Energy Harvesting for System of Coupled Oscillators under External Excitation in the Vicinity of Resonance 1:1

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ABSTRACT

Vibration energy is abundantly present in many natural and artificial systems and can be assembled by various devices,mainly using piezoelectric and electromagnetic means. In the present article, the electromechanical system with two degrees of freedom is considered. To the main mass, whose vibrations are to be reduced, an additional element (dynamical vibration absorber or DVA) is attached. The DVA consists of a spring, damping and piezoelectric elements for energy harvesting. The goal is to reduce the maximal possible responses of the main structure and at the same time collect energy from the vibration *of the system. An analytical approach is proposed to find the solution of the problem. We show that the piezoelectric element allows effective energy harvesting and at the same has a very limited influence on reducing the amplitude of oscillations of the main mass. The theoretical results are confirmed by numerical experiments.*

1 INTRODUCTION

In various environments, there are many different sources from which energy can be obtained. Among the main types we highlight the ambient solar energy, thermal sources (based on temperature variations), radio frequency energy and kinetic energy. In this last direction much research has been done over the past two decades [2–5]. In particular, this is caused by a significant reduction in the size and energy use of modern electronic devices, which prompts researchers and industry to open circuits for introducing into these systems numerous power supplies that can collect energy from the environment for an unlimited time. Also kinetic energy source has some advantages over other harvesting technologies. Kinetic motion energy sources are available in most environments, and due to Microelectromechanical Systems (MEMS) technology, kinetic energy harvesters can be micro-scaled with great success and used in a wide range of applications.

There are three common conversion mechanisms for kinetic energy harvesting, such as piezoelectric, electromagnetic and electrostatic. Among other things, piezoelectricity is one of the most attractive transfer mechanisms during mechanical energy conversion. The advantages of using piezoelectric materials in mechanical vibration systems include their higher specific power and ease of implementation. Piezoelectric materials have the ability to generate electrical stress during deformation due to vibrations (direct impact), and, on the other hand, they are deformed when exposed to external stress (reverse impact). For the energy collection mechanism, this is a direct piezoelectric effect that allows the material to absorb the mechanical energy of vibration from its main structure or environment and converts it into electrical energy, and thus forms the basis of the vibration-based piezoelectric energy collection area.

Models using various types of oscillations were considered by many researchers [6–10]. Among models considered are: a cantilever beam carrying a tip mass [11], tuned auxiliary structure [12], rotational motion system [13], stall-induced oscillations of airfoils [14, 15]. In the paper [16] the energy harvesting from dynamic vibration pendulum absorber was studied; works [17] and [18] considered devices based on a nonlinear vibration absorber. Dual-mass vibration energy harvester with force excitation was investigated in works [19, 20]. Such device is able to harvest more energy than the traditional single degree-of-freedom one, when subjected to harmonic force or base displacement excitations. The efforts to increase various aspects of efficiency the harvester have been undertaken in papers [21–24].

In the present report we consider the 2–DoF mechanical system with piezoelectric element PZE attached under the external harmonic excitation in the vicinity of resonance. We are mainly interested in the mathematical side of the problem, in particular, the development of an approach that allows us to obtain analytical expressions to describe the desired parameters of the absorber and PZE. The paper is organized as following. In Section 2 the problem formulation and some auxiliary manipulations are given. In Section 3 the tuning methodology for reducing the maximal responses of the host structure is described. Section 4 deals with optimization of piezoelectric characteristics in order to maximize the harvested power.

2 FORMULATION OF THE PROBLEM

The primary structure is assumed to be a single degree of freedom system as shown in Fig. 1. The mass and stiffness of the primary structure are represented by m_p and k_p respectively, whereas the energy harvesting DVA has an mass, stiffness and damping as m_a, k_a, and c_a, respectively. The electrical capacitance and resistance are denoted by C_p and R_l, respectively. The parameter θ characterizing the coupling between the electrical and mechanical parts of the harvester. The dynamics of the primary mass (m_p), the absorber mass (m_a), and voltage flow can be expressed by three coupled ordinary differential equations in the following form

$$
\begin{aligned}
&m_p\ddot{x}_p + k_p x_p - k_a(x_a - x_p) - c_a(\dot{x}_a - \dot{x}_p) = F_0 \exp^{i\omega t},\\
&m_a\ddot{x}_a + c_a(\dot{x}_a - \dot{x}_p) + k_a(x_a - x_p) - \theta v = 0, \quad C_p\dot{v} + \frac{v}{R_l} + \dot{x}_a = 0,
\end{aligned} \tag{1}
$$

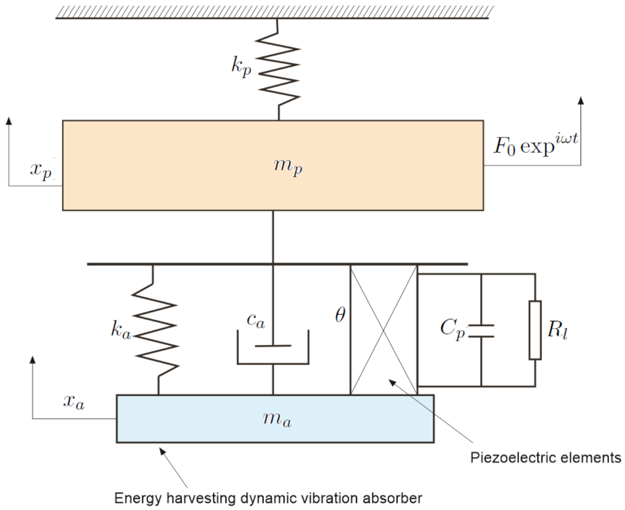

Fig. 1. Energy harvesting DVA attached to a primary system

where x_p and x_a are the displacements of the primary mass and absorber, respectively. The voltage across the load resistor is denoted by v. The electromechanical coupling and the mechanical force are modeled as proportional to the voltage across the piezoceramic in second equation. The third Equation (1) is obtained from the electrical circuit, where the voltage across the load resistance arises from the mechanical strain through the electromechanical coupling, and the capacitance of the piezoceramic C_p. The primary structure is assumed to be driven by a harmonic excitation with amplitude F_0.

In terms of dimensionless parameters the amplitudes of steady state harmonic responses are

$$\begin{aligned} X_p &= F_0 \frac{-(2\zeta\alpha+1)\mu g^2 + \mu\kappa + [-\mu\alpha g^3 + [(2\zeta+\kappa\alpha)\mu + p^2)g]i}{\Delta_{Re} + i\Delta_{Im}}, \\ X_a &= -F_0 \frac{\mu g^2 + (\alpha\mu g^3 - \beta g)i}{\Delta_{Re} + i\Delta_{Im}},\ V = F_0 \frac{\mu\sqrt{\beta}g(-2\zeta g + \kappa i)}{\Delta_{Re} + i\Delta_{Im}}, \end{aligned} \tag{2}$$

where $\mu = m_a/m_p$ is the mass ratio; $\omega_p = \sqrt{k_p/m_p}$, $\omega_a = \sqrt{k_a/m_a}$ – undamped natural frequencies of the primary system and the DVA considered separately; $\kappa = \omega_a^2/\omega_p^2$ – the tuning factor; $g = \omega/\omega_p$ – the forcing frequency ratio; $\zeta = c_a/(2m_a\omega_p)$ – the damping ratio;

$$\alpha = \omega_p R_l C_p,\ \beta = \theta^2 \frac{R_l}{m_p \omega_p}, \Delta_{Re} = -[2\zeta\alpha\mu^2 + (2\zeta\alpha + 1)\mu]g^4 + (1\kappa\mu^2 + [2\zeta(\alpha+\beta) + \\ + \beta(\kappa+1)]\mu)g^2 - \beta\kappa\mu,\ \Delta_{Im} = -\alpha\mu g^5 + [(2\zeta + \alpha\kappa)\mu^2 + [\alpha(\kappa+1) + 2\zeta]\mu + \beta]g^3 - \\ - [(\kappa(\alpha+\beta) + 2\zeta)\mu + \beta]. \tag{3}$$

Thus, from formulas (2), (3) we can write the relative responses of the primary mass and absorber as follows

$$\begin{aligned} \frac{|X_0|}{(X_0)_{st}} &= \sqrt{\frac{[-(2\zeta\alpha+1)\mu g^2 + \beta\kappa\mu]^2 + g^2[-\alpha\mu g^2 + (2\zeta + \alpha\kappa)\mu + \beta]^2}{\Delta_{Re}^2 + \Delta_{Im}^2}}, \\ \frac{|V|}{(X_0)_{st}} &= \mu g \sqrt{\frac{\beta(4\zeta^2 g^2 + \kappa^2)}{\Delta_{Re}^2 + \Delta_{Im}^2}}, \end{aligned} \tag{4}$$

where $(X_0)_{st} = F_0/k_0$ – static displacement of the primary mass.

3 MINIMIZING THE RESPONSES OF THE PRIMARY MASS

3.1 Preliminary Analysis

Our first goal is to minimize the peaks of the amplitude-frequency curve, that is, the choice of such parameters of the absorber and piezoelectric element, in which the maximal possible responses of the host system do not exceed a certain value under condition of proximity of frequencies ω and ω_0.

Let us consider a function

$$f(\mu, k, h, \gamma, \alpha, \beta) = \frac{a_3\gamma^3 + a_2\gamma^2 + a_1\gamma + a_0}{b_5\gamma^5 + b_4\gamma^4 + b_3\gamma^3 + b_2\gamma^2 + b_1\gamma + b_0}, \tag{5}$$

$$
\begin{aligned}
&a_3=\mu^2\alpha^2,\ a_2=\mu^2(\alpha^2h^2-2\alpha^2k+1)-2\mu\alpha\beta,\ a_1=\mu^2(h^2+\kappa^2\alpha^2-2\kappa)+\\
&+2\mu(h+\alpha\kappa)\beta+\beta^2,\ a_0=\mu^2\kappa^2,\ b_5=\mu^2\alpha^2,\ b_4=\mu^4\alpha^2h^2+2\mu^3\alpha^2(h^2-\kappa)+\\
&+\mu^2[\alpha^2(h^2-2\kappa-2)+1]-2\mu\alpha\beta,\ b_3=\mu^4(h^2+\alpha^2\kappa^2)-2\mu^3[h^2(\alpha^2-1+\alpha\beta)-\\
&-\alpha^2\kappa^2+\kappa(1-\alpha^2)]+\mu^2[h^2(1-2\alpha^2-2\alpha\beta)+\alpha^2\kappa^2+2\kappa(2\alpha^2+2\alpha\beta-1)-2]+\\
&+2\mu\beta(h+\alpha\kappa+2\alpha)+\beta^2,\ b_2=\mu^4\kappa^2-2\mu^3[h^2+\kappa^2(\alpha^2-1+\alpha\beta)-\kappa]+\\
&+\mu^2[h^2(-2+\alpha^2+2\alpha\beta+\beta^2)+\kappa^2(1-2\alpha^2-2\alpha\beta)-2\kappa(-2+\alpha^2+\alpha\beta)+1]+\\
&+2\mu\beta(2h-\alpha\kappa+\alpha-\kappa\beta)-2\beta^2,\ b_1=-2\mu^3\kappa^2+\mu^2\{h^2-\kappa^2[2-(\alpha+\beta)^2]-2\kappa\}+\\
&+2\mu\beta[h+\kappa(\alpha+\beta)]+\beta^2,\ b_0=\mu^2\kappa^2,
\end{aligned}
\tag{6}
$$

where $h=4\zeta^2,\ \gamma=g^2,\ f=(|X_0|/(X_0)_{st})^2$.

Suppose that values for parameters $\mu,h,\kappa,\alpha,\beta$ are selected. We are interested in the case when the ordinate values at the peaks of the amplitude curve $y=f(\gamma)$ coincide. The extremum points of the function are determined from the condition $df/d\gamma=0$ or $P_1(\gamma)=0$, where the polynomial $P_1(\gamma)$ is of the seventh degree.[1]

Theoretically, we can exclude the explicit presence of a variable γ by considering the condition $P_2(\xi)=0$, where $P_2(\xi)$ is the resultant of polynomials P_1 and P on γ. $P_2(xi)$ is a fifth-order polynomial on ξ, and the optimal values of the absorber and PZE are determined as the solution of the problem of the conditional extremum of an implicit function $P_2(\xi)=0$ with a restriction $Dis_{P_2}(\xi)=0$, where Dis_{P_2} is the discriminant of the polynomial P_2. The difficulty lies in the fact that the analytical expressions are too bulky (more than 10^6 signs) for computer processing and subsequent analysis. The direct numerical analysis based on grid search is quite expensive, because even with a given value of μ, four parameters remain for variation.

3.2 Mathematical Reformulation of the Task

With the aim to simplify the computational algorithm, we shall use the approach proposed in the article [26, 27]. Its idea is as follows. Suppose that the parameters $\mu,\kappa,h,\alpha,\beta$ are set, while

[1]Note that in the general case the fraction (5) is irreducible, and the roots of the polynomial $P_1(\gamma)$ cannot be written in explicit form.

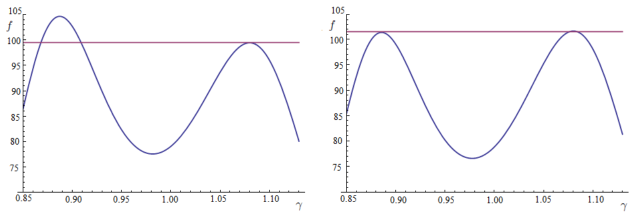

Fig. 2. Geometric representation

we are interested in a configuration in which the function $f(\gamma)$ takes the same values at the points of maximum γ_1, γ_2. Geometrically, this means that the curve $y = f(\gamma)$ has a common tangent at the points γ_1, γ_2 (Fig. 2).

Let the values of parameters $\mu, h, \kappa, \alpha, \beta$ are determined, and ξ be some fixed number. Then the equality $f = 1/\xi$ is equivalent to the following polynomial equation

$$P(\gamma) \triangleq b_5\gamma^5 + b_4\gamma^4 + (b_3 - \xi a_3)\gamma^3 + (b_2 - \xi a_2)\gamma^2 + (b_1 - \xi a_1)\gamma + b_0 - \xi a_0 = 0. \tag{7}$$

A consequence of the requirement of the equal peaks is the presence of two pairs of multiple (real) roots in the polynomial $P(\gamma)$, that is, it can be represented in the form

$$P(\gamma) = \mu^2(\alpha^2\gamma + L)(\gamma^2 - M\gamma + \kappa N)^2 > 0, M > 0,\ N > 0,\ M^2 - 4\kappa N > 0, \tag{8}$$

where L, M, N are some unknown parameters. Then, we conclude from (8) that expression

$$\begin{aligned}
&\gamma^4\{\alpha^2(h^2 - 2\kappa + 2M - 2) + 1 - 2\alpha\beta - L + 2\mu\alpha^2(h^2 - \kappa) + \mu^2 h^2\alpha^2\} + \gamma^3\{h^2(1 - 2\alpha^2) + \\
&+ 2h\beta + \alpha^2\kappa^2 + 2\kappa(2\alpha^2 - \alpha^2 N + \alpha\beta - 1) + 2LM - \alpha^2 M^2 + \alpha^2(1 - \xi) + 4\alpha\beta + \beta^2 + \\
&+ 2\mu[h^2(1 - \alpha^2 - \alpha\beta) + \kappa(\alpha^2\kappa + \alpha^2 + 2\alpha\beta - 1] + \mu^2[h^2(1 - 2\alpha\beta) + \alpha^2\kappa^2]\} - 4h\beta\gamma^2 + \\
&+ \gamma^2\{h^2[\alpha^2(1 - \xi) - 2] + \kappa^2(1 - 2\alpha^2) + 2\kappa[-LN + \alpha^2(MN + \xi - 1) - 2\alpha\beta + 2] - LM^2 -
\end{aligned} \tag{9}$$

$$-\xi+1+2\alpha\beta(\xi-1)-2\beta^2-2\mu[h^2(1-\alpha\beta)+\kappa^2(-1+\alpha^2+\alpha\beta)+\kappa(-1+2\alpha\beta+\beta^2)]+$$
$$+\mu^2[h^2(1-2\alpha\beta)+\alpha^2k^2]\}+\gamma\{(1-\xi)[h^2+2h\beta+\alpha^2\kappa^2+2\kappa(\alpha\beta-1)+\beta^2]+$$
$$+\kappa(2LMN-\alpha^2\kappa N^2-2\kappa)+2\mu\kappa[\kappa(-1+\alpha\beta)+\beta^2]+\mu^2\kappa^2\beta^2\}+\kappa^2(1-f-LN^2)$$

is equal to zero identically on γ, therefore the coefficients on $\gamma^s(s=\overline{0,4})$ should be equal to zero. From the coefficient on γ^4 we express

$$L=\mu^2h^2\alpha^2-2\mu\alpha^2(\alpha\beta-h^2+\kappa)+\alpha^2(h^2-2M-2\kappa-2)+1, \tag{10}$$

and from the constant term we get the target function in the following form

$$\xi=1-N^2[\mu^2h^2\alpha^2-2\mu\alpha^2(\alpha\beta-h^2+\kappa)+\alpha^2(h^2-2M-2\kappa-2)+1]. \tag{11}$$

After substituting the expressions for ξ and L we have three other conditions (the coefficients on γ^3,γ^2,γ)

$$\phi_1=\sum_{j=0}^{2}A_{1j}h^j=0,\ \phi_2=\sum_{j=0}^{4}A_{2j}h^j=0,\ \phi_3=\sum_{j=0}^{4}A_{3j}h^j=0, \tag{12}$$

where coefficients $A_{js}(\kappa,M,N,\alpha,\beta)$ are presented in Appendix A.

Assuming the value of μ to be known, we obtain the conditional extremum problem for the function ξ of six variables $h,\kappa,\alpha,\beta,M,N$ with three constraints (12). This problem can be solved in the usual way with the help of Lagrange multipliers. As an alternative, equalities (12) can be considered as an implicit definition of three functions on three independent arguments, for example, $M(h,\kappa,\alpha),\ N(h,\kappa,\alpha),\ \beta(h,\kappa,\alpha)$. Their partial derivatives on arguments h,κ,α can be expressed from the system

$$\frac{\partial\phi_j}{\partial\eta_s}+\frac{\partial\phi_j}{\partial M}\frac{\partial M}{\partial\eta_s}=0,\ \frac{\partial\phi_j}{\partial\eta_s}+\frac{\partial\phi_j}{\partial N}\frac{\partial N}{\partial\eta_s}=0,\ \frac{\partial\phi_j}{\partial\eta_s}+\frac{\partial\phi_j}{\partial\beta}\frac{\partial\beta}{\partial\eta_s}=0,\ \boldsymbol{\eta}=(h,\kappa,\alpha),\ j,s=1,2,3 \quad (13)$$

and substituted into the equalities

$$\frac{\partial f}{\partial\eta_j}+\frac{\partial f}{\partial M}\frac{\partial M}{\partial\eta_j}+\frac{\partial f}{\partial N}\frac{\partial N}{\partial\eta_j}+\frac{\partial f}{\partial\alpha}\frac{\partial\alpha}{\partial\eta_j}=0,\ j=1,2,3. \quad (14)$$

System (14) determines the stationary points $\boldsymbol{\eta}_0$ of the problem.

Remark. Both approaches described above are still very cumbersome from the point of view of the analytical representation, although less cumbersome than the calculations mentioned in the previous section. Although the number of variables and additional relations has increased, the expressions for ϕ_j are ”observable” (in contrast to P_2, Dis_{P_2}). This method of solution can be considered as a kind of ”diversification” task , when instead of two extremely bulky expressions, four ”moderately bulky” ones are considered - from the point of view of analysis, it is easier to reveal the existing relationships.

3.3 An Asymptotic Analysis

At the moment, we are not sure that the exact solution to the problem can be presented in explicit form. However, with the goal of applied orientation, we can use the asymptotic approach. Let us consider the following sums

$$h=h_0+\beta h_1+\cdots,\ \kappa=\kappa_0+\beta\kappa_1+\cdots,\ M=M_0+\beta M_1+\cdots,\ N=N_0+\beta N_1+\cdots. \quad (15)$$

where three dots means terms of higher order on β. The main difficulty is finding the zeroth approximation, since the system of equations 12 remains essentially nonlinear. Nevertheless, reducing the number of variables by one allows us to find additional relationships, which leads to the solution of the problem.

The expression for ϕ_1 is now linear on h^2, and ϕ_2, ϕ_3 are biquadratic ones. It is possible to express h^2 from first equality (12) and substitute into two others, however the resulting expressions are too large and their direct analysis is difficult. At the same time, the expression

$$\widetilde{\phi}_2(h^2) \triangleq \phi_2(h^2) - \alpha^2\phi_3(h^2) \tag{16}$$

is also linear with respect to h^2. Having written the resultant of polynomials $\phi_1(h^2), \widetilde{\phi}_2(h^2),$ we obtain the necessary condition for compatibility of the system (12)

$$res_1 = (\alpha^4\kappa N + \alpha^2 M + 1)^2[\mu_1 M^2 - 4M - \mu_1 N^2 + 2\mu_1\kappa N - \mu_1^3\kappa^2 - 2\mu_1\kappa(\mu_1 - 1) + 4] = 0. \tag{17}$$

Here we denote $\mu_1 = \mu + 1$ for convenience sake.

The first multiplier is obviously positive, hence

$$\mu_1 M^2 - 4M - \mu_1 N^2 + 2\mu_1\kappa N - \mu_1^3\kappa^2 - 2\mu_1\kappa(\mu_1 - 1) + 4 = 0. \tag{18}$$

As one can see, the obtained connection between M, N, κ and μ_1 does not contain the parameter α.[2] We can present M in the following form

$$M = \frac{1}{\mu_1}(2 \pm R),\ R = \sqrt{\mu_1^2 N^2 - 2\mu_1^2\kappa N + \mu_1^4\kappa^2 + 2\mu\mu_1^2\kappa - 4\mu}. \tag{19}$$

[2]This fact also follows from physical considerations, but we are interested in the mathematical confirmation of this fact.

Now we write the resultant of the polynomials $\phi_1(h^2),\ \phi_3(h^2)$ and substitute M in it. The resulting expression can be represented as polynomial on R:

$$res_2(R) = B_4R^4 + B_2R^2 + B_0 + R(B_3R^2 + B_1), \tag{20}$$

where coefficients B_j are presented in Appendix B.

Getting rid of irrationality (radical R) in equality $res_2 = 0$, we come to the necessary condition of solvability the system (12) (with $\beta = 0$) as $\sigma_1^2\sigma_2 = 0$, where

$$\begin{aligned}
&\sigma_1 = \mu_1^2\alpha^8N_0^4 + 4\mu_1\alpha^6N_0^2 - 2\alpha^4\{\mu_1^2N_0^2 - 4\mu_1^2\kappa_0N_0 - 2[\mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1-1) - 4\mu_1 + 3]+\\
&+ 4\mu_1\alpha^2 + \mu_1^2,\ \sigma_2 = N_0^5 - N_0^4[-2\kappa_0(\mu_1^2+1)+1] + 2\mu_1\kappa_0N_0^3[2\mu_1\kappa_0(\mu_1+2)-2]+\\
&+ 2\mu_1^2\kappa_0^2N_0^2(-2\mu_1^2\kappa_0 + \mu_1 + 2) + \mu_1^4\kappa_0^3N_0(\mu_1^2-4) + \mu_1^6\kappa_0^4.
\end{aligned} \tag{21}$$

The polynomial σ_2 has the fifth order on N_0 and the fourth order on κ_0, however with substitution $N_0 = \mu_1\kappa_0\widetilde{N}$ we have

$$\kappa_0 = -\frac{\mu_1^2\widetilde{N}^4 - 4\mu_1\widetilde{N}^3 + 2\widetilde{N}^2(\mu_1+2) - 4\widetilde{N} + 1}{\mu_1^2\widetilde{N}[\mu_1^2\widetilde{N}^4 - 2\widetilde{N}^3(\mu_1^2+1) + 2\widetilde{N}^2(\mu_1+2) - 4\widetilde{N} + 1]}. \tag{22}$$

At this point we we can consider $\widetilde{N}$ as an independent parameter (argument) and go back up the chain $\kappa_0(\widetilde{N},\mu_1) \longrightarrow M_0(\widetilde{N},\mu_1) \longrightarrow h_0(\widetilde{N},\mu_1) \longrightarrow \xi_0(\widetilde{N},\mu_1)$, and expression to be optimized is

$$\xi_0(\widetilde{N},\mu_1) = -\frac{4(\mu_1-1)^2\widetilde{N}^3[\mu_1^2\widetilde{N}^4 - \widetilde{N}^3(\mu_1+1)^2 + 2\widetilde{N}^2(\mu_1+2) - 4\widetilde{N} + 1]}{[\mu_1^2\widetilde{N}^4 - 2\widetilde{N}^3(\mu_1^2+1) + 2\widetilde{N}^2(\mu_1+2) - 4\widetilde{N} + 1]^2}. \tag{23}$$

It is easy to verify that $\partial\xi_0/\partial\mu_1 > 0$, that is, with an increase in μ, the value of ξ_0 increases (and f decreases), which is logically clear – as the mass of the absorber grows, its efficiency increases.

Then we can assume that value for μ is given, and only $\widetilde{N}$ should be chosen to maximize the value of ξ_0. The equation $d\xi_0/d\widetilde{N} = 0$ has six complex solutions, one negative (not suitable, because $\widetilde{N}$ should be positive) and one positive

$$\widetilde{N}^\star = \frac{\sqrt{3\mu_1+1}-1}{\mu_1} = \frac{\sqrt{4+3\mu}-1}{1+\mu}, \tag{24}$$

which brings the maximum for ξ_0

$$\xi_0 = \frac{\mu[(8+9\mu)^2(16+9\mu)+128(4+3\mu)^{3/2}]}{(64+80\mu+27\mu^2)^2} \tag{25}$$

After substitution of $\widetilde{N}^\star$ and subsequent simplifications we also have

$$\begin{aligned} M_0 &= \frac{2}{3}\frac{128+184\mu+69\mu^2+(32+84\mu+27\mu^2)\sqrt{4+3\mu}}{(1+\mu)(64+80\mu+27\mu^2)} \\ \kappa_0 &= \frac{8}{3}\frac{16+23\mu+9\mu^2+2(2+\mu)\sqrt{4+3\mu}}{(1+\mu)^2(64+80\mu+27\mu^2)}, \\ h_0^2 &= \frac{2}{3}\;\frac{64+248\mu+255\mu^2+81\mu^3-2(16+20\mu+9\mu^2)\sqrt{4+3\mu}}{(64+80\mu+27\mu^2)(1+\mu)^3}. \end{aligned} \tag{26}$$

To find the next group of coefficients h_1, κ_1, M_1, N_1, we equate to zero the coefficients in the expansions in ϕ_j. In contrast to the zeroth approximation, the resulting system is linear, so the three coefficients are expressed in terms of the fourth, for example

$$\kappa_1 = \kappa_{11}h_1 + \kappa_{10},\ M_1 = M_{11}h_1 + M_{10},\ N_1 = N_{11}h_1 + N_{10}, \tag{27}$$

where $\kappa_{10}, \cdots, N_{11}$ are rational functions of the arguments α, μ (due to cumbersomeness, we do not give them here).

Table 1. Values of $\xi_1(\alpha_i), \xi_0 = 0.991$

α_i	$\xi_1(\alpha_i),\ 10^{-3}$	α_i	$\xi_1(\alpha_i),\ 10^{-3}$
0.1	$1.53 - 0.046h_1$	3	$-5.29 - 1.97h_1$
0.3	$-0.4 - 0.063h_1$	8	$-2.55 - 13.75h_1$
0.5	$-2.24 - 0.097h_1$	12	$-1.82 - 30.88h_1$
1	$-5.42 - 0.26h_1$	20	$-1.2 - 85.76h_1$

Substituting the found expressions in (11), we obtain

$$\xi = \xi_0 + \beta[h_1\xi_{11}(\alpha,\mu) + \xi_{10}(\alpha,\mu)] + \cdots \tag{28}$$

A numerical analysis shows that, in the assumed range of changes of α, μ the quantity ξ_{11} is negative, therefore, the condition $\xi_1 > 0$ requires the positiveness of ξ_{10}. [3] However, this is only performed for small values of α (see Table 1). Thus, we conclude that the presence of PZE cannot increase the value of ξ, that is, reduce the maximum amplitude of oscillations of the primary mass. Also an increase in the value of β which is related to constant θ (remind that θ characterizes the coupling between the electrical and mechanical parts of the harvester), according to (28), may lead to increase of the magnitude of responses of the host system. By this reason it is likely to choose θ no more than of order 10^{-1}. This conclusion is an agreement with results of [7, 19].

3.4 Numerical Testing

The numerical experiments with varying the system parameters show a very insignificant effect of the characteristics of the piezoelectric element on the decrease in the amplitude of oscillations of the main mass. At the same time, if the parameter β has an order of magnitude greater than 10^{-2}, then the amplitude of the oscillations increases significantly (Fig. 3). Moreover, due to the continuous dependence of the function ξ on the parameter β, the optimal absorber values for system (1) will differ very slightly from the degenerate case $\beta = 0$, that is, the values of κ, h

[3] Generally speaking, the value for h_1 can be chosen negative, although not too large in module due to limitation $h_0 + \beta h_1 > 0$. However, such a choice cannot improve the situation.

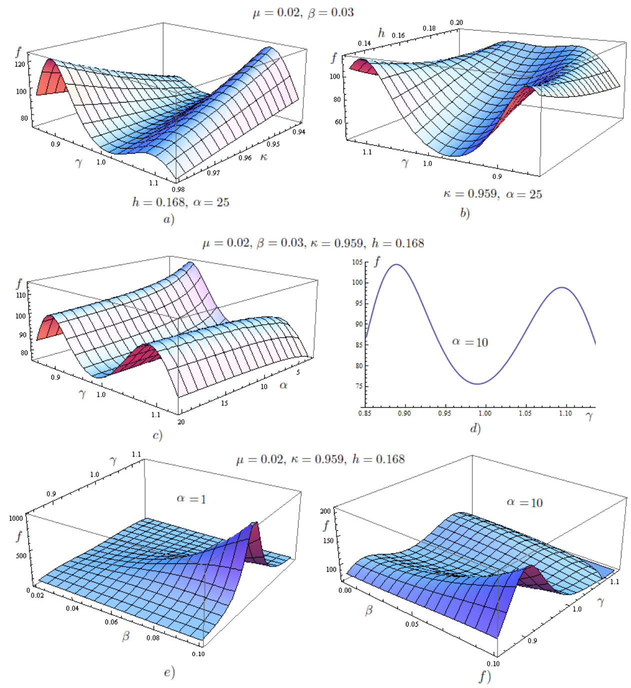

Fig. 3. The amplitude of the oscillations

determined according to formulas (26).

Thus, we can give the following recommendations for choosing system parameters:

– the parameters of the absorber, when known the mass ratio μ, are taken related to values of κ_0, h_0;

– the parameter β should be taken sufficiently small (for example 0.01 with $\mu = 0.02$);

– the value of parameter α practically does not affect the value of ξ (one can take an arbitrary value in the interval $[0.01, 10]$).

4 OPTIMIZATION THE HARVESTING POWER

Our second task is to determine the parameters of the piezoelectric element in order to maximize energy collection. We assume that parameters of the DVA are taken according to foregoing. Depending on the specified quality criterion, the procedure for determining these parameters may vary. We will consider the following options.

A) Provided that the frequency of the external action is unknown exactly, but close to the resonant one ($\gamma \in [\gamma^{(1)}, \gamma^{(2)}]$), select the parameters α and β so that the minimum value of the

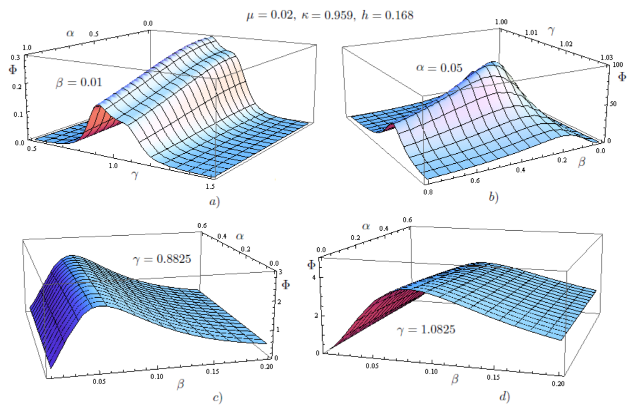

Fig. 4. The typical view of the surface (29)

objective function

$$\Phi(\alpha,\beta,\gamma) = \frac{V^2}{R_l} = \frac{\mu^2\beta(h^2\gamma+\kappa^2)}{b_5\gamma^5+b_4\gamma^4+b_3\gamma^3+b_2\gamma^2+b_1\gamma+b_0} \tag{29}$$

with respect to argument γ turned out to be the greatest possible (with respect to α,β). Mathematically, this task is easy enough. We have a system

$$\frac{\partial\Phi}{\partial\gamma}=0,\ \frac{\partial\Phi}{\partial\alpha}=0,\ \frac{\partial\Phi}{\partial\beta}=0. \tag{30}$$

The derivative $\partial\Phi/\partial\alpha$ is linear on α, and two others are quadratic on β. Their resultant is high order polynomial on γ, however it has no real roots in vicinity of $\gamma_0 = 1$. Hence, the lowest values on γ are taken on the sides of the interval. The relevant values of α,β are calculated accordingly to numeric values of $\mu,\gamma^{(1)},\gamma^{(2)}$. The typical view of the surface (29) is presented in Fig.4.

B) As another criterion, we can take the ”neutral” requirement of maximizing the averaged (with respect to γ) value of the function Φ over the interval $[\gamma^{(1)},\gamma^{(2)}]$. Taking into account that the expression considered is representable as the sum of tabular integrals, from a technical point of view, solving this problem does not seem complicated. The corresponding calculations can be performed analytically (a computational procedure is similar to that used in the previous case) or

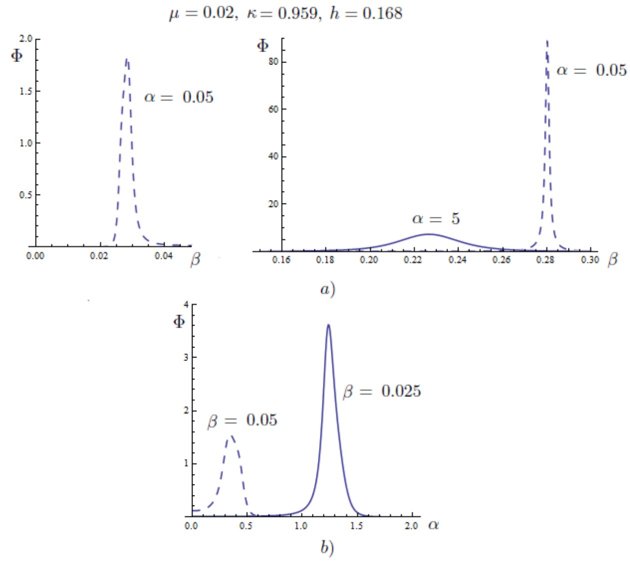

Fig. 5. The result of maximizing the averaged value of the function Φ

numerically. Results are presented in Fig.5.

5 CONCLUSIONS

In this paper, we consider an electro-mechanical system consisting of a primary element and a dynamic absorber and a piezoelectric element connected to it. The goal is to reduce the vibration of the primary structure and at the same time collect the energy through the interaction of the host system and the vibration absorber. An analytical and numerical study of the dynamics of the system is carried out. It is shown that the piezoelectric element practically does not improve the effect of the absorber in terms of reducing the oscillations of the main mass. At the same time, piezoelectric element with improper selected parameters can significantly increase the amplitude of these oscillations. The problem of collecting vibrational energy is also considered. Relations are found between the parameters of the piezoelectric element at which the energy collected has a maximum value.

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APPENDIX A: EXPRESSIONS FOR COEFFICIENTS $A_{JS}(\kappa, M, N, \alpha, \beta)$

$$A_{12} = -\mu_1(2\mu_1\alpha^2 M + \mu_1\alpha^4 N^2 + \mu_1 - 2\alpha^2 - 2\mu\alpha\beta),\ A_{11} = -2\beta,$$

$$A_{10} = -3\alpha^2 M^2 + 2M(-\alpha^4 N^2 + 2\mu_1\alpha^2\kappa + 2\alpha^2 - 1 + 2\alpha\beta) + \alpha^2 N^2(2\mu_1\alpha^2\kappa + 2\alpha^2 - 1 + 2\alpha\beta)+$$

$$+ 2\alpha^2\kappa N - \mu_1^2\alpha^2\kappa^2 + 2\kappa[\mu 1 - \alpha^2(\mu_1 + 1) - \alpha\beta(2\mu_1 - 1)] + 2 - 4\alpha\beta - \beta^2,$$

$$A_{24} = -\mu_1^2\alpha^4 N^2,\ A_{23} = 0,\ A_{22} = \mu_1^2\alpha^2 M^2 - 2\alpha^4 MN^2 + \alpha^2 N^2[2\mu_1\alpha^2[\kappa(\mu_1 + 1) - \mu_1^2 + 2\alpha^2 - 1+$$

$$+ 2\alpha\beta(\mu_1^2 + 1)] + 2\mu_1^2\alpha^2\kappa N + 2(\mu_1 + \alpha\beta) - \mu^2\beta^2,\ A_{21} = 4\beta,$$

$$A_{20} = -M^2(2\mu_1\alpha^2\kappa - 1 + 2\alpha^2 + 2\alpha\beta) + 2\alpha^2 MN[N(2\alpha^2\kappa - 1 + 2\alpha\beta) + \kappa] + N^2\{-4\mu_1\alpha^4\kappa^2+$$

$$+ 2\alpha^2\kappa[\mu_1 + 1 - 2\alpha^2 - 2\alpha\beta(\mu_1 + 1)] + 2\alpha^2 - 1 + 4\alpha\beta(\alpha^2 + 1) - 4\alpha^2\beta^2\} - 2\kappa N(2\mu_1\alpha^2\kappa + 2\alpha^2-$$

$$- 1 + 2\alpha\beta) + \mu_1\kappa^2(-\mu_1 + 2\alpha^2 + \mu\alpha\beta) - 2\kappa(\mu_1 + 1 - 2\mu_1\alpha\beta + \mu\beta^2) + 2\beta^2,$$

$$A_{34} = -\mu_1^2\alpha^2 N^2,\ A_{33} = -2\mu_1^2\alpha^2\beta N^2,$$

$$A_{32} = -2\alpha^2 MN(N + \mu_1^2\kappa) - N^2[(\mu_1^2\alpha^4\kappa^2 - 2\mu_1\alpha^2 k(\mu_1 + 1) + 1 - 2\alpha^2]+$$

$$+ 2\alpha\beta(\mu_1^2\alpha^2\kappa - 1) + \mu_1^2\alpha^2\beta^2,\ A_{31} = 2N^2\beta(-2\alpha^2 M + 2\mu_1\alpha^2\kappa - 1 + 2\alpha^2 + 2\alpha\beta),$$

$$A_{30} = -4\alpha^2\kappa M^2 N + 2MN\{-\alpha^2 N[\alpha^2\kappa^2 + 2\kappa(1 - \alpha\beta) + \beta^2] + \kappa(2\mu_1\alpha^2\kappa + 2\alpha^2 - 1 + 2\alpha\beta)\}+$$

$$+ N^2\{2\mu_1\alpha^4\kappa^3 + 2\alpha^2\kappa^2[\alpha^2 - 2\mu_1 + \alpha\beta(2\mu_1 + 1)] + 2\kappa[1 - 2\alpha^2 + \alpha\beta(2\alpha^2 - 3) + \alpha^2\beta^2(\mu_1 + 2)]+$$

$$+ \beta^2(2\alpha^2 - 1 + 2\alpha\beta)\} + \kappa[\kappa(2\mu_1 - 2\mu\alpha\beta - \mu^2\beta^2 - 2\mu\beta^2].$$

APPENDIX B: EXPRESSIONS FOR COEFFICIENTS B_J

$$B_4 = \alpha^6 N_0(3N_0 - 4\mu_1^2\kappa_0),\ B_3 = 2\mu_1 N_0^2\alpha^8(-N_0^2 + \mu_1^2\kappa_0 N_0 + \mu_1^2\kappa_0^2)+$$

$$+ 4\kappa_0^2 N_0\alpha^6[N_0(-\mu_1^2\kappa_0(\mu_1 - 1) + \mu_1 - 3) + 3\mu_1^2\kappa_0] + 2\mu_1 N_0\alpha^4(-4N_0 + 5\mu_1^2\kappa_0),$$

$$B_2 = -\mu_1^4\kappa_0^2\alpha^{10}N_0^4 + 2\mu_1\alpha^8 N_0^2[N_0^2(\mu_1^2 k(\mu_1 - 1) - \mu_1 + 2) - 2\mu_1^2\kappa_0 N_0 - \mu_1^2\kappa_0^2]+$$

$$+ \mu_1\alpha^6 N_0[5\mu_1 N_0^3 - 4\mu_1\kappa_0 N_0^2(\mu_1^2 - 1) - N_0(11\mu_1^3\kappa_0^2 - 4) + 4\mu_1^2\kappa_0(\mu_1^3\kappa_0^2+$$

$$+ 2\mu_1\kappa_0(\mu_1 - 1) - 4) + 8\mu_1\kappa_0] + 2\mu_1\alpha^4[N_0^2(5\mu_1^2\kappa_0(\mu_1 - 1) - 5\mu_1 + 17) - 18\mu_1^2\kappa_0 N_0 + 4\mu_1^4\kappa_0^2]+$$

$$+ \mu_1^2 N_0\alpha^2(7N_0 - 8\mu_1^2\kappa_0), B_1 = B_{14}\alpha^8 + B_{13}\alpha^6 + B_{12}\alpha^4 + B_{11}\alpha^2 + B_{10},$$

$$B_{14} = -4\mu_1^3 N_0^5\kappa_0 + 4\mu_1 N_0^4[\mu_1^4\kappa_0^2 + \mu_1^2\kappa_0(\mu_1 - 1) - 2\mu_1 + 2] + 2\mu_1^3\kappa_0 N_0^3[\mu_1^2\kappa_0^2(-\mu_1^2 + 2)-$$

$$- 2\mu_1^2\kappa_0(\mu_1 - 1) + 4(\mu_1 - 1)] - 2\mu_1^3\kappa_0^2 N_0^2[\mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1 - 1) - 4\mu_1 + 4],$$

$$B_{13} = -4\mu_1^2 N_0^4[\mu_1^2\kappa_0(\mu_1 - 1) + 3 - \mu_1] - 4\mu_1^2\kappa_0 N_0^3[2\mu_1^2\kappa_0(\mu_1 - 1) - 3\mu_1^2 - 2\mu_1 + 6]+$$

$$+ 4N_0^2[\mu_1^6\kappa_0^3(\mu_1 - 1) + \mu_1^4\kappa_0^2(-5\mu_1 + 11) + 2\mu_1^2\kappa_0(-3\mu_1^2 + 8\mu_1 - 5) + 4(\mu_1^2 - 4\mu_1 + 3)]-$$

$$- 12\mu_1^2\kappa_0 N_0[\mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1 - 1) - 4(\mu_1 - 1)],$$

$$B_{12} = -4\mu_1^3 N_0^4 + 2\mu_1^5\kappa_0 N_0^3 + 4\mu_1 N_0^2[\mu_1^4\kappa_0^2 - 4\mu_1^2\kappa_0(\mu_1 - 1) + 4\mu_1 - 10]+$$

$$+ 2\mu_1^3\kappa_0 N_0[-\mu_1^4\kappa_0^2 - 2\mu_1^2\kappa_0(\mu_1 - 1) + 4(\mu_1 + 4)] - 16\mu_1^5\kappa_0^2,$$

$$B_{11} = -8\mu_1^2\{N_0^2[\mu_1^2\kappa_0(\mu_1 - 1) - \mu_1 + 3] - 3\mu_1^2\kappa_0 N_0 + \mu_1^4\kappa_0^2\},\ B_{10} = 2\mu_1^3 N_0(-N_0 + \mu_1^2\kappa_0),$$

$$B_0 = B_{05}\alpha^{10} + B_{04}\alpha^8 + B_{03}\alpha^6 + B_{02}\alpha^4 + B_{01}\alpha^2 + B_{00},$$

$$B_{05} = \mu_1^4 N_0^4\{\mu_1^2 N_0^2 - 2\mu_1^2\kappa_0 N_0 + \mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1 - 1) - 4(\mu_1 - 1)\},$$

$$B_{04} = 4\mu_1^3 N_0^5\kappa_0(\mu_1^2\kappa_0(\mu_1 - 1) - \mu_1 + 2) - 2\mu_1 N_0^4[\mu_1^6\kappa_0^3(\mu_1 - 1) + \mu_1^4\kappa_0^2(\mu_1^2 - 5\mu_1 + 7)-$$

$$- 2\mu_1^2\kappa_0(3\mu_1^2 - 7\mu_1 + 4) + 4(\mu_1^2 - 3\mu_1 + 2)] + 4\mu_1^3\kappa_0 N_0^3[\mu_1^2\kappa_0^2(\mu_1^2 - 1) + 2\mu_1^2\kappa_0(\mu_1 - 1)-$$

$$- 4(\mu_1 - 1)] + 2\mu_1^3\kappa_0^2 N_0^2[\mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1 - 1) - 4(\mu_1 - 1)],$$

$$B_{03} = 2\mu_1^4 N_0^5\kappa_0 + 2\mu_1^2 N_0^4[-2\mu_1^2\kappa_0^2 + \mu_1^2\kappa_0(\mu_1 - 1) + 2] + 2\mu_1^2\kappa_0 N_0^3[\mu_1^4\kappa_0^2 + 8\mu_1^2\kappa_0(\mu_1 - 1)-$$

$$- 4(\mu_1^2 + 3\mu_1 - 4)] - 2N_0^2[3\mu_1^6\kappa_0^3(\mu_1 - 1) + 2\mu_1^4\kappa_0^2(\mu_1^2 - 8\mu_1 + 10) - 4\mu_1^2\kappa_0(5\mu_1^2 - 11\mu_1 + 6)+$$

$$+ 8(2\mu_1^2 - 5\mu_1 + 3)] + 8\mu_1^2\kappa_0 N_0[\mu_1^4\kappa_0^2 + 2\mu_1^2\kappa_0(\mu_1 - 1) - 4(\mu_1 - 1)],$$

$$B_{02} = 2\mu_1^3 N_0^4[\mu_1^2\kappa_0(\mu_1 - 1) - \mu_1 + 3] + 4\mu_1^3\kappa_0 N_0^3[\mu_1^2\kappa_0(\mu_1 - 1) - \mu_1^2 - \mu_1 + 1]+$$

$$+ 2\mu_1 N_0^2[\mu_1^6\kappa_0^3(1 - \mu_1) + \mu_1^4\kappa_0^2(5\mu_1 - 7) + 2\mu_1^2\kappa_0(3\mu_1^2 - 4\mu_1 + 1) + 4(-\mu_1^2 + \mu_1 + 1)]+$$

$$+ 4\mu_1^4\kappa_0 N_0[\mu_1^3\kappa_0^2 + 2\mu_1\kappa_0(\mu_1 - 1) - 4] + 8\mu_1^5\kappa_0^2,$$

$$B_{01} = \mu_1^4 N_0^4 - 2\mu_1^4\kappa_0 N_0^3 + \mu_1^2 N_0^2[\mu_1^4\kappa_0^2 + 10\mu_1^2\kappa_0(\mu_1 - 1) + 4(-3\mu_1 + 5)] - 16\mu_1^4\kappa_0 N_0 + 8\mu_1^6\kappa_0^2,$$

$$B_{00} = 2\mu_1^3 N_0^2[\mu_1^2\kappa_0(\mu_1 - 1) - \mu_1 + 2] - 4\mu_1^5\kappa_0 N_0 + 2\mu_1^7\kappa_0^2.$$