

23rd EURO Working Group on Transportation Meeting, EWGT 2020, 16-18 September 2020, Paphos, Cyprus

A simulation assessment of shockwave detection and damping algorithms based on magnetometers and probe vehicle data

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Abstract

Nowadays, connected cars are uncommon on our streets, but their percentage is expected to grow uninterruptedly. These devices would provide a lot of data in real-time that can be used for road operators to improve the traffic. This research is focused on one of these applications, which is shockwave damping on freeways. This work evaluates two shockwave detection methods that use probe vehicle data and fixed sensors data and one mitigation algorithm that uses variable speed limits to resolve shockwaves. This paper analyses through microscopic traffic simulation the performance of the selected algorithms and how their parameters affect them in several scenarios. Also, the effect of the penetration rate of probe vehicle data is evaluated. Finally, the best algorithm with the best parameters configuration is applied to a realistic model of the AP7 freeway in Girona (Spain). The obtained results show that the algorithms applied greatly reduce the total travel time in this network.

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Peer-review under responsibility of the scientific committee of the 23rd Euro Working Group on Transportation Meeting

Keywords: shockwave detection; shockwave damping; probe vehicle data; floating car data, variable speed limits; traffic simulation.

1. Introduction

A shockwave is a traffic phenomenon that happens in high-density roads (non-urban) when a perturbation occurs, like an abrupt change of lane or an accidental breaking of a vehicle. One key difference between shockwaves and regular jams is that although shockwaves can be produced by bottlenecks that reduce the capacity of a road or because of a jam, they can remain in the road much after this bottleneck has been resolved. Nevertheless, it is remarkable the fact that shockwaves do not require a physical bottleneck to appear. The presence of a shockwave can reduce the maximum flow of a road up to 30% according to Hegyi et al. (2008). Thus, mitigating them would greatly improve the efficiency of freeways which would provide great benefits from an economical and environmental point of view.

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This paper is organized as follows. First, Section 2 summarizes a literature review about shockwave detection and mitigation. Then, Section 3 and Section 4 present the detection and mitigation algorithms evaluated in this paper. In Section 5, the simulation framework and the computational experiments are described. Finally, Section 6 presents the final conclusions and some future research.

2. Related work

2.1. Shockwave detection

Shockwaves are characterized by having significantly lower speeds and higher densities than the rest of the road. The boundaries of a shockwave (head and tail) are discontinuous transitions from a high density zone (congested traffic), inside the shockwave, to a zone with a lower density (free traffic), outside the shockwave (Treiber and Kesting (2013)). There are mainly two ways to detect shockwaves. They use these characteristics to identify them. The first one consists of using the data available to estimate the state of the flow (speed, density and flow) and then search shockwaves in the state obtained. They search in the estimated state of the road for regions with the first characteristics, low speed and high densities. Other methods directly use the data available, particularly probe vehicle data (PVD) to detect the head and the tail of shockwaves.

One method that uses the first approach is the one proposed in Hegyi et al. (2008). They use Adaptive Smoothing Method (ASM), presented in Treiber and Helbing (2002), to determine the state of the road. This is done using fixed sensors data. This data is weighted and the speed and flow at discretized positions is computed. This method has been extended in Van Lint and Hoogendoorn (2010) to merge data from different sources, including PVD.

To detect the head and tail of a shockwave one example is the algorithm proposed in Izadpanah et al. (2009). We explain it in detail in the detection algorithms section. Another example is Li et al. (2017) that reconstructs the shockwave speed extending the PVD measures available to all vehicles. They solve an undetermined system of equations using orthogonal matching pursuit assuming a triangular fundamental diagram. Also, Rempe et al. (2017) forecast the position of shockwave fronts in a short time horizon (10 min) by estimating it using Lighthill-Whitham-Richards theory. They estimate the flow as a weighted average of the measures, like ASM.

2.2. Shockwave damping

Once shockwaves have been detected, several strategies have been proposed to mitigate them. Usually, the following actions are considered: setting variable speed limits (VSL) and ramp metering. SPECIALIST is an algorithm proposed in Hegyi et al. (2008) that mitigates shockwaves. We explain it later in Section 3. Another type of algorithms proposed are the ones based on model predictive control (MPC). It consists of finding the most appropriate configuration of VSL by solving an optimization problem. To evaluate the objective function a simulation that begins at the current state of the network with the given VSL is executed. Since this simulation is executed many times, it must be fast. Thus, it is done with macrosimulation models, which are faster than microsimulations. To represent the shockwaves in macrosimulation models, the authors of these papers usually modify a well-known macrosimulation model. For instance, this is done in Hegyi et al. (2005), where the authors modify the METANET model. Another model, the cell transmission model, is used in an MPC paper by Han et al. (2017).

Other methods have been proposed. We can find one in Motamedidehkordi et al. (2016). In this paper the authors consider that the driving behavior is modified when the drivers receive a Traffic Jam Ahead Warning (TJAW). Horn (2013) proposed a new driving model for autonomous vehicles that could reduce perturbations such as shockwaves. The model consists of a driving model where the distance to the vehicle behind is also considered (in addition to the leading vehicle, such as in car following models). A different approach is proposed in Behl and Mangharam (2010). They propose to use pace cars. A pace car is a vehicle that drives at a fixed speed and cannot be overtaken.

The main objective of this work is to perform shockwave damping and mitigation on freeways using data from fixed sensors and probe vehicle data. We use ASM based detection and the algorithm proposed in Izadpanah et al. (2009) and compare them using a microscopic traffic simulation framework. We also test the SPECIALIST mitigation algorithm with different parameters. Moreover, we test the best detection algorithm with the best parameters on a real freeway.

3. Detection algorithms

3.1. Algorithm proposed in Izadpanah et al. (2009)

The detection algorithm proposed by Izadpanah et al. (2009) only requires PVD measures and consists of analysing the trajectory data of each individual connected vehicle. The authors want to detect the inflection points, which are great changes in the speed of vehicles that appear in their trajectories. They can be visually seen in the position-time plot as the changes of slope of the line. The idea is that many of these points that represent a significant change of speed can indicate that there is a shockwave front there. Thus, after detecting inflection points for all vehicles, these are clustered to identify shockwave fronts.

The algorithm used for detecting the inflection points is based on the computation of a two-piecewise regression by testing all middle points of the trajectory and compute two regressions, one on the left and other on the right. Then, the difference in their errors is compared with a linear regression to see if it is significant or not, i.e. it is an inflection point or not. To detect all inflection points, two-piecewise algorithm is used with different chunks of the data that are updated with new data and start on the last inflection point detected.

The clustering used is not one of the usual clusterings, where points are grouped by proximity. In this case, we expect the inflection points to follow linear patterns because it is assumed that shockwaves propagate with constant speed. Thus, we perform a clustering in which we aim at putting in the same cluster points that follow a linear pattern. This is called clusterwise linear regression. To obtain this, the algorithm used aims at minimizing the sum of the squared errors of linear regression in each cluster.

To do this clusterwise linear regression, we implement the method explained in Gitman et al. (2018), which is not the same one proposed in the original paper but follows the same main idea. First, each point is assigned to a cluster randomly. The number of clusters is fixed and several values are tried. Then, we compute the linear regression for all clusters. Once we have all the models, we compute for each point the error with each model and assign it to the cluster that has the lowest error, with a normalization factor. We iterate this step until we achieve converge in the total error.

3.2. ASM based detection

The ASM algorithm is described in Treiber and Helbing (2002) and allows to estimate the value of a traffic measure (speed, density or flow) in any location of a one lane section using one source of data. We only use it to estimate the speed, so from now on we focus on speed, but any of the other two measures could be estimated instead.

A shockwave is a region of more than a certain length (a parameter) where vehicles have a speed smaller than a fixed speed (another parameter) that moves on the road with a certain speed. This movement is measured in its boundaries, the head and the tail. The parameters must be calibrated to obtain a quick detection but avoid having false detections. As is done in Hegyi et al. (2008), we directly apply the definition to detect shockwaves by getting through the vector of speed estimations and counting the length of regions where the speed is smaller than the speed threshold. This vector of estimated speeds is computed as follows. At each step, the method checks if the PVD is updated or not according to some fixed message frequency of the PVD. It does the same with sensor data, that are updated every certain time. If at least one of the two sources has new speed estimations, the detector performs the ASM fusion algorithm to merge both estimations into a single speed estimation at each discrete point.

We extend this method to ensure that shockwaves must move. We maintain a list of the detected shockwaves that is updated when new measurements arrive and apply the previous algorithm to the new data to retrieve possible new shockwaves because we do not know if they are shockwaves or just jams. We update the position of the jams previously detected to an estimation of where they should be now, knowing the interval of time between measurements. Then, we find the new detected shockwaves that are close to these estimated positions. If a shockwave has a new measurement close, we consider that it has not vanished, and we update its position. If it does not, it is vanished and removed from the list of shockwaves. The measurements that do not match any previously detected shockwave are added to the list of detected shockwaves. If in the next step another measurement confirms them as shockwaves, they are maintained. Otherwise, they are dropped. This way, a region with a low density must be detected by two consecutive measures, in a displaced position the second time, to be considered a shockwave. This allows to distinguish shockwaves from jams. The distance between the measured and the estimated position of the shockwave

is taken as the maximum distance between tails (expected and measured) and the distance between heads (also expected and measured).

4. Mitigation algorithm (SPECIALIST)

This algorithm was proposed in Hegyi et al. (2008) and later extended in Hegyi et al. (2013) to take in to account PVD measures. It is based on shockwave theory developed by Lighthill-Whitham-Richards, presented in Lighthill and Whitham (1955).

This algorithm makes use of fixed sensors and probe vehicle data. The objective of the algorithm is to create a region upstream of the shockwave with a lower speed and flow that makes the tail of the shockwave propagate at a lower speed than the head. Thus, eventually the head will meet the tail and the shockwave will be resolved. After this, the shockwave is resolved, and although a region with a higher density and lower speed remains, this region is assumed to have a higher flow than the shockwave, so the traffic is improved. To achieve this, the algorithm detects the shockwave using the first detection algorithm explained (ASM based) and then, mitigates it using variable speed limits (VSL) on an upstream region of the shockwave. These VSL are applied using variable message signs (VMS).

The procedure to apply this algorithm is the following:

- The shockwave is detected and located with some method (it can be any algorithm).
- All the traffic states are determined, by direct measure and using the parameters of the algorithm.
- Check if the shockwave is solvable. Sometimes, the traffic states are in a configuration where the shockwave is not solvable. This is explained in Hegyi et al. (2008). If the shockwave is not solvable, the algorithm ends here.
- If it is solvable, the VSL region of application is computed with and the VMS are activated.
- Once the shockwave is no longer detected (it has been mitigated) the VSL are released progressively.

5. Computational experiments

5.1. Simulation framework

This paper analyses through microscopic traffic simulation the performance of the selected algorithms and how their parameters affect them. Also, the effect of the penetration rate of connected vehicles is evaluated. The selected microscopic traffic simulator is Aimsun Next 8.3 (Aimsun (2019)), which can model the interactions for each vehicle and collect data from them individually. We emulate that each connected vehicle sends position and speed data every certain period of time.

5.2. Scenarios

We test the proposed algorithms in two scenarios. The first one is a dummy network, a straight freeway of 10 km with 3 lanes. The second one corresponds to a section of the AP7 freeway in Catalonia (Spain) used in the European project C-Roads Spain, which was built and calibrated by the company Aimsun SLU. This is another reason to use Aimsun Next for the proposed simulation framework. This freeway, in the section tested, has also 3 lanes. Each network was simulated with different PVD penetration rates, giving several scenarios.

To generate a shockwave, we created a high input flow in the networks. Specifically, we put enough traffic to saturate the freeway with as much traffic as possible. Then, we stop a car enough time to create the desired shockwave. In the dummy network, there is enough to stop a car for 5 seconds, but in the AP7 to generate a shockwave we need to stop it for 20 seconds. In the dummy network the simulations executed last 30 min and the shockwaves are simulated at the 15 min. In the AP7 network the simulations last 2 hours and the shockwave is generated after 1 hour of simulation. In both scenarios, we used a configuration of variable message signals spaced 1 km through all the freeway. In addition, we set fixed sensors, which are magnetometers, each 500 m and they update their data each 15 seconds. Both parameters are fixed for all scenarios.

5.3. Design of experiments

With respect to the ASM detection algorithm, this paper presents the experiments related with the parameters that we consider more relevant, which are the speed and the length thresholds. The considered values are the following: 20, 30, 40 and 50 km/h for the speed threshold and 100, 200, 300, 500 and 700 m for the length threshold.

It is important to note that, a more complete design of experiments is presented by the authors in Garriga (2019), which includes other factors like the kernel sizes, the discretization size or the sending time.

In this paper, we study the variations of the results with respect to a reference scenario, which is the medium sized kernel (200m, 20s) for congested and free flow (parameters of ASM), a speed threshold of 30 km/h, a length threshold of 100m, a discretization of 100 m and a sending time of 5s. Each of the described parameters configuration is simulated with the following PVD penetration rates: 5, 10, 20, 50, 70 and 100%. In addition, each scenario is simulated twice, once considering that fixed sensors are available as well as the PVD and another considering that only PVD is available.

Regarding the detection algorithm proposed in Izadpanah et al. (2009), the clustering parameters are relevant when there is more than one shockwave, which is not the case of our scenarios. The weight is also fixed for the same reason. Thus, we only tested the parameters threshold and sending time, with the values 20, 30 and 40 km/h and 1, 5 and 10 s, correspondingly.

Despite having only these two parameters, we did not perform a factorial design because this algorithm is very slow, and each simulation takes a lot of time, approx. 170 min. Thus, we did like in the ASM detection, we use a reference scenario, where the threshold is 20 km/h and the sending time is 5 s. We run all experiments for the PVD penetration rates 5, 10, 20 and 50%. We do not run the simulations with a 70 and 100% of penetration rate because of the required computational time for this algorithm. This algorithm does not use fixed sensors, so they are not used in its experiments.

To evaluate the mitigation algorithm, we need to combine it with a detection algorithm. In this paper, we propose to use the one that obtained a better performance in the experiments (see performance metrics). The design of experiments proposed considers the following parameters and values (see Hegyi et al. (2008)):

- ρ_4 : 18, 22 and 26 veh/km/lane
- ρ_5 : 14, 18 and 22 veh/km/lane
- q_5 : 1500, 2000 and 2500 veh/h
- Variable Speed Limit (VSL): 40, 50 and 60 km/h

In this case, we perform a full factorial design because there are some combinations that do not fulfil certain traffic theory constraints; thus, the number of experiments is affordable (180 exp.). These configurations include all the PVD penetration rates considered (5, 10, 20, 50, 70 and 100%).

5.4. Performance metrics

Regarding the shockwave detection, we did not have a database of shockwave measures to calibrate our algorithms. Thus, we had to generate our shockwaves, as explained before. The only things that we can control about a shockwave are its generation time and its starting position. Thus, what we can measure is if the shockwave is well detected at its beginning. Thus, we measure the delay in the detection for the detection algorithms. This is how many seconds elapse between the generation time of the shockwave and the detection time of the algorithm.

With respect to the shockwave mitigation, we can measure if the SPECIALIST algorithm has mitigated the shockwave or not by checking if it is still being detected and we measure the improvement on the network. The proposed metric for this is the total time spent (TTS), which measures the total time that cars have spent inside the network, which is the sum of all travel times for all cars. In addition, the last instantaneous TTS (LITTS), which is the average TTS of vehicles that arrive at the end of the network in the last minute of the simulation. The TTS measures how the networks are affected in the short term by the mitigation algorithm, because we run short simulation of 30 min. The LITTS measures how the state of the network is affected more in the long term, because we see which TTS the vehicles tend to have after applying the algorithm. It is a measure that helps to determine in which state the network is left, while the TTS represents in which state the network has been. Both metrics are represented divided by a reference value which is the metric obtained with the given scenario when the shockwave is not mitigated.

5.5. Analysis of the results

Table 1 contains the delay obtained with ASM detection without using fixed sensors. We see that the speed threshold does not affect the delay significantly. On the contrary, the length threshold does. The larger it is, the bigger is the delay, as it was expected.

Table 1. Delay obtained in the dummy scenario with ASM regarding speed threshold (km/h) and length threshold (m) without sensors.

Penetration rate (%)	Speed threshold (km/h)				Length threshold (m)				
	20 km/h	30 km/h	40 km/h	50 km/h	100 m	200 m	300 m	500 m	700 m
5	15	15	15	15	15	115	140	220	340
10	15	15	15	15	15	80	100	195	320
20	35	20	15	10	20	85	115	205	295
50	20	40	20	20	20	80	135	225	320
70	20	20	20	20	20	75	150	205	305
100	20	15	15	15	15	75	125	240	310

Table 2 contains the delay obtained with ASM detection using fixed sensors. We observe that the results are not very different from the ones obtained without fixed sensors. Thus, we observe that using fixed sensors does not improve the detection delay.

Table 2. Delay obtained in the dummy scenario with ASM regarding speed threshold (km/h) and length threshold (m) with sensor data.

Penetration rate (%)	Speed threshold (km/h)				Length threshold (m)				
	20 km/h	30 km/h	40 km/h	50 km/h	100 m	200 m	300 m	500 m	700 m
5	15	15	15	15	15	15	75	175	260
10	15	15	15	15	15	15	100	105	260
20	35	20	20	15	20	70	130	220	305
50	75	20	20	20	20	80	130	205	320
70	70	20	20	20	20	75	125	205	305
100	70	20	15	15	20	80	125	245	310

In Table 3 we have the results obtained with the detection algorithm proposed in Izadpanah et al. (2009). We observe that if the speed threshold is larger the delay is larger, as could be expected because less inflection points are detected. We also observe that a lower sending time provides better results. Comparing with the ASM detection, we observe clearly that these are much worse. The lowest delay achieved is between 150-200s, while ASM detection has a delay around 10 times lower.

Table 3. Delay obtained in the dummy scenario with Izadpanah et al. (2009) proposed detection regarding speed threshold (km/h) and sending time (s).

Penetration rate (%)	Speed threshold (km/h)			Sending time (s)		
	10 km/h	20 km/h	30 km/h	1 s	5 s	10 s
5	415	1205	1220	991	1205	1580
10	320	1195	1195	985	1195	1550
20	300	1140	1140	941	1140	1480
50	160	420	1130	312	420	1480

The SPECIALIST algorithm was activated with 171 parameters configurations. In 126 of them the shockwaves are mitigated. Table 4 represents the average TTS and LITTS obtained with all the scenarios with the given speed, because

we have a full factorial design, so each speed is executed with several combinations of parameters. Some combinations of penetration rate and VSL do not have measures because the shockwave is not mitigated with any combination of the parameters. We observe that in general the effect of the mitigation is not appreciable because the values obtained are almost equal to the ones obtained with the shockwave without mitigation. The only exception is the LITTS for a 60 km/h VSL, where we observe that a reduction is achieved.

Table 4. Results obtained with SPECIALIST.

Penetration rate (%)	VSL (km/h)					
	40 km/h		50 km/h		60 km/h	
	TTS	LITTS	TTS	LITTS	TTS	LITTS
5	1.0152	1.0287	1.0157	1.0081	-	-
10	1.0222	1.0034	1.0168	1.0033	-	-
20	-	-	-	-	1.0276	0.8870
50	1.0206	1.0244	1.0078	1.0424	1.0025	0.9846
70	1.0195	1.0223	1.0179	1.0168	-	-
100	-	-	1.0161	1.0252	1.0271	0.8817

The results obtained in the AP7 network are presented in Table 5. In this network the best parameters are used with the best detection algorithm, which is the ASM based detection, so there are no parameters in this table. We observe that the results are very similar independently of the penetration rate. The TTS is reduced a 18% and the last instantaneous TTS is reduced a 16.5%. These results are much better than the ones observed in the dummy network. In addition, the shockwave is mitigated in all six scenarios.

Table 5. Results obtained in the AP7 scenario.

Penetration rate (%)	TTS	LITTS
5	0.8189	0.8320
10	0.8185	0.8632
20	0.8189	0.8320
50	0.8186	0.8320
70	0.8188	0.8320
100	0.8186	0.8320

6. Conclusions and future research

The objective of the paper was to implement two shockwave detection algorithms, test how they are affected by their parameters and compare them. Then the best one was used to test the effect of the parameters of a mitigation algorithm, the SPECIALIST. Finally, with the best parameters obtained, we tested the mitigation algorithm in a real freeway.

On the one hand, the results obtained show that the performance of the algorithms is affected by its parameters as expected, except for the speed threshold of the ASM that did not have a significant effect. On the other hand, the effect of the penetration rate and the sensors is quite unclear. We expected that the presence of sensors would improve the delay, but this is not observed. In the detection algorithm proposed in Izadpanah et al. (2009), a higher penetration rate obtains a lower delay, but this is not observed in the ASM detection. In this algorithm, the delay is not significantly affected by the penetration rate.

We have observed that ASM detection works very well, the detection takes approximately 10 s. These are similar results to the ones obtained by Hegyi et al. (2013). This fast detection is very good and key to mitigate the shockwave. On the contrary, Izadpanah et al. (2009) proposed detection algorithm did not work that well. We were able to detect

shockwaves, but the delay in the detection was much higher. Moreover, the time to compute this algorithm is much higher than the one that ASM detection requires.

Regarding the mitigation algorithm, we did not observe a considerable improvement on the dummy network, particularly regarding the TTS. There was a greater improvement on the last instantaneous TTS, but the significant thing is that we observe that the shockwaves in the real freeway are mitigated. The improvement of the TTS in the real network is a 18%, so this is a very good improvement.

There are many ways to extend this work. For instance, a major metric that we did not analyze is the false positives rate. We have worked with a fixed type of shockwave and a fixed flow for each network, but this could be extended to creating shockwaves with different sizes in scenarios with various flows. Moreover, there are some relevant factors that we have considered fixed but that could modify the detection and the mitigation. These are the distance between fixed sensors, that we fixed to 500 m, this could be a parameter of study. Another one is the distance between the variable message signs. Also related with the VMS, we could study the performance of the mitigation in a scenario where not all vehicles comply with the speed limit. Further research on why the penetration rate does not seem to behave as expected in ASM detection would be worth investigating.

7. Acknowledgements

Throughout this work, the authors have benefited from the support of inLab FIB team at Universitat Politècnica de Catalunya. This research was funded by Secretaria d'Universitats i Recerca de la Generalitat de Catalunya (2017-SGR-1749).

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