Cite this article
Sanabra-Loewe M, Capellà-Llovera J, Ramírez-Anaya S and Pujadas-Gispert E
A path to more versatile code provisions for slab deflection control.
Proceedings of the Institution of Civil Engineers – Structures and Buildings,
https://doi.org/10.1680/jstbu.20.00205

Research Article
Paper 2000205
Received 17/08/2020;
Accepted 11/01/2021

Keywords: codes of practice & standards/design methods & aids/ slabs & plates

Structures and Buildings

ICE Publishing

A path to more versatile code provisions for slab deflection control

Marc Sanabra-Loewe PhD
Lecturer, Department of Architectural Technology, Polytechnic University of Catalonia (Barcelona Tech), Barcelona, Spain (Orcid:0000-0002-7920-2427) (corresponding author: marc.sanabra@upc.edu)

Joaquín Capellà-Llovera PhD
Tenured Lecturer, Department of Architectural Technology, Polytechnic University of Catalonia (Barcelona Tech), Barcelona, Spain

Sandra Ramírez-Anaya MSc
Architect, BIS Structures, Barcelona, Spain

Ester Pujadas-Gispert PhD
Post-doctoral researcher, Innovative Structural Design (ISD), Eindhoven University of Technology, Eindhoven, The Netherlands (Orcid:0000-0002-3882-5863)

An assessment of the methods of codes ACI 318 and Eurocode 2 is performed to establish a minimum slenderness for reinforced concrete slabs, which is a simplified method for controlling the deflection of slabs. This assessment is performed by comparing these methods with each other and with two further methods that have inspired, among others, the Australian standard for concrete structures. The variables evaluated include those that most influence deflection control (span, load and allowable deflection). The study cases include spans that range from 6 m (20 ft) to 12 m (40 ft) and loads that range from 5 kN/m^2 (104 lb/ft^2) to 15 kN/m^2 (313 lb/ft^2). The results of the study reveal some of the key advantages and shortcomings of the ACI 318 and Eurocode 2 provisions. Suggestions are provided to improve the current provisions of the codes. Methods that are not too complicated but more refined can help the code provisions to become more versatile. The aim of this work is to minimise potential problems caused by designing excessively thick slabs (heavy and pollutant) or excessively slender slabs (too deformable).

Notation

b	breadth of the rectangular section
d	dead load
E_c	elastic modulus of concrete
f_{ck}	characteristic cylinder compressive strength (MPa)
f_r	modulus of rupture
h	depth of slab
I_e	effective moment of inertia
K	boundary condition factor
k_{AR}, k_{DP}, k_{SS}	coefficients applying to two-way slabs
L, l	span
L/h	span/depth ratio
l/d	span/effective depth ratio
l_n	net span
l_n/h	net span/depth ratio
M^*	factored moment
M_a	largest moment in the load history
M_{cr}	cracking moment
M_k	characteristic moment
$W_{l(var)}$	variable portion of the live load
W_s	sustained load
α	stiffness coefficient
α_m	modified stiffness coefficient
α_0	initial stiffness coefficient
β_{ES}	ratio applying to two-way slabs
γ_c	partial factor for concrete
γ_s	partial factor for steel
Δ_{inc}	incremental deflection
$(\Delta_{inc}/l)_{allow}$	allowable incremental deflection ratio
κ	boundary condition coefficient
λ	multiplier for long-term deflections under sustained loads
ρ	reinforcement ratio
ρ_0	reference reinforcement ratio (one-way)

1. Introduction

In recent decades, several authors have questioned the advantages of the simplified deflection-control methods that are currently provided in some codes for the design of reinforced concrete members (Beal, 2009; Bondy, 2005; Eren and Dancygier, 2020; Pecić and Marinković, 2011; Scanlon and Lee, 2006; Vollum, 2009). Some of these authors have stated that simplified deflection-control methods, such as the ACI 318 method, lack transparency and scarcely define the ranges for proper application. Others have pointed out several shortcomings of Eurocode 2. Besides, some of the authors of the current provisions of Eurocode 2 have proposed a way to increase the flexibility and transparency of its provisions (Pérez *et al.*, 2017; Pérez and Corres, 2014). In parallel, a new approach to the problem has been pioneered by the Australian codes (SA, 2001), which is based on deflection-control methods that are similar to the methods discussed in this study: methods that are independent of the reinforcement ratio of a slab.

1.1 Motivation for this study

When designing a reinforced concrete member under flexure, establishing an initial geometry of the cross-section of the member is usually required in the early stages of design; this is different from what happens when designing a steel member or a timber member under flexure. This difference is due to the fact that reinforced concrete is a composite material that cracks under the tension caused by flexure and its design is not as linear as that of a steel member or a timber member. In the case of concrete, the required amount of steel reinforcement can only be determined based on a pre-established geometry of the concrete cross-section. In addition, deflections can only be calculated when the amount of steel reinforcement is known. Thus, deflection calculations are typically performed at the end of the design process. However, deflection checks typically govern the design of most reinforced concrete members under flexure. Given the importance of deflections, they should be considered from the earliest design stages even if they cannot be actually calculated at that time, as the amount reinforcement of the member is an unknown. This paradox is solved by establishing an initial geometry of the cross-section of the member that implicitly considers allowable deflection requirements. For this reason, picking a correct initial geometry for the cross-section can be considered a deflection-control method.

The selection of an initial geometry in the early stages of the design of a slab is currently achieved by establishing its depth (h). Expressing the depth (h) as a function of span (L) by the concept of slenderness of the slab (L/h) is a common approach.

The codes ACI 318 (ACI, 2014) and Eurocode 2, EN 1992-1-1 (CEN, 2004) currently have provisions to establish a minimum slenderness of the slab and state that following these provisions may be sufficient to guarantee that the required deflection limits of the same codes are satisfied. This approach of codes is very helpful because it allows a designer to establish the depth of a slab in the early stages of design and solve the previously mentioned paradox.

However, in some cases in current practice, these provisions of codes may cause inappropriate use, which may happen when designers disregard the results of detailed deflection calculations in the case where they contradict the depths given by the minimum slenderness provisions provided in the codes.

For example, in countries such as Spain, some insurance companies ask designers to use the minimum slenderness provisions of Eurocode 2, even if thinner slabs that satisfy the deflection limits allowed by the code can be employed. Selecting slabs that are thicker than those strictly necessary means that floors are more expensive, heavier and more polluting than strictly necessary; which may, in turn, cause unnecessarily large, expensive and pollutant-generating foundations.

The opposite situation may also occur, such as designing excessively slender and deformable slabs, as Bondy (2005) demonstrated when following minimum slenderness provisions for slabs, as established in the ACI 318 code.

2. Design of the study

To assess the performance of the simplified deflection-control methods for one-way and two-way slabs, as outlined in ACI 318 (ACI, 2014) and in Eurocode 2 (CEN, 2004), this study compares these methods to two other methods that are based on analytical simplified computation of deflections. The latter methods are based on research initiated by Andrew Scanlon in the early 1980s (Scanlon and Murray, 1982). In contrast, the deflection-control method, which is referred to in this study as the 'long Rangan–Scanlon method', was proposed by Scanlon in 1999 (Scanlon and Choi, 1999) based on previous studies by Rangan (1982) and Gilbert (1985). Furthermore, a method developed by Scanlon and Lee (2006) is referred to in the present study as the 'short method of Rangan–Scanlon'.

3. Description of the deflection-control methods studied

3.1 Method of ACI 318

The ACI 318 method (ACI, 2014) was developed by the American Concrete Institute (ACI) in the mid-1900s and has not undergone many changes since that time. This method is primarily based on experience (ACI, 2014; Bondy, 2005). The method proposes to determine the depth (h) of a member based on a simple constant ratio:

$$L/h = n (\text{or } L_n/h = n) \quad (1)$$

where n takes different values depending on the boundary conditions, the type of structural member, the density of concrete and the strength of steel.

This method and similar methods will be termed 'constant slenderness methods' (CS methods) in this study.

In the code, one-way slabs and two-way slabs are treated separately. For one-way slabs, the code provisions are given in section 7.3 and table 7.3.1.1, only for those slabs that are '*not* supporting or attached to partitions or other construction likely to be damaged by large deflections' (ACI, 2014: p. 83). These slabs are referred to in this study as slabs that are 'not supporting non-structural damageable elements'. For two-way slabs, provisions are given in section 8.3 and table 8.3.1.1, both for slabs that may and for slabs that may not 'support or be attached to non-structural elements likely to be damaged by large deflections' (ACI, 2014).

According to the ACI 318 provisions, the slenderness of slabs thus obtained serves as an alternative method to control deflections as opposed to the detailed calculations of deflections. By way of detailed deflection computation examples, Bondy (2005) showed that the *L*/*h* ratios offered by the ACI may produce excessive slenderness of slabs, which do not conform to the ACI allowable deflection requirements.

Thus, although this 'CS' method has the advantage of being simple and has been validated by extensive experience with reasonably good results, it is not applicable in all cases. This situation may be linked to the fact that the results of this simplified deflection-control method do not consider the influence of several factors that do have a significant influence on deflection (refer to Table 1).

3.2 Methods of Eurocode 2 (CEN, 2004)

In section 7.4.2 of Eurocode 2 (CEN, 2004), which is referred to as 'Cases where calculations may be omitted', provisions are established to control deflections and avoid actual deflection computations. In this study, these provisions are regarded as two distinct methods, even if the code presents them as one sole consistent method. On the one hand, the main method of Eurocode 2, is classified as a 'reinforcement-dependent slenderness method' (RDS) in this study. On the other hand, there is a 'CS method', which is similar to that of ACI 318. The Eurocode 2 presents this latter method only as a design aid of the main method.

The Eurocode 2 main method, which is based on Corres and others (Corres *et al*., 2003), is designed to control deflection by establishing the slenderness of members and considers a larger number of factors than the ACI method (refer to Table 1). Thus, a better degree of accuracy in the results is expected.

The formula to calculate the slenderness (*l*/*d*) for members with low reinforcement ratios, such as slabs, is described in section 7.4.2 of the Eurocode 2, where it is numbered 7.16.a, but numbered as Equation 2 here.

2. $$\frac{l}{d} = K\left[11 + 1.5\sqrt{f_{\text{ck}}}\frac{\rho_0}{\rho} + 3.2\sqrt{f_{\text{ck}}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] \text{ if } \rho \leq \rho_0 \quad \text{with } \rho_0 = \sqrt{f_{\text{ck}}} \times 10^{-3}$$

where *l*/*d* is the span/effective depth ratio; *K* is the boundary condition factor (refer to table 7.4N for details) (CEN, 2004); ρ_0 (defined above) is the reference reinforcement ratio (one-way); ρ is the one-way required tension reinforcement ratio at mid-span to withstand service loads; and f_{ck} is the characteristic cylinder compressive strength (MPa).

The method appears to be simple: use the closed-form Equation 2. However, this method requires previous knowledge of the actual steel reinforcement ratio of the member, which is typically unknown in the early stages of design, when depth is also an unknown. Therefore, the iterative nature of the calculation for reinforced concrete, as previously mentioned, cannot be solved unless the code has provided reference values for reinforcement ratios. The only reference to reinforcement ratio that has been found by the authors is included in the Spanish code EHE (CPH, 2008), which is consistent with Eurocode 2. The EHE code does not explicitly suggest reinforcement ratios that should be applied in Equation 2, but suggests that the reinforcement ratio $\rho = 0.50\%$ can be used for slabs when using table 50.2.2.1 of EHE, which is said to be consistent with the results of Equation 2 of the Eurocode 2. This study tests the

Table 1. Factors that influence the establishment of the depth of slabs in the early stages of design that are considered in several deflection-control methods

Factors	ACI (2014)	CEN (2004)[a]	'Short method of Rangan–Scanlon' (2006)	'Long method of Rangan–Scanlon' (1999–2014)
Span	Yes	Yes	Yes	Yes
Boundary conditions	Yes	Yes	Yes	Yes
Total loads	—	Indirectly	Yes	Yes
Sustained loads	—	Indirectly	Yes	Yes
Load history	—	Indirectly	Indirectly	Indirectly
Allowable deflection	—	—	Yes	Yes
Yield strength of steel	Indirectly	Yes	Indirectly	Yes
Required amount of reinf. (M+)	—	Yes	Indirectly	Yes
Actual amount of reinf. (M+)	—	Yes	—	—
Required amount of reinf. (M–)	—	Only cantilevers	Indirectly	Yes
Actual amount of reinf. (M–)	—	Only cantilevers	—	—
Density of concrete	Indirectly	—	Yes	Yes
Elastic modulus of concrete	Indirectly	Indirectly	Yes	Yes
Concrete compressive strength	—	Yes	Indirectly	Yes
Tensile strength of concrete	—	—	Indirectly	Yes
Moment redistribution	—	—	—	—
Safety introduction method	—	—	Indirectly	Yes

[a]The support document Corres *et al*. (2003) is considered to obtain the column of data for Eurocode 2

reinforcement ratio $\rho = 0.50\%$ as a possible constant ratio to avoid iterative computations when using Equation 2.

In addition to this main method of Eurocode 2, a second method exists: the 'CS method' described in table 7.4.N of the code. In general terms, this method is similar to that of the ACI code but the slenderness values obtained when using each method differ in most cases.

The Eurocode 2 presents this second method as a design aid for the main method. However, the present study could not find consistency between the two methods. Thus, they were analysed separately.

It must be pointed out that some of the main authors of the current provisions of Eurocode 2 have proposed an evolved version (Pérez *et al.*, 2017) of the formulas currently included in Eurocode 2 that can certainly increase the consistency between the formula and the table. The evolved formula is designed to include a larger number of factors influencing deflection control, including several of the factors that this study considers. Therefore, there is currently an interest in optimising the deflection control provisions and thereby improving future codes and structures.

3.3 'Short method of Rangan–Scanlon' (1999–2006)

This method considers an extensive range of factors that influence deflection computation (refer to Table 1). The 'short method of Rangan–Scanlon' is classified in this study as a 'reinforcement-independent slenderness method', which is also termed the RIS method. The independence of the amount of reinforcement is a relevant advantage of this method, given that the reinforcement ratio is unknown in the early stages of design.

One of Scanlon's last works on this subject (Scanlon and Lee, 2006) describes the method as derived from an earlier work of Rangan (1982). Among its primary advantages is the rationality of the method. The slenderness of slabs is obtained by using a formula derived from classical formulas of deflection of one-way members.

The method focuses on computing incremental deflection, which is considered to be the governing factor in the design of slabs:

3. $$\Delta_{\text{inc}} = \frac{\kappa l^4}{384 E_c I_e}\left(\lambda W_s + W_{l(\text{var})}\right)$$

where Δ_{inc} is the incremental deflection; κ is the boundary condition coefficient (= 5 for simply supported, = 1,4 for continuous members, = 2 for one continuous end, and = 48 for cantilevers); l is the span; E_c is the elastic modulus of concrete; I_e is the effective moment of inertia (after Branson); λ is the multiplier for long-term deflections under sustained loads; W_s is the sustained load; and $W_{l(\text{var})}$ is the variable portion of the live load.

Scanlon's proposal in 2006 (Scanlon and Lee, 2006) is a generalisation, for a variety of one- and two-way floors, of a method that he developed in 1999 (Scanlon and Choi, 1999) for one-way slabs (described as the 'long method of Rangan–Scanlon'). To transform the original one-way formulation of 1999 into the more general two-way formula of 2006, Scanlon incorporated the crossing beam method (ACI, 1995; Scanlon and Murray, 1982) and works by Timoshenko (Timoshenko and Woinowsky-Krieger, 1959), Branson (1963), Rangan (1982) and Gilbert (1985).

Equation 4 is obtained by transforming I_e of Equation 3 into $\alpha I_b (= \alpha b h^3/12)$, dividing Equation 3 by l and setting the right-hand side of the resulting equation as equal to $(\Delta_{\text{inc}}/l)_{\text{allow}}$. Finally, l^3/h^3 is isolated, and both sides of the equation are cube-rooted.

4. $$\frac{l_n}{h} = \{\beta_{\text{ES}}\}\left[\left(\frac{\Delta_{\text{inc}}}{l}\right)_{\text{allow}} \frac{32\alpha E_c b}{\kappa\left(\lambda W_S + W_{L(\text{var})}\right)} \frac{\{k_{\text{DP}}\}}{\{k_{\text{AR}} k_{\text{SS}}\}}\right]^{1/3}$$

where Δ_{inc}, l, E_c, I_e, λ, W_s and $W_{l(\text{var})}$ have the same meaning defined for Equation 3; l_n/h is the net span/depth ratio; $(\Delta_{\text{inc}}/l)_{\text{allow}}$ is the allowable incremental deflection divided by the span; α is the stiffness coefficient (typically between 0 and 1); and b is the breadth of the rectangular section.

The ratio $\{\beta_{\text{ES}}\}$ and the coefficients enclosed in curly braces $\{k_{\text{AR}}, k_{\text{DP}}, k_{\text{SS}}\}$ apply only to two-way slabs as they convert an equation for one-way deflection into an equation that considers two-way deflection (refer to section A1.4.2 of the Appendix and Scanlon and Lee (2006) for more detail).

According to Scanlon, α can be taken as a constant, which is one of his major contributions to Rangan's formulations. Scanlon proposed a value of 0.4 in 1999 (Scanlon and Choi, 1999) and later proposed a value of 0.52 in 2006 (Scanlon and Lee, 2006) based on a new parametric study. The latter value is used in the present study, since it is the value recommended by Scanlon in his latest study.

For simplicity, given the extensive experience of Scanlon in this field, in the present study, the values of all coefficients of Equations 3 and 4 are considered adequate (more details provided in A1.4 of the Appendix).

Despite its advantages, this method has a disadvantage: it requires iteration because the weight of the slab (which is a part of the sustained load W_s) influences the determination of depth. However, iterations typically converge very quickly. For example, when the depth obtained by the CS method of ACI

318 is considered for the first iteration, more than three iterations in total are rarely necessary (Scanlon and Lee, 2006).

3.4 'Long method of Rangan–Scanlon'

In this study the 'long method of Rangan–Scanlon' refers to a method that was developed by Scanlon earlier than the 'short method', as described in the previous section. This method can either be used for the same purpose as the methods mentioned above (as is done throughout this study), or as a method complementary to the 'short method' in order to improve it and prepare it to be fully operational, as clarified next.

Scanlon investigated the value of the coefficient α in detail in his 1999 paper (Scanlon and Choi, 1999), in which he performed an extensive parametric study. This study did not consider α as a constant but instead included the calculation of the effective moment of inertia (I_e) in the iteration process. Subsequently, the value of α was calculated based on I_e:$\alpha = I_e/I_g$. As a result of the systematic use of this 'long method', in 2006 Scanlon (Scanlon and Lee, 2006) proposed the use of a constant value of α and developed what in this study is termed the 'short method of Rangan–Scanlon'.

However, an analysis of the 'long method of Rangan–Scanlon' (Scanlon and Choi, 1999) shows that the value of α is not constant but has small variations that fundamentally depend on the reinforcement ratio and the tensile strength of the concrete. Consequently, the 'long method' cannot be strictly considered a 'RIS method'. In this study, however, the 'long method' is grouped with the 'short method' because it is considered a complement to it that will facilitate the future development of a more definitive version of the 'short method'.

In this study, the 'long method' of 1999 (Scanlon and Choi, 1999) has been slightly modified, because the values of α and L_n/h are formulated as a function of the main variables that affect α. This modification has been performed using the following iterative process.

(*a*) Compute the slenderness of the slab using the 'short method' and the initial stiffness coefficient, $\alpha_0 = 0.52$.
(*b*) Compute the required reinforcement ratio for a unit width of the one-way slab (or a unit with of the support band of the two-way slab).
(*c*) Compute the effective moment of inertia (I_e) and the corresponding stiffness coefficient, which is referred to as the modified stiffness coefficient $\alpha_m = I_e/I_b$.
(*d*) Compare the value of α_m that is obtained in step (*c*) with the value of α_0 that is obtained in step (*a*). If $\alpha_0 = \alpha_m$, the slenderness can be considered adequate, and α_0 is considered valid. Otherwise, the process is repeated until convergence with α_0 equal to the α_m computed in step (*c*).

Despite the work-intensive nature of this version of the 'long method', it is useful in this study because it provides slenderness values that are more accurate than would be obtained by the 'short method'.

A considerable number of values of α have been obtained using this method for the slabs included in this study. Although they are not listed here, the values vary between 0.25 and 0.45, with most of the results concentrated between 0.30 and 0.40. This interval is slightly different from the value 0.52 that was proposed by Scanlon in 2006 (Scanlon and Lee, 2006). Although applying α as a constant is very useful because it enables the use of the 'short method of Rangan–Scanlon', the value of α in the 'short method' should not always take the same value for all members and situations. The values of this coefficient α should be further evaluated and tabulated as a function of some key variables that control deflection (Sanabra Loewe and Scanlon, 2014).

Improvements to the classical equations to calculate deflections are permanently being studied to consider, among other factors, the behaviour of non-rectangular cross-sections (Shaaban and Mustafa, 2019) and the behaviour of new reinforcement solutions (Ge *et al.*, 2020), or to perfect the calculation of the effective moment of inertia (Nguyen *et al.*, 2020).

Thanks to the rationality of the method of Rangan–Scanlon, it may be adapted to these advances or to future needs, basically by finding proper values of coefficient α (and potentially of other parameters of the equation).

4. Comparative study of the deflection-control methods

A comparative study has been performed to understand better the four deflection-control methods described above and their sensitivities to different parameters. The study is divided into two parts. In the first part, the depths obtained with the different methods are compared considering different typical boundary conditions (simply supported and continuous). In the second part, the sensitivity of the methods is assessed with reference to several key parameters that influence the deflection control (magnitude and duration of superimposed loads, allowable deflection limit and moment redistribution).

The methods analysed were developed for different codes that are not directly comparable. Consequently, in this study, materials (concrete and steel) and deflection-control limits are selected to make comparisons possible while maintaining compatibility with the corresponding codes. (Details are provided in Sections A1.1, A1.2 and A1.3 of the Appendix.)

4.1 First part of the study

In total, eight variants of the previously discussed methods are examined, as listed below.

- Method 1: the ACI method, which is a CS method.
- Method 2: four variants of the Eurocode 2 methods, as listed next.
  - Method 2a: the CS method, following Table 7.4N of the Eurocode.
  - Method 2b: Equation 2, which is a 'RDS method.' The required tensile reinforcement ratio at mid-span (ρ), which is employed in Equation 2, is based on computing moments considering an elastic behaviour (no moment redistribution). A different ρ is obtained for each case, depending on the corresponding diagram of moments.
  - Method 2c: equivalent to method 2b, but considering a moment redistribution of 15% of the elastic negative moments on bearings.
  - Method 2d: Equation 2 with a constant tensile reinforcement ratio: $\rho = 0.50\%$.
- Method 3: the 'short method of Rangan–Scanlon' with $\alpha = 0.52$ ('RIS method').
- Method 4: two variants of the 'long method of Rangan–Scanlon', which is considered a RIS method even if it is not, in the strictest sense.
  - Method 4a: the required amount of tensile reinforcement is based on computing moments considering an elastic behaviour (no moment redistribution).
  - Method 4b: the same as method 4a considering a limited moment redistribution of 15% of the negative elastic moments.

Results of the first part of the study are displayed in Figures 1 and 2. Figure 1 shows the results for simply supported one-way slabs. Figure 2 shows the results for the interior panels of two-way slabs. Some differences exist in the design and behaviour of one-way and two-way slabs.

4.2 Analysis of results of the first part of the study

Regarding the two CS methods 1 and 2a, a certain level of agreement is obtained for simply supported one-way slabs (Figure 1), where Eurocode 2 depths are only 12% smaller than ACI depths. For the interior panels of slabs (Figure 2), the depths of Eurocode 2 are 31% larger than those of ACI. This difference is relevant considering that both codes establish that these depths entitle a designer to avoid actual deflection computations. While actual reinforced concrete deflection computations are not too controversial, such a difference in the two methods, which claim to 'replace' actual deflection calculations, can only mean that both codes are taking different choices, even if these choices are not explicitly acknowledged. These choices will be further explained in the analysis of the second part of the study.

Concerning the sets of data that correspond to Eurocode 2 methods, numerous revealing observations are noted, as shown in Figures 1 and 2. The three sets of data based on the RDS method (methods 2b, 2c and 2d) consistently yield very different results when compared. In both figures, methods 2b and 2d yield the most different depths. The depths of

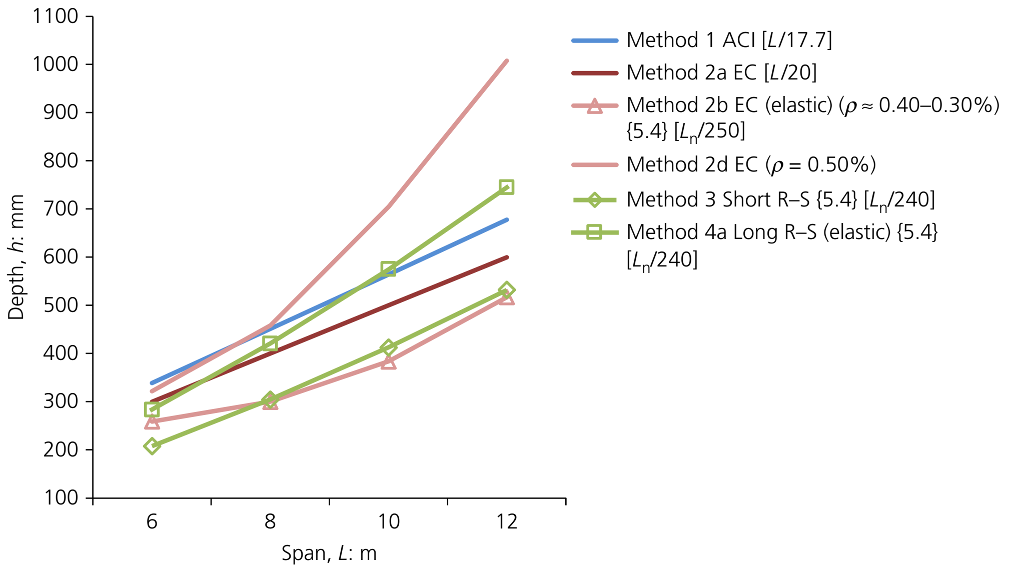

Figure 1. Depth (h) plotted against span (L) for simply supported one-way slabs that are 'not supporting non-structural damageable elements' (100 mm = 3.94 in; 2 m = 6.56 ft. Note on annotation in key: ACI refers to ACI 318; EC refers to Eurocode 2; short R–S refers to short Rangan–Scanlon method; long R–S refers to long Rangan–Scanlon method; {5.4} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 40% of superimposed loads; [L_n/…] indicates the allowable incremental deflection of a series; 'elastic' indicates moment diagrams without moments redistribution; (ρ = …%) indicates reinforcement ratio or range of reinforcement ratios of a series)

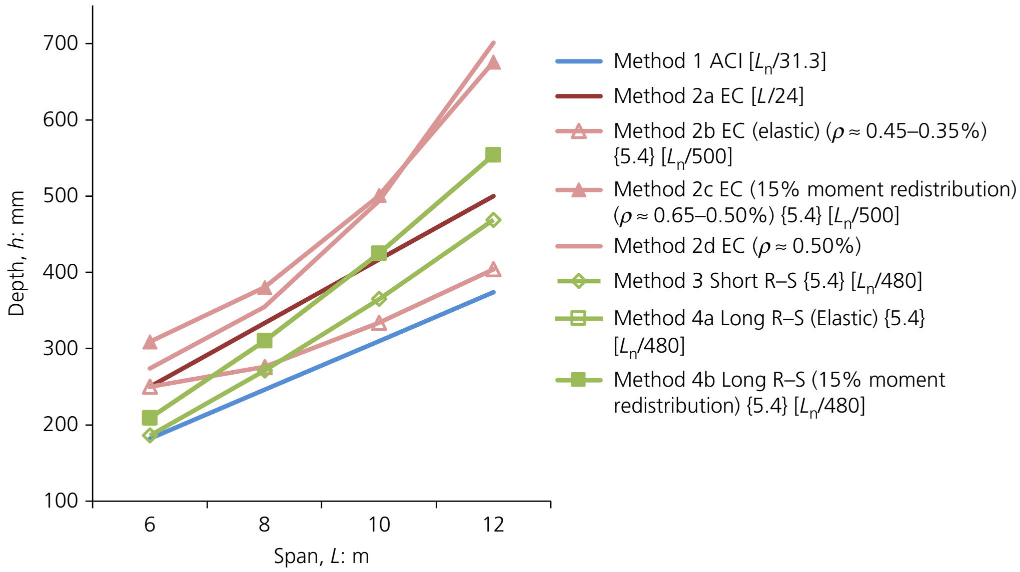

Figure 2. Depth (*h*) plotted against span (*L*) for interior panels of two-way slabs that are 'supporting non-structural damageable elements.' (100 mm = 3.94 in; 2 m = 6.56 ft. Note on annotation in key: ACI refers to ACI 318; EC refers to Eurocode 2; short R–S refers to short Rangan–Scanlon method; long R–S refers to long Rangan–Scanlon method; {5.4} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 40% of superimposed loads; [Ln/480] and [Ln/500] indicate the allowable incremental deflection of a series; 'elastic' indicates moment diagrams without moments redistribution; (15% moment redistribution) indicates moments diagrams that consider a 15% of negative moment redistribution; (ρ = …%) indicates reinforcement ratio or range of reinforcement ratios of a series)

method 2d, on average, are more than 50% larger than those of method 2b, while the ratios of reinforcement of method 2d are only approximately 25% larger than those of method 2b. This finding raises questions regarding the applicability of the method based on Equation 2 without some guidance about the amount of reinforcement, given the considerable influence of this variable. For example, watching the results of series under method 2d for the constant ratio $\rho = 0.50\%$, the large depths that are yielded indicate that the ratio $\rho = 0.50\%$ can be considered an upper limit instead of a mean value of ρ for slabs. So, the ratio $\rho = 0.50\%$ seems too conservative to be utilised on a general basis to obtain the initial dimension of the depth of slabs. Thus, the series of method 2d with a constant reinforcement ratio of $\rho = 0.50\%$ is disregarded for the second part of the study.

A notable difference in the results between the sets of data for method 2b and the sets of data for method 2c is observed (Figure 2), particularly for larger spans (10 m (33 ft) or more). A 15% moment redistribution, which is the difference between method 2b and method 2c, causes an increase of 40% in positive reinforcement and an increase of almost 80% in the depth of the slab. This finding shows the enormous influence of an accurate computation of the actual redistribution of moments, which is surprising for a method that establishes the depth of a slab in the early stages of its design. To avoid the high influence of the redistribution of moments on the prediction of depth, one possible approach is suggested: Equation 2 could include the amount of negative moment reinforcement at the bearings, not focusing the formula exclusively on the amount of positive moment reinforcement at the midspan.

Considering the four sets of data of Eurocode 2 as a whole, methods 2a, 2b, 2c and 2d in Figures 1 and 2, none of the three sets of data based on the RDS method (methods 2b, 2c and 2d) shows a clear relation with the depths yielded by the CS method (method 2a) of the same Eurocode 2. These results could raise questions regarding the consistency of the two methods (RDS and CS), which are presented as a sole method in this code.

Additional observations of the sets of data of the 'long method of Rangan–Scanlon' (methods 4a and 4b) are feasible. For example, no significant difference is observed between the results of the 'long method' for elastic diagrams of moments (method 4a) and the results of the 'long method' for diagrams with a moments redistribution of 15% of the elastic negative moment (method 4b). The results obtained in the two cases are similar. In Figure 2, method 4a cannot be distinguished from method 4b because they are overlapped. Although this situation might be considered a shortcoming of the method, the results are consistent with the fact that Scanlon does not describe the effect of the moments redistribution in the method.

Beyond this limitation, the series of the 'long method' (methods 4a and 4b) are the series that show better agreement with the results of the two CS methods (methods 1 and 2a), as shown in both figures (Figures 1 and 2), especially for short spans. This situation may be a certain proof of its right guess, given the vast experience of the CS methods. Additional details in this are provided in the second part of the study.

The results of the 'short method of Rangan–Scanlon' (method 3) consistently yield depths that are smaller than those of the 'long method' (method 4a). For simply supported one-way slabs (Figure 1), approximately 30% smaller depths are obtained; for interior panels of two-way slabs (Figure 2), approximately 10% smaller depths are obtained. This finding may suggest that the value of alpha ($\alpha = 0.52$) proposed by Scanlon in 2006 (Scanlon and Lee, 2006) and applied in this study in the 'short method of Rangan–Scanlon' is slightly unconservative. Further investigation may be needed to obtain proper values of α depending on the main variables that influence deflection calculation and deflection control (Sanabra Loewe and Scanlon, 2014). Considering that the value of α selected for the 'short method' in this study does not seem to be perfectly tuned, the 'short method' is not employed in the second part of the study.

4.3 Second part of the study

In this part of the study, an assessment is made of the sensitivity of the RDS method of the Eurocode 2 (methods 2b and 2c) and the sensitivity of the 'long method of Rangan–Scanlon' (method 4a), which is considered an RIS method, to certain key variables: the magnitude and duration of the superimposed loads, the allowable deflection limit and the moment redistribution. As a background the data of series from methods 2b, 2c and 4a are displayed, which are the depths yielded by the CS methods of ACI (method 1) and Eurocode 2 (method 2a). To avoid surcharging the figures with information, the focus has been the 'long method of Rangan–Scanlon' (method 4a), while series of data of method 2b and method 2c, of Eurocode 2, have been limited to the most representative cases.

4.4 Analysis of results of the second part of the study

In Figure 3, devoted to studying the influence of the magnitude of the superimposed loads in the interior panels of two-way slabs, it can be observed how the 'long method of Rangan–Scanlon' – two curves (method 4a) – regardless of the span, provides an increase in the depths of the slabs of +20% to +25% as the superimposed loads are increased 200%, from 5 kN/m^2 to 15 kN/m^2. Similar increases are observed for the RDS method of Eurocode 2 (methods 2b and 2c). For superimposed loads of 15 kN/m^2, the curves for method 2b and method 2c of Eurocode 2 in Figure 3 are above and below the curve (method 4a) of the 'long method of Rangan–Scanlon'. Curves for method 2b and method 2c of Eurocode 2 show similar results for superimposed loads of 5 kN/m^2. However, the results of the Eurocode 2 for 5 kN/m^2 are not displayed in Figure 3 to avoid excess of data in the figure.

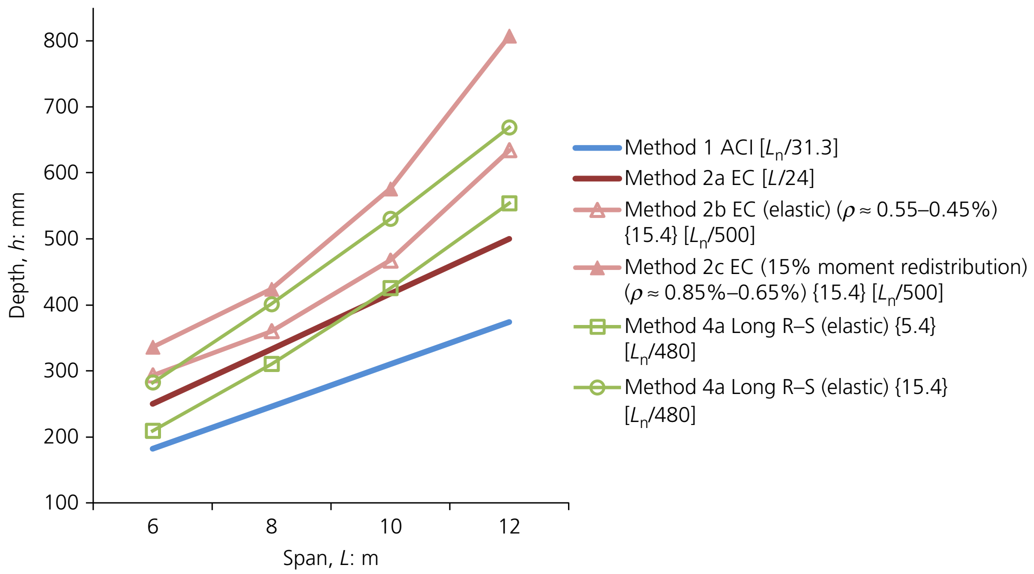

Figure 3. Depth (*h*) plotted against span (*L*) of interior panels of two-way slabs 'supporting non-structural damageable elements'; to assess the variation in depths yielded by method 2b, method 2c and method 4a to changes in the magnitude of superimposed loads and to changes in the moments redistribution. (100 mm = 3.94 in; 2 m = 6.56 ft. Note on annotation in key: ACI refers to ACI 318; EC refers to Eurocode 2; long R–S refers to long Rangan–Scanlon method; {5.4} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 40% of superimposed loads; {15.4} is the load case where total superimposed load is 15 kN/m^2, where sustained loads are 40% of superimposed loads; [Ln/480] and [Ln/500] indicate the allowable incremental deflection of a series; 'elastic' indicates moment diagrams without moments redistribution; (15% moment redistribution) indicates moments diagrams that consider 15% of negative moment redistribution; (ρ = …%) indicates reinforcement ratio or range of reinforcement ratios of a series)

In Figure 3, the method of Eurocode 2 (methods 2b and 2c) is dependent on knowing in advance the amount of moment redistribution, which is consistent with the findings in the first part of the study. A 15% moment redistribution increases the positive reinforcement by 50%, and causes an increase in the depth of floors of 30%.

Beyond the particular issue with the moment redistribution that the method of Eurocode 2 has, both methods (Eurocode 2 and the 'long method of Rangan–Scanlon') yield reasonable increases in depth when the loads are increased. This result contrasts with constant depth methods of the ACI (method 1) and Eurocode 2 (method 2a), which have no criteria or poor criteria for addressing the magnitude of the load.

Comparing the results of the CS methods of the ACI (method 1) and Eurocode 2 (method 2a) with those of the other two more refined methods, in particular with the 'long Rangan–Scanlon method', several observations are noted. First, the depths of the ACI method (method 1) seem slightly unconservative or tuned only for short spans (around 6 m) and light loads (less than 5 kN/m^2). This result is consistent with the findings of Bondy[1] and the comment in section R8.3.1 of the ACI code, which acknowledges that the constant slenderness given in the code may not be suitable in the case of 'unusually heavy' loads. However, 'usual' or 'unusual' loads are not defined. Second, the depths of Eurocode 2 (method 2a) seem well tuned for moderate spans (just under 10 m) with superimposed loads of 5 kN/m^2, similar to those in the lower bound curve of the 'long Rangan–Scanlon method' (method 4a) in Figure 3. In the same figure, for short spans (6 m), Eurocode 2 seems to be tuned for higher loads (nearer 10 kN/m^2). This result is consistent with note 1 in table 7.4N of Eurocode 2, which is the table that defines method 2a. Note 1 acknowledges that the values in table 7.4N 'have been chosen to be generally conservative and calculation may frequently show that thinner members are possible'.

The two short comments in the previously mentioned codes, in addition to the comparison with the results of the refined methods, would put forward the very different choices taken by the two codes (31% in difference of depths) to address the same problem: the poor capacity of CS methods to adjust the results to an increase in the superimposed loads.

In both codes, the constant depth ratios (methods 1 and 2a) are primarily tuned for short or moderate spans (less than 10 m). These ratios seem to disregard the fact that the increase in span exponentially affects the deflection and should cause increases in depth accordingly, just as more refined methods are able to do: the RDS method of Eurocode 2 (methods 2b and 2c) and the 'long method of Rangan–Scanlon' (method 4a).

The ACI seems to be tuned for short spans (around 6 m) and light loads, while Eurocode 2 seems to be tuned for moderate spans (around 8 m) and moderately high loads. Thus, following the ACI provisions without calculating deflections may produce economic floors for domestic spans and loads, while following the Eurocode 2 provisions may produce uneconomic floors for domestic spans and loads. However, possibly the ACI does not clearly warn of potential excessive deflection for larger spans or larger loads, and Eurocode 2 does not clearly warn of potentially expensive and unnecessarily polluting floors for domestic loads and spans.

Figure 4 shows that an increase in the proportion of sustained superimposed load without varying the total load has a negligible influence on the depths of slabs that are obtained with the 'long method' (method 4a). The RDS method of Eurocode 2 (methods 2b and 2c) continues yielding very different depths depending on the redistribution of moments. The results of the RDS method of Eurocode 2 (methods 2b and 2c) are approximately above and below the results of the 'long method of Rangan–Scanlon' (method 4a).

Figure 5 shows that a less restrictive deflection limitation (e.g. $L_n/250$ for roofs) reduces the depths obtained with the 'long method of Rangan–Scanlon' (method 4a). Allowing a double deflection causes a 20% reduction in the depth of the slab, which seems reasonable. In contrast, the RDS method of Eurocode 2 (methods 2b and 2c) does not adequately address this situation. This is explained because a relaxation of the allowable deflection typically causes a reduction in depth, while superimposed loads are kept constant. As a result, the amount of reinforcement is increased. In the RDS method of Eurocode 2, additional reinforcement causes an increase in depth (refer to Figure 5), which is not the desired result.

5. Conclusions and recommendations for codes provisions

Establishing the depth of a slab as the first step of its design is a common practice as a means to control its deflection. This decision in the early stages of design has a major importance from both the economic point of view and the environmental point of view, as well as from the serviceability point of view. Thus, improvements in this simple early phase of design can have a relevant influence and should be regarded by code developers as a way to improve the efficiency and sustainability of structures. Improving this part of the codes should not be too labour intensive from the scientific point of view, but could be highly influential on general practice.

This study compares three families of simplified deflection-control methods for the design of one-way and two-way reinforced concrete slabs, including the provisions of the ACI 318 code and the Eurocode 2 code for minimum depth provisions. In these codes, the depth (h) of the slab is established as related to the span (L) using the concept of slenderness (L/h).

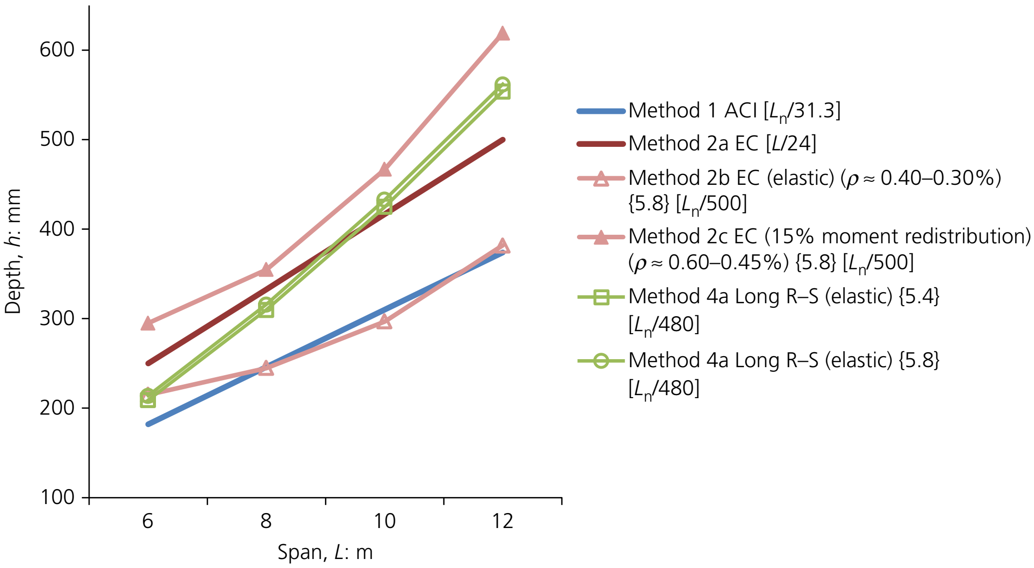

Figure 4. Depth (h) plotted against span (L) of interior panels of two-way slabs 'supporting non-structural damageable elements'; to assess the variation of depths yielded by method 2b, method 2c and method 4a to changes in the proportion of sustained superimposed loads and to changes in the moments redistribution. (100 mm = 3.94 in; 2 m = 6.56 ft. Note on annotation in key: ACI refers to ACI 318; EC refers to Eurocode 2; long R–S refers to long Rangan–Scanlon method; {5.4} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 40% of superimposed loads; {5.8} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 80% of superimposed loads; [Ln/480] and [Ln/500] indicate the allowable incremental deflection of a series; 'elastic' indicates moment diagrams without moments redistribution; (15% moment redistribution) indicates moments diagrams that consider 15% of negative moment redistribution; (ρ = …%) indicates reinforcement ratio or range of reinforcement ratios of a series)

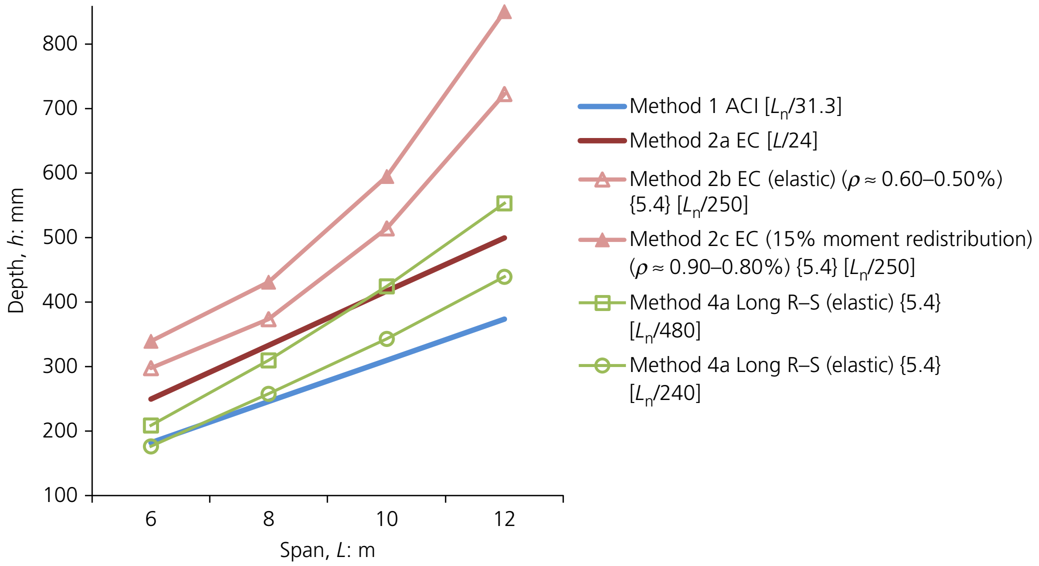

Figure 5. Depth (h) plotted against span (L) of interior panels of two-way slabs 'not supporting non-structural damageable elements'; to assess the variation in depths yielded by (2b), (2c} and (4a} to changes in the moments redistribution and different limits imposed on incremental deflection. (100 mm = 3.94 in; 2 m = 6.56 ft. Note on annotation in key: ACI refers to ACI 318; EC refers to Eurocode 2; long R–S refers to long Rangan–Scanlon method; {5.4} is the load case where total superimposed load is 5 kN/m^2, where sustained loads are 40% of superimposed loads; [Ln/240], [Ln/250] and [Ln/480] indicate the allowable incremental deflection of a series; 'elastic' indicates moment diagrams without moments redistribution; (15% moment redistribution) indicates moments diagrams that consider 15% of negative moment redistribution; (ρ = …%) indicates reinforcement ratio or range of reinforcement ratios of a series)

The following three families of deflection-control methods are examined

(*a*) two CS methods, which are detailed in standard codes: ACI 318 and Eurocode 2
(*b*) one 'RDS method', which is included in Eurocode 2
(*c*) two variants of a 'RIS method', which is based on the work of Rangan, Scanlon and others and termed in this study 'methods of Rangan–Scanlon'.

Constant slenderness methods have been shown to be reasonably satisfactory for half a century. One of their main assets is extreme simplicity. In contrast, this study shows their limited capacity to address changes in some of the main variables that influence deflection control: load intensity, span and allowable deflection limits. Some of these shortcomings are implicitly acknowledged in brief comments in the codes. These limitations of CS methods force each code to take a 'choice'. Thus, each code tunes its CS method to particular cases, which are considered to be more relevant by that particular code. An example of the 'choice' of codes when using the CS method is demonstrated by the fact that the Eurocode 2 depths for interior panels of two-way slabs are 31% larger than those of the ACI. This is because the ACI specifies a constant slenderness that is tuned for very short spans (around 6 m) and very light loads (domestic loads), while Eurocode 2 specifies a slenderness value that is tuned for moderate spans (around 8 m) and moderate loads (around 5 kN/m^2).

A first step to improving code provisions could be to replace vague sentences on the intensity of loads by clear statements establishing what load or range of loads each provision is valid for. As a complement, some easy rule could be used to adjust the set of slenderness values provided in the code to the intensity of loads, such as 'given an increase of loads of x%, the slenderness provided should be reduced by y%'. In addition, a range of spans could be established for the use of the code provisions and, eventually, the constant slenderness (L/h) could be modified by some expression of the shape $h = ct + L/h$, where ct is a constant used to allow the increase of the slope of the slenderness plot, to adapt it better to the exponential influence of span. As an example of the possibilities to increase the flexibility of CS methods in the codes, the ACI code shows how addressing the variation of allowable deflection can be done by offering two sets of constant slenderness, one for slabs 'supporting damageable elements,' and one for slabs 'not supporting damageable elements.'

Instead of trying to amend their too simple code provisions by adding correction factors to CS methods, some codes (Australian standard and Eurocode 2) are starting to incorporate methods that consider at the same time a larger number of factors that influence deflection control. For example, Eurocode 2 includes a 'RDS method' and the Australian standard uses a 'RIS method' that is related to the two 'methods of Rangan–Scanlon' discussed in this study. Several of these more refined methods reviewed in this study show promising results. These more refined methods (RDS and RIS) have shown consistent variations in depth that are associated with variations in span and load.

The 'RDS method' ofEurocode 2 has a particularly high sensitivity to the variations in the amount of reinforcement at midspan. Also, the RDS is revealed to be too sensitive to the variations in the percentage of moments redistribution. Both problems cause difficulties in the use of the method in practice. To address these two problems, the amount of negative reinforcement could be included in the Eurocode 2 formulas and the designer should be given sufficient criteria to determine the amount of reinforcement to use as a function of the main variables that influence deflection. Finally, the 'RDS method' of Eurocode 2 is only tuned for a certain allowable deflection and cannot address the changes in this variable. In conclusion, it is found that this method of the Eurocode still needs a considerable amount of study to make it useable in real practice. Moreover, its equations are basically empirical and are not easy to modify rationally. That is why the authors of the current study suggest that Eurocode 2 could consider abandoning the effort of amending this method, and undertake a change to adopt a more rational 'RIS method' derived from the 'methods of Rangan–Scanlon'.

The 'methods of Rangan–Scanlon' are based on classical equations for deflection, and are thus supported by decades of tests and scientific literature. Consequently, they could be rationally amended to adapt the results of future research on the formulations of deflection, including for example new cross-sections, or new reinforcing solutions. Moreover, these methods are designed to be almost insensitive to the actual amount of reinforcement in slabs because they address this variable using a stiffness coefficient α ($= I_e/I_g$) that is almost a constant for most slabs. This approach is very convenient because the reinforcement ratio is typically an unknown in the early stages of design. This study has revealed that these methods provide reasonable depths for variations in the main variables: span, intensity of load and allowable deflection. However, the value (or range of values) of the α coefficient to be used in the 'methods of Rangan–Scanlon' are not yet completely established, and will require additional investigation. Once appropriate values of α are determined, criteria that enable a designer to select α as a function of the main variables that influence deflection will be needed.

6. Recommendations for practice

As a result of this study some recommendations are yielded that can be used by practitioners.

- In general terms, given that currently it is general practice to use computers to design structures and deflection calculations can be easily performed, one of the main applications of minimum slenderness provisions in the codes is to take a depth to start the computer-aided design. While a code provision recommending an unconservative depth should easily be detected when deflection is checked using computers, an excessively conservative depth will not be detected. For this reason, those code provisions that have been found to yield excessive depths are those that the practitioner should consider more seriously in order to increase the economy and sustainability of future designs.

- Regarding the use of the ACI-318 provisions:
  - For one-way simply supported slabs 'not supporting damageable elements' and for interior panels of two-way slabs 'not supporting damageable elements' the code provisions for slabs should give economical slabs that perform well, as long as the slabs are not loaded beyond 5 kN/m^2. For larger loads, it is recommended not to establish the depth of slabs with the CS method of the code as a replacement for proper calculation of deflections. For two-way slabs 'supporting damageable elements' the code provisions were found to be unconservative even for 5 kN/m^2 loads. Proper calculation of deflections is also recommended in these cases to establish the depth of slabs.
- Regarding the Eurocode 2 provisions:
  - Despite the fact that the code presents the two methods (table 7.4.N and equation 7.16.a) as one sole method, no basis was found to consider these two methods as two parts of the same method.
  - The constant-slenderness method (table 7.4.N) should be preferred over the 'RIS method' (formula 7.16.a), given that the latter is found not to be operational unless the code is modified to give more guidance on how to use it.
  - When using the CS method for slabs 'not supporting damageable elements', the slenderness for one-way simply supported slabs may give good results for low loads (under 5 kN/m^2), but for larger loads, proper deflection calculation should not be avoided. Contrastingly, for interior panels of two-way slabs, the code provisions are found to be excessively conservative for low loads (equal or under 5 kN/m^2). They yield uneconomical and unnecessarily heavy slabs.
  - When using the CS method for interior panels of two-way slabs 'supporting damageable elements', the code provisions yield excessively conservative depths for spans under or equal to 8 m and loads under or equal to 5 kN/m^2. These spans and loads are very common, so large amounts of uneconomical and not sustainable floors could the outcome of following this provision. It is recommended in this case to take slabs at least 30% shallower than the code provision, and based on that initial assumption compute the deflections properly. For loads around 10 kN/m^2 and spans under or equal to 8 m, the code provisions can be used.
- In general, to deal with depths of slabs with loads larger than those mentioned above, the following approximate rule can be used: as the load grows 100%, the depth should grow around 25%.
- Proper deflection calculations using computers are always recommended in any case.

7. Research significance

Construction of concrete floors is currently one major source of concrete consumption. The depth of a slab is the key parameter to control the material consumption in the slab and its weight. Selecting the depth of a slab is a precondition to commence its design and deeply influences its final performance as well as the volume of material used in its construction.

This paper includes information that may lead to improvements in the current methods used to establish the depth of slabs in the early phases of the design, to facilitate designing slabs with the correct depth – not thinner or thicker than strictly necessary. It is demonstrated how the two main families of methods included in ACI 318 and Eurocode 2 can be superseded by a simple but rational method, termed here the 'short method of Rangan–Scanlon'. In contrast with the Eurocode 2 method, it is shown that one key advantage of the method presented is its independence of the reinforcement ratio of the slab. It is also demonstrated that, unlike the CS methods of ACI 318 and Eurocode 2, the rationality of the method presented allows consideration of a large number of factors that influence the depth of the slab, including span, load, allowable deflection, incremental deflection and modulus of elasticity of the concrete. Despite the potential of the method, the present paper also suggests that some additional investigation is still needed before it can be used in general practice.

Appendix

This Appendix presents the general criteria used in calculations in this study to obtain the depths represented in the plots displayed in Figures 1–5.

After the general calculation criteria, the criteria used to determine the depths of slabs following the method of the ACI, the method of Eurocode 2 and the 'long method of Rangan–Scanlon' are provided.

A1.1 General criteria for calculations

A1.1.1 Geometric definition

- To convert L to L_n (ACI 318), which is referred to as L_{eff} in Eurocode 2, the supports are assumed to have a width of 300 mm (11.8 in) (parallel to the span).
- In two-way slabs, spans in each of the two directions are considered equal. Spans that differed in two directions were not studied in any case.

- To relate depth (h) to effective depth (d), an effective cover of 30 mm, measured from the centre of gravity of the reinforcement to the exterior face of the concrete, is assumed.

A1.1.2 Materials properties

- $f_{ck} = 30$ MPa ($f'_c = 4350$ psi) for concrete; $f_y = 500$ MPa (72.5 ksi) for steel.
- The density of concrete is considered equal to 2548 kg/m^3, which corresponds to a specific weight of 25 kN/m^3 (160 lb/ft^3).

A1.1.3 Deflection calculation and allowable deflection

- Throughout this study, the term 'incremental deflection' is used for the part of the total deflection that occurs 'after attachment of non-structural elements' to the structure (after the ACI 318 text).
- The contribution of compression reinforcement is not considered, which tends to be very low for members with a low amount of compressed reinforcement, such as slabs.
- To simplify calculations, the amount of reinforcement (ρ) obtained in all calculations does not consider the actual diameters of the available reinforcement bars. Considering a whole number of actual diameters or reinforcement bars would typically cause rounding up of the amount of reinforcement of slabs in an uneven way.
- Calculation of the time-dependent deflections assumes that $\lambda = 2$.
- To control the deflections of the members that are 'not supporting non-structural damageable elements', the total deflection is limited to $L_n/250$ for the Eurocode 2 method (after section 7.4.1 [4]). For the Rangan–Scanlon methods, table 24.2.2 of the ACI is followed with a maximum incremental deflection of $L_n/240$. These two criteria are not strictly comparable. For example, the results that are produced by the 'long method of Rangan–Scanlon' have been verified for load case 5.4 (refer to its definition below), and the deflection control for the Eurocode 2 method is significantly more demanding than the deflection control for the ACI method. The Eurocode 2 method requires depths that are 10–20% larger.
- For the deflection control of members that support 'non-structural damageable elements', the time-dependent deflection under sustained loads is limited to $L_n/500$ for the Eurocode 2 method (after section 7.4.1[5]), while the ACI requirements are followed for the Rangan–Scanlon methods, with a maximum incremental deflection of $L_n/480$ (after section 24.2). In this case, almost no difference exists between the requirements of the two codes. For example, the results that are produced by the 'long method of Rangan–Scanlon' for load case 5.4 show that, in all cases, the difference between the depth requirements of the two codes is less than 3%.

A1.1.4 Description of load cases

- The following load cases are utilised in this study.
  (*a*) Load case 5.4. The total superimposed load (SDL + LL) is 5 kN/m^2 (104 lb/in^2), and 40% of the load (2.00 kN/m^2 (41.8 lb/in^2)) is a sustained load (SDL + LL$_s$). This load case corresponds to light loads with a small proportion of sustained loads, which is quite common in actual practice. This case is the most common load case in this study.

  SW = 25 kN/m^3 h (160 lb/ft^3 h); SDL = 0.71 kN/m^2 (14.8 lb/in^2); LL = 4.29 kN/m^2 (89.6 lb/in^2) with $\psi_2 = 0.3$ (30% of LL is a sustained load: LL$_s$ = 1.29 kN/m^2 (26.9 lb/in^2)).

  (*b*) Load case 5.8. The total superimposed load (SDL + LL) is 5 kN/m^2 (104 lb/in^2), and 80% of this load (4.00 kN/m^2 (83.5 lb/in^2)) is a sustained load (SDL + LL$_s$). This load case corresponds to light loads with a large proportion (80%) of the sustained load.

  SW = 25 kN/m^3 h (160 lb/ft^3 h); SDL = 2.50 kN/m^2 (52.2 lb/in^2); LL = 2.50 kN/m^2 (52.2 lb/in^2) with $\psi_2 = 0.6$ (60% of LL is a sustained load: LL$_s$ = 1.50 kN/m^2 (31.3 lb/in^2)).

  (*c*) Load case 15.4. The total superimposed load (SDL + LL) is 15 kN/m^2 (313 lb/in^2), and 40% of the load (6.00 kN/m^2 (125 lb/in^2)) is a sustained load (SDL + LL$_s$). This load case corresponds to intensive loads with a low proportion of the sustained load.

  SW = 25 kN/m^3 h (160 lb/ft^3 h); SDL = 2.13 kN/m^2 (44.4 lb/in^2); LL = 12.87 kN/m^2 (269 lb/in^2) with $\psi_2 = 0.3$ (30% of LL is a sustained load: LL$_s$ = 3.87 kN/m^2 (80.8 lb/in^2)).

A1.2 Depths according to the ACI 318 method

- The ratios L/n and L_n/n that are used for the slenderness in the ACI 318 method in this study are not directly provided in tables of the code because steel grades used in the USA differ from those in Europe. Thus, small adjustments to the slenderness values provided by the ACI 318 tables were necessary. For simply supported slabs, the ACI 318 method establishes a slenderness of $L/20$ for G60 steel ($f_y = 60\,000$ psi). For other steels, the slenderness can be multiplied by $(0.4 + f_y/100\,000)$ (where f_y is in psi). For two-way slabs on isolated supports, the values in table 8.3.1.1 are interpolated, after footnote [2] of the table.

A1.3 Depths according to the Eurocode 2 method

- In the text of the study only formula 7.16.a is shown, here named Equation 2, which is to be used for members with a low reinforcement ratio ($\rho \le (f_{ck})^{1/2}/1000 = 0.50\%$, for

f_{ck} = 25 MPa (3625 psi)), such as most common slabs. This formula is employed in the present study to compute the depth of most slabs with Eurocode 2 on variants (method 2b and method 2c). For cases with a high ratio reinforcement (ρ>0.50%), formula 7.16.b of Eurocode 2 has been employed. The results obtained by either of these formulas have been adjusted, after paragraph 7.4.2, considering the span and the type of structural member.

- Before the Eurocode 2 method is utilised for variants method 2b and method 2c, the reinforcement ratio (ρ) must be determined to use expression 7.16.a (2) or 7.16.b. To obtain the reinforcement ratio ρ, the geometry of the slab has to be established. The slenderness that is used to find ρ has been determined using the 'long method of Rangan–Scanlon'.

To compute the cracking moment (M_{cr}) instead of the modulus of rupture (f_r), as applied by Scanlon after the ACI 318 standard, the flexural tensile strength of concrete is utilised following Eurocode 2:

$$f_{ctm,fl} = (1.6 - h)0.3\sqrt[3]{f_{ck}^2} \geq 0.3\sqrt[3]{f_{ck}^2} \text{ (with } h \text{ in m)} \quad (5)$$

A1.4 Depths according to the 'Long method of Rangan–Scanlon'

A1.4.1 Criteria relative to computing the reinforcement ratio (ρ) and the effective moment of inertia (I_e)

- For the ultimate limit state calculations, the coefficients of security are γ_c = 1.5 and γ_s = 1.15; the factored moment is M^* = 1.5M_k; and the span is taken as the clear span (L_n).
- To compute the cracked moment of inertia (I_{cr}), M_a is taken equal to M_k. (For additional details, refer to the following comments about the load history.)
- To determine the reinforcement ratio in slabs, the typical distribution of the direct method is used to calculate the stresses in two-way slabs on isolated supports, which can be divided into equivalent frames. These reinforcement ratios are only relevant at the support strips because the method of Rangan–Scanlon only considers the deflections at these strips. The negative moment is considered equal to 1.5 × (2/3) × M_0, where M_0 is half of the total moment of the equivalent frame. For consistency, the positive moment is considered equal to 1.2 × (1/3) × M_0.
- The method of Branson (1963) is used to compute the effective moment of inertia at critical sections (I_e^+, I_e^-). Bischoff and Scanlon demonstrated that, in general terms, the Branson expression should only be applied for heavily reinforced members (ρ>1%) (Bischoff and Scanlon, 2007). However, Scanlon and Bischoff suggested that the Branson method may be used in combination with the 'methods of Rangan–Scanlon' if the tensile strength of concrete recommended by the authors is employed (Sanabra Loewe and Scanlon, 2014).[16] Following these recommendations, when using the 'long method of Rangan–Scanlon' in this study – method 4a and method 4b – a constant tensile strength of concrete (f_r), which is equal to $0.33(f'_c)^{1/2}$, is employed.

A1.4.2 Criteria related to Equation 4 of Scanlon and Lee for two-way slabs

- The transparency and relative simplicity of the 'long method of Rangan–Scanlon', makes it possible in this study to introduce minor adjustments to the method, which was originally proposed by Scanlon in 1999 (Scanlon and Choi, 1999). These adjustments have been performed to include the advances of Scanlon in his 2006 work (Scanlon and Lee, 2006).
- The modulus of elasticity of concrete E_c is computed according to the ACI code ($4700\cdot(f'_c)^{1/2}$ = 23 500 N/mm^2 (3408 ksi)), because this value is the value employed by Scanlon in his studies. Using this value instead of the value of the Eurocode 2 code produces similar depths that are slightly larger (never exceeding an extra 5% depth). Using the modulus of elasticity in the Eurocode 2 code yields slightly higher reinforcement ratios (ρ) and slightly lower values of the coefficient of stiffness (α).
- In his studies, Scanlon typically utilises a default load history in which he assumes that concrete at an early age is subjected to the most critical loads that it will ever have to undergo because each slab will have to support the self-weight of two floors shored on the slab during the erection of the structure. Although this criterion or similar ones (Vollum and Hossain, 2002) may be realistic in some cases, it was not applied in this study because it produces excessively conservative results.
- In Equation 4, coefficient k_{SS} (= 1.35) increases the stresses in the support strips of two-way slabs on isolated supports. Coefficient k_{DP} (= 1.35) is utilised when dropping capitals exist (which is not the case in this study). β_{ES} is the ratio of the longer span to the shorter span when spans in the two directions differ (which is not the case in this study). k_{AR} is a coefficient complementary to β_{ES}, as defined in Scanlon and Lee (2006). For this study, k_{AR} is taken as equal to 0.6 in all cases.
- The values of the coefficients k_{SS}, k_{DP} and k_{AR} of Equation 4 are justified in Scanlon and Lee (2006). Adjustments of these values may be provided by future studies. However, additional refinement of the values of these coefficients may have a very limited influence on the results because these coefficients are cube-rooted in Equation 4.

REFERENCES

ACI (American Concrete Institute) (1995) *ACI 435R-95: Control of Deflection in Concrete Structures*. American Concrete Institute, Farmington Hills, MI, USA.

ACI (2014) *ACI 318-08: Building Code Requirements for Structural Concrete and Commentary*. American Concrete Institute, Farmington Hills, MI, USA.

Beal AN (2009) Eurocode 2: Span/depth ratios for RC slabs and beams. *The Structural Engineer* **87(20)**: 35–40.

Bischoff PH and Scanlon A (2007) Effective moment of inertia for calculating deflections of concrete members containing steel reinforcement and fiber-reinforced polymer reinforcement. *ACI Structural Journal* **104(1)**: 68–75.

Bondy KB (2005) Code deflection requirements – time for a change? In *Serviceability of Concrete* (Barth F, Frosch R, Nassif H and Scanlon A (eds)). American Concrete Institute, Farmington Hills, MI, USA, SP-225, pp. 133–145.

Branson DE (1963) *Instantaneous and Time-Dependent Deflections of Simple and Continuous Reinforced Concrete Beams*. Bureau of Public Roads, Montgomery, AL, USA.

CEN (2004) *EN 1992-1-1: Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings*. European Committee for Standardisation, Brussels, Belgium.

Corres H, Pérez A, López JC and Edtbauer J (2003) *PrEN Chapter 7-Serviceability Limit States. Deflections: Supporting Document*. The European Concrete Platform ASBL. Eurocode 2 Commentary. European Concrete Platform, Brussels, Belgium. https://doi.org/10.13140/2.1.4146.0804.

CPH (Comisión Permanente del Hormigón) (2008) *Instrucción de Hormigón Estructural (EHE-08)*. Ministerio de Fomento, Madrid, Spain.

Eren T and Dancygier AN (2020) Evaluation of span-to-depth ratio provisions for deflection control of one-way RC construction. *Structures* **25(6)**: 696–707, https://doi.org/10.1016/j.istruc.2020.03.035.

Ge W, Song W, Ashour AF, Lu W and Cao D (2020) Flexural performance of FRP/steel hybrid reinforced engineered cementitious composite beams. *Journal of Building Engineering* **31(9)**: 1–17, https://doi.org/10.1016/j.jobe.2020.101329.

Gilbert RI (1985) Deflection control of slabs using allowable span to depth ratios. *ACI Journal Proceedings* **82(1)**: 67–72.

Nguyen HD, Zhang Q, Choi E and Duan W (2020) An improved deflection model for FRP RC beams using an artificial intelligence-based approach. *Engineering Structures* **219(19)**: 1–12, https://doi.org/10.1016/j.engstruct.2020.110793.

Pecić N and Marinković S (2011) Design aspects of Eurocode 2 methods for deflection control. In *Fib Symposium Prague 2011: Concrete Engineering for Excellence and Efficiency* (Sruma V (ed.)). Czech Concrete Society, Prague, Czech Republic, vol. 1: pp. 195–198.

Pérez A and Corres H (2014) EN 1992. Problems in its application and suggestions for improvement. *Hormigón y Acero* **65(272)**: 113–122, https://doi.org/10.1016/s0439-5689(14)70003-8.

Pérez A, Medoza J and Corres H (2017) Slenderness limits for deflection control: a new formulation for flexural reinforced concrete elements. *Structural Concrete* **18(1)**: 118–127, https://doi.org/10.1002/suco.201600062.

Rangan BV (1982) Control of beam deflections by allowable span-depth ratios. *ACI Journal Proceedings* **79(5)**: 372–377.

SA (Standards Australia) (2001) *Committee BD-002: Concrete Structures*. Standards Australia, Sydney, Australia.

Sanabra Loewe M and Scanlon A (2014) Reinforced concrete predimensioning to enhance optimization. In *37th IABSE Symposium Report* (Pulido MDG (ed.)). IABSE, Madrid, Spain, pp. 1626–1633.

Scanlon A and Choi BS (1999) Evaluation of ACI 318 minimum thickness requirements for one-way slabs. *ACI Structural Journal* **96(4)**: 616–621, https://doi.org/10.14359/699.

Scanlon A and Lee YH (2006) Unified span-to-depth ratio equation for nonprestressed concrete beams and slabs. *ACI Structural Journal* **103(1)**: 142–148.

Scanlon A and Murray DW (1982) Practical calculation of two-way slab deflections. *Concrete International* **4(11)**: 43–50.

Shaaban IG and Mustafa TS (2019) Towards efficient structural and serviceability design of high-strength concrete T-beams. *Proceedings of the Institution of Civil Engineers – Structures and Buildings* 1–13, https://doi.org/10.1680/jstbu.19.00081.

Timoshenko S and Woinowsky-Krieger S (1959) *Theory of Plates and Shells*. McGraw-Hill Book Co., New York, NY, USA.

Vollum RL (2009) Comparison of deflection calculations and span-to-depth ratios in BS 8110 and Eurocode 2. *Magazine of Concrete Research* **61(6)**: 465–476, https://doi.org/10.1680/macr.2009.61.6.465.

Vollum RL and Hossain TR (2002) Are existing span-to-depth rules conservative for flat slabs? *Magazine of Concrete Research* **54(6)**: 411–421, https://doi.org/10.1680/macr.2002.54.6.411.