THE SCALE EFFECT

Changing the scale entails much more than changing the size. The proportional dead weight, rigidity, surface, volume and weight also change with the size of the structure. The performance and features of a given solution change when the scale is varied. Nature never changes the scale: different sizes result in different shapes, materials or proportions.

“I have sketched a bone whose natural length has been increased three times and whose thickness has been multiplied until, for an animal of the corresponding size, it could perform the same functions that the small bone performs for the smaller animal. You can see from the figures how the big bone has been deformed and loses all proportion. It is clear then that, if one intends to maintain the same proportions in the members of a giant as in those of an ordinary man, one must find a harder and more resistant material to make the bones, or one must accept a relative decrease in force with respect to that of men of average height; for if its height is increased indefinitely, it will fall and be crushed by its own weight. On the other hand, if the size of a body is decreased, the strength of that body does not decrease in the same proportion; in fact, the smaller the body, the greater its relative strength” (Galileo Galilei, 1638).

A child is not and adult scaled down. The size changes, so do the proportions.

1, 2, 3 Albrecht Dürer, 1528: Four books on human proportion. The Proportions of an Infant. Dürer defined the child’s overall height as 1 unit so that the image could easily be rescaled to any size. Each number on the drawing indicates a body dimension as a fraction of that unit. On the lateral view of the arm, for example, the width at the shoulder is one tenth of the child’s total height, whereas at the elbow it is one sixteenth and at the wrist, one twenty-third.

4 The Mother of God of Tenderness (S XIII. The State Russian Museum, St.Petersburg). Reciprocally, an adult is not just an enlargement.

6, 7: An adult and a child represented with the same height. Note the change in shape: the child has a much larger head and shorter neck, arms and legs. In addition, its brain represents 10% of the body weight, while that of the adult only represents 2%.

The scale effect is particularly evident in the connections.
1 The type of cable termination, for example, depends on the size, because the ratio surface/cross-section varies.

2 Doubling the diameter, the lateral area compared to the cross section decreases by half. Therefore, the surface available to transmit the friction load has decreased by half.

3 For small cable diameters, friction or clamp type terminations are sufficient. They have a greater surface of friction related to the cross-sectional area than do intermediate and thick diameter cables.

4 As increasing the diameter of the cable decreases the lateral surface of friction related to the cross-sectional area, the intermediate diameters (up to 36 mm) require pressed type terminations. There is not enough lateral surface of friction to transmit the load intermittently through clips. The entire length of the connection is needed.

5 Speltered terminals are used for larger sizes because thick cables still have less surface of friction related to the cross-sectional area than do intermediate and small diameter cables. So they can not transmit the load through friction at all.

*Form and scale*

A cube (density $\gamma$) measures $a \times a \times a$ and rests on a pedestal $d \times d$.

Stress on the pedestal: $\sigma = a \cdot a \cdot \gamma / (d \cdot d) \rightarrow d^2/a^2 = a \cdot \gamma / \sigma$. 

Ratio of the width $a$ of the cube and that of the pedestal: $\frac{d}{a} = \sqrt{\frac{\gamma}{\sigma}} \cdot \sqrt{a}$

It is not constant. It depends on $\sqrt{a}$

**The smaller a sustained form, the more thin can be the structure that sustains it.**

A cube $a \times a \times a$ rests on a pedestal whose transversal section (in plan) is $d \times d$. The ratio between the width of the cube $a$ and that of the pedestal $b$ is not constant. If, for example, $\gamma = 2,5$ T/m$^3$ and $\sigma_{ad} = 250$ kp/cm$^2$: $a = 1.000 \rightarrow d = 1.000$; $a = 100 \rightarrow d = 31,62$; $a = 10 \rightarrow d = 1$;

Conclusion: **the smaller a sustained form, the more thin can be the structure that sustains it.**

“The bridges may be enlarged as necessary making their parts stronger in proportion” (A.Palladio, 1570). “A small dog could probably carry on its back two or three dogs of the same size; but a horse could not carry even one of its own size” (Galileo Galilei, 1638).
It is hazardous to scale up a structural system used for small and medium sized structures.

1, 2: R. Taillibert & St. du Château with L.Stromeyer, 1965: Convertible roof for an open air theatre in Cannes: 800 m²; Ø 33 m; operation time 12 min. It was the first constructed example of a convertible roof with an exterior slanted mast and a centrally bunched roof skin. Eight cables describe an 18º slanted cone. They come together in a point suspended from the top of the mast, forming one of the three guy directions of the mast. At the same time, the cables are trolley cables to support and transport the retractable membrane.

3, 4: R.Taillibert with L.Stromeyer, 1966: Convertible roof, Boulevard Carnot swimming pool, Paris: 1800 m²; 62 x 32 m; operation time 12 min. A convertible roof was planned in order that the pool could be used all the year around, regardless of the weather, thereby increasing the utilization potential of the facility. A solution was chosen following the design of Cannes, for which good experimental values were available. Thus, the same solution was adopted for a roof 2,25 times larger. Nevertheless, operation was very difficult and restricted twice a year.

Next page: R.Taillibert with Lavalin and sbp, 1987, Montreal Stadium. 20.000 m² of convertible PVC coated Kevlar fabric roof, anchored on the prestressed concrete roof, and suspended from a 168 m high inclined tower including a panoramic funicular. Operation time 30 minutes. It required a staff of five for about two hours. Design snow load: 280 kp/m² (Canadian code). Deflection accepted (at a design stage): 5 m. Snow accumulation: 1.000 kp/m² + ponding + small angles effect and curvature variations → it collapsed and was replaced by a non retractable textile roof that also tore.

“The geometry used was adapted from previous smaller examples (Cannes 800 m² and Paris 1.800 m²) where the variations of snow distribution, wind effect, changes of angles, uncertainties on the elastic modulus of the cables, long term creeping, tolerances and non-linear behaviour were not as significant” (M.Majowiecki, 2000).
Moreover, sensitivity to small variations depends on the size: 10 times 1 KN equals 10 KN. (The difference is 9 KN). 10 times 100 KN equals 1000 KN. (The difference is 900 KN). Unexpected 9 KN are easier to redistribute than (or not so catastrophic as) 900 KN.

A.Gaudí (1833 - 1926) et al. (from 1926): Temple of the Sagrada Familia, Barcelona.

The design of the temple of the Sagrada Familia was based on the incomplete chapel of the Colònia Güell (1908-1914), designed by the same architect (3).
Several aspects are specially relevant regarding the change of scale:

4 The size has been more than doubled, meaning that the weight has increased more than $2^3$ times.

On the other hand, the original design was based on a reversed gravity model (1, 2) that did not take into account the horizontal seismic and wind loads, especially relevant in this case due to the increased lateral surface (squared) and weight (raised to the cube).

In addition, the Colònía Güell chapel was made of stone and brick adequate materials to resist massive compression that in the Sagrada Familia Temple had to be completed with reinforced concrete during Gaudí’s time. As a result, the current construction has to resort to concreted reinforcements and post-tensioning (5, 6).

5 Dar Al-Handasah with Projacs and gmp Architeckten, 2022: Al Bayt Stadium, Al Khor City, Qatar. The $770 \times 10^6 \text{€}$ Al Bayt Stadium in Al Khor City is the most astonishing spectacular height of change of scale. Its design is based on the Bayt Al Sha’ar, a black and white tent (1) used traditionally by nomadic people in Qatar as a welcome symbol of hospitality for desert travelers.

Especially surprising are the dimensions (311 x 273 m), the change of scale with respect to the original model, the trussed steel structure above the reinforced concrete (4), the retractable roof, the 6 different kinds of membrane (2, 3) including the one specially customized, and the services, comprising conventional energy-consumer air conditioning. The steel structure doubles and envelopes the reinforced concrete structure of the stands and air conditioned will be installed even though the model in which the designed is inspired is conditioned without energy consumption. The lack of adequacy to the original model will be revealed by the perimeter ties along with their anchors, which are not necessary at all.
The scale effect is also shown by the paradox of textile structure design (Moncrieff, Gründig & Ströbel, 1999):

“Smaller structures are more difficult to pattern than larger ones. This is due to the fact that with larger structures the limitation of the maximum fabric width necessitates the use of many cloths. The 3 D to 2 D patterning distortion is therefore low for each cloth. With smaller structures the maximum fabric width is large in comparison to the structures overall dimensions. Strips of fabric in a smaller structure experience a much higher distortion.”


2, 3 J.B.Pascual with J.Llorens and Arqintegral, 2004: Shadows for the “Xian Warriors” exhibition, Barcelona.