FÍSICA

APRÈN UPC

Apunts

Laser models and dynamics

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Assignatura: Laser systems and applications

Titulacions: Màster Universitari Erasmus Mundus en Enginyeria Fotònica, Nanofotònica I Biofotònica (pla 2010)
Màster Universitari en Fotònica (pla 2013)

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Laser Models and Dynamics

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SCHEDULE OF THE COURSE

Semiconductor light sources

- 1 (11/12/2020) Introduction. Light-matter interactions.
- 2 (15/12/2020) LEDs and semiconductor optical amplifiers.
- 3 (18/12/2020) Diode lasers.

Laser Material Processing

- 4 (22/12/2020) High power laser sources and performance improving novel trends
- 5 (12/1/2021) Laser-based material macro processing.
- 6 (15/1/2020) Laser-based material micro processing.

Small lasers, biomedical lasers and applications

- 7 (19/1/2021) Small lasers.
- 8 (22/1/2021) Biomedical lasers.

Laser models

9 (26/1/2021) Laser turn-on and modulation response.

10 (29/1/2021) Optical injection, optical feedback, polarization.

- 11 (2/2/2021) Students' presentations.
- 12 (5/2/2021) Students' presentations.
- 9/2/2021: Exam

Lecturers: C. Masoller, M. Botey

Learning objectives

- Understand the simplest class B laser model that explains:
  - The turn-on of a laser diode.
  - The response to current modulation.
- Become familiar with a more advanced laser model that allows to understand the effects of optical perturbations.
- Acquire a basic knowledge of models that describe multi-mode emission and the polarization dynamics of VCSELs.

Bibliography:

Semiconductor Lasers : Stability, Instability and Chaos, J. Ohtsubo (Springer, 3er ed. 2013)
EBook available:
http://recursos.biblioteca.upc.edu/login?url=http://dx.doi.org/10.1007/978-3-642-30147-6

Outline

- Introduction: class A, B and C lasers
- Rate equations governing class B lasers
  - Multimode extension
- Dynamical effects induced by a time-varying pump current
  - Laser turn-on
  - Periodic modulation
- Rate equation governing the optical field of a diode laser
- Dynamical effects induced by optical perturbations
  - Optical injection
  - Optical feedback
- Rate equations governing the polarization of a VCSEL

The simplest model of a semiconductor laser consists of two rate equations.

- One equation governs the number of photons in the cavity (S) and the other one governs the number of carriers (pairs of electrons and holes, N).
- Lasers that are governed by two rate equations are **class B lasers**. Other class B lasers are ruby, Nd:YAG, and CO_2 lasers.
- "Free-running": diode lasers display a stable output (only transient relaxation oscillations).
- But when they are perturbed, because of nonlinearly and the α-factor (specific of semiconductor materials) diode lasers can display complex dynamical behavior.

τ_n = Carrier lifetime
τ_p = Photon lifetime

Typical values for common class B lasers

Laser	τp (s)	τn (s)	$\gamma = \tau p/\tau n$
CO_2	10^{-8}	4×10^{-6}	2.5×10^{-3}
solid state (Nd^{3+}:YAG)	10^{-6}	2.5×10^{-4}	4×10^{-3}
semiconductor (GaAs)	10^{-12}	10^{-9}	10^{-3}

Other types of lasers

- **Class A** (Visible He-Ne lasers, Ar-ion lasers, dye lasers): governed by one rate equation for the optical field (the material variables can be adiabatically eliminated), no oscillations.

- **Class C** (infrared He-Ne lasers): governed by three rate equations (N, S, P=macroscopic atomic polarization), display sustained oscillations and even a chaotic output. No commercial applications.

Dynamics of Class C, B and A lasers

$$E = \sqrt{S} = \left| E_x + iE_y \right|$$

S. Wieczorek et al. / Physics Reports 416 (2005) 1–128

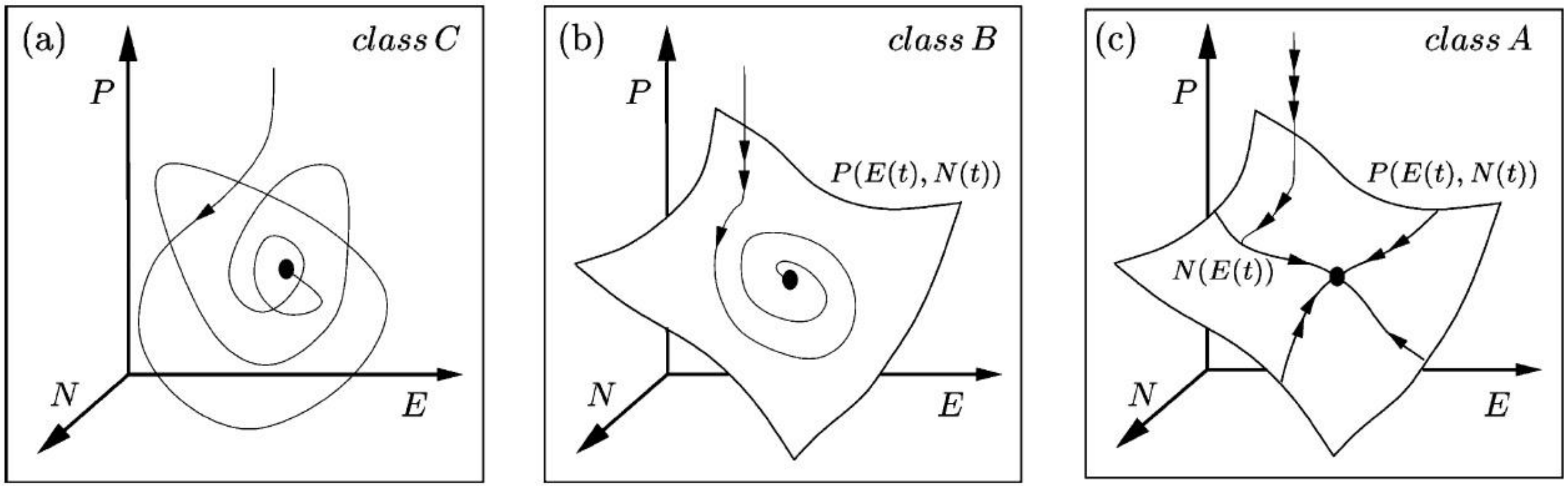


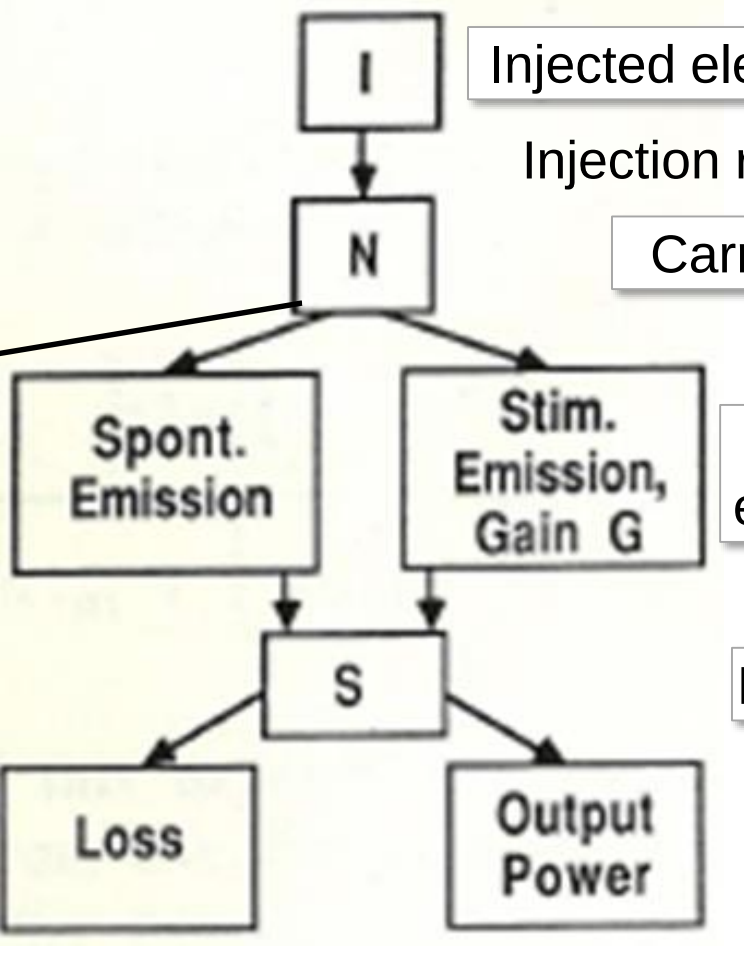
Fig. 1. Sketches of a typical trajectory approaching a stable fixed point in class-C, class-B, and class-A free-running lasers.

infrared He-Ne lasers

Semiconductor, ruby, Nd:YAG, CO_2 lasers

Visible He-Ne, Ar-ion, dye lasers

Diode lasers convert electrical power into optical power



Injected electrical current

Injection rate: (I/e)/V

Carrier density

Non-radiative recombination

Gain: stimulated emission-absorption

Photon density

Rate equation for the carrier density N

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS$$

I/(eV) : injection rate (number of electrons injected per unit volume and per unit time; I is the injected current in Amperes).

N/τ_N : carriers lost due to spontaneous emission and non-radiative processes.

GS : carriers lost due to stimulated emission – absorption. The gain G is a function (of N, S, temperature, λ, etc.) that "encodes" the information about the active medium ("bulk", MQWs, QDs, etc.)

Rate equation for the photon density S

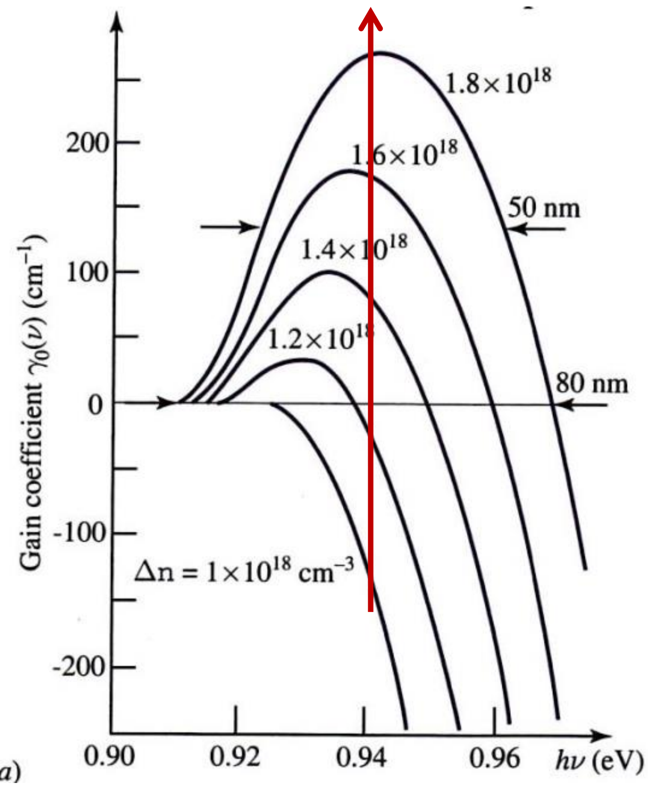
$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

- GS: Stimulated emission - absorption
- $\frac{S}{\tau_p}$: Cavity losses
- $\frac{\beta_{sp} N}{\tau_N}$: Spontaneous emission

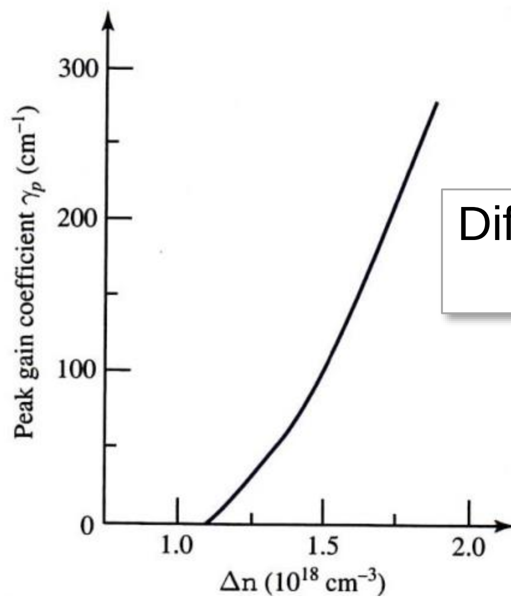
τ_p : Photon lifetime in the cavity.

β_{sp} : Spontaneous emission factor.

Simple model for the gain of semiconductor material



RT InGaASP laser



$$G = G_N\left(N - N_0\right)$$ DH

$$G = G_N N_0 \ln\left(N / N_0\right)$$ QW

Differential gain coefficient

Carrier density at **transparency**

G_N depends on the wavelength

Threshold condition: gain = loss

$$G_N\left(N_{th} - N_0\right) = \frac{1}{\tau_p}$$

Simplest model for class-B lasers: two coupled and nonlinear ordinary differential equations (ODEs)

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau_N} - GS$$

Typical parameters of diode lasers

τ_p	**1 ps**
τ_N	1 ns
β_{sp}	10^{-4}

- Important simplification: spatial effects neglected.
- Additional nonlinearities from: $\frac{1}{\tau_N} = A + BN + CN^2$ $\quad G = \frac{G_N (N - N_0)}{1 + \varepsilon S}$
- Gain saturation (ε) is a simple way to include phenomenologically spatial effects such as carrier diffusion and optical and thermal inhomogeneities.
- Above threshold we shall see that N is nearly constant ("clamped") so the variation of τ_N with N can be neglected.
- These equations allow to understand the LI curve and the laser's modulation response.
- To understand the intensity noise and the line-width (the optical spectrum), we need an equation for the complex optical field E ($S=|E|^2$) (more latter).

Role of spontaneous emission

$$\frac{dS}{dt} = \left(G - \frac{1}{\tau_p} \right) S + \frac{\beta_{sp} N}{\tau_N}$$

- If at t=0 there are no “seed” photons in the cavity: S(0) = 0
- Then, without noise (β_{sp}=0): if S=0 at t=0 $\Rightarrow$ dS/dt=0

  $\Rightarrow$ S remains 0 (regardless of the value N).
- Without spontaneous emission there is no “seed” for stimulated emission and the laser does not turn.
- Above threshold stimulated emission dominates and spontaneous emission can be neglected (not true for small lasers and nanolasers).

Normalized equations, neglecting β_{sp}

- Dimensionless variable: $$N' = \frac{N - N_0}{N_{th} - N_0}$$
- Using the threshold condition: $$G_N\left(N_{th} - N_0\right) = \frac{1}{\tau_p}$$
- We obtain: $$\frac{dS}{dt} = \frac{1}{\tau_p}\left(N' - 1\right)S$$
$$\frac{dN'}{dt} = \frac{1}{\tau_N}\left(\mu - N' - N'S\right)$$
- Here μ is the pump current parameter, proportional to I/I_{th}
- Normalizing the equations eliminates two parameters (G_N, N_o).
- In the following we drop the " ' "

Steady state solutions

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}(\mu - N - NS)$$

$$\frac{dS}{dt} = 0 \Rightarrow \begin{cases} S = 0 \\ N = 1 \end{cases}$$

$$\frac{dN}{dt} = 0 \Rightarrow \begin{cases} S = 0 \rightarrow N = \mu \\ N = 1 \rightarrow S = \mu - 1 \end{cases}$$

Laser off

Stable if $\mu<1$: $S=0$, $N=\mu$

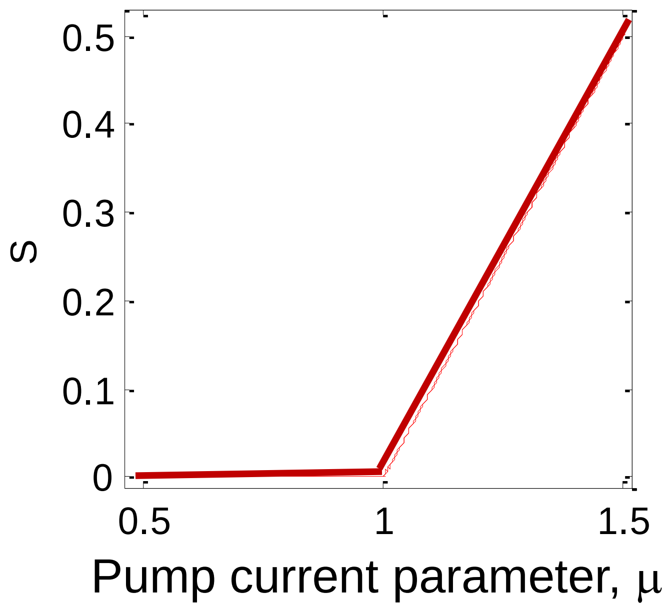
Laser on

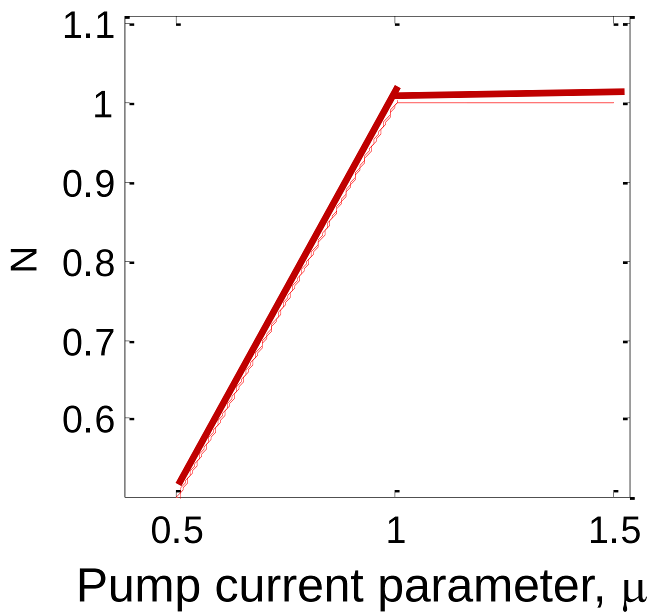
Stable if $\mu>1$: $S = \mu-1$, $N = 1$

$\mu_{th} = 1$

Above threshold the carriers are "**clamped**".

Photons and carriers vs. the pump current parameter





off

S=0

on

$S=\mu-1$

$\mu_{th} = 1$

off

$N=\mu$

on

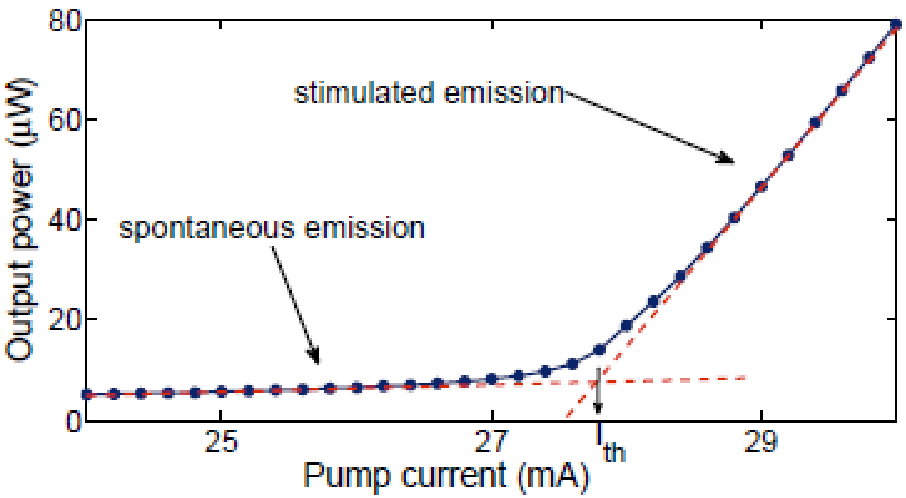
N=1

Above threshold the carriers are "**clamped**".

Experimental LI curve

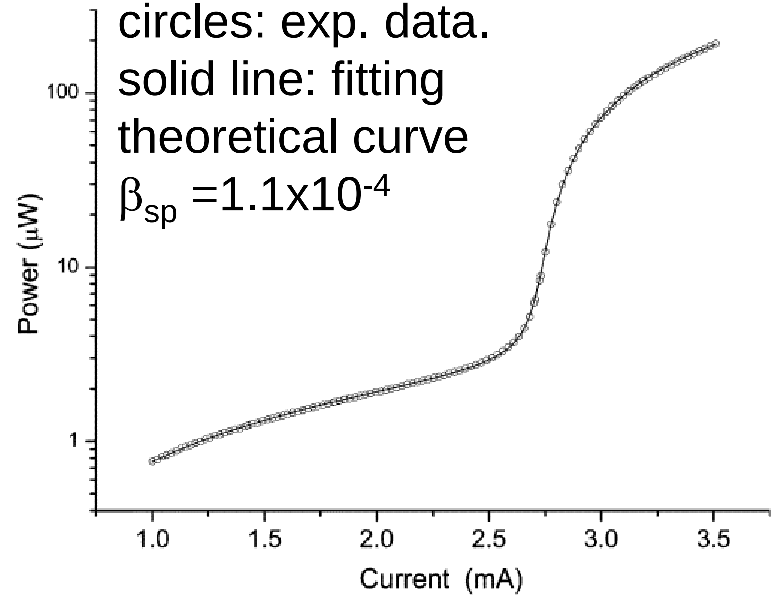
The LI curve is reproduced by the model when the spontaneous emission factor, β_{sp}, is not neglected.

$$\frac{dS}{dt} = GS - \frac{S}{\tau_p} + \frac{\beta_{sp} N}{\tau_N}$$



EEL (λ=670 nm) with MQW active region.

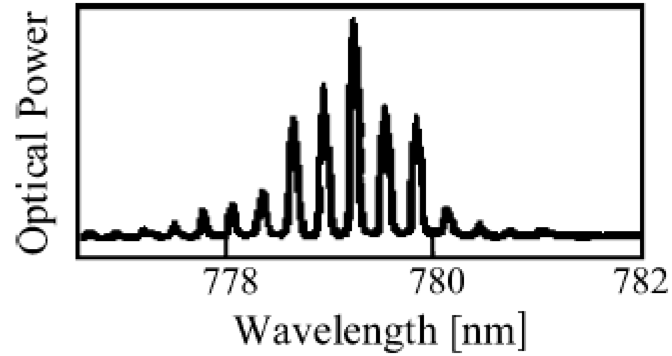
A. Aragoneses PhD thesis (UPC 2014).



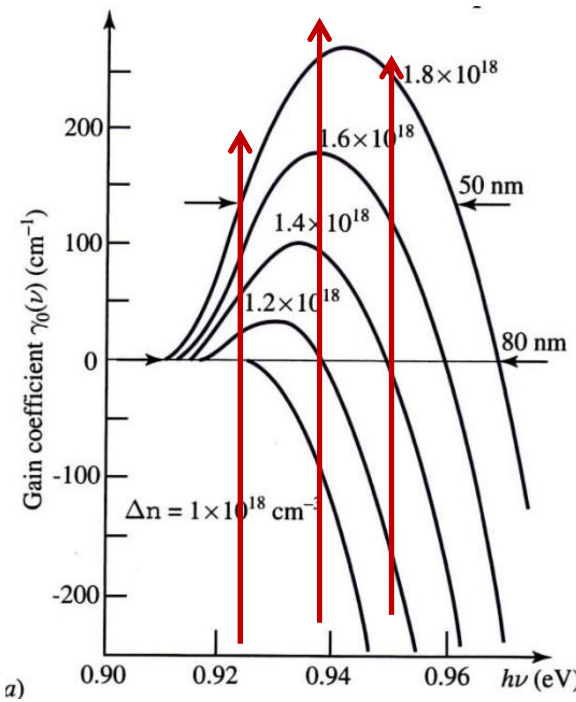
VCSEL (λ=770 nm) with an active region composed by 3 8-nm QWs.

S. Barland et al, IEEE J. Quantum Electron. 41, 1235 (2005).

Extension for a multi-mode laser



A rate equation for the carrier density (n) + a rate equation for the photon density of each mode.



Gain coefficient for mode *j*:

$$G_{\mathrm{n},j} = G_{\mathrm{n}} \left\{ 1 - \left(\frac{j}{M} \right)^2 \right\}$$

Net gain - loss

Spontaneous emission

$$\frac{\mathrm{d}S_j(t)}{\mathrm{d}t} = [G_{\mathrm{n},j}\{n(t) - n_{\mathrm{th},j}\}]S_j(t) + R_{\mathrm{sp}}(\omega_j)$$

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{J(t)}{ed} - \frac{n(t)}{\tau_{\mathrm{s}}} - \sum_{j=-M}^{M} G_{\mathrm{n},j}\{n(t) - n_0\}S_j(t)$$

Carrier injection

Carrier loss

Carrier recombination due (stimulated emission-absorption)

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  - **Periodic modulation**
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  - Optical feedback
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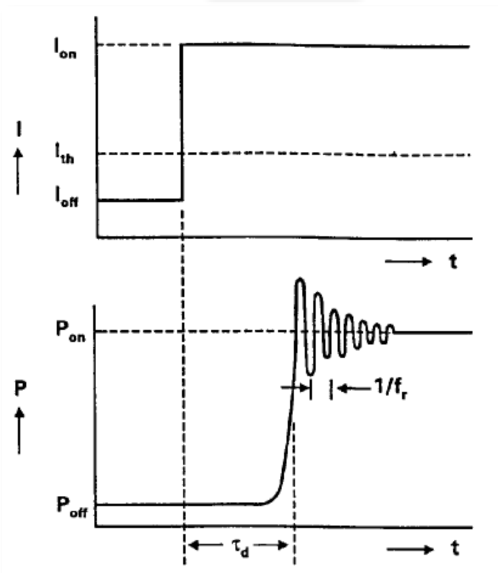
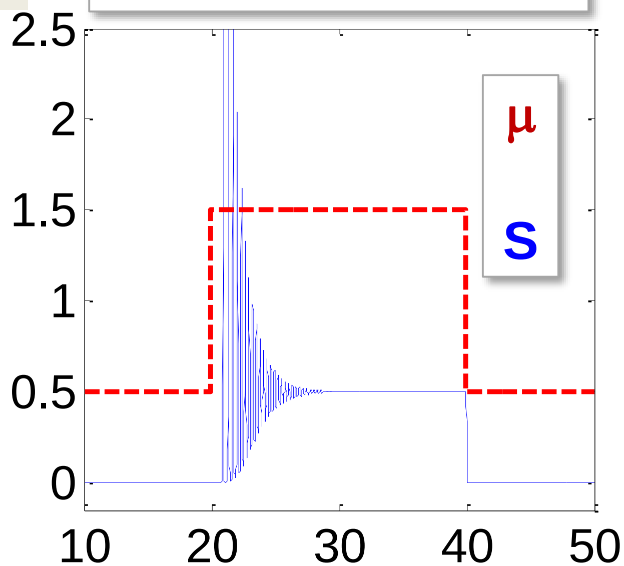
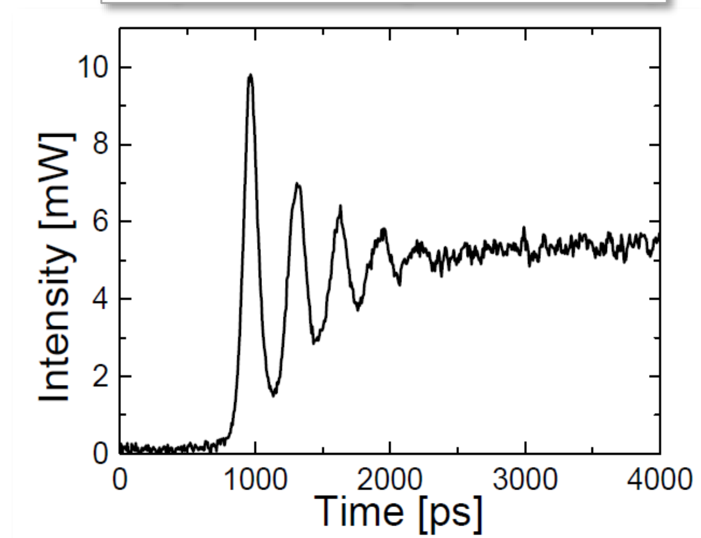
Laser turn on with a pump current step: turn-on delay and relaxation oscillations

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - NS\right)$$

μ (t)







A linear stability analysis of the rate equations allows to calculate the **angular relaxation oscillation frequency** (important parameter for lasers used in optical communications)

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

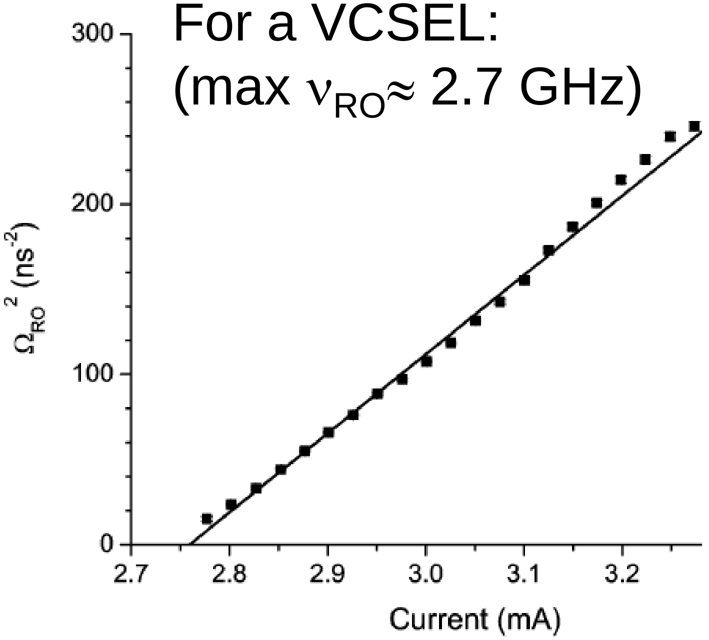
Variation of the RO frequency with the pump current (and therefore, with the laser output intensity)

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

Laser on: $S = \mu - 1$ $\Rightarrow$

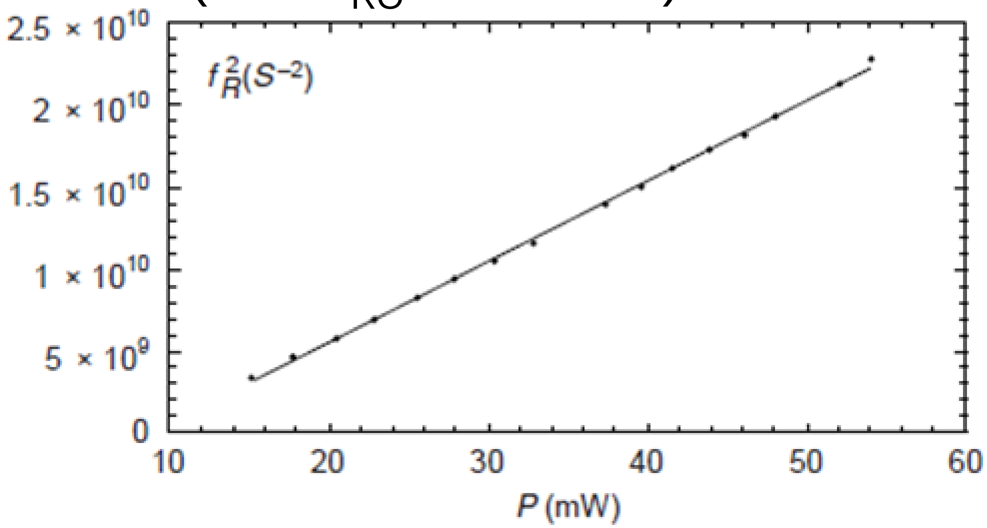
$$\omega_{RO} = \sqrt{\frac{S}{\tau_p \tau_N}}$$

For a VCSEL:
(max $\nu_{RO} \approx$ 2.7 GHz)



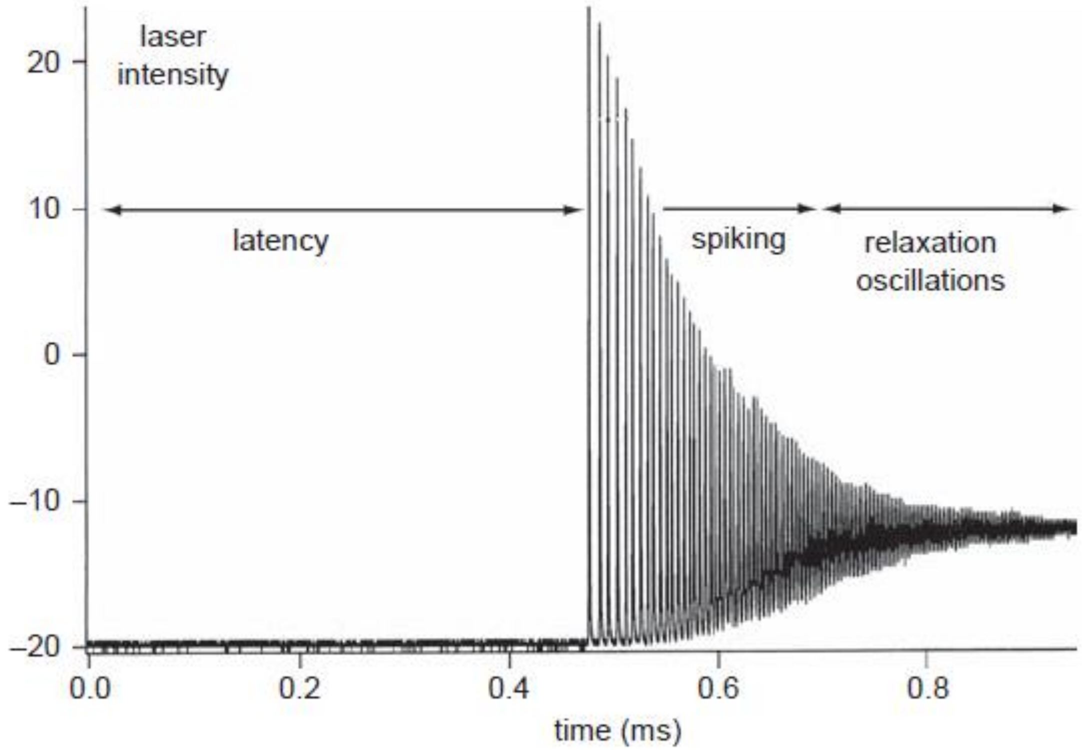
S. Barland et al, IEEE J. Quantum Electron. 41, 1235 (2005).

For an erbium doped fiber laser:
(max $\nu_{RO} \approx$ 158 KHz)



T. Erneux and P. Glorieux, Laser Dynamics (Cambridge University Press 2010)

Turn-on of a Nd^{3+}:YAG laser

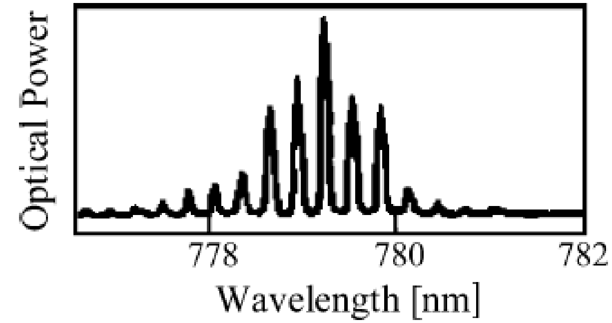


Note the time-scale: for diode lasers is a few ns

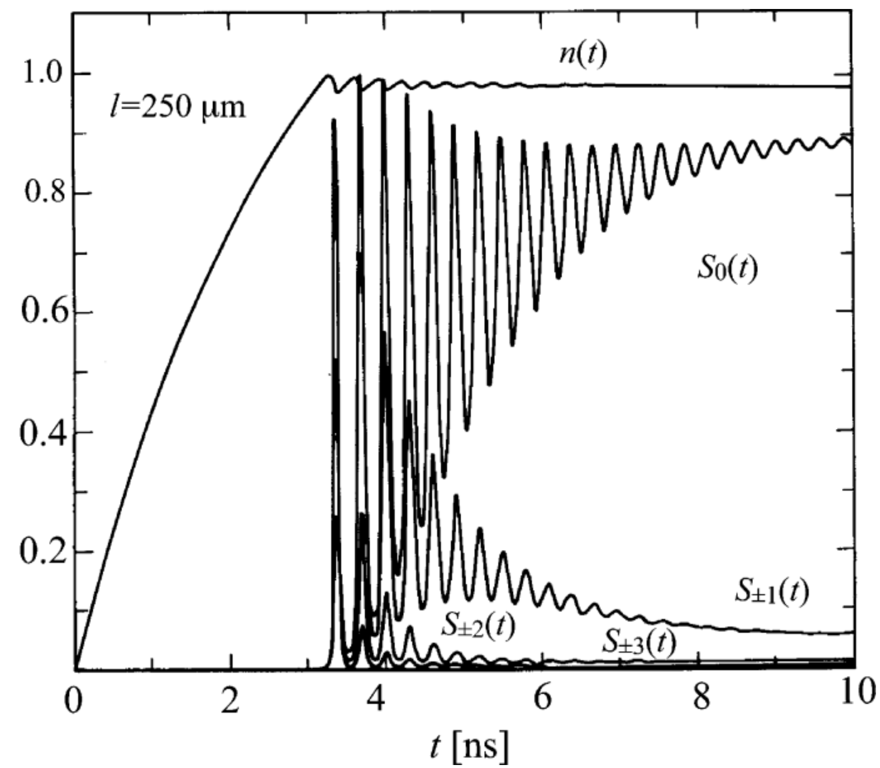
Source: T. Erneux and P. Glorieux, Laser Dynamics (Cambridge University Press 2010)

Turn-on of a multi-mode laser

Parabolic gain profile: $G_{n,j} = G_n \left\{ 1 - \left(\frac{j}{M} \right)^2 \right\}$

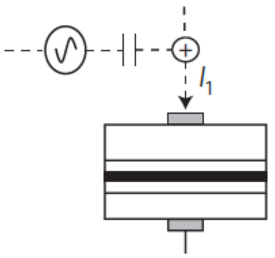


- Winner takes all: after transient "mode-competition", the mode with maximum gain wins.
- But non-transient mode competition has been observed in many semiconductor lasers.
- More advanced gain models allow to understand the origin of mode competition.

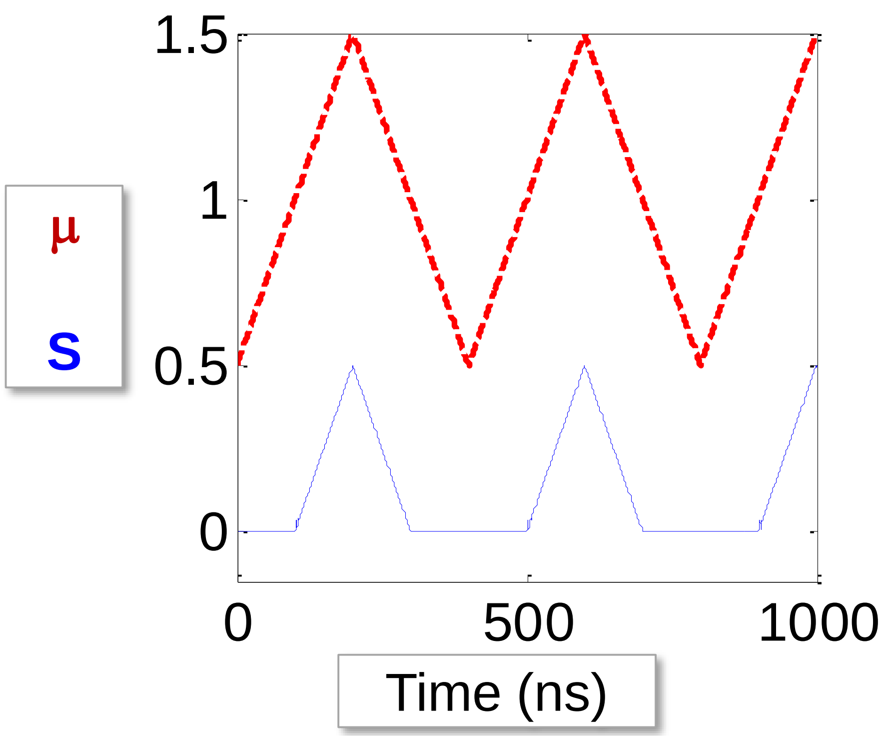


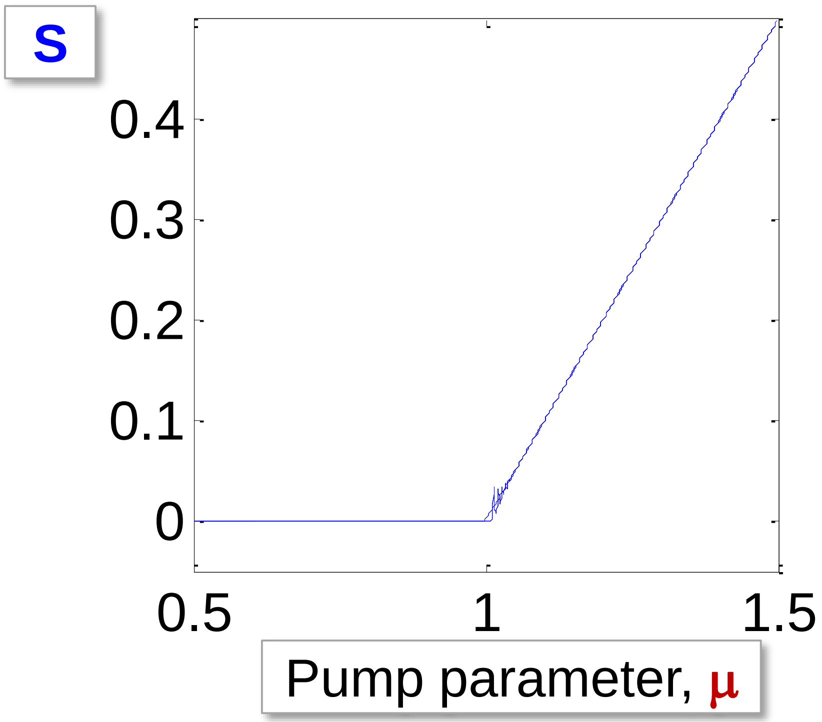
J. Ohtsubo, Semiconductor lasers (Springer 2013)

What happens if we turn on-and-off the laser using a triangular signal?

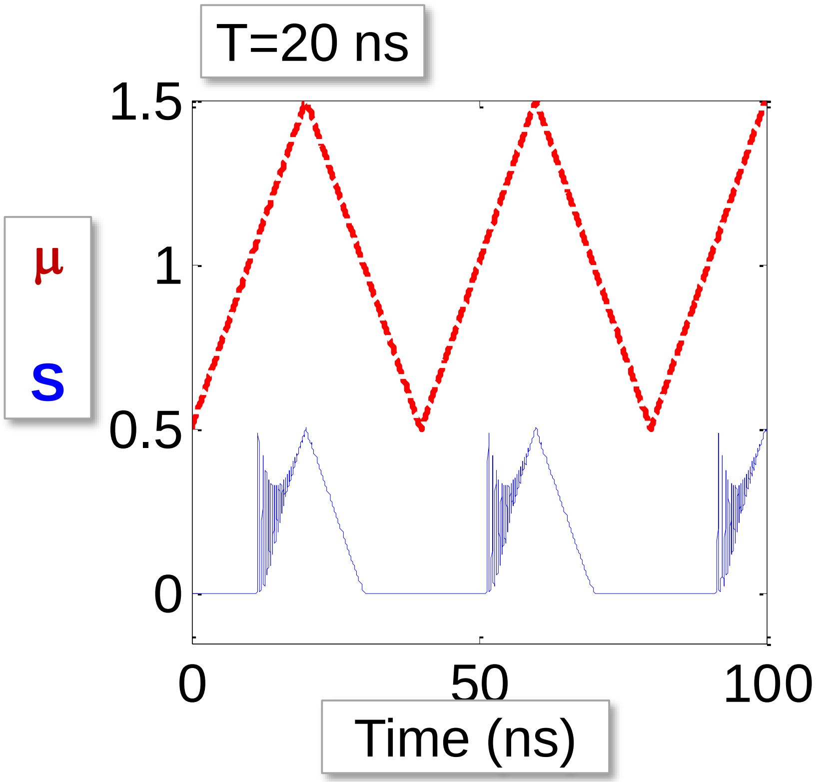


When the current varies "slowly" with respect to the laser characteristic response time (the relaxation oscillations):

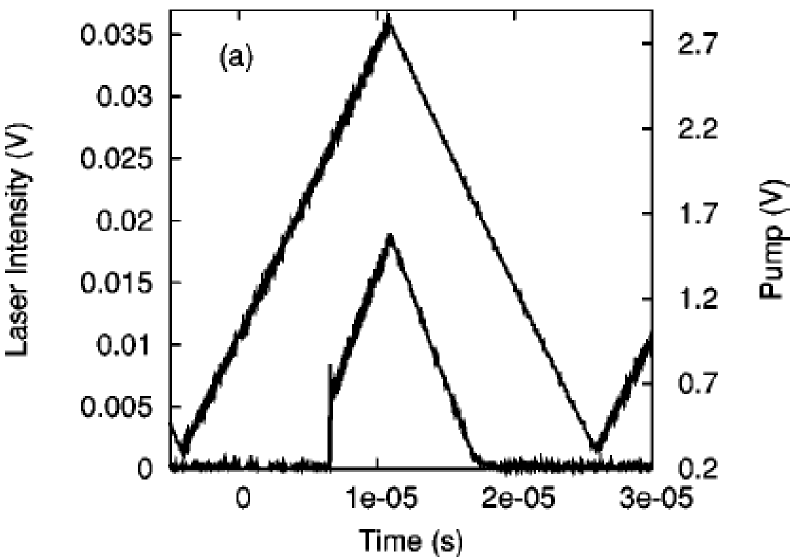




But when there is a fast variation of the laser current: delay in the turn-on and oscillations



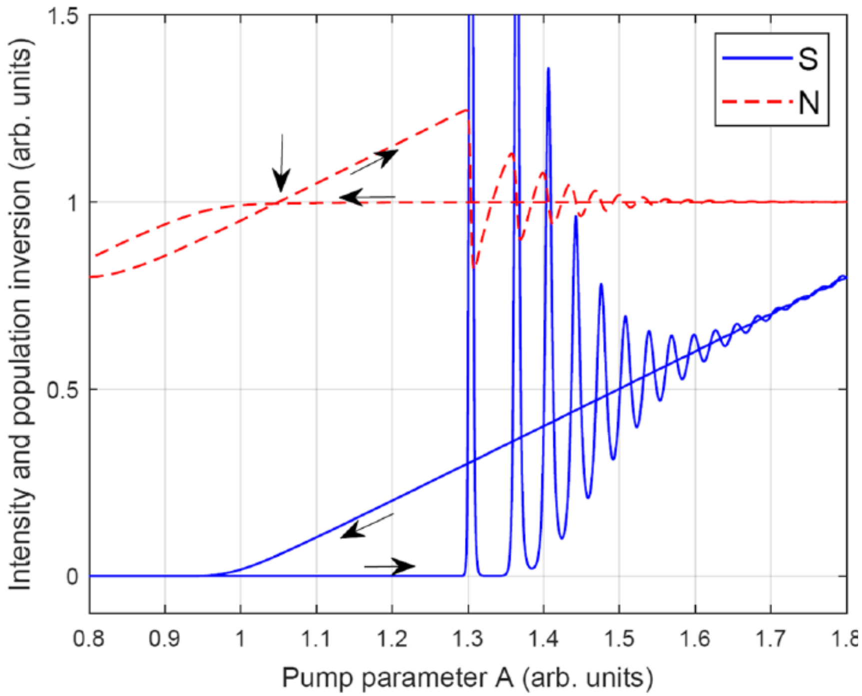
Experiments



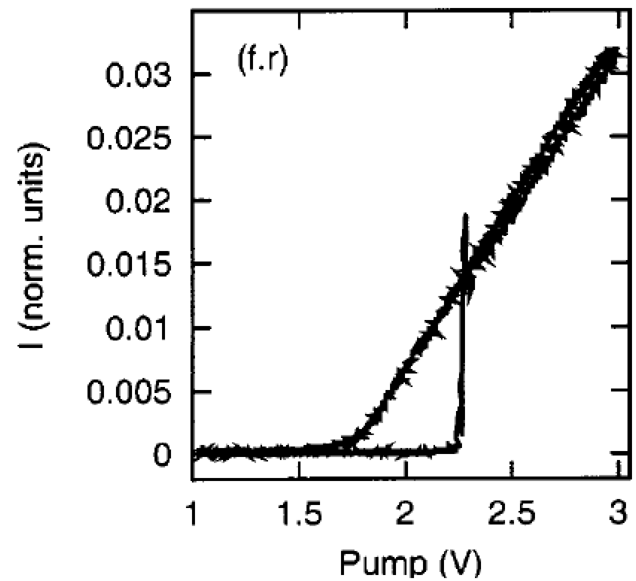
Tredicce et al, Am. J. Phys., Vol. 72, No. 6, 2004

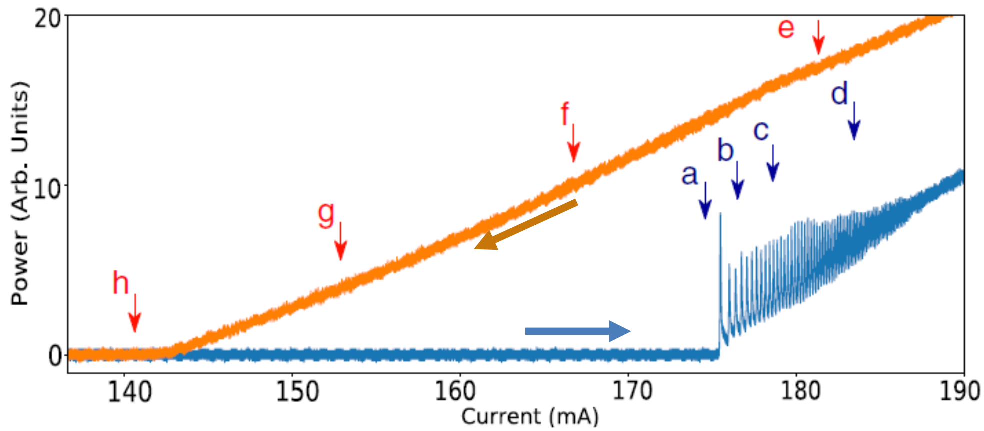
Dynamical hysteresis

Simulations



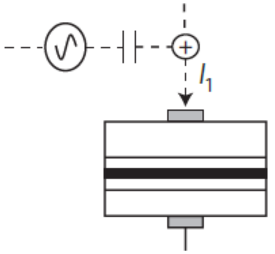
Experiments with different lasers



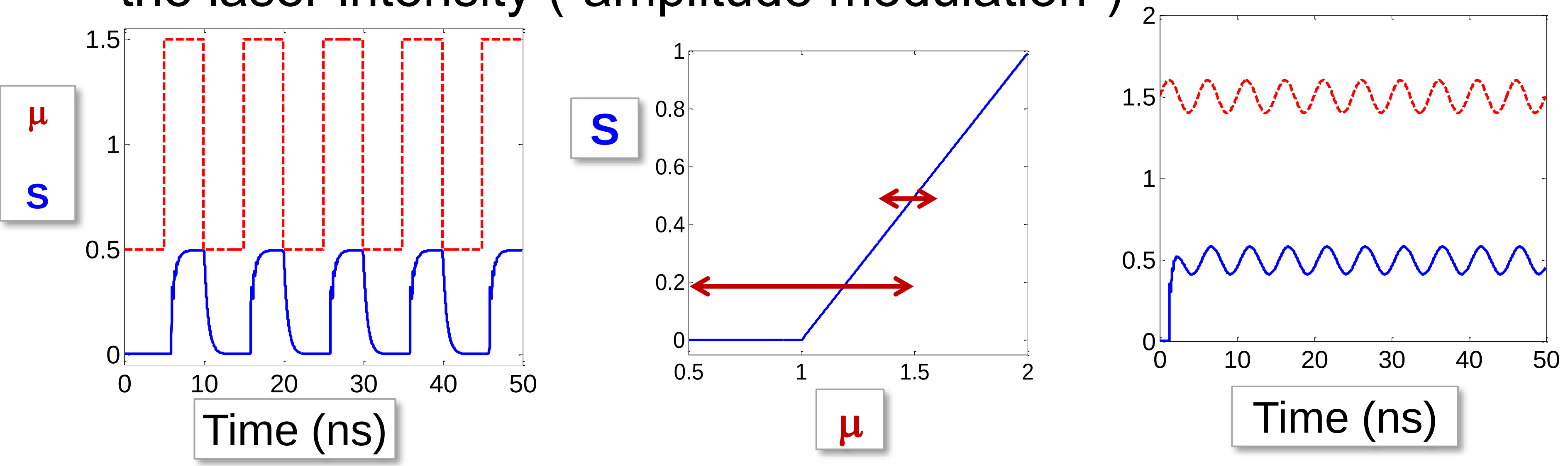


J. Tredicce et al, Am. J. Phys. 72, 779 (2004);
M. Marconi et al, Phys. Rev. Lett. 125, 134102 (2020)

The intensity of a laser can be modulated by modulating the laser current ("direct modulation").



Current modulation allows to encode information in the laser intensity ("amplitude modulation")



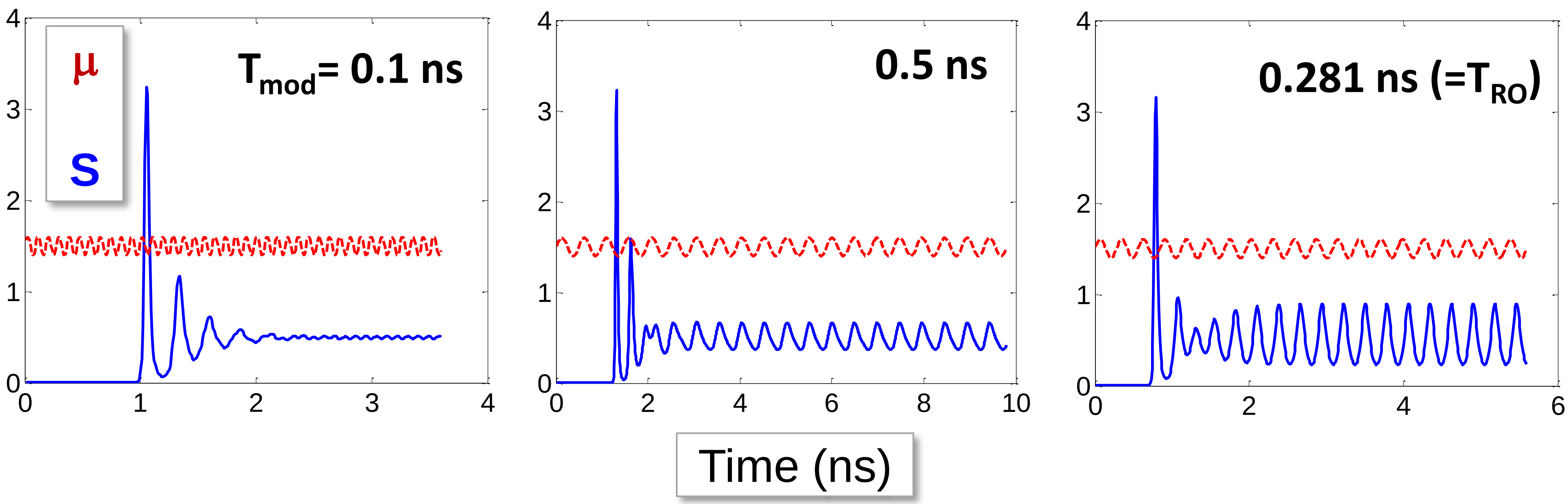
Small-amplitude sinusoidal modulation: role of the modulation frequency

$\mu = \mu_{dc} + A \sin \omega_{mod} t$

$\mu_{dc} = 1.5$, $A=0.1$

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

For $\mu=1.5$: $\nu_{RO} = 3.56$ GHz

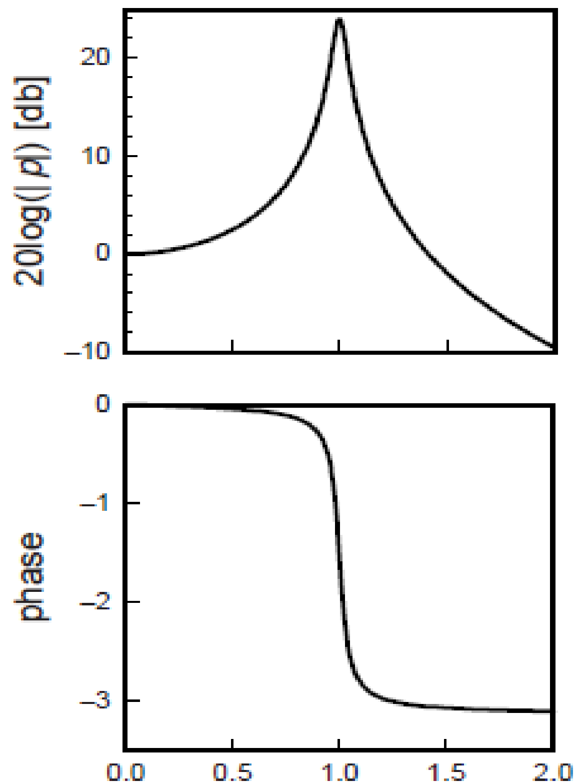


The laser intensity (**S**) oscillates at the same frequency of the pump current (**μ**), but the current and the intensity are not necessarily in phase.

Small-signal modulation response

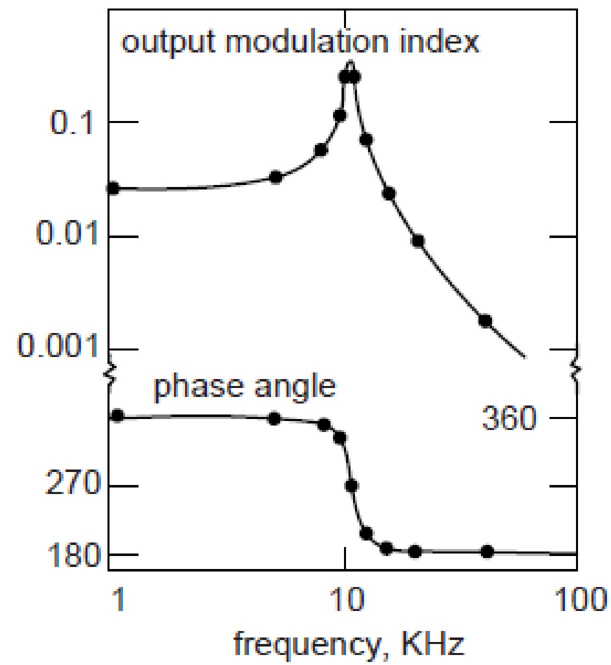
Oscillation amplitude

Analytical expressions can be calculated from the linearization of the rate equations.



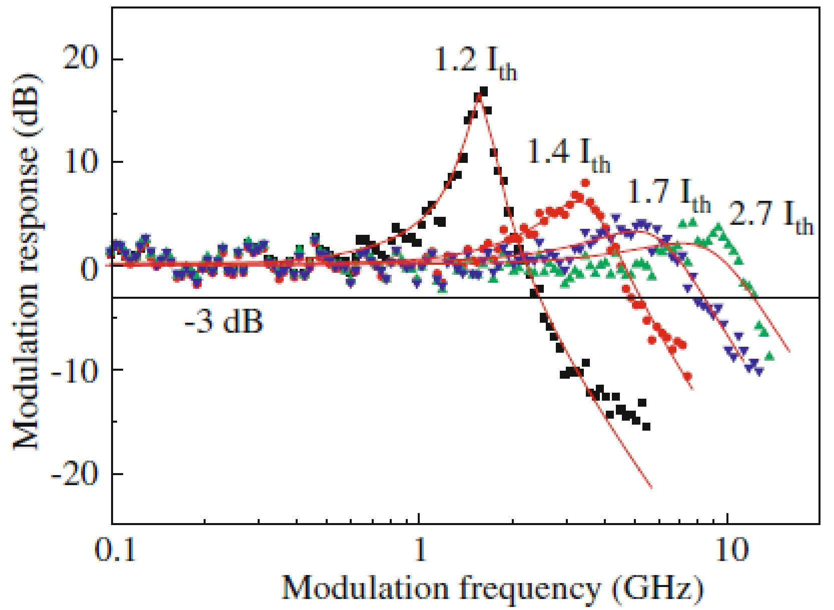
$\omega_{mod}/\ \omega_{RO}$

diode-pumped Nd^{3+}:YAG laser



Modulation frequency (KHz)

The modulation response has a peak at $\omega_{mod} = \omega_{RO}$



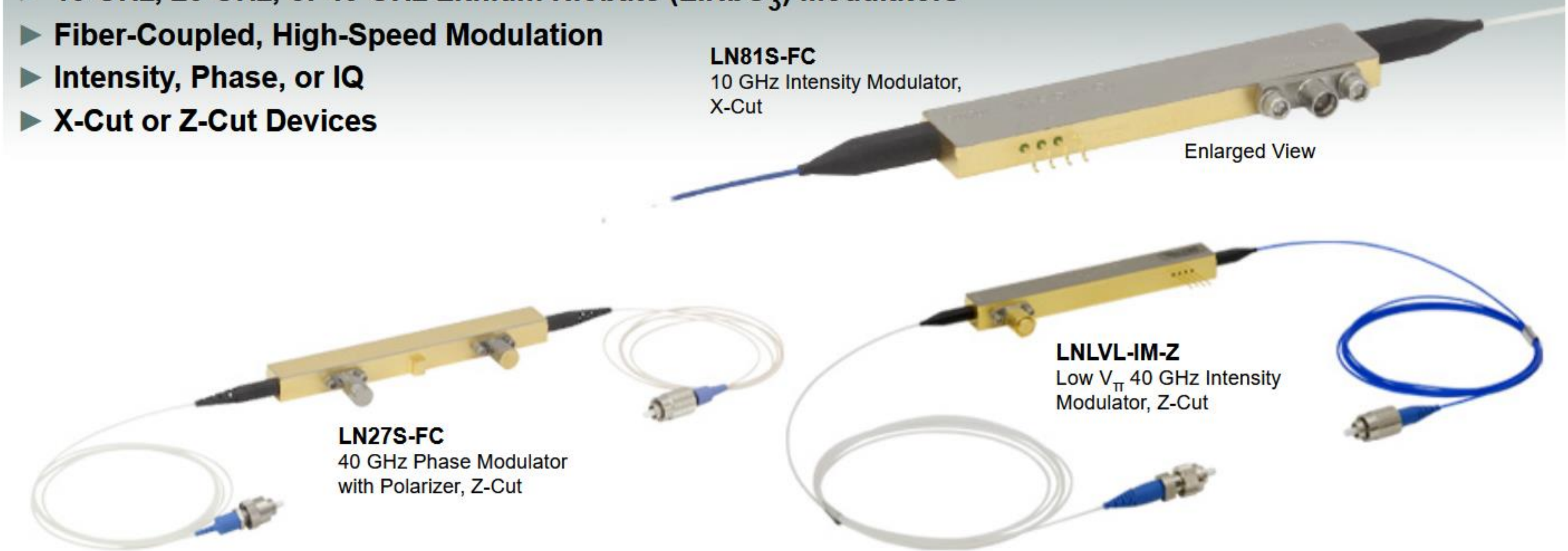
$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

- This peak limits the speed at which the laser intensity can be modulated "directly" up to ≈10 GHz.
- Which is way too slow to meet present requirements of high-speed optical telecommunication systems.
- Solution? The cw laser intensity is modulated with an external electro-optic modulator.

R. Michalzik Ed., VCSELs (Springer 2013).

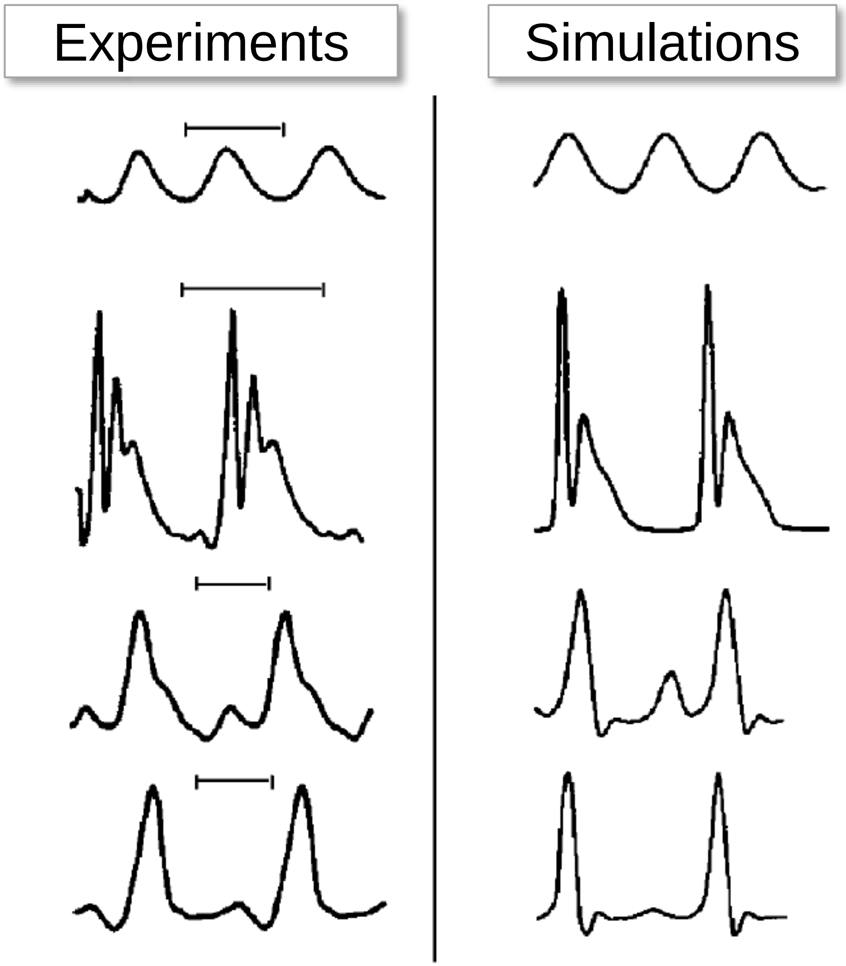
Lithium Niobate Electro-Optic Modulators, Fiber-Coupled

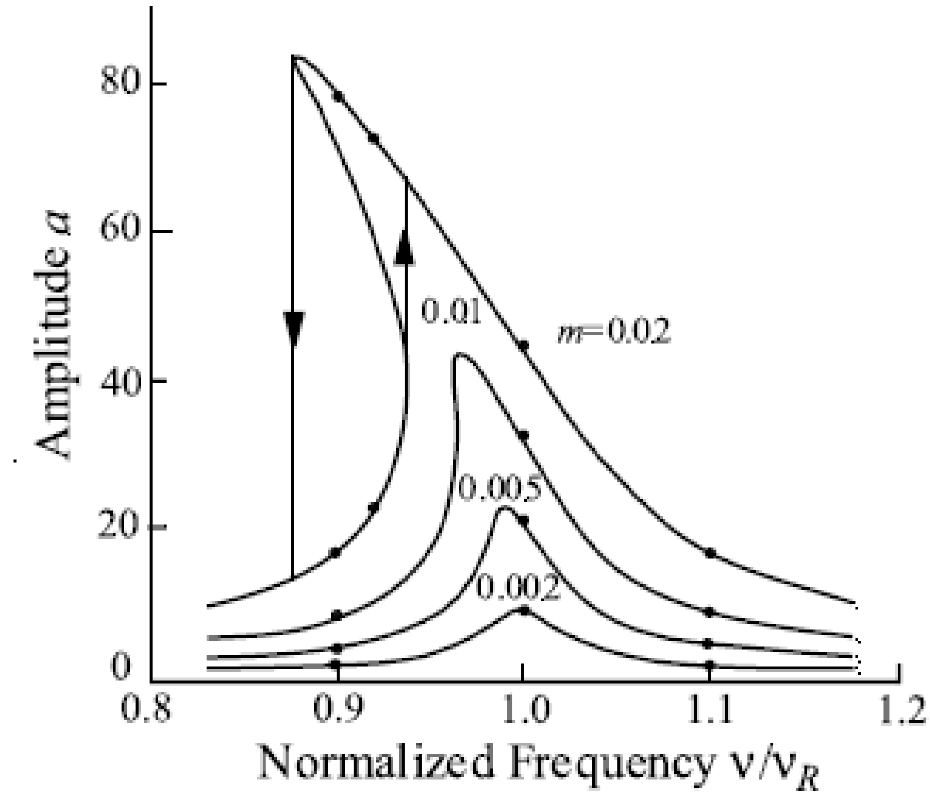
- 10 GHz, 20 GHz, or 40 GHz Lithium Niobate ($LiNbO_3$) Modulators
- Fiber-Coupled, High-Speed Modulation
- Intensity, Phase, or IQ
- X-Cut or Z-Cut Devices



Source: Thorlabs, wikipedia

With large-signal current response: complex intensity dynamics, including bistability and non-sinusoidal oscillations





$$\mu(t) = \mu_0\left[1 + m\sin(\omega_{\text{mod}}t)\right]$$

J. Ohtsubo, Semiconductor lasers (Springer 2013)

Summary

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S + \frac{\beta_{sp} N}{\tau_N}$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2\right)$$

These two ordinary differential equations for the photon and carrier densities allow to understand the laser dynamics when the injection current varies in time:

- The turn on delay followed by relaxation oscillations
- The LI curve
- The modulation response

These equations use the simplest gain model: $G = G_N(N-N_o)$.

More advanced gain models are needed in order to understand multi-mode emission.

Outline

- Introduction: class A, B and C lasers
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  - Periodic modulation
- Rate equation governing the optical field of a diode laser
- Dynamical effects induced by optical perturbations
  - Optical injection
  - Optical feedback
- Rate equations governing the polarization of a VCSEL

Complex rate equation for the slowly-varying optical field, *E*, in a single-mode semiconductor laser

Photon density: $S = |\mathrm{E}|^2$

$\mathrm{E}(t) = E(t)e^{i\omega_0 t}$

$S = |E|^2$

Slowly-varying complex field: $E(t) = E_x(t) + iE_y(t)$

$$\frac{dS}{dt} = \frac{1}{\tau_p}(N-1)S + \frac{\beta_{sp}N}{\tau_N} \Rightarrow \frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E + \sqrt{\frac{\beta_{sp}N}{\tau_N}}\xi$$

α factor: an effect unique to semiconductor lasers

stochastic term represents the spontaneous emission of photons: complex, uncorrelated, Gaussian white noise.

$$\xi = \xi_x + i\xi_y$$

$$k = \frac{1}{2\tau_p}, \quad D = \frac{\beta_{sp}N_0}{\tau_N}$$

$$\frac{dE_x}{dt} = k(N-1)(E_x - \alpha E_y) + \sqrt{D}\xi_x$$

$$\frac{dE_y}{dt} = k(N-1)(\alpha E_x + E_y) + \sqrt{D}\xi_y$$

Carrier density equation: (unchanged)

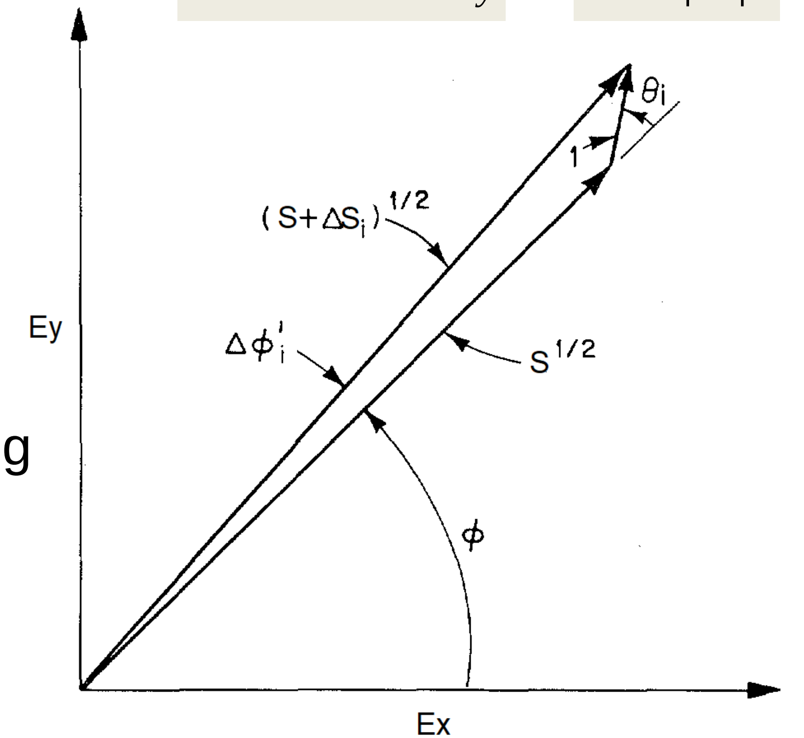
$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2\right)$$

C. H. Henry, "*Theory of the Linewidth of Semiconductor Lasers*“, IEEE J. Quantum Electron.18, 259 (1982).

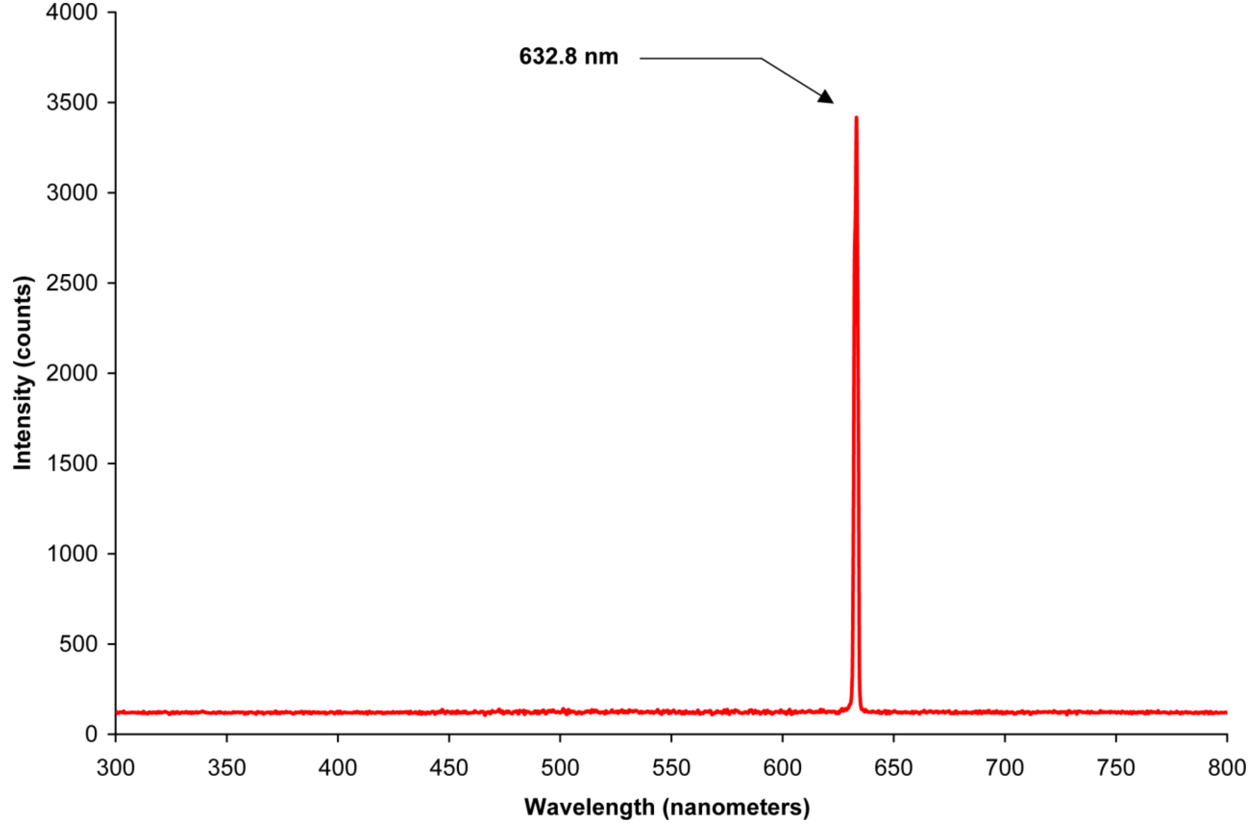
Which physics does the α -factor represent?

$$E = E_x + iE_y \qquad S = |E|^2$$

- In each round-trip, photons generated by spontaneous emission are added to the circulating field.
- These photons randomly change **the amplitude and the phase** of the field.
- Amplitude fluctuations are damped: the power returns to the steady state.
- For phase fluctuations, there is no restoring force $\Rightarrow$ the phase undergoes a random walk, which is the origin of the **finite linewidth of any laser** given by the Schawlow-Townes formula ($\Delta\nu \sim 1/P_{out}$).



- But the linewidth of a semiconductor lasers is significantly higher than the predicted by the ST formula.
- Typical semiconductor laser linewidths are in the MHz range.
- With frequency selective optical feedback (diffraction grating), it can be reduced to the kHz range.



Spectrum of a HeNe laser illustrating its very high spectral purity (limited by the measuring apparatus). The 0.002 nm bandwidth is over 10000 times narrower than the spectral width of a LED.

Reminder:

Spectral width of an LED: $\Delta\lambda \approx 1.45\ \lambda_p^2\ k_B T$ (when $k_B T$ in eV and λ_p in μm).

$\lambda_p = 1\ \mu\text{m}$ at T= 300 K (k_B=8.6 x 10^{-5} eV/K): $\Delta\lambda \approx$ 37 nm

Source: Wikipedia

Why semiconductor lasers have large linewidths?

- Due to the dependence of the refractive index (n) on the carrier density (N).
- Changes in the intensity S (with respect to steady-state value, due to spontaneous emission) produce changes in N which in turn cause changes complex susceptibility (the gain and the refractive index).
- The change of the refractive index results in an **additional** change of the phase of the field (in addition to the change $\Delta\phi$ illustrated before).
- The variation of the phase changes the instantaneous frequency:

$$E(t) = \sqrt{S(t)}e^{i\phi(t)} = \sqrt{S(t)}e^{i\omega(t)t}$$

- Summary: **amplitude-phase coupling** $\Delta S \rightarrow \Delta N \rightarrow \Delta n \rightarrow \Delta\phi \rightarrow \Delta\omega$
- Henry introduced the factor (α) to account for the extra change in the phase of the field due to the variation of the refractive index, Δn.
- The **linewidth enhancement factor α** is a very important parameter of a semiconductor laser.
- Typically for a "bulk" active region **$\alpha \approx 4.5$-6**; MQWs **$\alpha \approx 3$-4.5**; QDs **$\alpha < 2$.**

C. H. Henry, "*Theory of the Linewidth of Semiconductor Lasers*", IEEE J. Quantum Electron.18, 259 (1982).

Rate equations for the optical field and the carrier density in a semiconductor laser.

$$\frac{dE}{dt} = k(1+i\alpha)(N-1)E + \sqrt{D}\xi$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2\right)$$

$$\begin{cases} \dfrac{dE_x}{dt} = k(N-1)(E_x - \alpha E_y) + \sqrt{D}\xi_x \\ \dfrac{dE_y}{dt} = k(N-1)(\alpha E_x + E_y) + \sqrt{D}\xi_y \end{cases}$$

Typical parameters:

α= 3, τ_p = 1 ps (k=1/2τ_p);

τ_N = 1 ns, D=10^{-4} ns^{-1}

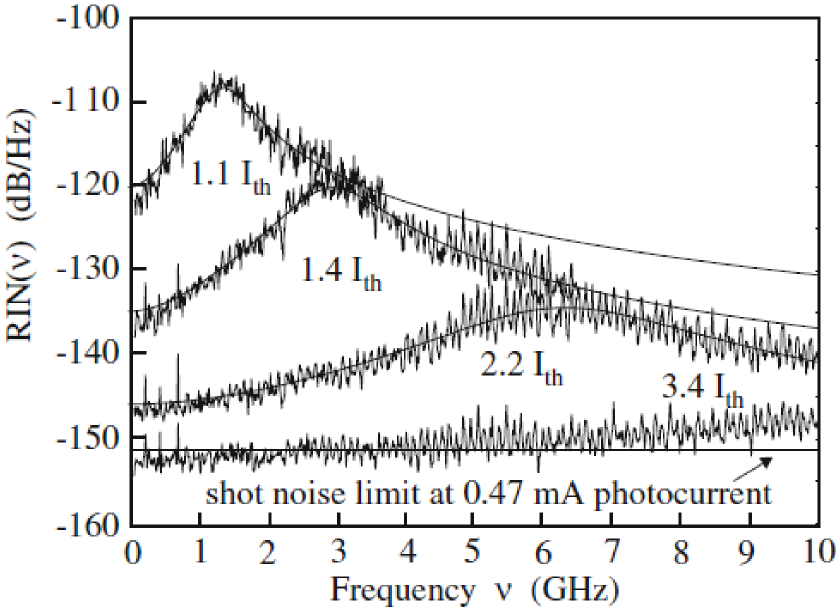
The stable steady-state solutions are the same as before:

$\mu \leq 1$ $E \approx 0$, $N = \mu$

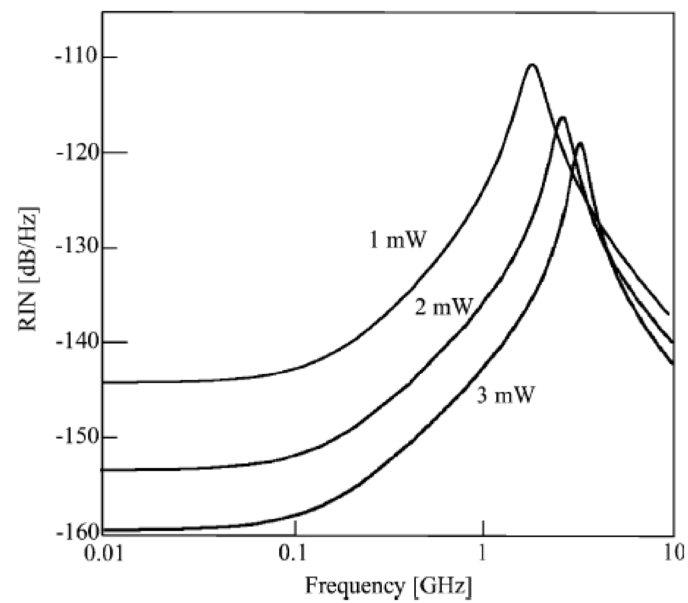
$\mu \geq 1$ $E^2 \approx \mu - 1$, $N = 1$

This model explains the Relative Intensity Noise (RIN), which is the Fourier transform of S(t)

Experimental



Numerical



The RIN has a peak at the relaxation oscillation frequency

$$\omega_{RO} = \sqrt{\frac{\mu - 1}{\tau_p \tau_N}}$$

$$\omega_{RO} = \sqrt{\frac{S}{\tau_p \tau_N}}$$

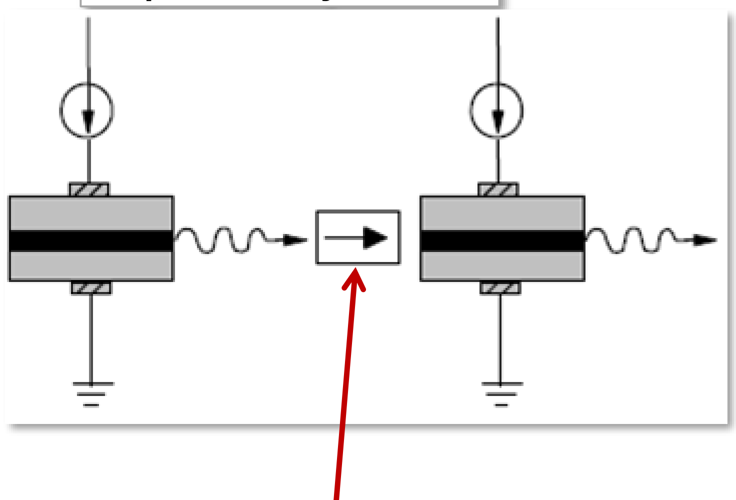
Laser on: $S = \mu - 1$

J. Ohtsubo, Semiconductor lasers (Springer 2013); R. Michalzik Ed., VCSELs (Springer 2013).

Optical perturbations

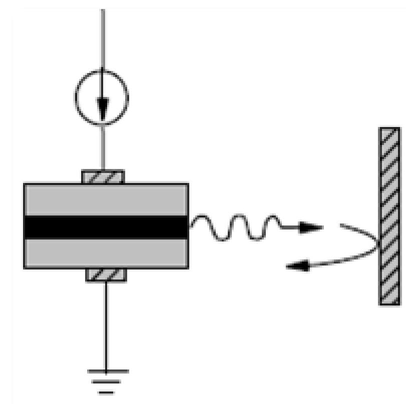
"Solitary" "free-running" semiconductor lasers emit a stable output (after turn-on transient relaxation oscillations), but they can be easily perturbed by external light and can display regular or irregular oscillations.

Optical injection

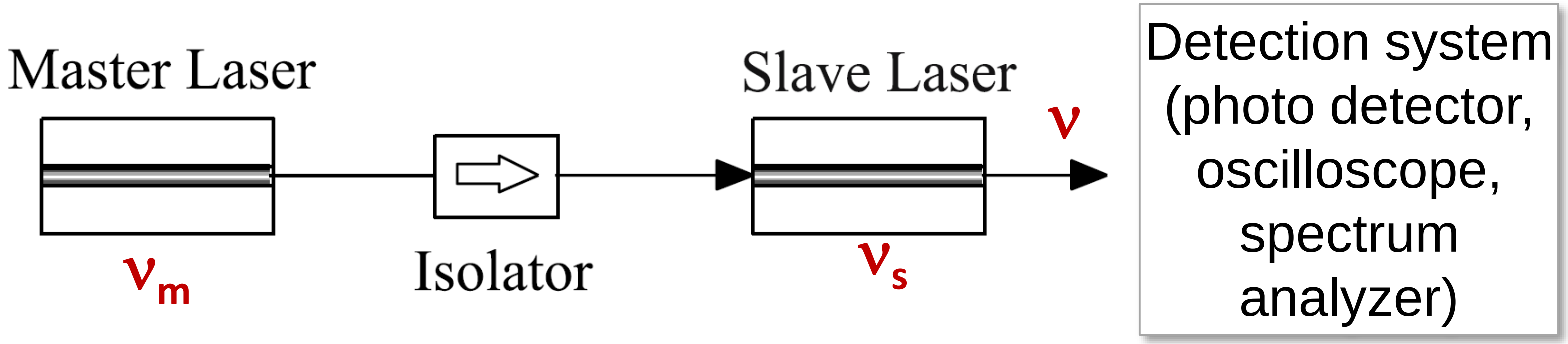


Optical isolator

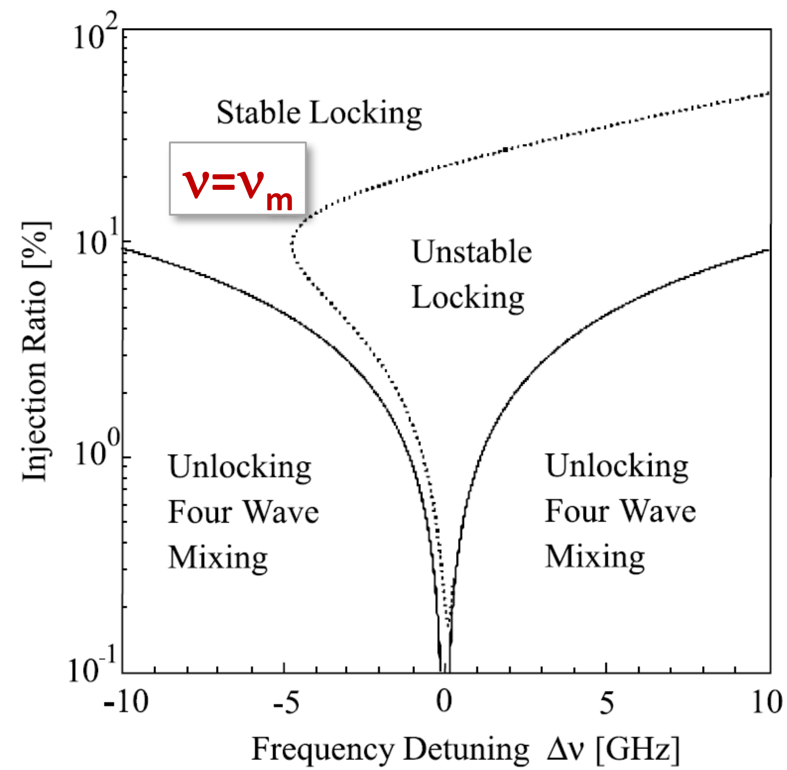
Optical feedback



Optical Injection



- Two Parameters:
  - Injection ratio
  - Frequency detuning $\Delta\nu = \nu_s - \nu_m$
- Dynamical regimes:
  - Stable locking (cw output)
  - Periodic oscillations
  - Chaos
  - Beating (no interaction)



Model for a single-mode optically injected laser

Optical field $\mathrm{E}(t) = E(t) \exp(i\omega_s t)$; $E(t)$ = slowly varying amplitude

Without injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E + \sqrt{D}\xi \qquad D = \frac{\beta_{sp} N_0}{\tau_N}$$

With injection:

$$\frac{dE}{dt} = \frac{1}{2\tau_p}(1+i\alpha)(N-1)E \quad + \underbrace{i\Delta\omega E + \sqrt{P_{\text{inj}}}}_{} + \sqrt{D}\xi(t)$$

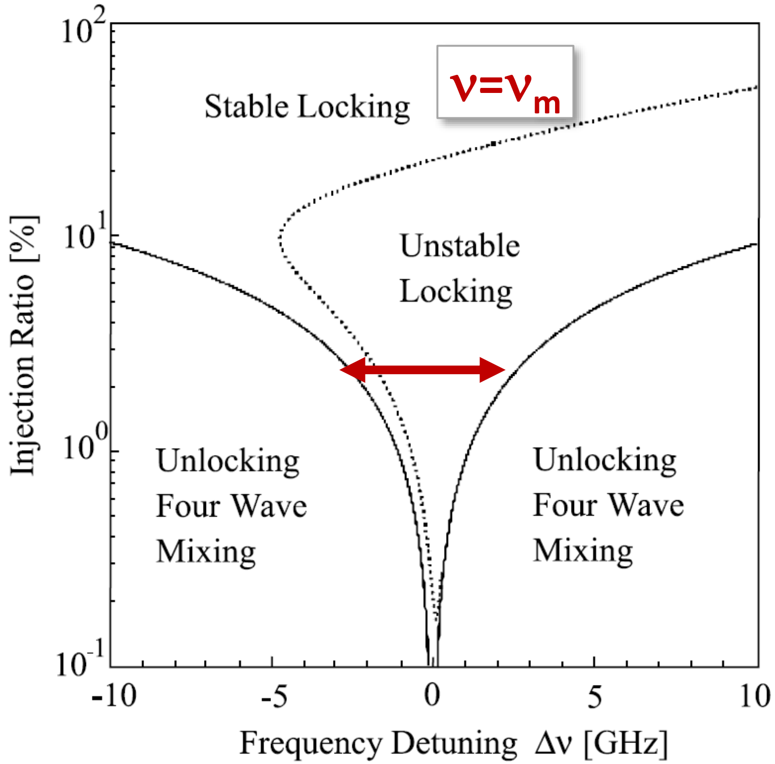
$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2\right)$$

optical injection
from master laser

P_{inj}: relative injection strength

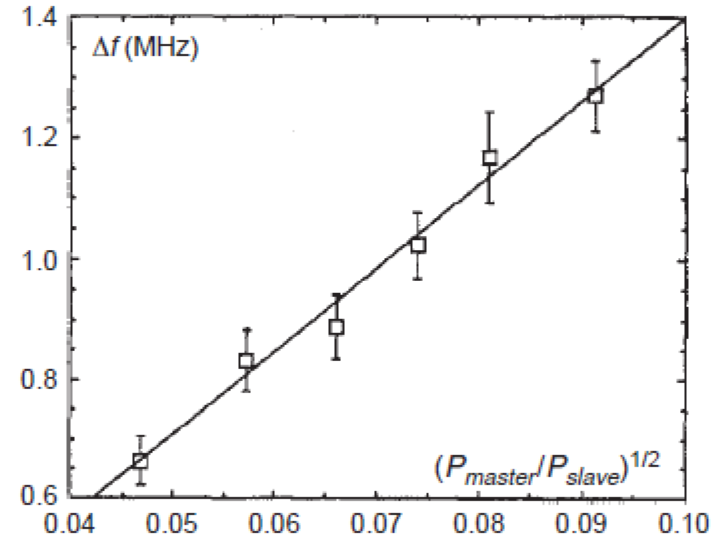
$\Delta\omega=\omega_s-\omega_m$: detuning

How wide the injection locking region is? (where the injected laser emits the same optical frequency as the master laser)



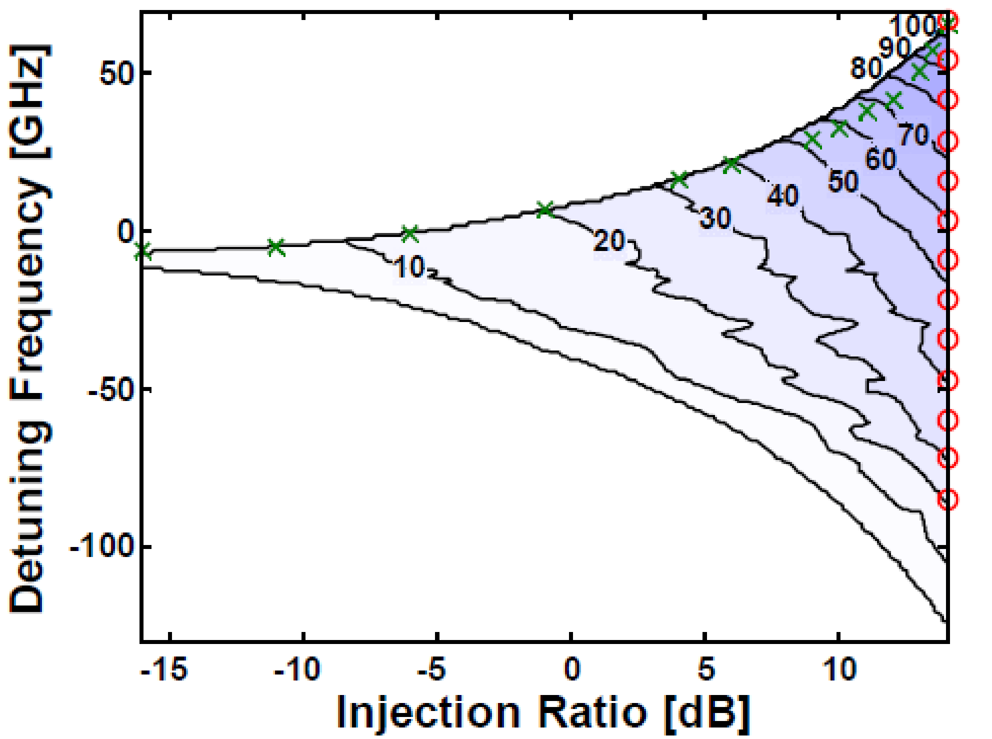
Model prediction: the locking range is proportional to the relative injection strength.

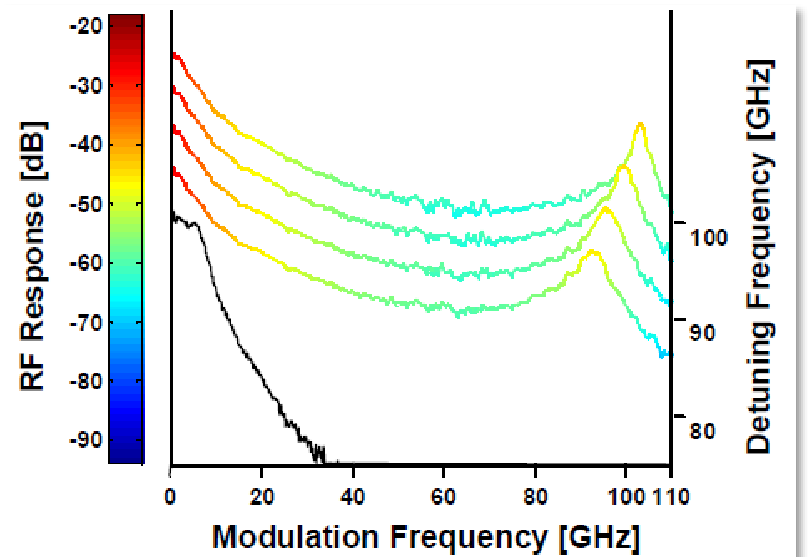
Experimental verification



Nd^{3+}:YAG laser

Injection locking increases the relaxation oscillation frequency and the modulation bandwidth



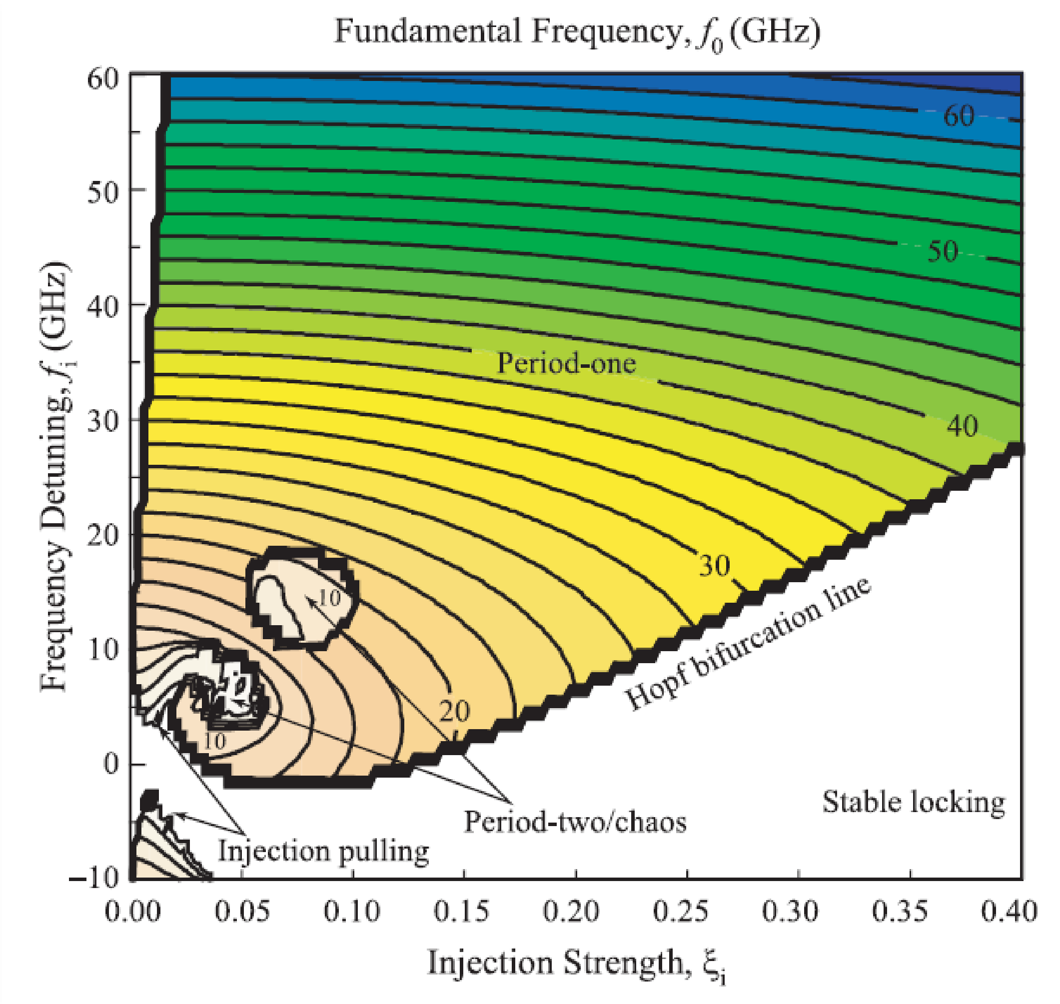


(a)

E. K. Lau et. al, Opt. Express 16, 6609 (2008).

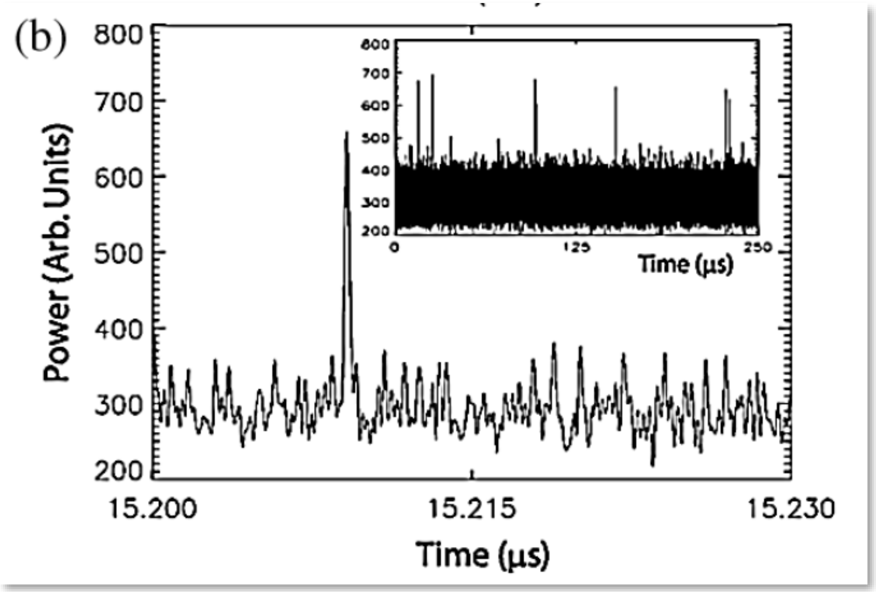
Outside the injection locking region: intensity oscillations

- Regular or irregular oscillations.
- In the region where the intensity oscillations are regular, their frequency can be controlled by tuning the injection strength and the detuning.

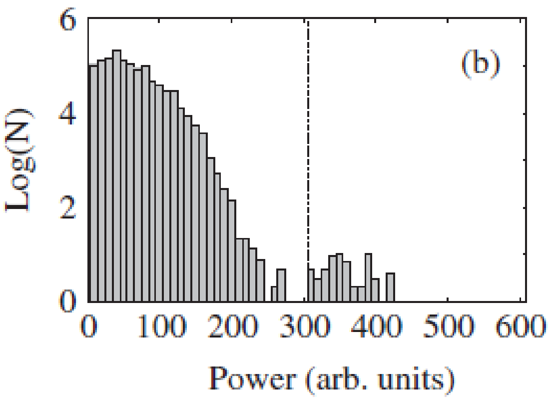


S-C Chan et. al, Optics Express 15, 14921 (2007).

In the parameter regions where the intensity oscillations are irregular, occasionally, very high intensity pulses can be emitted (known as "optical rogue waves")



How high an optical rogue wave is?
Histogram of pulse heights:



Rogue wave if the height of the optical pulse is:
$I > \langle I \rangle + 8\sigma$

- If the high pulses can be controlled (i.e., generated on demand), they can have interesting applications for biomedical imaging and sensing.
- A research line in our lab is devoted to understand the mechanisms that generate and control optical rogue waves in optically injected diode lasers.

C. Bonatto et al, PRL 107, 053901 (2011), Optics & Photonics News February 2012, Research Highlight in Nature Photonics DOI:10.1038/nphoton.2011.240

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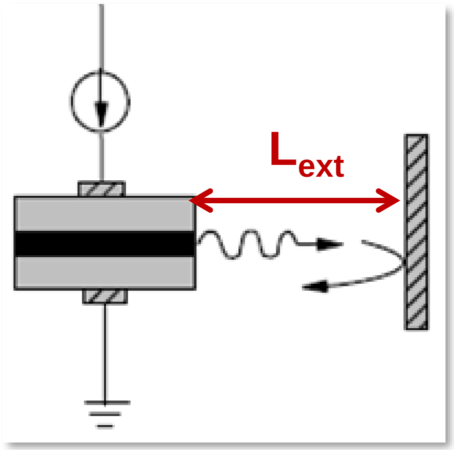
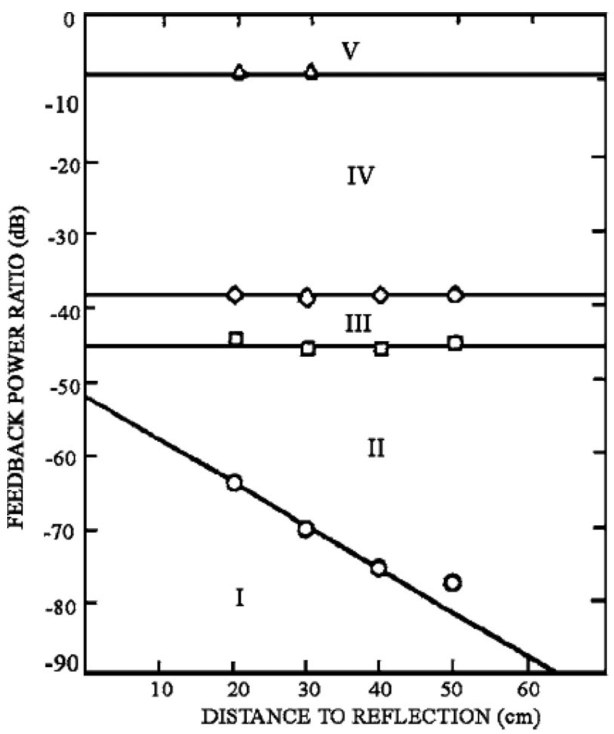
  - Optical feedback
- Rate equations governing the polarization of a VCSEL

Optical feedback

- Three Parameters:
  - Injection ratio
  - Delay time $\tau = 2L_{ext}/c$
  - Feedback phase: the accumulated phase of the returning field, $\omega\tau$
- Feedback regimes
  - **Regime I**: line narrowing or broadening depending on the feedback phase,
  - **Regime II**: mode-hopping,
  - **Regime III**: single-mode narrow-line,
  - **Regime IV**: “coherence collapse”,
  - **Regime V**: single-mode operation in an extended cavity mode (not in a mode of the laser cavity).

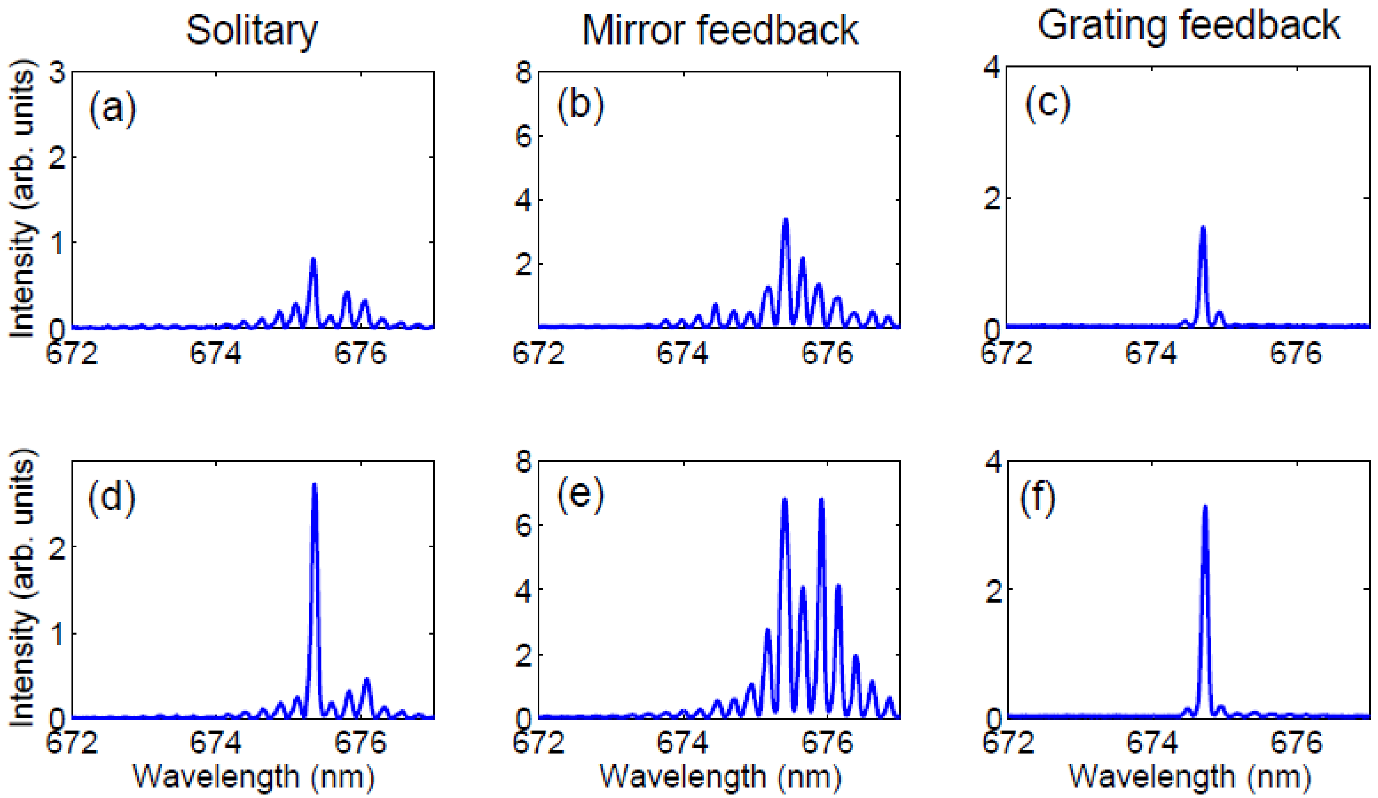


Tkach and Chraplyvy diagram



R. W. Tkach and A. R. Chraplyvy, IEEE J. Lightw. Technol. 4, 1655 (1986).

An example from our lab: effect of optical feedback in the optical spectrum of a multi-mode diode laser



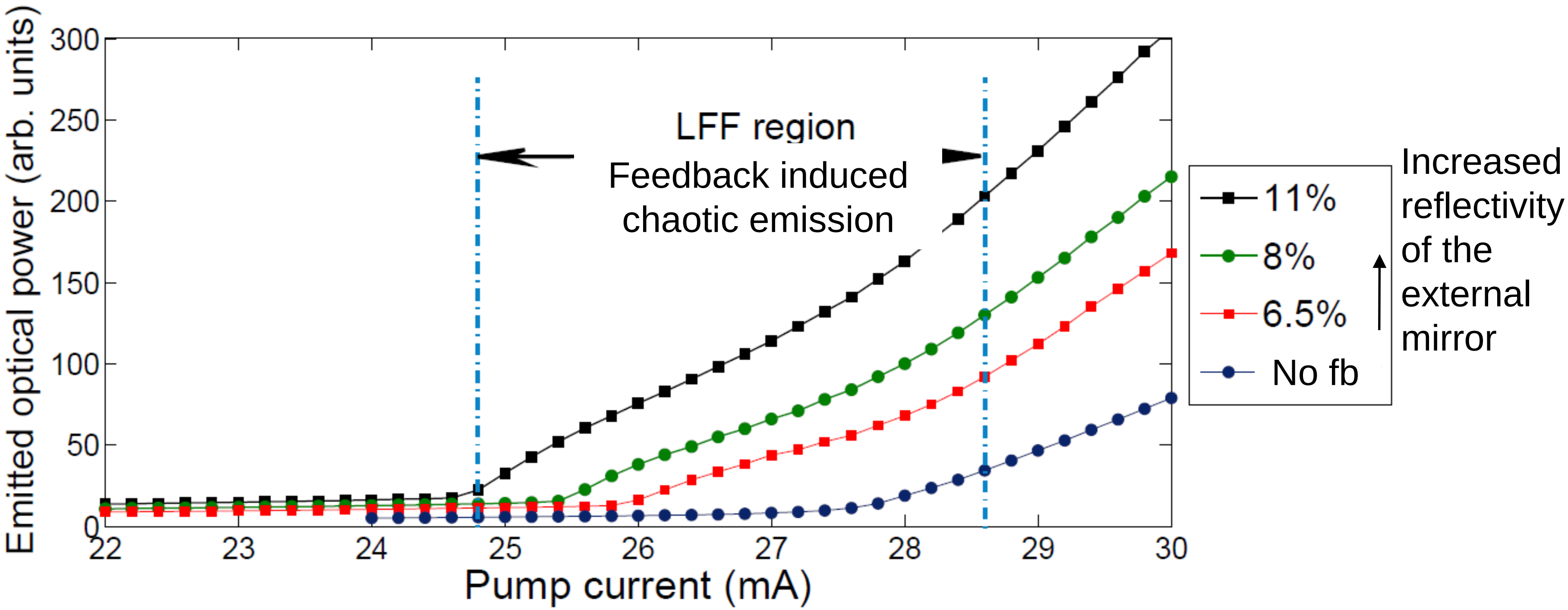
with optical feedback from a normal mirror the linewidth increases;

with frequency-selective optical feedback (from a diffraction grating) the linewidth decreases.

Figure 2.9: (a,d) Optical spectrum of a laser of a nominal wavelength of 675 nm, without feedback. (b,e) Optical spectrum of the laser with feedback from a mirror. (c,f) Optical spectrum of the laser with feedback from a difraction grating. Top row corresponds to low pump current. Bottom row corresponds to high pump current.

Source: A. Aragoneses PhD thesis (UPC 2014).

Reminder: effect of optical feedback on the LI curve



- The amount of feedback is experimentally quantified by the % of threshold reduction:

$$\text{feedback strength} = \frac{I_{th,sol} - I_{th,fb}}{I_{th,sol}} \times 100$$

Source: A. Aragoneses PhD thesis (UPC 2014).

Simplest model describing a single-mode diode laser with weak optical feedback (known as Lang and Kobayashi model)

Optical field $\mathrm{E}(t)=E(t)\exp(i\omega_0 t)$; $E(t)$="slowly varying" amplitude.

Without feedback: $$\frac{dE}{dt}=k(1+i\alpha)(N-1)E+\sqrt{D}\xi$$

With feedback:

$$\frac{dE}{dt}=k(1+i\alpha)(N-1)E+\eta\,E(t-\tau)e^{-i\omega_0\tau}+\sqrt{D}\xi$$

$$\frac{dN}{dt}=\frac{1}{\tau_N}\left(\mu-N-N|E|^2\right)$$

Single feedback reflection (multiple reflections neglected)

Control parameters:

η = feedback strength

τ = feedback delay time $\tau=\frac{2L_{ext}}{c}$

μ = pump current parameter

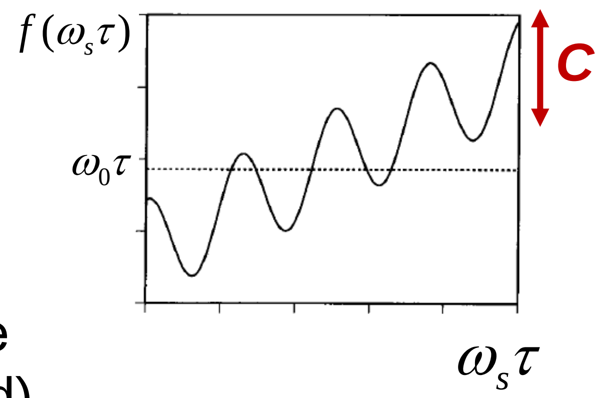
This model explains *many* of the effects induced by optical feedback.

R. Lang and K. Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980).

Steady state solutions of the Lang and Kobayashi model

$\mathrm{E}(t) = E(t) \exp(i\omega_0 t)$. Steady state solution: $\mathrm{E}(t) = E_s \exp(i\omega_s t)$, $N(t) = N_s$

$$\omega_0\tau = \omega_s\tau + C\sin\left(\omega_s\tau + \arctan\alpha\right) = f(\omega_s\tau)$$



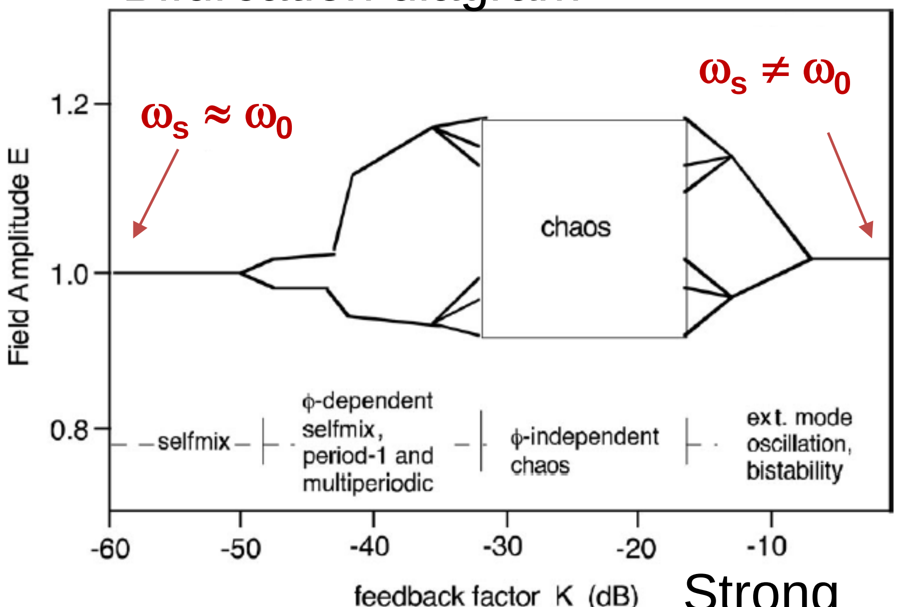
- Effective feedback strength: $C = \eta\tau\sqrt{1+\alpha^2}$
- For **C > 1**: several solutions! stable or unstable “external cavity modes” (constructive or destructive interference of the laser field and the feedback field).

$$N_s = 1 - \eta\cos(\omega_s\tau)/k$$

- Without noise: emission in the maximum gain mode (ω_s that maximizes N_s).
- With spontaneous emission noise: mode-switching.
- Very weak feedback (small η) from a distant reflector (large τ) leads to a large enhancement of the laser linewidth. This is known as “coherence collapse regime” in which the laser emits a chaotic output.
- A third important parameter determines the characteristics of the chaotic output: the pump current.

Revisiting the feedback-induced regimes

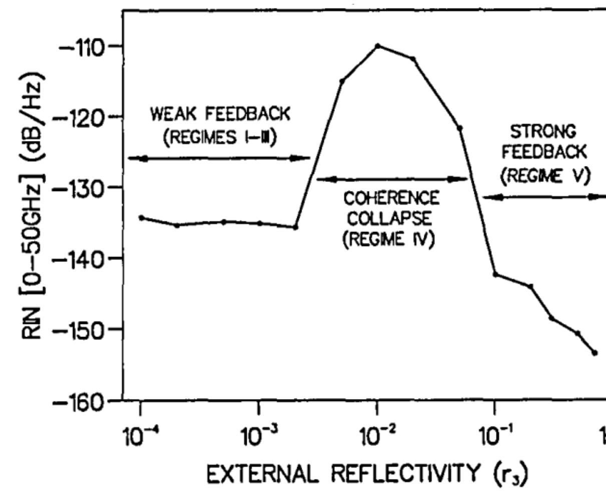
Bifurcation diagram



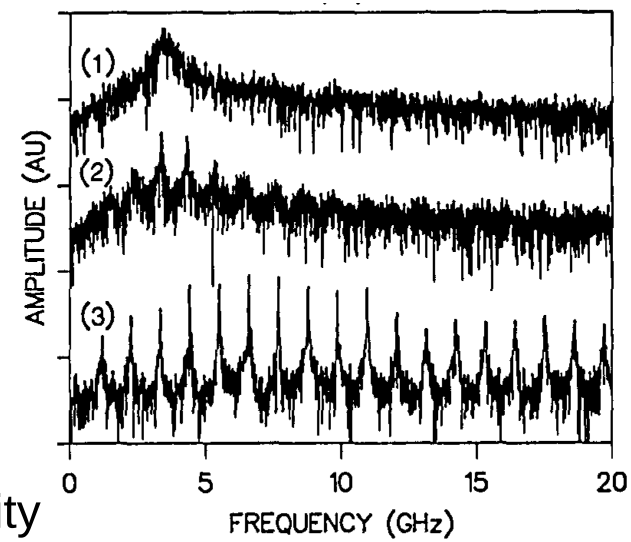
Weak feedback

Strong feedback: Stable emission in the max. gain mode

Width of the relative intensity noise (RIN)

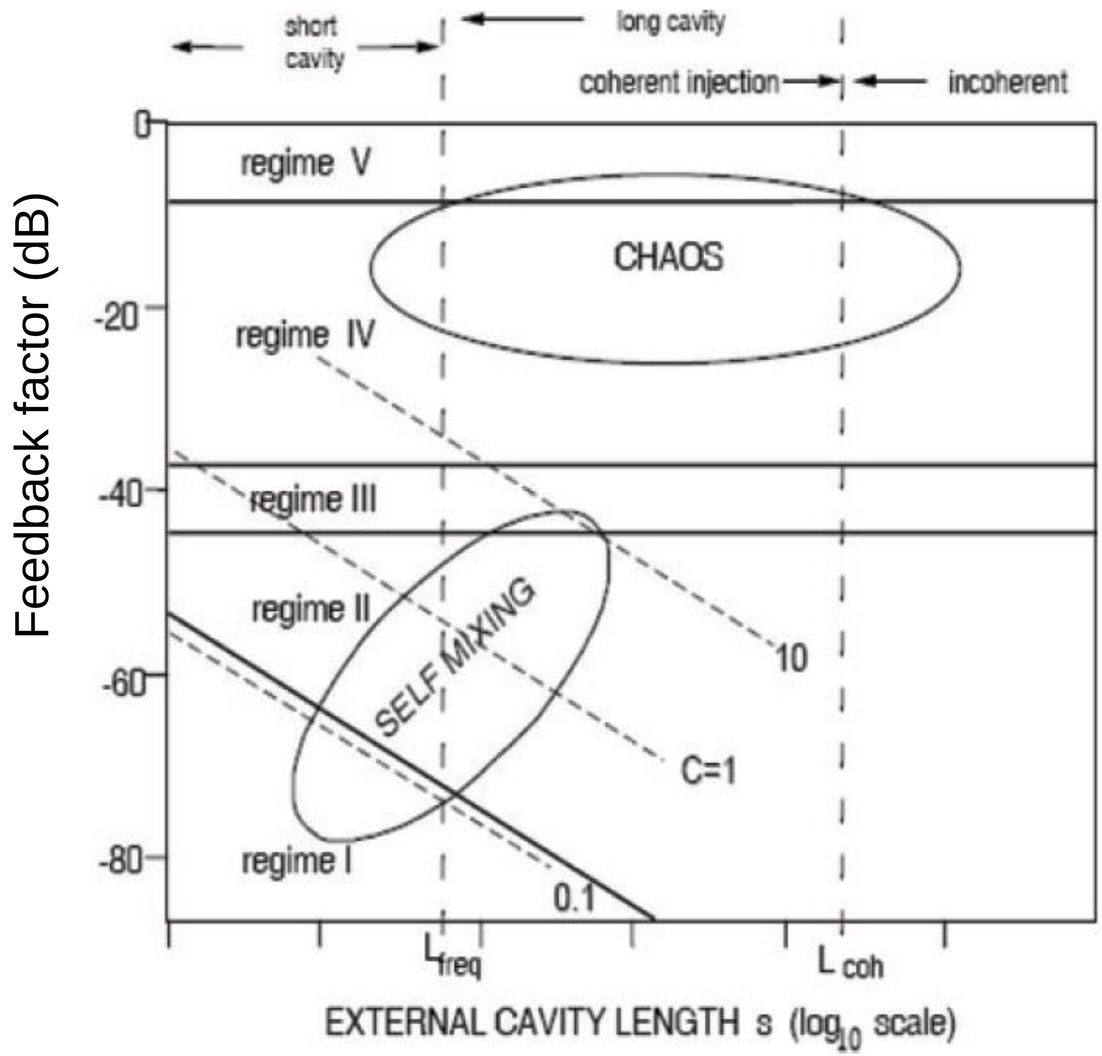


Increasing feedback strength (spectra separated vertically for clarity). Peaks at the external-cavity frequency: $1/\tau$



Langley et al, Opt. Lett. 19, 2137 (1994).
S. Donati, Laser and Photonics Rev. 6, 393 (2012).

Revisiting the feedback-induced regimes



$$C = \eta \tau \sqrt{1 + \alpha^2}$$

S. Donati and R-H Horng, J. Sel. Top. Quantum Electron. 19 1500309 (2013).

Incoherent optical feedback

- When the external cavity is long (longer than the coherence length of the laser light) the feedback light has an incoherent coupling with the light in the laser cavity.
- The situation is similar when the feedback light has a polarization that is orthogonal to the polarization of the intra-cavity field.
- In both cases, the feedback light does not interfere with the intra-cavity field, but interacts with the carriers.

$$\frac{dE}{dt} = k(1+i\alpha)(N-1)E + \sqrt{D}\xi$$

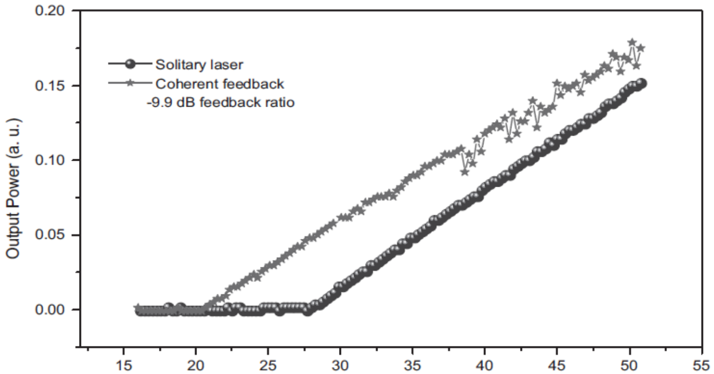
$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2 - \underbrace{N\eta|E(t-\tau)|^2}_{\text{feedback}}\right)$$

- Steady-state solution when $\mu<1$: S=0, N=μ
- Steady-state solution when $\mu>1$: N=1, S=(μ-1)/(1+η)

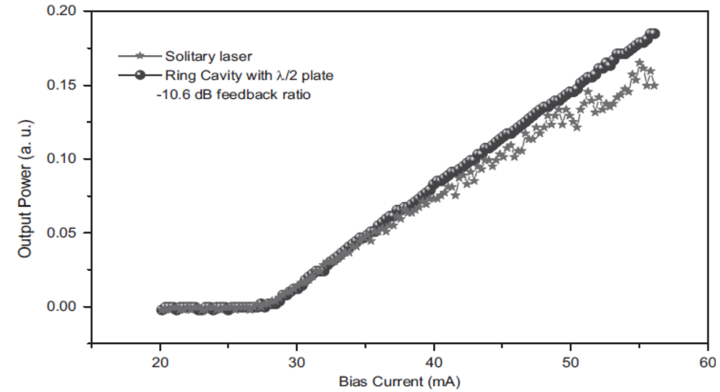
  $\Rightarrow$ incoherent feedback does not reduce the threshold but it decreases of the slope of the LI curve.
- Depending on (η, τ, μ) stable emission can become unstable and the laser intensity can display regular or irregular oscillations.

Comparison of coherent and incoherent optical feedback effects on the LI curve and intensity dynamics induced by strong polarization-rotated feedback

Coherent feedback



Incoherent feedback

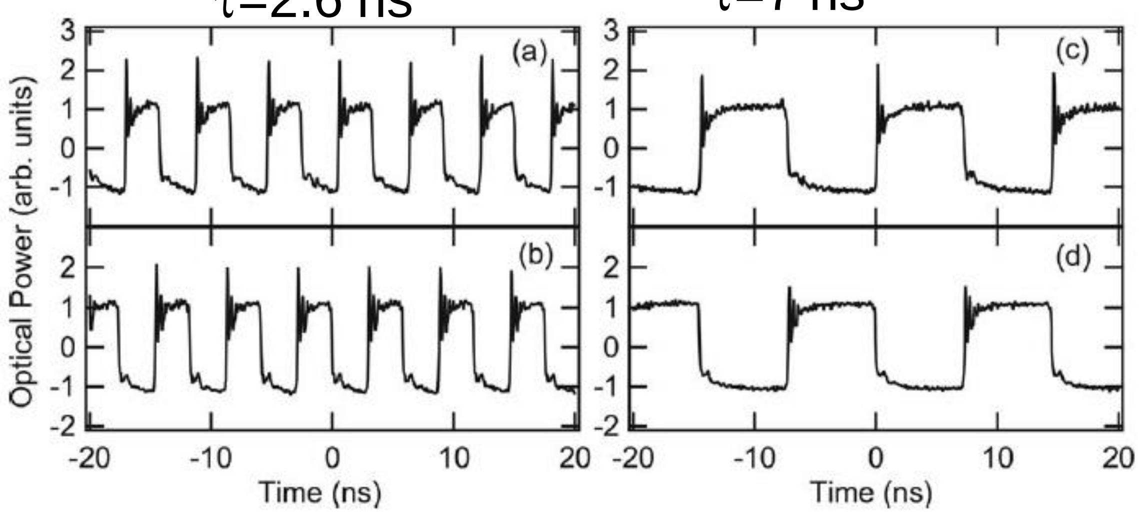


τ=2.6 ns

τ=7 ns

Horizontal polarization

Vertical polarization



R. Ju et al, IEE Proc. Optoelectron. 153, 131 (2006); Gavrielides et al, Opt. Lett. 31, 2006 (2006).

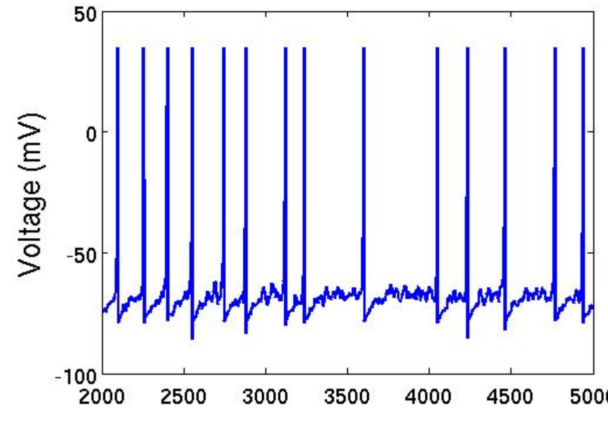
Reminder: work carried out in our lab in Terrassa

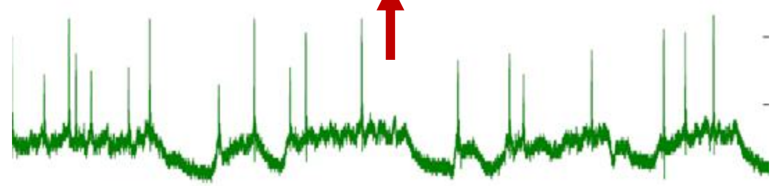
Can we build optical neurons using inexpensive diode lasers?

Spikes emitted by a diode laser with **optical feedback**

Time (μs)

Neuronal spikes (simulated and experimental)

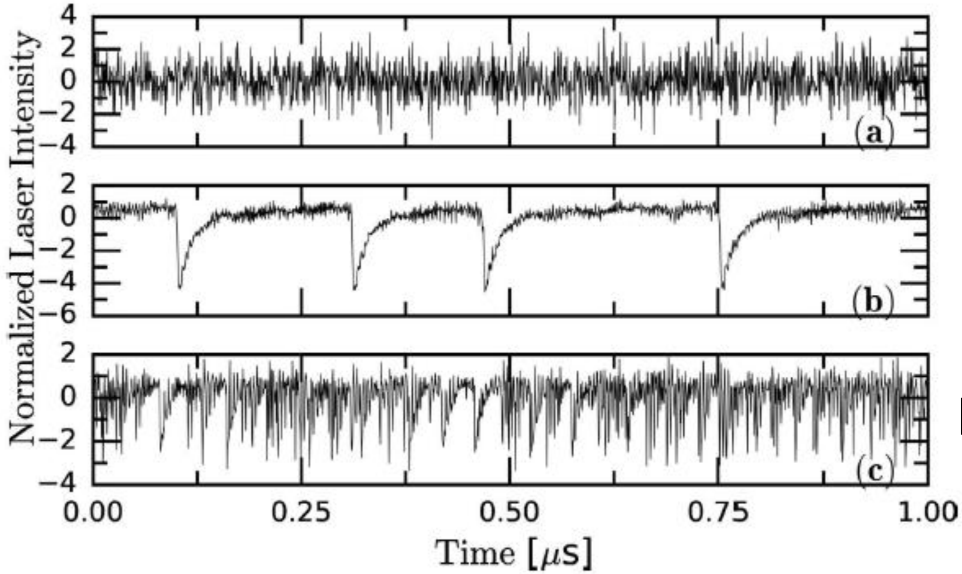




Optical spikes can be (at least) 3 orders of magnitude faster than the spikes of biological neurons! (important for photonic neuromorphic computing).

Which is the origin of the optical spikes? How do they depend on the laser's parameters?

- Near threshold the intensity fluctuations are random (spontaneous emission "noise" dominates); as the pump current increases, the optical spikes appear gradually.
- The phenomenon (known as "low-frequency fluctuations") occurs, near threshold, in EELs (single-mode or multimode), VCSELs, QCLs.



I/I_{th} = 0.95

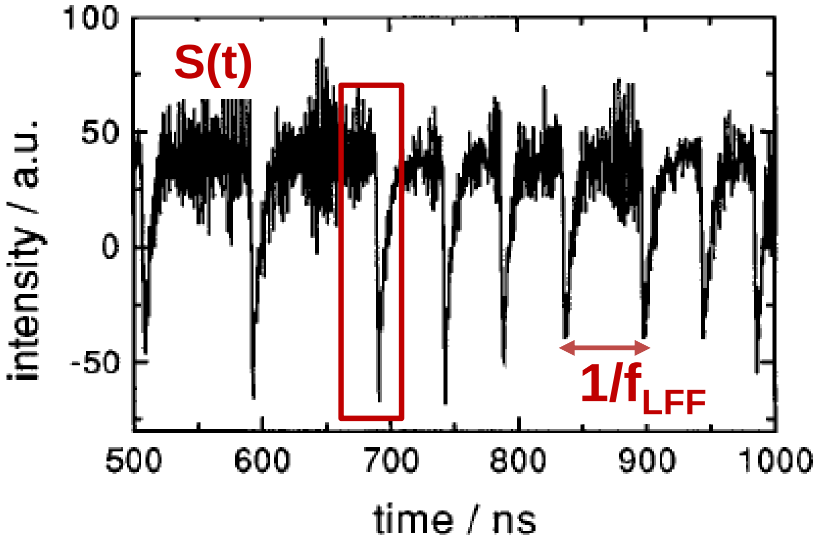
I/I_{th} = 1.02

I/I_{th} = 1.2

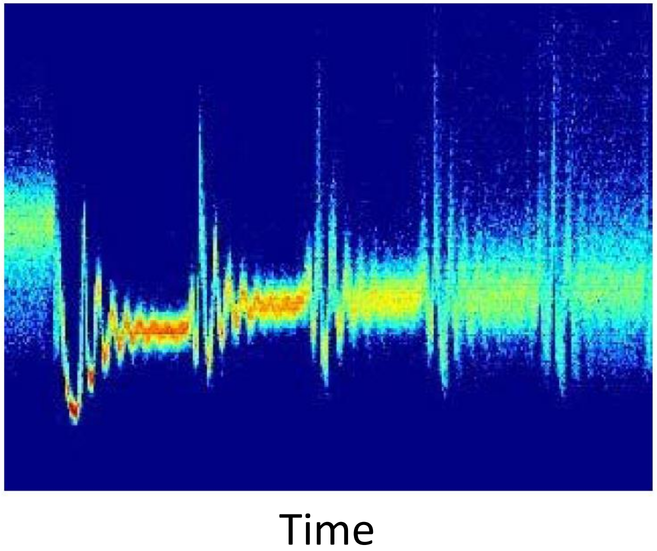
A video showing the laser intensity as the pump current increases can be found here: https://youtu.be/nltBQG_IIWQ.

C. Quintero-Quiroz et al, Sci. Rep. 6, 37510 (2016).

The low-frequency fluctuations (LFF) are oscillations of the laser intensity that have different time-scales



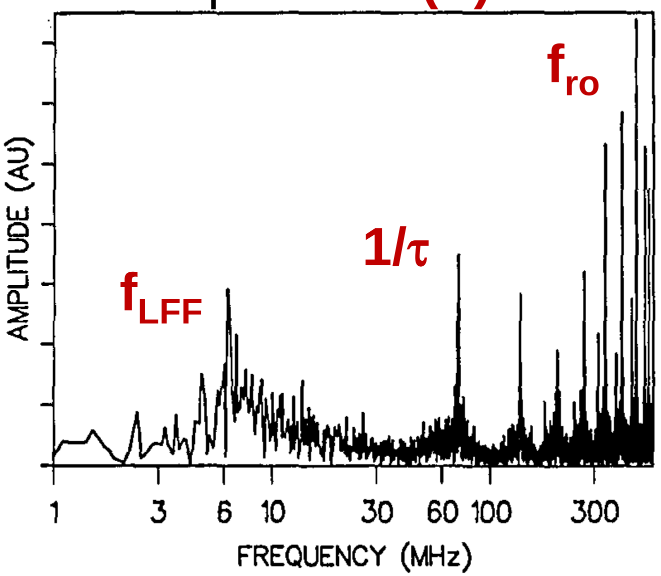
The laser intensity after a drop: steps of duration τ with relaxation oscillations



$$\tau = \frac{2L_{ext}}{c}$$

If L_{ext} = 1 m
$\Rightarrow \tau$ = 6.7 ns

RIN spectra: **S(ω)**

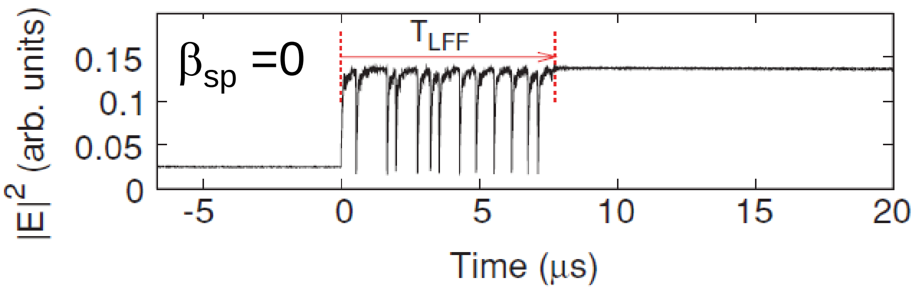


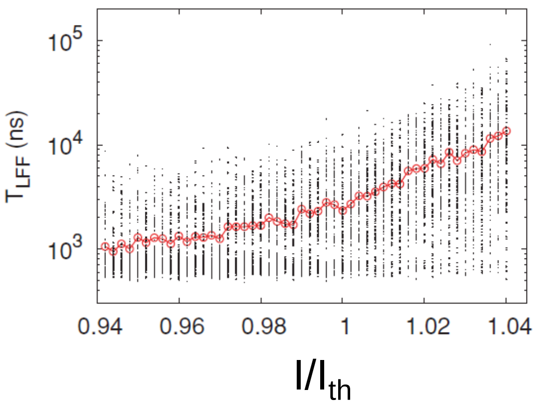
Three main frequencies:
- relaxation oscillations f_{ro}
- the “steps”: external cavity frequency $1/\tau$
- the “drops”: f_{LFF}

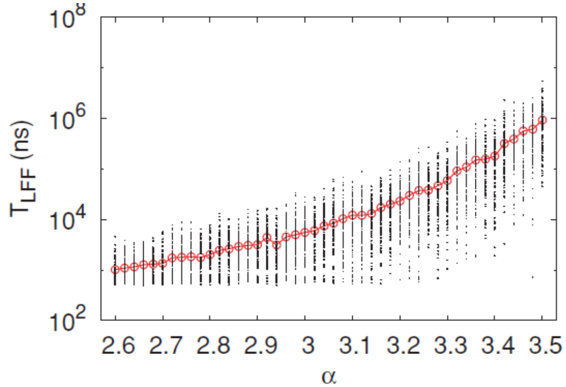
M. Sciamanna (PhD Thesis 2004).
Langley et al, Opt. Lett. 19, 2137 (1994).

Does the LK model explain the low-frequency fluctuations (LFF)? (i.e., the optical spikes emitted near the lasing threshold)

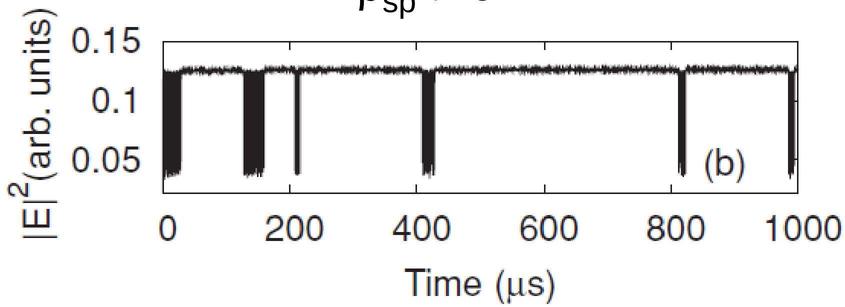
In simulations of the LK model without spontaneous emission noise the LFF is a transient phenomenon (the optical spikes die out).



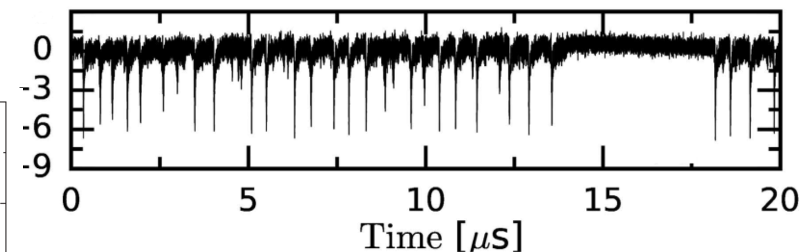




But noise “re-starts” the spikes.



Good agreement with observations

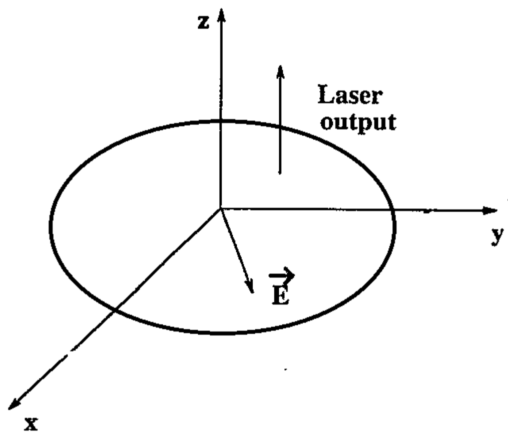


Torcini et al, Phys. Rev. A 74, 063801 (2006);
J. Zamora-Munt et al, Phys. Rev. A 81, 033820 (2010).

Outline

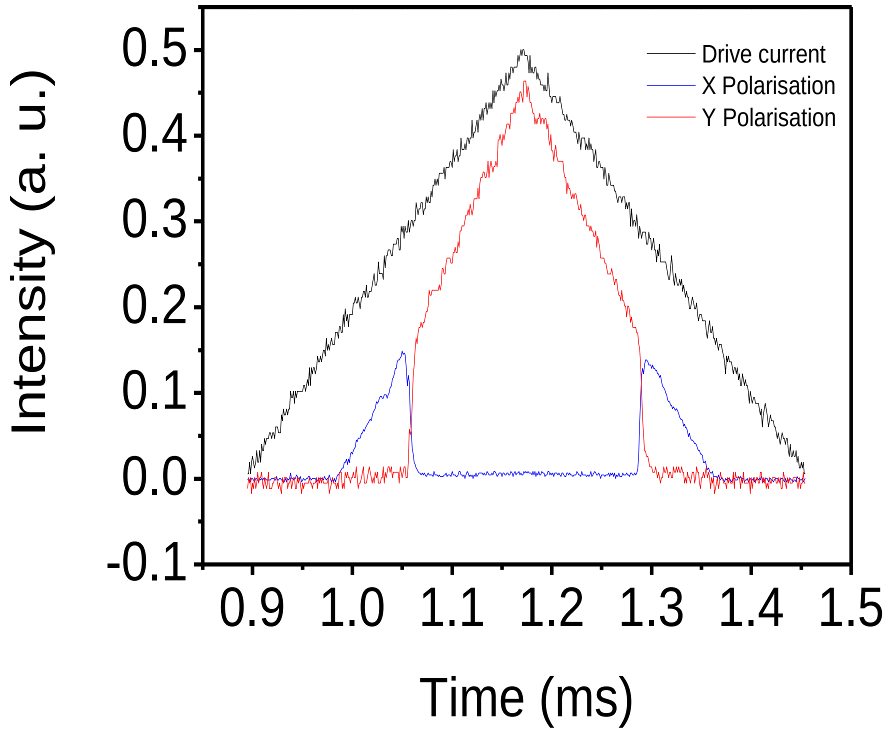
- Introduction: class A, B and C lasers
- Rate equations governing class B lasers
  - Multimode extension
- Dynamical effects induced by a time-varying pump current
  - Laser turn-on
  - Periodic modulation
- Rate equation governing the optical field of a diode laser
- Dynamical effects induced by optical perturbations
  - Optical injection
  - Optical feedback
- Rate equations governing the polarization of a VCSEL

Some VCSELs display polarization switching (PS)



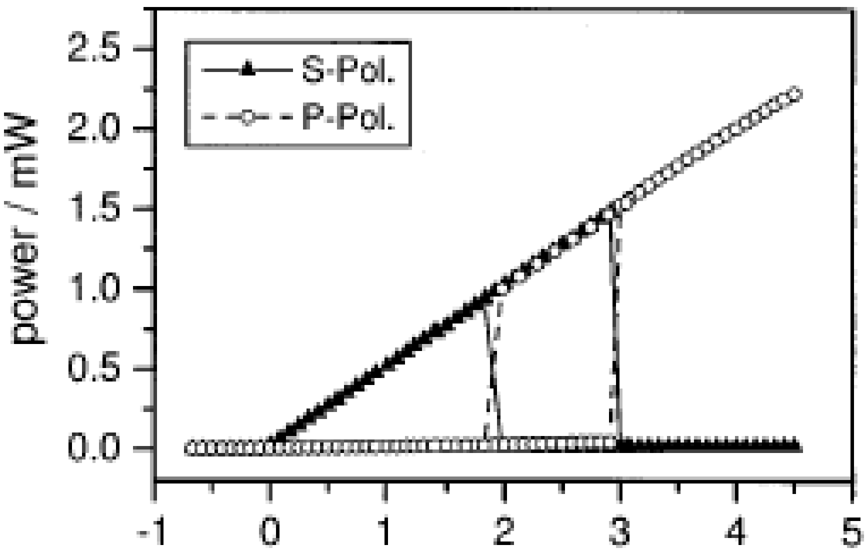
$$\mathbf{E} = [F_x(x,y,t)\hat{\boldsymbol{x}} + F_y(x,y,t)\hat{\boldsymbol{y}}]\, e^{iKz - i\nu t} + \text{c.c.},$$

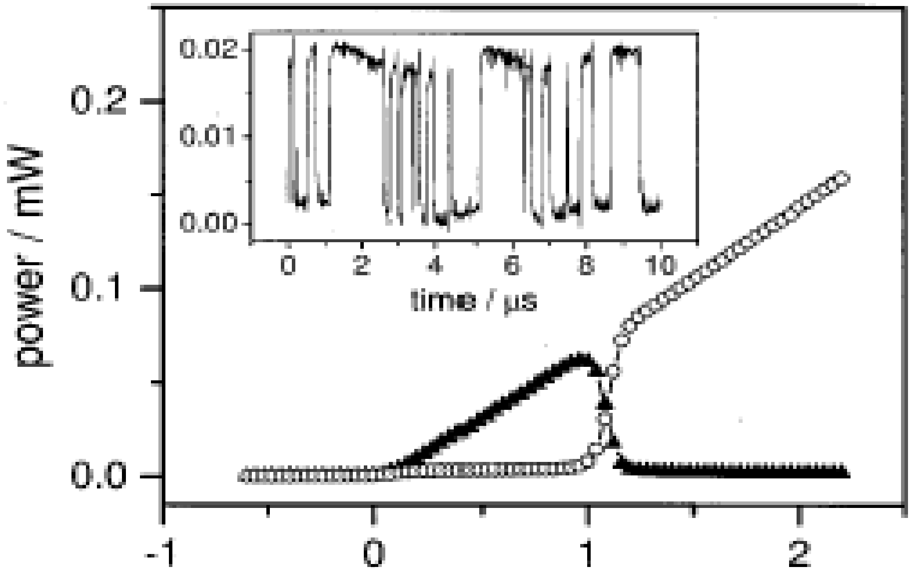
- Due to the VCSEL circular cavity geometry two linear orthogonal polarization modes have similar gain and losses (similar but not equal –due to dichroism).
- The refractive index is slightly different for the two polarization modes (birefringence).
- Often a polarization switching occurs as the pump current increases or decreases.



Source: Y. Hong, Bangor University, Wales, UK

Polarization-resolved LI curve

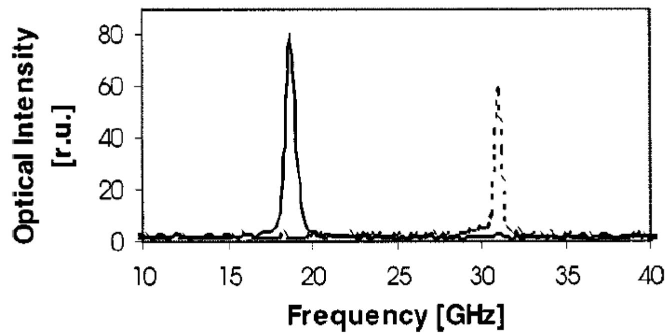




- Often **hysteresis** (the PS points for increasing and for decreasing current are different).
- The total power (**$S=S_x+S_y$**) varies monotonically with the pump current.

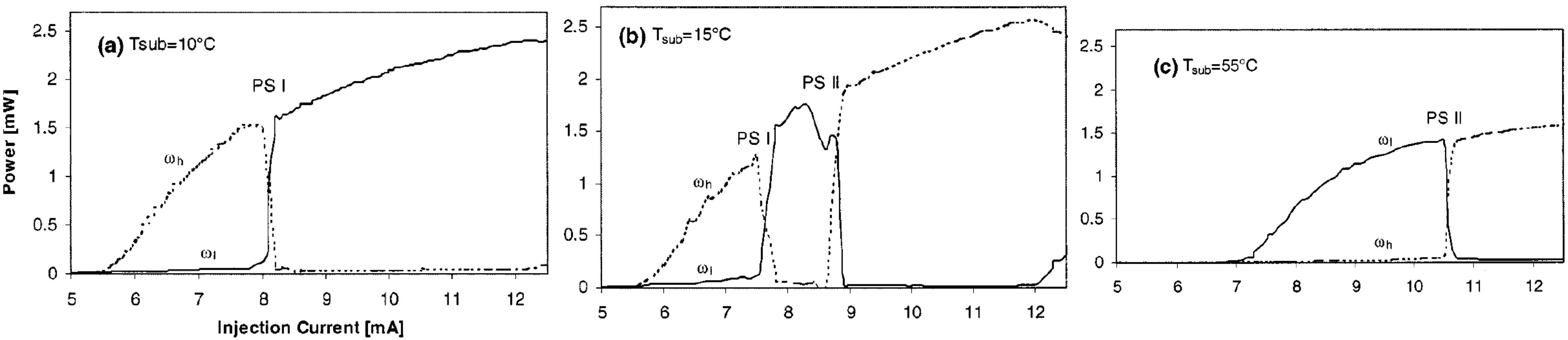
J. Kaiser et al, JOSAB 19, 672 (2002)

Current-driven PS



Birefringence: the polarizations have different optical frequencies.

PS Type I: from the high freq. (dash). to the low freq. (solid) polarization.

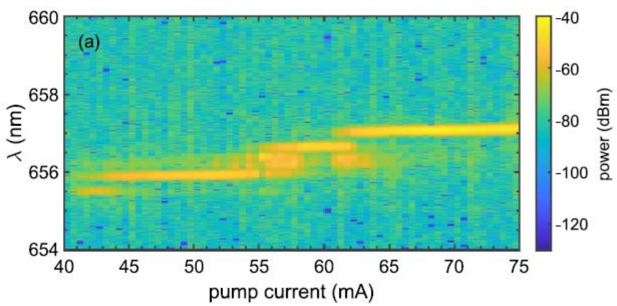


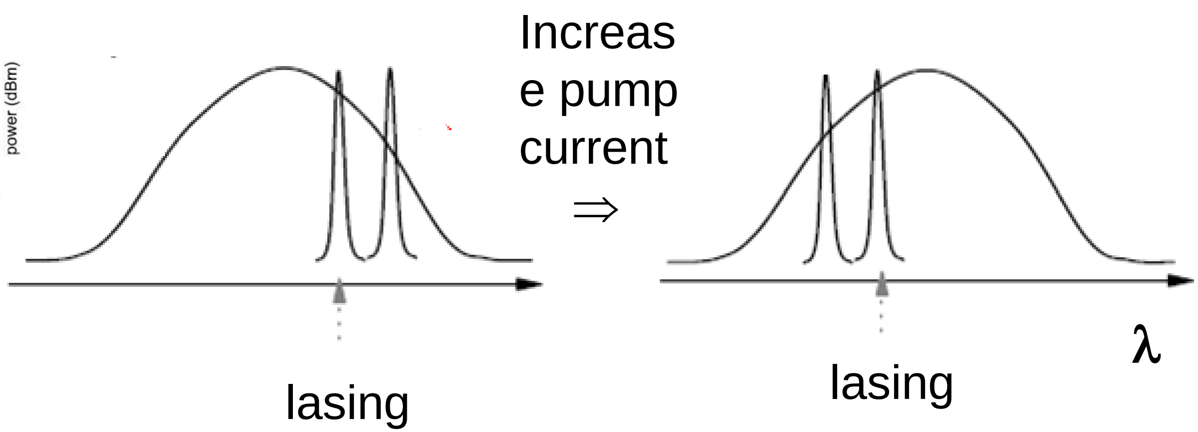
PS Type II: from the low freq. to the high freq. polarization.

Several models have been proposed to explain this behavior.

B. Ryvkin et al, J. Opt. Soc. Am. B 16, 2106 (1997).

Thermal shift of the gain curve





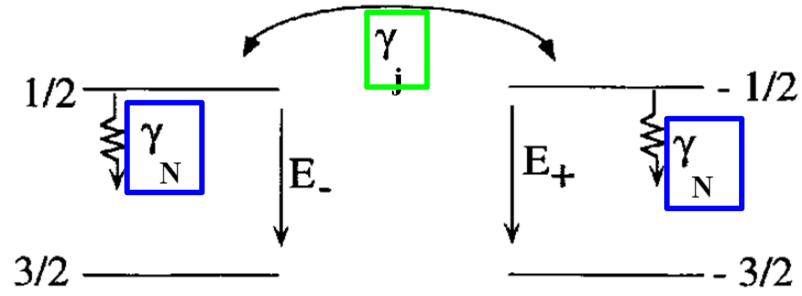
When the pump current increases ⇒ Joule heating ⇒ different thermal shift of the gain curve and of the cavity modes

- This mechanism *only* explains the high freq. → low freq. (PS type I)
- PS has been observed even when the temperature of the VCSEL active region is kept constant.

How can we explain this observation?

The spin-flip model

Two carrier populations, N_+ and N_-, represent electrons and holes with **spin up or spin down**, which recombine to generate photons with **right or left circular polarization.**



Slowly varying amplitudes of the orthogonal linearly polarized components of the optical field:

$$E_x = (E_+ + E_-)/\sqrt{2}$$

$$E_y = -i(E_+ - E_-)/\sqrt{2}$$

$$\frac{dE_\pm}{dt} = \kappa(1+i\alpha)(N_\pm - 1)E_\pm - (\gamma_a + i\gamma_p)E_\mp + \sqrt{D}\xi_\pm$$

dichroism (γ_a)

birefringence (γ_p)

$$\frac{dN_\pm}{dt} = \gamma_N\left(\mu_\pm - N_\pm - N_\pm |E_\pm|^2\right) - \gamma_j(N_\pm - N_\mp)$$

spin-flip rate (γ_j): mixes up the two carrier populations

Reminder: equations for a single-mode laser

$$\frac{dE}{dt} = k(1+i\alpha)(N-1)E + \sqrt{D}\xi$$

$$\frac{dN}{dt} = \frac{1}{\tau_N}\left(\mu - N - N|E|^2\right)$$

The model does not take into account thermal effects.

San Miguel et al, Phys. Rev. A 52, 1728 (1995);
Martin Regalado et al, IEEE J. Quantum Electron. 33, 765 (1997).

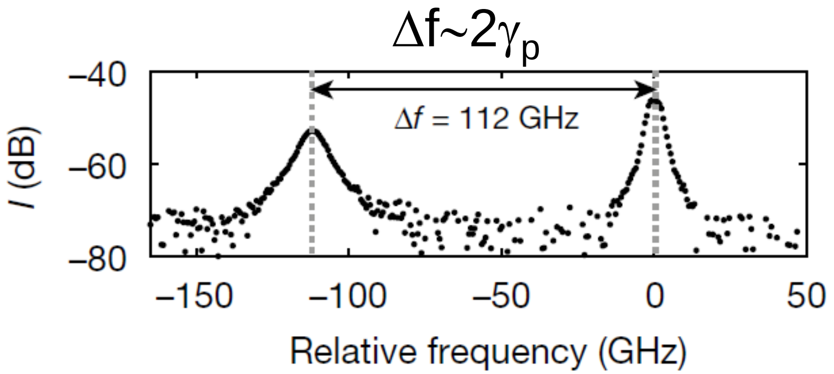
Steady state solutions of the spin-flip model

X-polarization: $E_x = \sqrt{\mu - 1}\, e^{-i\gamma_p t} \qquad E_y = 0$

Y-polarization: $E_x = 0 \quad E_y = \sqrt{\mu - 1}\, e^{i\gamma_p t}$

For both solutions: $N = N_+ + N_- = 1$

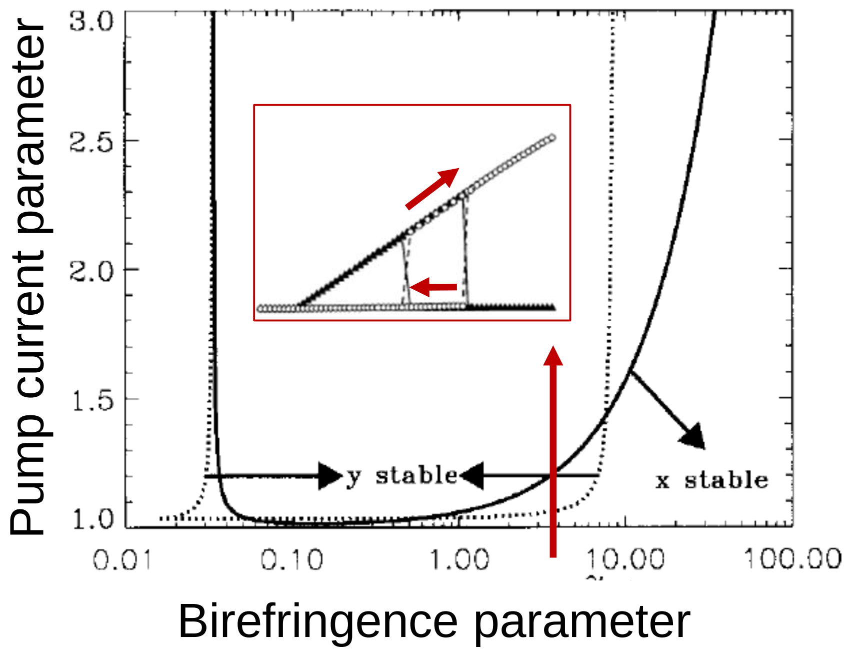
$n = N_+ - N_- = 0$



From a linear stability analysis there are three possibilities depending on the parameters:

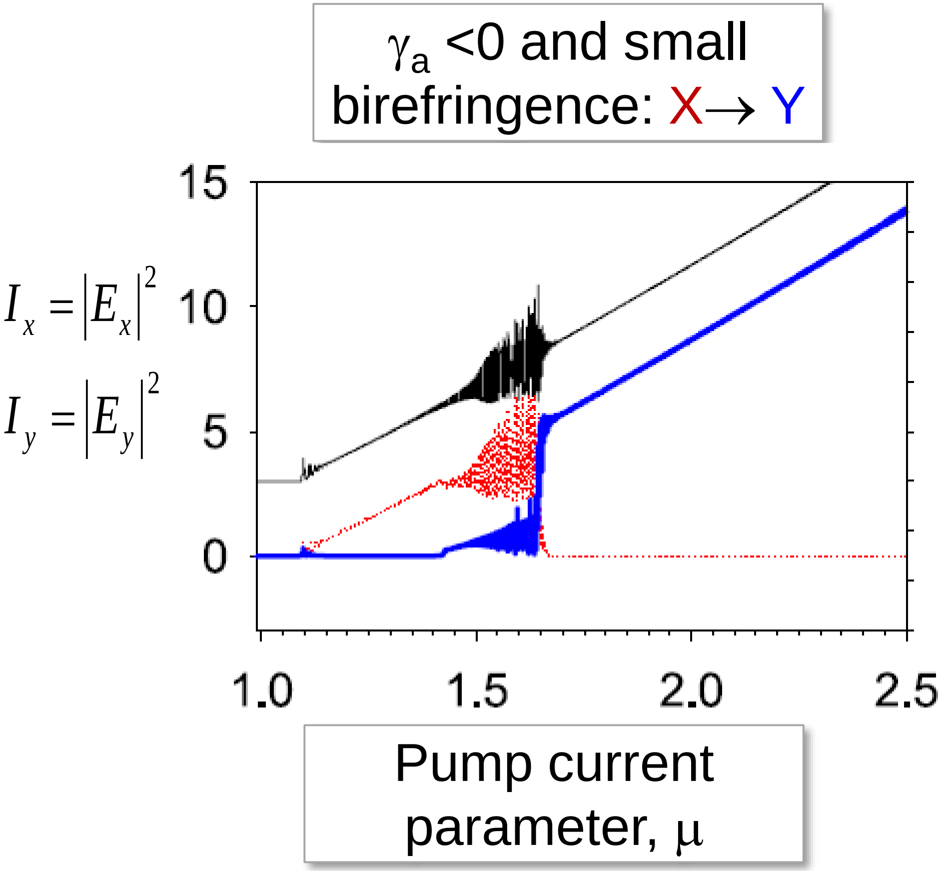
- only one solution is stable
- both solutions are stable
- none of them is stable.

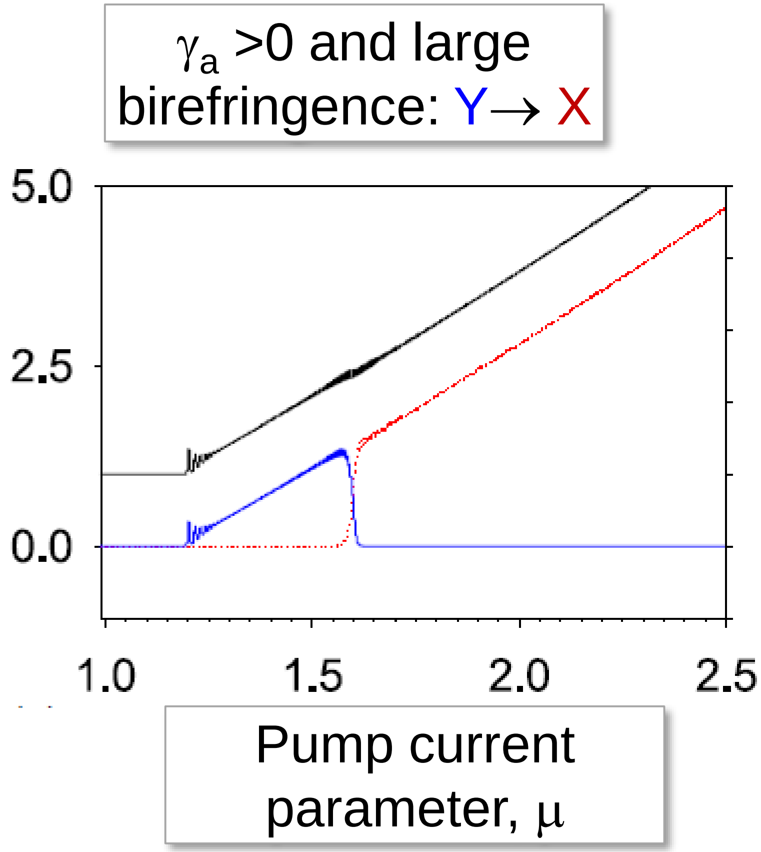
Example (follow red arrow): x is stable very close to the threshold ($\mu \approx > 1$); for higher μ both x and y are stable, and for even higher μ, only y stable.



Martin Regalado et al, IEEE J. Quantum Electron. 33, 765 (1997).
M. Lindemann et al, "Ultrafast spin-lasers", Nature 568, 212 (2019).

The spin-flip model explains two types of non-thermal polarization switching





The model also explains the stochastic PS.

Martin Regalado et al, IEEE J. Quantum Electron. 33, 765 (1997).
M. S. Torre et al, Phys. Rev. A 74, 043808 (2006).

The spin-flip model has been extended to explain the polarization dynamics of highly-birefringent VCSELs

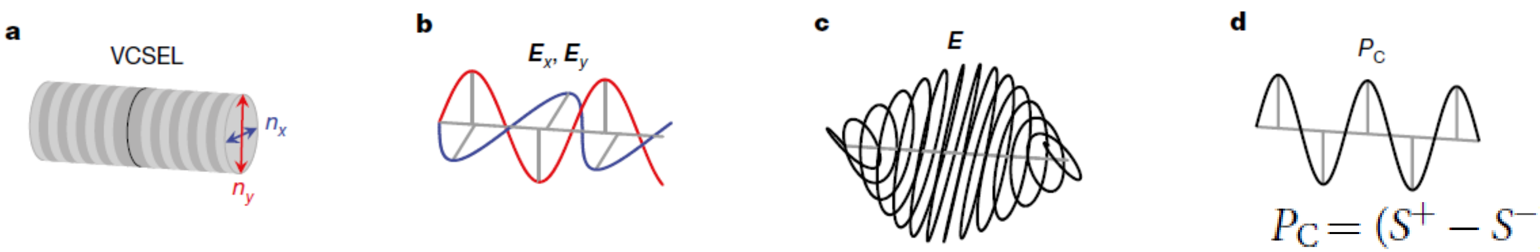
$$\dot{E}^{\pm} = [1/(2\tau_p)](1+i\alpha)(N \pm n - 1)E^{\pm} - (\gamma_a + i\gamma_p)E^{\mp} - (\epsilon_a + i\epsilon_p)\ |E^{\pm}|^2 E^{\pm} \quad (5)$$

$$\dot{N} = \gamma[J_-(t) + J_+(t)] - \gamma N - \gamma(N+n)|E^+|^2 - \gamma(N-n)|E^-|^2 \quad (6)$$

$$N = N_+ + N_-$$

$$n = N_+ - N_-$$

$$\dot{n} = \gamma[J_-(t) - J_+(t)] - \gamma_s n - \gamma(N+n)|E^+|^2 + \gamma(N-n)|E^-|^2 \quad (7)$$

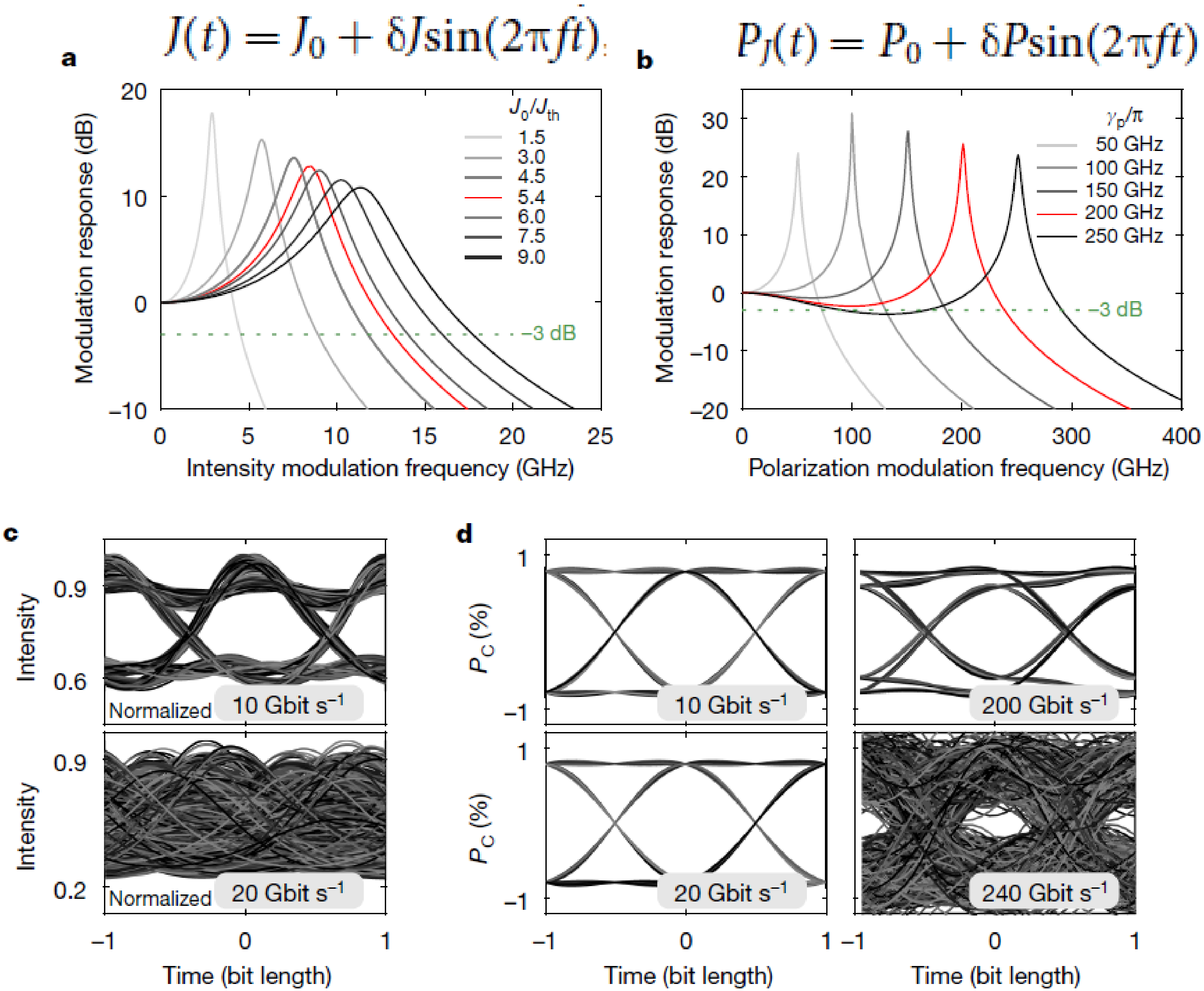


$$P_C = (S^+ - S^-)/(S^+ + S^-)$$

The polarization modes experience different refractive index (due to birefringence) are thus have different frequency.
The beat frequency between the two modes leads to a periodic evolution of the total field $\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y$, which leads to periodic polarization oscillations.

M. Lindemann et al, "Ultrafast spin-lasers", Nature 568, 212 (2019).

Comparison of intensity and polarization modulation



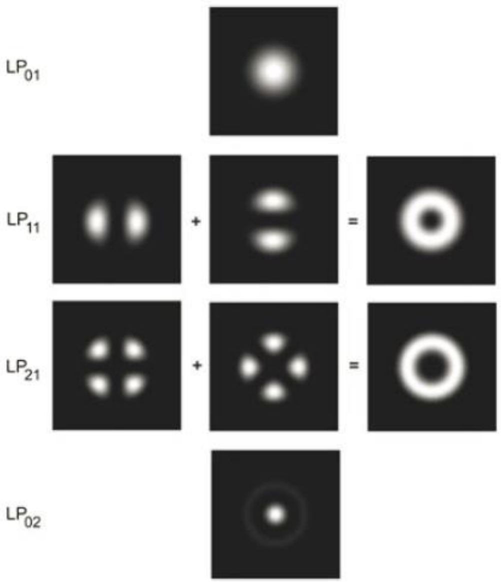
The quality of digital data transfer can be quantified with an "**eye diagram**": a binary input signal is simulated by 2^{10} pseudorandom bits and the output time traces are superimposed. A central opening allows to distinguish between '0' and '1' .

"eye" closed ☹ "eye" open ☺

M. Lindemann et al, "Ultrafast spin-lasers", Nature 568, 212 (2019).

VCSELs also display complex transverse mode behavior.

The VCSEL starts lasing on the fundamental model (LP01) but as the pump current increases, higher order modes turn on, whose polarization is often orthogonal to the polarization of the fundamental mode.



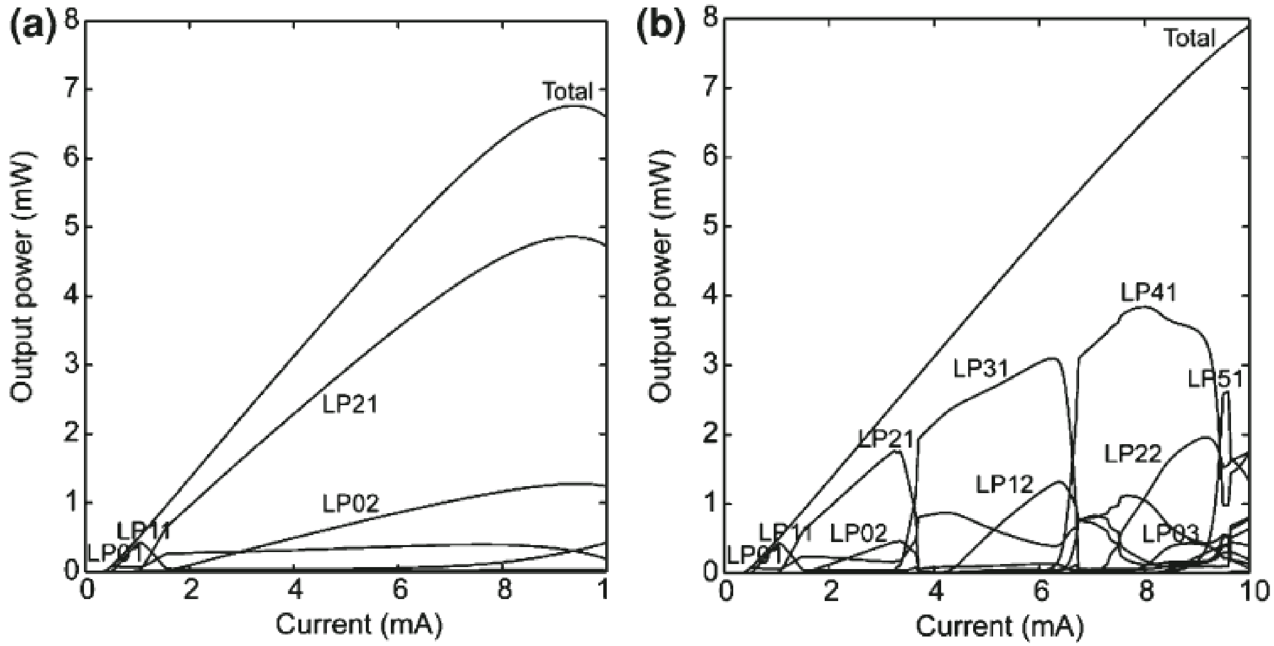


Fig. 4.4 Mode-resolved power-current characteristics for a 7 µm oxide-confined 850 nm VCSEL without (**a**), and with (**b**) effects of carriers and temperature accounted for

Larsson and Gustavsson, Chapter 4 in VCSELs Ed. Michalzik (Springer 2013)

Summary

- Optical injection
  - Two parameters: the detuning (ν_m-ν_s) and the relative injection strength.
  - In the injection locking region the laser emits a stable output and the emission frequency is the same as the injected light (ν_s=ν_m).
  - OI can generate intensity oscillations (regular or chaotic).
- Optical feedback
  - Three parameters: the relative feedback strength, feedback delay time and feedback phase.
  - Coherent feedback reduces the laser threshold.
  - It can reduce the laser linewidth.
  - OF can also generate a chaotic output.
- VCSEL polarization
  - Due to their circular cavity geometry the polarization emitted by a VCSEL is determined by “residual” anisotropies (dichroism and birefringence).
  - Some VCSELs display polarization dynamics (switching, bistability and oscillations) that can be due to thermal or to non-thermal effects.

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