Analytical characterisation of axial shortening due to creep of reinforced concrete columns in tall buildings

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ABSTRACT

This paper proposes a new methodology to analyse how creep affects to reinforced concrete columns of tall buildings with central core that range between 20 and 40 floors. Creep shortening is due to compression stresses and are the different stress values between central core and columns which produces a differential shortening and a relevant bending moment increase in main structural elements. In the first place, it is shown how axial shortening due to creep affects to a 30-storey tall building based on the formulation included in Eurocode. All parameters that influence the behaviour under this phenomenon have been fixed. Afterwards, the polynomial that characterizes creep in this building is defined and, in the last place, it is analysed how variables such as concrete strength or the amount of reinforcement affect to long term shortening. These variables are included in the in the initial algorithm with different correction parameters.

1. Introduction

First references related to the concrete creep phenomenon date from more than one century ago. These references analyse the effect that time has on the behaviour of reinforced concrete beams [1]. Throughout this period, several mathematical models were used to characterize the behaviour of concrete under compression stress along time. However, at the beginning, these investigations could not be translated into constitutive models of the material to consider the effects of this phenomenon in the global analysis of the structure. The main reason is that the use of computers for analysis and design of large and complex structures was not generalized until the end of the 20th century.

Although the first objective of the existing researches associated to the physical creep phenomenon was focused on the consequences that it has on the behaviour of reinforced concrete beams, and especially in prestressed concrete beams [2,3] y [4], it is also essential to characterize the shortening of reinforced concrete columns on tall buildings, especially when the building has a central core, due to the different stresses between core and façade columns.

The reason is that generally the compression stresses produced by gravitational loads in the central core walls is much lower than the one produced in columns. The result is a differential shortening between them along time. This differential shortening will have several consequences that must be taken into account in the structural design of the building, such as the unloading of the columns, the overload of the core and the increase in the bending moments and shear forces in the beams and the floor slabs.

The first calculation method to analyse the effects of shortening of concrete columns in tall buildings due to the phenomena of creep and shrinkage of concrete was presented by M. Fintel and F.R. Khan in 1969 [5]. This method includes fundamental aspects to analyse shortening due to creep effect on concrete columns. Some of these factors are the construction speed of the building and the amount of reinforcement of columns. Subsequently several authors have proposed different methods to analyse the shortening of reinforced concrete columns in tall buildings [6,7,8,9].

The relevance of bending moment increases on beams due to column shortening is evaluated by Jayasinghe and Jayasena in [8], where the additional elastic moment that should be considered in the structural design is calculated, without plastic redistribution. It shows that the 30-storey tall building has the most relevant increase in the bending moments of the main beams, with additional "$M/(b \cdot h^2)$" that reaches a maximum value of 3.53 kN/m^2 in the 25th storey.

Although there are several constitutive models that intend to characterize the creep phenomenon in concrete elements under a constant load, such as those contained in Eurocode EC-2 [10] and ACI [11] and [12], the definition of an equation that includes all the physical parameters that affect creep entails a great difficulty. Several parameters are associated with the viscoelastic response of the material which affects its long-term response, such as the strength and the environmental conditions, for example temperature and relative humidity. In addition, the inclusion of these physical phenomenon implies, not only the incorporation of these parameters that characterize the creep of the concrete, but the construction process of the building.

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Several numerical methods have already been developed to analyse the shortening of reinforced concrete columns in the design of tall buildings. Some of these methods are general enough to be applied to any concrete tall building [10]. These methods make it possible a structural design that considers the effects derived from shortening of columns, such as the increasing of bending moments in beams and slabs.

Nevertheless, in the first stages of design in this kind of buildings it is necessary to develop several conceptual models to understand the behaviour of the structural framework. It is the reason why the present work shows a simplified analytical method to evaluate axial shortening due to creep of reinforced concrete columns to make it possible the inclusion of this effect since the initial stages of design. Shrinkage is not considered since it is not affected by the stress in the columns.

This method involves defining a set of polynomials without the requisite to include all the physical phenomena. The objective is not to analyse infinite cases which would involve an endless number of algorithms and data for any type of building and any external parameter. The main aim is to analyse a specific case study and, subsequently, to evaluate the influence that different variables have on the overall response of the concrete columns in front of long-term strain due to creep phenomena.

The study analyses a 30-storey tall building using specific parameters to define how this physical phenomenon affects a reinforced concrete column chain, with the method included in [10]. The initial analysis is developed over a 30-storey building with an usual construction process. These 30-storey building is a common layout in current cities and also it is for this height when it is produced an important increase in beam bending moments.

Once this first analysis is concluded, the study is extended to buildings with 20 and 40 storeys to analyse the influence of parameters, such as the amount of reinforcement, the concrete strength, the relative humidity and the notional size of columns, on column shortening due to the creep.

Finally, a set of polynomials is presented. This set can be applied to tall buildings of 20, 30 and 40 storeys with the use of correction coefficients, regardless of how the previous four parameters are modified. It is possible to know, thanks to this set of polynomials, for general cases, the response that the reinforced concrete columns under constant load will have over time.

2. Theory

According to Eurocode EC-2 Annex B [10], the deformation of a concrete element under compression stress, without the contribution of the reinforcement, can be defined with the expression (1), where the first term of the parenthesis corresponds to elastic shortening and the second to creep.

$$\varepsilon_{c\sigma}\left(t,t_0\right) = \sigma\left(t_0\right)\left(\frac{1}{E_{c,t_0}} + \frac{\varphi\left(t,t_0\right)}{E_{c28}}\right) \tag{1}$$

$\varepsilon_{c\sigma}$ is the concrete strain; $\sigma\left(t_0\right)$ is the compression stress applied to the concrete member; E is the tangent Young's modulus of concrete defined for a specific time; and $\varphi\left(t,t_0\right)$ is the creep coefficient.

In this constitutive model, the calculation of the strain of the concrete element is based on a creep coefficient that depends on the load history. Specifically, on concrete age at application time for each load percentage. For each specific load, creep coefficient is defined as follows (2).

$$\varphi\left(t,t_0\right) = \varphi_0\beta_c\left(t-t_0\right) \tag{2}$$

The notional creep coefficient (creep at infinite time) is defined according to equation (3).

$$\varphi_0 = \varphi_{HR}\beta\left(f_{cm}\right)\beta\left(t_0\right) \tag{3}$$

Where φ_{HR} is the coefficient of influence of relative humidity; $\beta\left(f_{cm}\right)$ is the factor that considers the effect of the concrete strength on the notional coefficient of creep; $\beta\left(t_0\right)$ is the load age influence factor $\left(t_0\right)$ on the notional creep coefficient; and $\beta_c\left(t-t_0\right)$ is the function that describes the development of creep over time.

Another relevant aspect to define the axial shortening due to creep of concrete columns in a tall building at infinite time is the contribution of the amount of reinforcement. For its calculation, the effect of creep can be understood as a loss of stiffness in the concrete material. For a given stress, the total shortening experienced by the concrete is characterized according to (4).

$$\varepsilon = \sigma\frac{1+\varphi}{E} \tag{4}$$

This is equivalent to consider that it is a material without creep, but that has a real elastic Young's modulus E′, of value E′ = E/(1 + φ).

As the concrete material and the reinforcement must bear the total axial stress with compatible strains, the equalities contained in (5) and (6) are fulfilled:

$$N = A_c\hat{A}·E_c\hat{A}·\varepsilon_{elast} + A_s\hat{A}·E_s\hat{A}·\varepsilon_{elast} = A_c\hat{A}·E_c\hat{A}·\varepsilon_{tot} \\ + A_s\hat{A}·E_s\hat{A}·\varepsilon_{tot} \tag{5}$$

$$N = \varepsilon_{elast}\hat{A}·\left(A_c\hat{A}·E_c + \xi\hat{A}·A_c\hat{A}·E_s\right) \\ = \varepsilon_{elast}\hat{A}·A_c\hat{A}·\left(E_c + \xi\hat{A}·E_s\right) \\ = \varepsilon_{tot}\hat{A}·A_c\hat{A}·\left(E_c + \xi\hat{A}·E_s\right) \tag{6}$$

Where A_c is the area of the concrete section; E_c is the tangent Young's modulus of the concrete; ε is the elastic strain; A_s is the area of reinforcement; E_s is the Young's modulus of the steel; and ξ is the reinforcement quantity.

From Eq. (6) it is possible to differentiate the elastic strain (7) from the total strain (8), considering the latter as the sum of the instantaneous elastic strain and the strain that occurs over time due to creep.

$$\varepsilon_{elast} = \frac{N}{A_c\hat{A}·\left(E_c + \xi\hat{A}·E_s\right)} \tag{7}$$

$$\varepsilon_{tot} = \frac{N}{A_c\hat{A}·\left(E_c' + \xi\hat{A}·E_s\right)} \tag{8}$$

Thus, the creep strain is:

$$\varepsilon_{fl} = \varepsilon_{tot} - \varepsilon_{elast} = \frac{N}{A_c}\left(\frac{1}{E_c' + \xi E_s} - \frac{1}{E_C + \xi E_s}\right) \tag{9}$$

By operating (9), the creep shortening can be determined as specified in (10).

$$\Delta L = \varepsilon_{fl}\hat{A}·L = \frac{N\hat{A}·L}{A_c}\hat{A}·\left(\frac{1}{\frac{E_c}{1+\varphi} + \xi E_s} - \frac{1}{E_c + \xi E_s}\right) \tag{10}$$

3. Case study and numerical development

This section presents the case study that has been used in the work and the numerical development to obtain the polynomial law that defines creep shortening of columns in the case study building.

3.1. Case study parameters

Fig. 1 and Table 1 describes the geometry and the fundamental parameters from the typology of the structure and the global geometry of the building that is analysed, and from the properties of the material, the relative humidity, the dimension of the concrete columns, etc. Regarding the column dimensions and its variation in height, it is proposed a reduction of 15 000 mm² on each floor. This gradual reduction
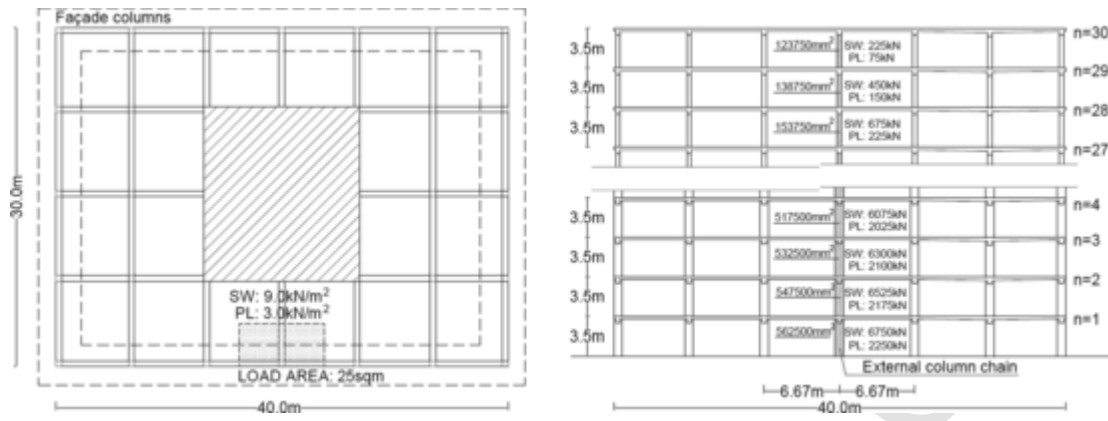
Fig. 1. Geometry of the case study.

Table 1
Fundamental parameters considered in the initial case study.

Parameter	Considered value
Typology	Central core
Number of storeys	30
Storey height	3.5 m
Area of influence of the columns	25 m^2
Concrete strength	40 MPa
Self-weight slab	225 kN (75%)
Other dead loads	75 kN (25%)
Relative humidity	70%
Notional size of the column	300 mm
Section of the first-floor concrete column	562 500 mm^2
Section of the last floor concrete column	123 750 mm^2
Section reduction between floors	15 000 mm^2
Slenderness of the column chain	4.54
Area density	114.38 (cm^2/n storeys)
Amount of reinforcement	1% of A_c
Mechanical reinforcement ratio	16%

in the column area in each storey is the most unfavourable situation for creep shortening analysis. Any other different sizing criteria, as for example, the definition of a common column size for a group of storeys would be more favourable. The differential shortening between the central core and the façade columns is produced because of a different magnitude in the concrete stresses of both elements. If the columns are oversized in some storeys the differential stresses will be minimised and consequently the columns shortening will be reduced too. Slenderness of the column chain, understood as the relationship between the maximum section and the minimum section, and the area density, defined as the average section area divided by the number of storeys, are the parameters that are taken as a reference that will be used later to extend the analysis to tall buildings with different number of storeys. These parameters can be used for any building height ranging between 20 and 40 storeys.

Regarding concrete strength, it affects three components of the problem: the coefficients φ_{HR}, β (f_{cm}), and β_c (t-t_0). The first coefficient, φ_{HR}, related to the influence of relative humidity on creep, produces a reduction on the notional creep coefficient if the concrete strength increases over 35 MPa (A.18). The second one introduces a correction in creep, increasing the notional creep coefficient if the concrete strength is reduced (A.21). The last coefficient that is affected by concrete strength is β_c (t-t_0). This coefficient considers the variation of creep over time after loads have been applied, modifying the notional creep coefficient. However, the influence of concrete strength in this coefficient is less relevant.

In addition, it is necessary to consider the influence that the tangent Young's modulus of the material will have on the creep shortening of the columns.

As for the notional size of the member, understood as twice the concrete cross-sectional area divided by the perimeter of the column, for a 600×600 mm^2 section column this parameter is 300 mm.

For a $400 \times 1\ 000$ mm^2 section column notional size is 286 mm, whilst for a 400×400 mm^2 section it is reduced to 200 mm. For this reason, it seems appropriate, for the usual concrete column chains in this type of buildings, to fix this parameter to a value of 300 mm to develop the initial case study. Subsequently, its incidence in shortening is evaluated.

3.2. Construction process

A specific construction process has been defined (which is also a typical solution for this type of building). The construction progress is two weeks per storey. The pouring of concrete for the columns of each storey is considered to take place at the intermediate point of construction of the storey. This is to say, after finishing any storey, the pouring concrete for the columns is one week later, and concrete for the upper slab is poured two weeks later.

The floor formwork is removed and propped 14 days after the concrete pouring, and just before concrete pouring on the upper floor. It means that each column is first loaded three weeks after pouring its concrete, and that from that moment its load increases in two-week periods.

No specific hypothesis is formulated regarding the number of floors that the propping of each new slab supports (two or three). The load concentrations that the construction process produces in the floors (and in the columns that support them) are limited to very short periods (between two and four weeks, depending on the specific construction progress). In these short periods, the modifications on creep shortening are limited.

The application of quasi-permanent loads (basically, finishes) begins once the construction of the structure is concluded, with a construction progress of two weeks per floor. Specifically, during the two weeks that the upper floor remains propped, the ground floor finishes are concluded. Finishes on the first floor are concluded two weeks after the formwork is removed on upper floor. This way, the work progress involves loading rates of two weeks from the moment that the finishes loads are applied. However, a reduction in the execution time of the finishes would have an insignificant influence in the evaluation of creep in columns. Although finishes were executed at a pace of one floor per week, the fact of their placement from the week 63 minimizes the influence of the construction speed in the determination of creep shortening in columns. In this way, the most important points of the

progress of loading the columns and the effect of creep shortening on the final response of the structure are as follows:

First floor slab is connected to the columns under that floor a week after its concrete pouring. These columns will receive their first load three weeks later (just when the floor formwork is removed and it is propped). All the long-term creep shortening on these columns will mean the descent of first floor nodes, not recoverable in any way.

Three weeks later, second floor slab will be connected to the columns. Between three and five weeks since the beginning first floor columns will be affected by creep shortening under one floor load. Meanwhile, columns on the second floor are not loaded yet. Five weeks later, the second floor slab formwork will be removed and propped. Concrete will be poured on the third floor and it will be connected to columns. The creep shortening of the first-floor columns between weeks three and five after construction (from 0 to 2 after loading) will be corrected by levelling the formwork before concrete is poured in that floor.

When the process is repeated for the fourth floor (seven weeks later), different corrections are included:

(a) Creep shortening experienced by first floor columns will be corrected for a load applied three weeks after its construction and for the period of five to seven weeks (2 to 4 weeks after applying the load), plus another load on the same column, applied at five weeks, and also for the period of five to seven weeks (0 to 2 weeks after the load application),
(b) Creep shortening experienced by second floor columns, for a load applied three weeks after construction and for the period of three to five weeks.

The creep shortening that will be corrected when pouring concrete on any storey floor will be:

(a) The one experienced by the column located two floors below ("n-2"), for a load of a slab applied three weeks earlier, during a period of three to five weeks after its construction (0 to 2 after applying the load).
(b) The one experienced by the column located three floors below ("n-3") for a load of a slab applied:

- three weeks earlier, during the period of five to seven weeks (2 to 4 after loading it),
- five weeks ago, for the same period after construction (0 to 2 weeks after loading it).

And so on.

3.3. Creep analysis

Firstly, it is necessary to calculate long term creep for each column. This creep is due to loads with the same value but applied in two-week increments. When the construction of the structure is complete, the remaining dead loads will begin to be applied. For any storey column there will be a time that nothing will happen until the finishes of the first floor it supports are completed; thereafter, the column will receive new loads every two weeks until the construction of the building is completed.

Creep at infinite time due to any load is provided by (3). In (3) φ_{HR} and β (f_{cm}) are constant values for the columns of all the floors. The third component, β (t_0), depends on the age of application of the load. Provided that, in the case with more components (the column of the first floor), loads are applied with a cadence of two weeks (from the third week since the construction of the column), until week n° 121 (30 self-weight of slabs, one every two weeks, followed by quasi-permanent loads, applied on each one of the 30 floors, also with a rhythm of every two weeks and consecutive to the previous ones), it is necessary to calculate 60 creep coefficients from week 3 to week 121; that is, from day

21 to 847. Between days 21 and 427, a self-weight load will be applied every 14 days, and between days 441 and 847, the quasi-permanent loads of a storey will be applied, also every 14 days. The results for the 60 coefficients mentioned are shown in Fig. 2.

Long-term creep coefficient for the column of the upper floor (30th), can be calculated considering that 75% of its load is applied at 21 days, and the remaining 25% in 441 days (60 weeks later). Its global creep coefficient (to be applied to the total load) will be obtained by weighting 21 days at 75%, and 441 days at 25%. In the case of the column of the 29th floor, as it supports two slabs, half self-weight is applied at 21 days, and the other half at 35 days. Half of permanent loads is applied at 441 days, and the other half at 455 days. In general, for any storey "31-n", the "n" coefficients for the construction periods of the structure should be added, dividing the result by "n" and multiplying it by 0.75. The sum of the "n" input coefficients of quasi-permanent loads should be added to this value, also divided by "n" and now multiplied by 0.25. Coefficients for all the columns are shown in Fig. 3.

From this point, it is necessary to include the influence of the amount of reinforcement in the process to determine the long-term creep shortening.

It should be clarified that throughout this process it is assumed that, in the general creep Eq. (1), E (c, t_0) is equal to E_{c28}. It must be considered that in (1) the variability of the tangent Young's modulus, E, only affects elastic strains, but not those of creep (these are referenced with respect to E_{c28}). In addition, this error is usually assumed in the conventional analysis of any structure, since, except in absolutely exceptional cases, this analysis is always carried out based on the value of the Young's modulus at 28 days, without including the incidence of the evolution of this variable over time.

Based on the development of this methodology, Fig. 4 shows the creep shortening of each column for the total 30 storeys of the struc-
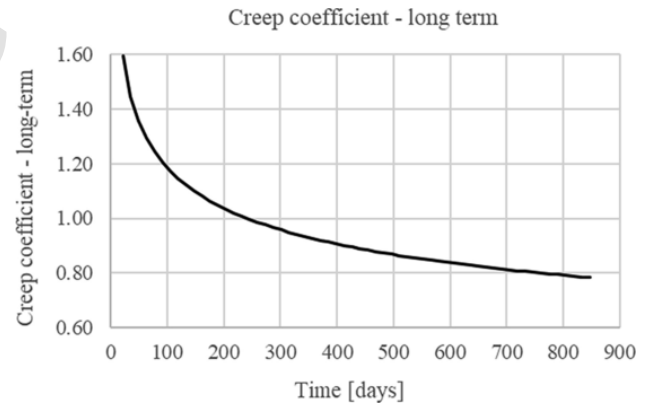

Fig. 2. Long term creep coefficient as a function of the age of application of the load.
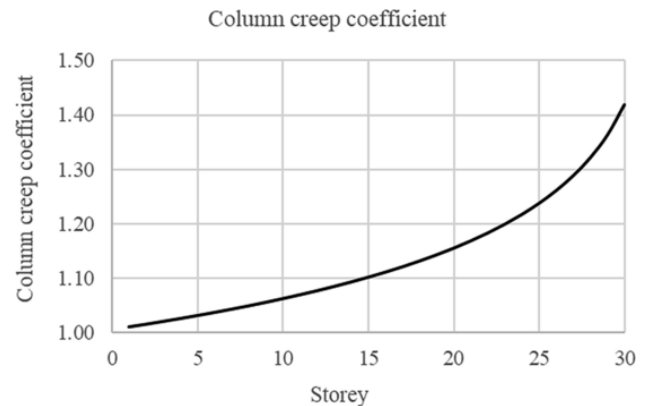

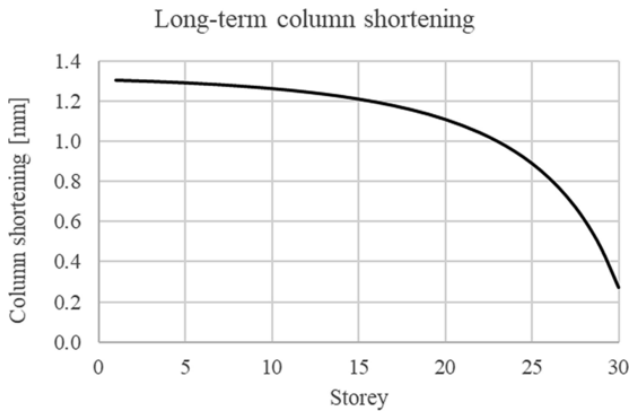
Fig. 3. Global column creep coefficient in each floor.

Fig. 4. Long-term creep experienced by the columns, without considering the correction during the construction process.

Fig. 5. Accumulated creep coefficients for the column of each storey, corresponding to the two-week evaluation period.

Fig. 6. Creep shortening of the column, depending on its area, in two-week periods.

ture, including the long-term creep coefficients, and considering the contribution of the amount of reinforcement of the concrete column. This long-term column shortening is referred to the maximum shortening that can be produced in the column of each storey at infinite time, considering the age of application of loads depending on the construction process.

As it has been previously stated, the values that are corrected at the moment the floor is constructed are those of creep shortenings experienced during the previous two weeks by all the columns below that have already been loaded during these two weeks (this excludes those of the storey immediately below). In the case of the storey "n-2" (two floors below the one being built), only the load of one storey (self-weight of the structure) will have been applied three weeks after the construction of the column, and it is evaluated during the first and second weeks (fourth and fifth after the construction of the column). On the contrary, in the "n-3" column there will be a load applied at three weeks that is evaluated in the period of the third and fourth week, and another load at five weeks (the same than on the upper floor), which is evaluated for the period of the first and the second.

In summary, for a column that has X floors above (the last one is the slab that is just being constructed), the number of loads to consider will be "X-1".

It means that to calculate the shortenings that are corrected in each storey it is necessary to define a set of creep coefficients that includes the combination of different ages of application of the load, from three weeks and with two-week sequences, and different evaluation periods, in pairs of weeks (1–2, 3–4, 5–6, etc.).

Thus, for the column that is located two storeys below the one being built ("n-2"), only a coefficient that corresponds to three weeks of load application age and 1–2 weeks of evaluation period is applied. For the "n-3" column, two coefficients should be added; the first one corresponds to an age of five weeks and an evaluation period of 1–2 weeks. The second one is related to 3 weeks of age and 3–4 weeks of period.

Finally, the creep shortening that a column of any storey experiences during a given evaluation period of two weeks is obtained according to the expression previously used for the long-term creep calculations, which includes the influence of the amount of reinforcement, now applying the load related to each step (225kN).

The accumulated creep coefficients for the columns of each storey, corresponding to the two-week evaluation period, are shown in Fig. 5. The value of the creep coefficient at two weeks tends to stabilize around the value of 0.7.

Fig. 6 shows the creep shortening that the column will experience depending on its area. This parameter also tends to stabilize, in this case in the value of 132/A, where A is the area of the column, in cm^2, obtaining the column shortening in millimetres.

From this point, it is possible to proceed to determine the creep shortenings that cannot be corrected through the construction process.
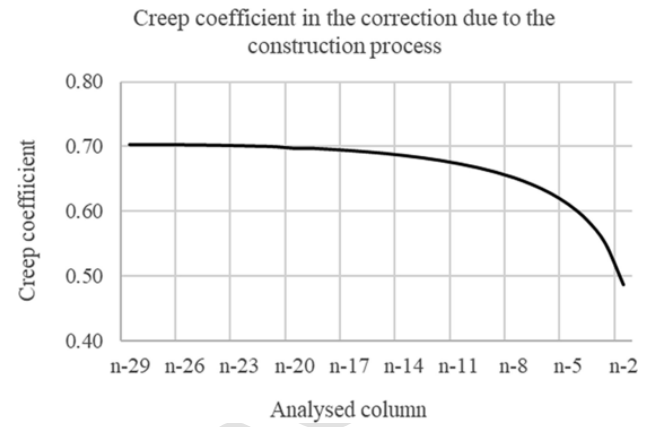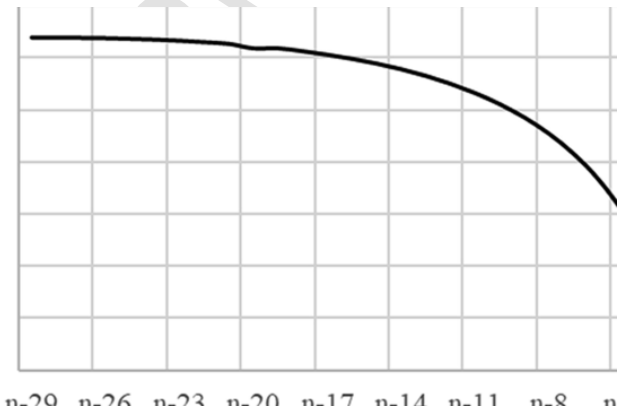
They are obtained for each floor as the difference between the long term shortening and the shortenings experienced by every floor between n-2 and 1 during the two weeks previous to its slab concrete pouring. In the case the first and second floor, their shortening will directly correspond to the long-term value. In the case of the third one, it will be necessary to subtract the first floor shortening. In the case of the fourth, second ("n-2"), and first ("n-3") floor shortening, and so on, would be suppressed from long-term shortening. Total creep shortening experienced by the columns of all the storeys of the building are shown in Fig. 7.

As can be seen, the last values are elongations. This does not mean that creep lengthens the column, but that the total value of shortenings
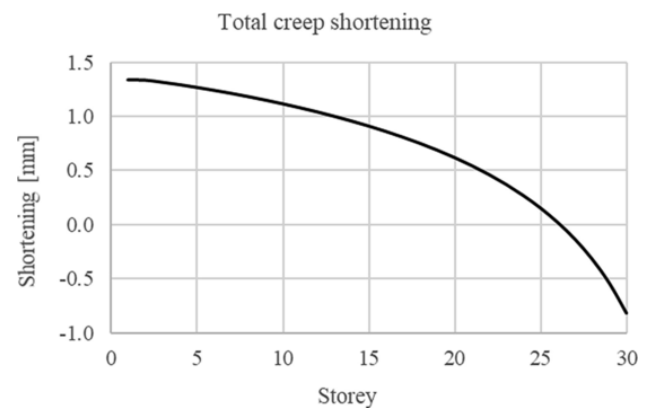

Fig. 7. Law of creep shortening, including the correction produced during the construction process.

experienced by the lower storeys during the two weeks prior to their construction, which are corrected when these storeys are constructed, are greater than the long-term creep shortening than the column will experience, because of the low loads it supports and its compression stresses.

Based on the approach that has been exposed here, it is possible to represent graphically the total variation of the storey height as a function of the shortening that each column experiences (Fig. 8), and to obtain a function that expresses this law of vertical displacement due to creep. (11).

This displacement law adjusts to a 4th degree polynomial, without independent term, since the displacement at the base of the building is zero (11), where "n" is the analysed storey.

$$Y_{30} = -3.17Â \cdot 10^{-5} Â \cdot (n)^4 + 1.09Â \cdot 10^{-3} Â \cdot (n)^3$$
$$- 2.94Â \cdot 10^{-2} Â \cdot (n)^2 + 1.47Â \cdot n \qquad (11)$$

4. Analysis of results and discussion

The study is extended to 20 and 40-storey tall buildings, and the algorithm that defines the shortening of concrete columns for buildings of this number of storeys is presented.

Subsequently, different parameters are included as variables, evaluating what influence these variables have on the shortening of the columns due to long term creep phenomenon. The entire study of the influence of variables takes as a reference the 30-storey tall building.

4.1. Study extension for tall buildings from 20 to 40-storeys

Once the 4th degree polynomial that characterizes the vertical displacements in the columns for the 30-storey tall buildings has been determined, the study is extended to define the 4th degree polynomials corresponding to the 20-storey buildings (12) and 40-storey buildings (13).

$$Y_{20} = -1.12Â \cdot 10^{-4} Â \cdot (n)^4 + 2.67Â \cdot 10^{-3} Â \cdot (n)^3$$
$$- 4.55Â \cdot 10^{-2} Â \cdot (n)^2 + 1.58Â \cdot n \qquad (12)$$

$$Y_{40} = -1.29Â \cdot 10^{-5} Â \cdot (n)^4 + 5.78Â \cdot 10^{-4} Â \cdot (n)^3$$
$$- 2.15Â \cdot 10^{-2} Â \cdot (n)^2 + 1.39Â \cdot n \qquad (13)$$

The results related to creep shortening depending on the number of storeys are shown in Fig. 9.

4.2. Influence of the amount of reinforcement in creep shortening

A fundamental parameter in the evaluation of creep shortening is the total amount of longitudinal reinforcement. It should be taken into account that the entire initial study has been carried out considering a
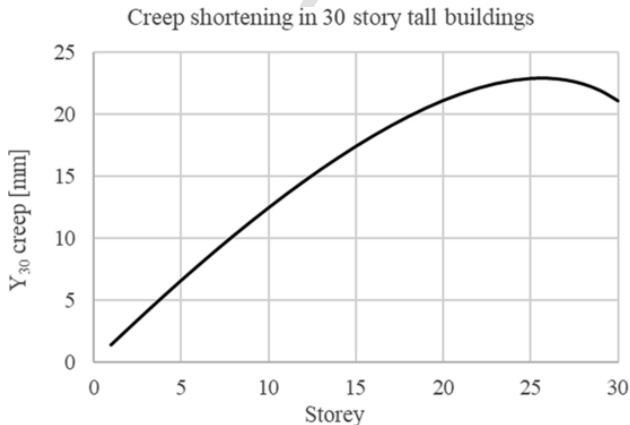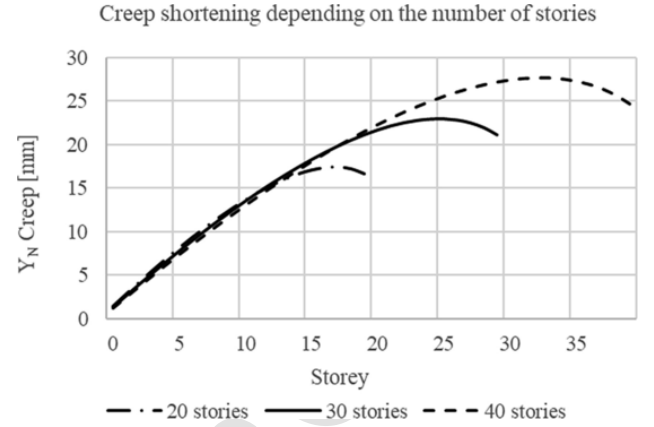

Fig. 9. Laws of vertical displacements for tall buildings of 20, 30 and 40 storeys.

relatively low amount of reinforcement, of 1% of A_c. This assumption is on the safe side to calculate the maximum values of column shortening.

However, it is also interesting to analyse how the variations in the amount of longitudinal reinforcement can affect the long-term shortening of concrete columns. The shortening variation is determined based on different amounts of reinforcement, analysing values between 0.5% of A_c and 4% of A_c and comparing them with the initial reference value of 1% (Fig. 10). Note that the proposed methodology to analyse creep shortening is based on the elastic behaviour of the reinforcement steel and it must be guaranteed that the amount of reinforcement defined in the project is under the steel yield point.

In this process, it must be noticed that the creep coefficients already determined are not affected by the total amount of longitudinal reinforcement that is considered, so all the calculations previously developed for this purpose are still valid, in both their procedure and in the values obtained. However, the shortening experienced by the reinforced concrete column does change. Similarly, the creep coefficients defined for two-week periods are not modified. However, the shortening that their effect causes and that includes the corrections derived from the construction process itself are affected.

In order to consider the influence that the amount of reinforcement has, a new factor is added into the initial equation. Its objective is to correct the dispersion that causes to increase or decrease the reinforcement in the shortening of the column. For the determination of the algorithm, in the 30-floor tall building the creep experienced in the 20th floor is taken as a reference, since it is the floor that best compensates the deviations of different signs, both in the upper and the bottom parts of the building.

Fig. 8. Law of absolute vertical displacements by floor.

Fig. 10. Law of vertical displacement in a 30-storey tall building depending on the amount of reinforcement.

The long-term descents that occur in storey 20 are firstly evaluated, depending on how the amount of reinforcement varies. Table 2 shows the results that link each one of the amounts analysed and the absolute and relative displacements in the storey 20. The amount of reinforcement of 1% is considered as a reference.

The graphic representation of these results is used to determine a behaviour law that can be assimilated to a very flat parabolic curve, with an increase in the ordinates when the decrease in the abscissa occurs (14), including in this equation the total amount of reinforcement as a percentage.

$$C_\xi = 0.0138\hat{A}\cdot\xi^2 - 0.184\hat{A}\cdot\xi + 1.17 \tag{14}$$

By including the coefficient C_ξ in (11), (12) or (13), a new algorithm is obtained to analyse the creep in tall buildings between 20 and 40 storeys, with quantities of reinforcement ranging between a minimum of 0.5% and maximum of 4%.

From the analysis of the results represented in Fig. 9, it can be concluded that the total amount of longitudinal reinforcement has a significant influence on the shortening that occurs along time in concrete columns. The increase of reinforcement from 1% to 2% of A_c produces a reduction in shortening of approximately 18%. It should be considered that, although the initial case study takes a small amount of reinforcement as a reference, as this is on the safe side, columns of tall buildings are characterized by higher amounts, close to 3% of A_c on many occasions. This fact is favourable regarding the maximum shortenings produced in the long-term period.

In addition, it is important to consider that, depending on the design loads, the minimum amount of reinforcement can range between both values, 0.5% and 1.0%. The minimum value of 0.5% makes possible to obtain a linear interpolation value.

4.3. Influence of the concrete strength

A second relevant parameter to define creep is the concrete strength. As it has been previously indicated, the concrete strength affects three components of the problem: the coefficients β (f_{cm}), φ_{HR} and a β_c (t-t_0). It means that, in order to define the creep variability based on the concrete strength, it is necessary to redefine all the long term creep coefficients and the creep coefficients for two-week periods that are used to apply the correction occurred during the construction process

In addition, related to the concrete strength, the influence that the variation in the tangent Young's modulus has in the shortening evaluation of columns must be considered.

If the process is repeated again, now taking as a reference the tower of 30 storeys and the initial amount of longitudinal reinforcement of 1% of A_c, but considering concrete strengths of 30, 35 and 50 MPa, and compared with the initial reference value of 40 MPa, significant differences in the long-term creep shortening of columns can be appreciated (Fig. 11).

Following the same analysis procedure that has been used to evaluate the influence of the amount of reinforcement to determine a correc-
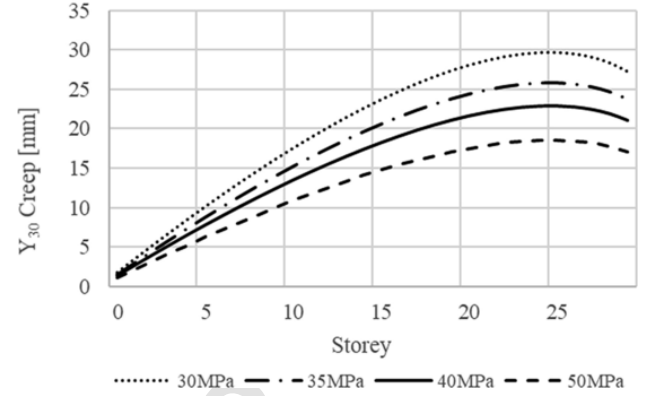

Fig. 11. Law of vertical displacement in a 30-storey tall building depending on concrete strength.

tion coefficient in the initial algorithm, storey 20 is taken as a reference. Since this is again the one that is more balanced for the deviations above and below it. To determine this new correction coefficient, the descents that occur in storey 20 at long-term are first evaluated, depending on strength of the concrete variation. Table 3 includes the absolute and the relative vertical displacement produced in the storey 20 related to the concrete strength variation, taking as a reference the value of 40 MPa. The graphic representation of the values included in Table 3 is used to define an equation assimilable to a very flat parabolic curve, according to (15).

$$C_{fck} = 0.00055\hat{A}\cdot f_{ck}^2 - 0.06858\hat{A}\cdot f_{ck} + 2.86 \tag{15}$$

The correction coefficient C_{fck} can be included in the general polynomials (11), (12) or (13) to consider the influence that the modification of the concrete strength has. Increases in the concrete strength of 25% compared to the value used in the initial case study imply that the maximum shortening in the reference building, is also reduced by 25%.

4.4. Influence of the relative humidity

A third aspect that has a relevant influence on the creep experienced by the concrete columns is the relative humidity. The initial case study is carried out considering a relative humidity of 70%. However, a reduction in relative humidity has an unfavourable effect on the physical phenomenon of creep. The incidence of humidity variation in the range between 80% and 50% is evaluated, considering the usual lower limit in indoor areas (Fig. 12).

For the determination of the corrective coefficient of relative humidity, the descents that occur in storey 20 at long-term period depending on how the relative humidity varies are determined again, as it has been done for the other variables. Table 4 shows the results that correspond to the absolute and relative values of vertical displacement experienced by storey 20, considering the relative humidity of 70% as a reference.

Table 2
Variation of creep shortening in storey 20, depending on the amount of reinforcement.

Amount of reinforcement [% of A_c]	Displacement in Storey 20 [mm]	Relative displacement related to an amount of 1% of A_c
0.5	22.89	1.08
1.0	21.10	1.00
2.0	18.13	0.86
3.5	14.74	0.70
4.0	13.83	0.66

Table 3
Variation of creep shortening in Storey 20, depending on the concrete strength.

Concrete Strength [MPa]	Displacement in Storey 20 [mm]	Relative displacement related to a strength of 40 MPa
30	27.38	1.30
35	23.86	1.13
40	21.10	1.00
50	17.09	0.81

Creep shortening depending on relative humidity

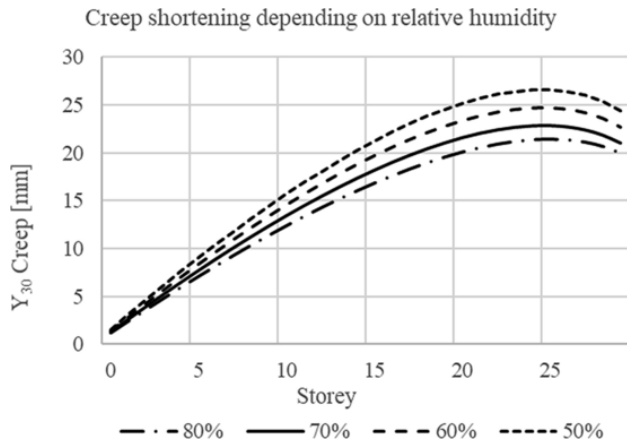**Fig. 12.** Law of vertical displacement in a 30-storey tall building, depending on the relative humidity.

Creep shortening depending on notional size

Fig. 13. Law of vertical displacement in a 30-storey tall building based on the notional size of the column.

Table 4
Variation of creep shortening in the Storey 20, depending on relative humidity.

Relative humidity (HR) [%]	Displacement in Storey 20 [mm]	Relative displacement related to a relative humidity of 70%
50	24.54	1.16
60	22.82	1.08
70	21.10	1.00
80	19.58	0.93

The graphic representation of these results is used to define the equation of a line that characterizes the correction coefficient C_{HR}, according to (16).

$$C_{HR} = -0.00786 \cdot HR + 1.55 \tag{16}$$

Any reduction in the relative humidity will represent an increase in long-term creep shortening. The influence of different humidity values is evaluated, taking as a reference a common humidity in coastal areas and islands of moderate size. In environments where the relative humidity is 50% instead of 70%, the long term shortening of the concrete columns is increased by 15%.

4.5. Influence of the notional size of the column

The notional size taken in the study for the concrete columns has been a typical value for tall building columns of 300 mm. In any case, it is highly unlikely this variable to be under 150 mm (minimum values close to 200 mm are usually considered) or over 400 mm. Fig. 13 shows the variation that occurs in creep shortening when the notional size varies between 150 mm and 400 mm.

When the creep shortening variation that occurs in the 30-storey building is evaluated, it can be seen how the maximum variation does not represent errors greater than 3.5% in any case with relation to the initial scenario, within the range 200–400 mm. Furthermore, it is important to notice that the greatest errors occur in the lower storeys, where small numerical variations represent a greater percentage of error. From the 10th floor, the error does not exceed 3% in any case.

Therefore, and considering that the errors that occur in the upper half of the building, where the accumulated total shortenings are greater, do not exceed 3% in any case. This margin of error can be considered as acceptable in the calculation of the creep shortening in concrete columns, unless a very accurate calculation is required, which surely does not make sense because this level of precision should also
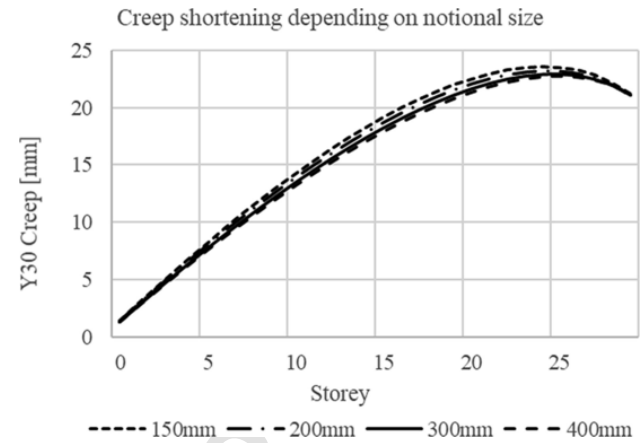
be extended to other aspects of the analysis and design, such as the foundation-structure interaction.

For this reason, it is not considered as necessary to include a new correction coefficient that would add more complexity to an algorithm that aims to approximate a complex phenomenon by means of a simplified method of analysis.

5. Conclusions

This study can be used to analyse the creep shortening that occurs in concrete column chains in buildings with 20, 30 and 40 storeys. The study begins from an initial case study of a 30-storey high building, which sets fundamental variables for determining creep shortening, such as the construction process, total amount of reinforcement, concrete strength, relative humidity or the notional size. In addition, the parameters defined as slenderness of the column chain, as well as area density are fixed.

A first approximation to the phenomenon of creep shortening in the initial case study makes it possible to define a 4th degree polynomial that reproduces perfectly the vertical displacements of the concrete columns on each floor due to long-term creep phenomenon, considering a common construction process. Subsequently, the same analysis methodology is applied, but extended to buildings of 20 and 40 storeys, so that a typical range of tall buildings in Europe is covered, ranging from 70 m to 140 m approximately. It can be seen how, for the three analysed heights, the change from shortening to lengthening once the correction is applied occurs at approximately the 85% of the height of the building. Thus, in the 20-storey building, the transition takes place between floors 17th and 18th. In the 30-storey building, it happens between 25th and 26th and, finally, in the 40-storey building, it occurs approximately on the 34th floor.

The influence that four parameters have on the creep calculation are analysed, specifically the total amount of reinforcement, the concrete strength, the relative humidity and the notional size of the column. These parameters are evaluated using the same procedure, always starting from the initial case of 30 storeys. Its applicability has been verified for buildings of 20 and 40 storeys. Firstly, the exact creep shortening experienced by the columns along time is determined when one of the variables is modified. Once the variation experienced in each case is obtained, the displacement for each storey is related by means of a function, which is incorporated as a correction coefficient into the initial equation.

The correction coefficients that are included in the initial equation to consider the incidence of these parameters correspond to very simple algorithms. The algorithms related to the amount of reinforcement and concrete strength are second degree equations. The relative humidity algorithm is a linear equation. Depending on the degree of precision required, the second-degree equations can even be assimilated to linear

equations due to the low curvature they have in the study interval. However, the notional size is not included as a correction coefficient for creep shortening, due to the minimal variations it produces within the usual range, between 200 mm and 400 mm. In conclusion, an equation is determined to evaluate, for tall buildings with 20, 30 and 40 storeys, the long-term creep shortening in concrete columns. This equation is defined as the initial 4th degree polynomial, Y_N, in which the three correction terms are added (17).

$$Y_{Creep} = Y_N \hat{A} \cdot C_\xi \hat{A} \cdot C_{fck} \hat{A} \cdot C_{HR} \tag{17}$$

As Y_{creep} is the total vertical displacement that is produce in each floor, Y_N is the 4th degree polynomial that represents the behaviour of the different storeys for the building of 20, 30 and 40 storeys, C_ξ is the correction coefficient for the amount of reinforcement, C_{fck} is the correction coefficient for the concrete strength, and C_{HR} the correction coefficient for the relative humidity. The entire procedure here presented is applicable to any concrete column length and to any load that affects the column chain. Both variables have a linear relationship in Eq. (10). It means that any variation in these parameters with respect to those considered here requires, for their inclusion, only a multiplying factor that modifies the total creep shortening of the columns.

CRediT authorship contribution statement

Carlos Muñoz Blanc: Conceptualization, Methodology, Investigation. **Agustín Obiol Sánchez:** Conceptualization, Supervision. **Inma Fortea Navarro:** Investigation, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A:. Notional creep coefficient

Fig. 2 included in 3.3 represents the notional creep coefficient, defined according to Eq. (3) developed in [10], as follows (A.1):

$$\varphi_0 = \varphi_{HR} \beta \left(f_{cm} \right) \beta \left(t_0 \right) \tag{A.1}$$

Where φ_{HR} is the coefficient of influence of relative humidity that depends on the concrete strength (A.2); $\beta \left(f_{cm} \right)$ is the factor that considers the effect of the concrete strength on the notional coefficient of creep (A.5); and $\beta \left(t_0 \right)$ is the load age influence factor $\left(t_0 \right)$ on the notional creep coefficient (A.6). Considering a concrete strength of 40 MPa, φ_{HR} is defined as follows:

$$\varphi_{HR} = \left[1 + \frac{1 - HR/100}{0.1 \hat{A} \cdot \left(h_0 \right)^{1/3}} \hat{A} \cdot \alpha_1 \right] \hat{A} \cdot \alpha_2 \tag{A.2}$$

Where HR is the relative humidity, h_0 is the notional size of the column, in mm, and α_1 and α_2 are coefficients that consider the influence of the concrete strength.

$$\alpha_1 = \left[\frac{35}{f_{cm}} \right]^{0.7} \tag{A.3}$$

$$\alpha_2 = \left[\frac{35}{f_{cm}} \right]^{0.2} \tag{A.4}$$

$$\beta_{(fcm)} = \frac{16.8}{f_{cm}^{0.5}} \tag{A.5}$$

$$\beta_{(t0)} = \frac{1}{0.1 + t_0^{0.2}} \tag{A.6}$$

Appendix B:. Global column creep coefficient

Fig. 3, included in 3.3, defines the creep coefficient used in equation (10) to evaluate the long-term (infinite time) creep shortening experienced by the columns, without considering the correction during the construction process. This creep coefficient is evaluated with equation (B.1). φ_0, included in both parts of the right side of the equation (B.1), is the notional creep coefficient, depending on the age of application of the load, where "j" is the storey that is evaluated and "n" is the number of storeys of the building.

$$\varphi_{col_j} = \left\{ 0.75 \hat{A} \cdot \frac{\sum_{i=j}^{n} \varphi_0(t)}{[(n+1) - j]} \right\}_{self-weight} \\ + \left\{ 0.25 \hat{A} \cdot \frac{\sum_{i=j}^{n} \varphi_0(t)}{[(n+1) - j]} \right\}_{finishes} \tag{B.1}$$

As an example, the development of this equation in a column of the 29th floor is, according to (B.2):

$$\varphi_{col_{29}} = \left\{ 0.75 \hat{A} \cdot \frac{\varphi_0 (21 days) + \varphi_0(35 days)}{[(30+1) - 29]} \right\} \\ + \left\{ 0.25 \hat{A} \cdot \frac{\varphi_0 (441 days) + \varphi_0(455 days)}{[(30+1) - 29]} \right\} \tag{B.2}$$

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