooperative) and the worst case scenario (opportunistic) in terms of platoon length. The specific dynamics of platoon formation have been neglected to achieve a simple but understandable model.

Additionally, the possibility to enforce a maximum platoon length is presented. This could be of interest due to technological limits on how long the platoon could be or enforced by a traffic administration. Enforcing a maximum of 20 vehicles per platoon has the strongest effect on the cooperative platoon (see Figure 2 a and c). Still when this is translated to lane capacities, the differences are only noticeable for both types of platoon at penetration rates that exceed 90% of CAVs. And speaking of capacity, here the main conclusions can be obtained. In any case the capacity is increased, being more noticeable the grater the penetration rate is, for a 50% penetration rate the increase is 22% for opportunistic and 48% for cooperative. Under the best possible conditions it achieves values in excess of 12000 veh/h/lane, which is 5,3 times the values achieved for RV with the same parameters.

Acknowledgements

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