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Title: Dynamic response of Pelton runners: numerical and experimental analysis in prototypes

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Abstract: With the growth of new renewable energies, hydropower has become crucial to stabilize the power grid. This has led turbines to working in more extreme loads and with more start/stop sequences. Pelton turbines are subjected to strong pulsating forces during operation due to the action of high-speed water jets. Consequently, buckets present large deformations and stresses, which are aggravated when the runner enters resonance. Therefore, the dynamic response of the turbine must be thoroughly studied and the most dangerous frequencies avoided.

In this paper, a detailed analysis of the modal behavior of Pelton turbines is presented. Four prototypes have been studied for such purpose. The first one has been modeled numerically and analyzed experimentally. Following a systematic investigation, the effect on the modal shapes and the frequencies of the different parts comprising the turbine has been evaluated. The single bucket and the whole runner have been studied.

The second turbine, which is installed in the same power plant, has been analyzed to determine the effect of the mechanical design on the frequencies.

Finally, two prototypes belonging to other power plants have been studied to see the effects of the hydrodynamic design on the modal behavior.

# Dynamic response of Pelton runners: numerical and experimental analysis in prototypes

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1 **Abstract**

2 Worldwide electricity generation is featured by the growth of intermittent renewable  
3 energies. In this context, hydropower plays a fundamental role because it provides  
4 flexibility to the power grid and ensures its stability. The new grid requirements have  
5 led hydropower units to working under a wider range of loads and with more start/stop  
6 sequences.

7 Pelton turbines are the most suitable for high heads and have a large regulation  
8 capacity. In operation, the runner is impinged by high-speed water jets, and, thus,  
9 subjected to strong pulsating forces. As a consequence, the structure presents large  
10 deformations and high stresses, especially on the buckets. This situation can be much  
11 aggravated if the frequency and shape of the excitation is resonant with those of the  
12 structure. Therefore, the modal response of the turbine must be thoroughly studied and  
13 the most dangerous frequencies avoided.

14 In this paper, a detailed analysis of the modal behavior of Pelton turbines is presented.  
15 Four prototypes have been studied for such purpose. The first one has been modeled  
16 numerically and analyzed experimentally. Following a systematic investigation, the  
17 effect on the modal shapes and the frequencies of the different parts comprising the  
18 turbine has been evaluated. The single bucket and the whole runner have been studied.

19 Finally, another turbine installed in the same power plant has been analyzed to  
20 determine the effect of the mechanical design on the frequencies. In addition, two  
21 prototypes belonging to other power plants have been studied to see the effects of the  
22 hydrodynamic design on the modal behavior.

23

24

25 **Keywords:** Hydropower, Pelton turbines, dynamic response

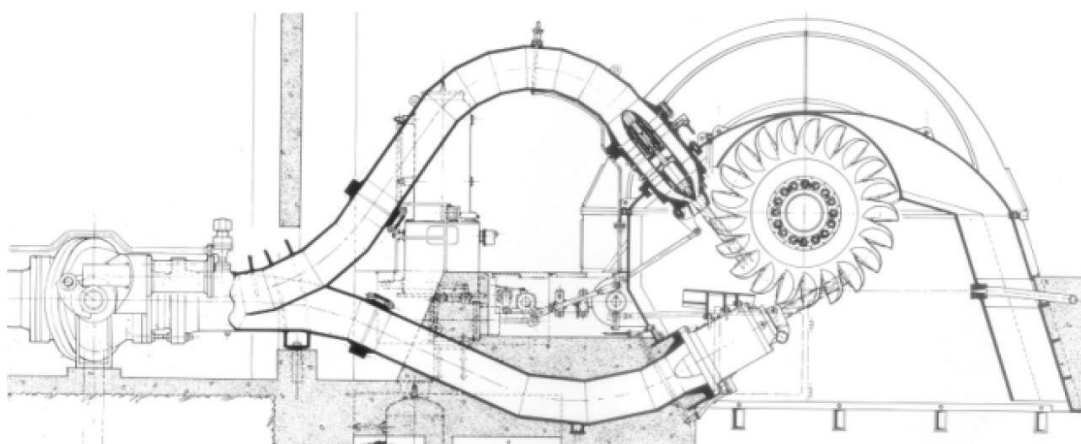
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27

28 **1. Introduction**

29 At present, new renewable energies are experiencing a strong development. These are  
30 predominantly constituted by wind power stations and solar farms, which, despite their  
31 environmental benefits, generate electricity in an irregular way and thus have a poor  
32 adaptability to the power grid demand. In such scenario, hydropower plants are  
33 becoming more important due to their ability to provide flexibility to the electric power  
34 system. It is out of question, then, that hydropower will play a big part in the future of  
35 energy generation [1]. To adapt to this increasingly fluctuating power need, turbines are  
36 required to respond faster to the grid demands and to undergo more frequent starts and  
37 stops. These harsher operating conditions have increased the action of the hydraulic  
38 forces on the structure of the turbines, which are more prone to suffering damage.  
39 Therefore, it is becoming more and more important to have a deep understanding of the  
40 structural response of hydraulic turbines when in operation.

41 Pelton turbines are among the most important types of turbines used in hydropower.  
42 They are action-type turbines with an efficiency that can exceed 90% over a wide  
43 operating range, thus being one of the most flexible types of turbine. They are installed  
44 in locations with high hydraulic heads, usually above 400 m, and operate with low  
45 discharges. The most powerful Pelton turbines are housed in the Bieudron power station  
46 in Switzerland, with three units operating under a maximum head of 1883 m and with a  
47 rated output of 423 MW each [2]. In Figure 1, a sketch of a horizontal Pelton turbine  
48 with two jets is represented. The main component of a Pelton turbine is the runner,  
49 which is a circular structure with several bucket-shaped pieces attached along its  
50 periphery. The role of the runner is to convert the hydraulic energy into mechanical  
51 energy by the impulse of the water. To do so, one or more nozzles are installed at the end  
52 of the penstock and directed tangentially to the runner. These convert almost all of the  
53 potential energy of the water into kinetic energy. The water comes out of the nozzles as a  
54 high-speed jet, which impinges the buckets of the runner perpendicularly. To take full  
55 advantage of the kinetic energy of the water, the buckets divide the jet into two nearly  
56 symmetrical streams thanks to a middle ridge and deflect them almost 180 degrees. The  
57 impulse performed on the buckets is then transmitted to the disk and finally to the shaft  
58 [3].



59  
60 *Figure 1. Cross section of a Pelton turbine*

61 Pelton runners may suffer different kinds of failures due to the continuous action of the  
62 water jets. The high velocity of the water particles may lead to cavitation pitting,

63 especially when there are small imperfections on the inner surface of the buckets or the  
64 design is not accurate enough [4]. In case of sand-laden water, the erosion of the bucket  
65 can become a serious problem and sometimes damage can be catastrophic [5][4].  
66 However, the most severe failures in Pelton turbines arise due to the fatigue of the  
67 material. The runner, and especially the buckets, undergoes very strong dynamic forces  
68 every time it is impinged by a water jet. This strong bending force is applied at a high  
69 rate, which is proportional to the rotational speed, the number of buckets and the  
70 number of jets. Since the structure of a bucket is similar to a cantilever beam, most of  
71 the stress is condensed at the base of the buckets, what makes it a critical part of the  
72 runner and prone to developing cracks. In case of wrong design or faulty casting of the  
73 buckets (which can leave imperfections in the material), the service life of the runner can  
74 be significantly reduced.

75 The effect of the water jets can be largely aggravated when the runner structure  
76 responds dynamically to these forces. This is known as resonance and happens when the  
77 frequency and distribution of the excitation force, i.e. the impingement of the jets, is near  
78 a natural frequency and mode shape of the runner. When this stage is reached, the  
79 deformation of the runner, especially in the bucket area, is amplified and the stresses  
80 largely increased. Therefore, it is crucial to avoid the resonance of the runner, and to do  
81 so, the modal behavior and excitation characteristics must be acutely analyzed when  
82 designing a new turbine [6]. After commissioning, to have a deep knowledge of the  
83 dynamics of the machine also makes it possible to monitor the state of the runner  
84 without needing to disassemble [7]. For all these reasons, the modal analysis of Pelton  
85 turbines is of paramount importance to ensure a long service life of the machines.

86

### 87 *1.1. Modal behavior of Pelton turbines*

88 Many publications can be found about the fluid dynamics of Pelton turbines [8–11].  
89 However, while some papers have been published regarding the structural response of  
90 Francis turbines and pump-turbines [12,13], there are a few studies addressing the topic  
91 on Pelton runners.

92 Pelton runners have typically suffered damage in the area of the buckets [14,15]. First,  
93 this was attributed to the stresses caused by the centrifugal forces and the impact of the  
94 jet, but with the trend to increase the runner size and the concentration of power, the  
95 importance of the bucket vibration was also acknowledged. One of the most relevant  
96 publications is from Grein et al. [16], who remarked the importance of the dynamic  
97 stresses as a controlling parameter for fatigue failure and identified the bucket vibration  
98 in circumferential direction as the most dangerous natural mode, whose amplification  
99 factor in case of resonance could reach up to x1000.

100 Some studies described the vibrational modes of Pelton buckets more accurately thanks  
101 to the development of Finite Element Methods and with experimental tests in models.  
102 Schmied et al. [17] identified the main modes of a Pelton bucket: bending mode in  
103 circumferential and axial direction, torsional mode, rim mode and radial mode. It also  
104 mentioned that as the number of nodal diameters of the runner increases, so does the  
105 resemblance to a pure bucket mode. Other authors, like Pesatori [18] and Sanvito [19],  
106 conducted simulations and experiments on buckets of a model runner and identified the  
107 mode shapes of the bucket but with limited boundary conditions. The influence of the

108 disk and mounting position was not considered and only the natural frequencies with  
109 higher number of node lines were determined.

110 A few studies can be found which used numerical models to assess the modal behavior  
111 and the stresses beared by prototype runners [20,21]. Nevertheless, the information is  
112 not discussed in detail. Angehrn and Dubas [22] carried out the modal and stress  
113 analysis of a 260 MW prototype runner, but mainly discussed the stress distribution  
114 endured by the structure.

115 In recent years, some authors have still addressed the dynamic behavior of Pelton  
116 runners. Mack et al. [23] performed a numerical and experimental study on a laboratory  
117 model to see the influence of water and mass variations on the natural frequencies of the  
118 runner. However, the study only focused on the circumferential bending modes and the  
119 influence of the shaft wasn't described. In other publications, numerical models of Pelton  
120 runners have been used to analyze some failures [24,25], but a detailed explanation of  
121 the runner modes is not provided.

122 In a nutshell, there are no publications providing a comprehensive analysis of the modal  
123 behavior of Pelton runners. The main bucket modes have been described, but the effect  
124 of attaching the buckets to the disk and the runner to the shaft is still to be discussed. In  
125 addition, most of the studies have relied on the use of numerical models and/or reduced  
126 scale models, but the modal analysis of real prototypes has not yet been published.

127 In this paper, the modal response of different Pelton turbines has been evaluated. Four  
128 different prototype runners were available to perform experimental tests. In the first  
129 part of the study, a numerical and experimental investigation of one of the prototype  
130 turbines has been carried out in a systematic way to determine the influence of the  
131 different components of the turbine. The study begins with the identification of the main  
132 modal shapes of a single bucket using Finite Element Method (FEM). Then the modal  
133 shapes and natural frequencies of the whole runner without any constraints are  
134 analyzed with another FEM numerical model. The results obtained are validated with  
135 experimental tests. At the end, the influence of every component of the turbine is  
136 discussed. In the second part of the study, another prototype installed in the same power  
137 plant (same operating conditions) but with some differences in the mechanical design  
138 has been analyzed experimentally to determine the variation in the frequencies between  
139 runners of the same specific speed  $n_s$ . Finally, the modal behavior of two other turbines  
140 belonging to different power plants (different  $n_s$ ) is evaluated.

141

## 142 *2. First Pelton turbine investigated*

143 The first Pelton turbine is located near a mountainous site in Catalonia (Spain). The  
144 machine has a horizontal setup with two equal runners attached on both ends of the  
145 shaft and is supported by two bearings (see Figure 2). Each runner is operated by a  
146 single jet arranged below the machine in horizontal direction. The characteristics of the  
147 turbine are listed in Table 1. To perform the first study, a new prototype runner was  
148 available outside the machine, hanging from a rope. Thanks to a 3D scanner, the  
149 geometry of the runner was virtually reproduced. From now on, this prototype will be  
150 referred to as 1-A.

151



Figure 2. View of the Pelton unit

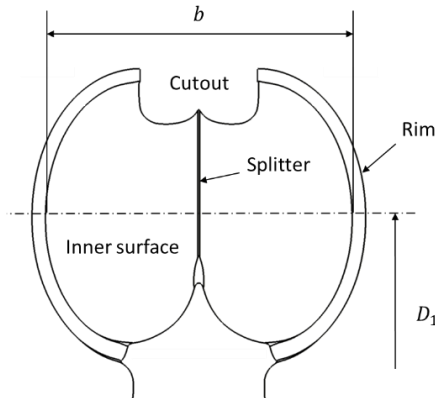
<b>Rated head</b>	770,45 m	<b>Nozzles</b>	2
<b>Rated output</b>	35 MW	<b>Jet diameter <math>d_j</math></b>	170 mm
<b>Speed</b>	600 min <sup>-1</sup>	<b>Specific speed <math>n_s</math></b>	19
<b>Runners</b>	2	<b>Groups</b>	2

Table 1. Characteristics of the turbine

152

153 The geometry of a Pelton runner is quite complex, especially the buckets. They must be  
 154 shaped to have maximum hydraulic efficiency, but also must guarantee enough  
 155 structural resistance to bear the forces applied to them during operation. Hence their  
 156 design must be a compromise between a hydraulic and a structural optimal shape.  
 157 However, the characteristics of a Pelton turbine are usually represented by two principal  
 158 dimensions: the pitch diameter  $D_1$ , which is defined as double the minimum distance  
 159 between the axis of the water jet and the runner axis, and the bucket width  $b$ . The ratio  
 160 between these two values is related to the specific speed of the turbine, and thus to the  
 161 head. A sketch of a Pelton bucket is shown in Figure 3 and the characteristics of the  
 162 studied runner are listed in Table 2.

163



<b>Pitch diameter <math>D_1</math></b>	1900 mm
<b>Bucket width <math>b</math></b>	547 mm
<b>No. of buckets</b>	22
<b>Ratio <math>b/D_1</math></b>	0,288
<b>Material</b>	Stainless steel Cr/Ni 13/4
<b>Year of construction</b>	2016

Table 2. Characteristics of runner 1-A

Figure 3. Main dimensions of a Pelton runner

164

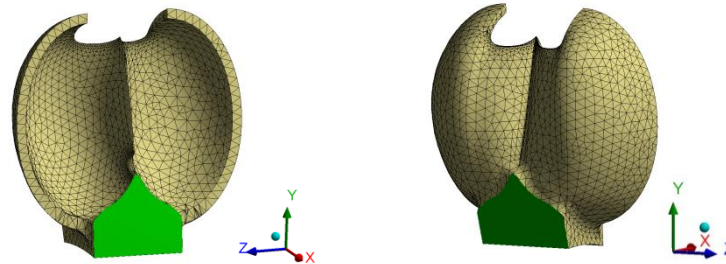
### 165 3. Modal analysis of a Pelton bucket

#### 166 3.1. Numerical model

167 The first step consisted in analyzing numerically the modal shapes of a single bucket.  
 168 The CAD geometry of the bucket was obtained from the 3D scanned model of the runner  
 169 and the simulation of the modal behavior was then carried out by means of Finite  
 170 Element Methods. The software used to this effect was the Modal module from ANSYS®  
 171 W [26]. First, the geometry of the bucket was meshed with 25000 elements after  
 172 performing a mesh sensitivity analysis. The areas with sharper edges and highest  
 173 curvatures, such as the cutout or the base of the middle ridge, had a more refined mesh.

174 In order to analyze the pure bucket modes without the effect of the disk, the boundary  
 175 conditions for this simulation were to fix the area connected to the rest of the runner (see  
 176 the green areas in Figure 4). The x-direction corresponds to the tangential direction of  
 177 the runner, y-direction to the radial and z-direction to the axial direction. The main  
 178 characteristics of the numerical model are listed in Table 3.

179



180

*Figure 4. Front and rear view of the meshed bucket*

181

*Table 3. Characteristics of the bucket model*

<b>Conditions</b>	Fixed base
<b>Material</b>	Stainless steel
<b>Density</b>	7500 kg/m <sup>3</sup>
<b>Young's modulus</b>	2x10 <sup>11</sup>
<b>Type of mesh elements</b>	Tetrahedral
<b>Number of mesh elements</b>	25000

182

### 183 *3.2. Mode shapes description*

184 Front and side views of the first eight mode shapes are displayed in Figure 5. The areas  
 185 with minimum displacement are represented in blue and the ones with maximum  
 186 displacement are represented in red. It is worth noting that the color scale is not the  
 187 same in all the pictures, since the program takes as a reference the maximum and  
 188 minimum values of every mode. The shape of the bucket without deformation is depicted  
 189 with the light wired figure. The first group of modes can be related to the first natural  
 190 mode of a cantilever beam, in which the structure tilts over the clamped base. In the first  
 191 and the second one, the bucket bends in the axial and tangential direction of the runner,  
 192 respectively. The bucket axial mode is found in the lowest frequency because the  
 193 stiffness is lower in the axial direction than in the tangential direction. For both mode  
 194 shapes the tips of the bucket deform in phase. In the third mode, or counter phase  
 195 tangential mode, the bucket halves bend tangentially in opposite phases, thus giving  
 196 certain torsion around the radial axis of the bucket. In this mode shape we can also see  
 197 that the splitter is deforming. The fourth natural mode involves the deformation of the  
 198 rims of the bucket, which deform in the axial direction in counter-phase, spreading and  
 199 contracting.

200 The following four modes are featured by the appearance of transversal nodes on the  
 201 bucket, what increases the stiffness, and thus the frequency. In the fifth and sixth modes  
 202 the tips of the bucket deform in the radial direction, with the same and opposed phases  
 203 on both sides, respectively. The deformation of the rims in the axial direction is also



204 relevant. The seventh mode is similar to the sixth mode, although it combines axial and  
 205 radial deformation of the tips and has a large twist in the splitter. In the eighth mode  
 206 two transversal nodes appear and the bucket tips deform in the radial direction. Since  
 207 the main dimensions of a bucket are standardized in relation to the jet diameter, we can  
 208 assume that the modes found in this case are representative of most Pelton runners,  
 209 with slight design variations, such as the attachment of ribs at the rear side of the  
 210 buckets to increase bending stiffness.

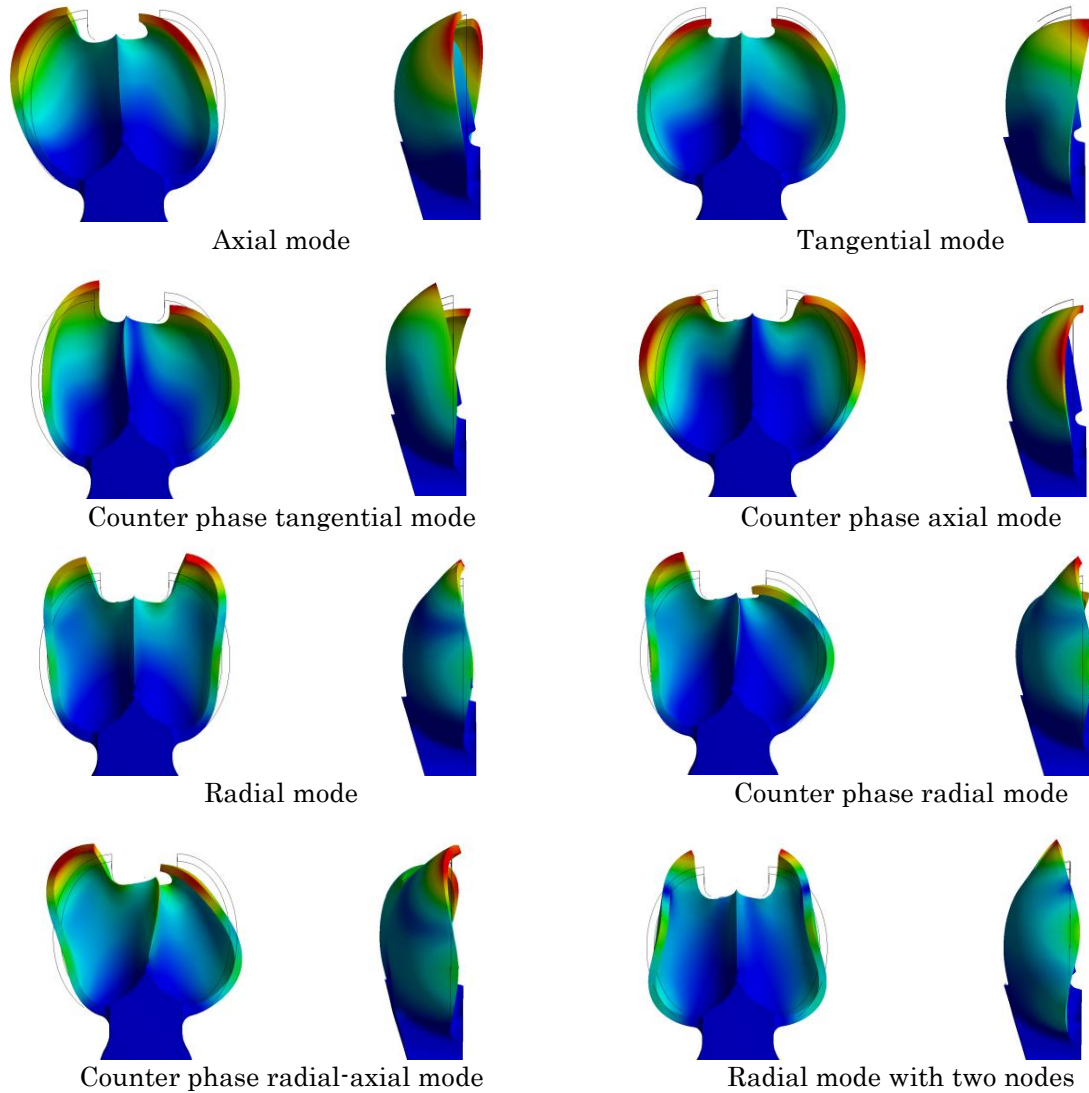


Figure 5. Pure bucket modes

## 211 4. Modal analysis of the Pelton runner

### 212 4.1. Numerical model

213 After performing the analysis of a single bucket the modal simulation of the whole  
 214 runner was carried out. The software used was the same as the single bucket simulation.  
 215 A mesh sensitivity analysis was done prior to the investigation to ensure a balance  
 216 between the stability of the simulation results, and reasonable time and computational

217 resources. The mesh density was increased in the sharp edges and the locations were  
 218 typically a high concentration of stresses occur. Since the object of this simulation was to  
 219 understand the modal behavior of the runner without the effect of the shaft, the  
 220 boundary conditions for this model were setting the structure as a free body, which  
 221 means that the runner had no constraints. The main characteristics of the final runner  
 222 model are listed in Table 4. Two views of the mesh are displayed in Figure 6.

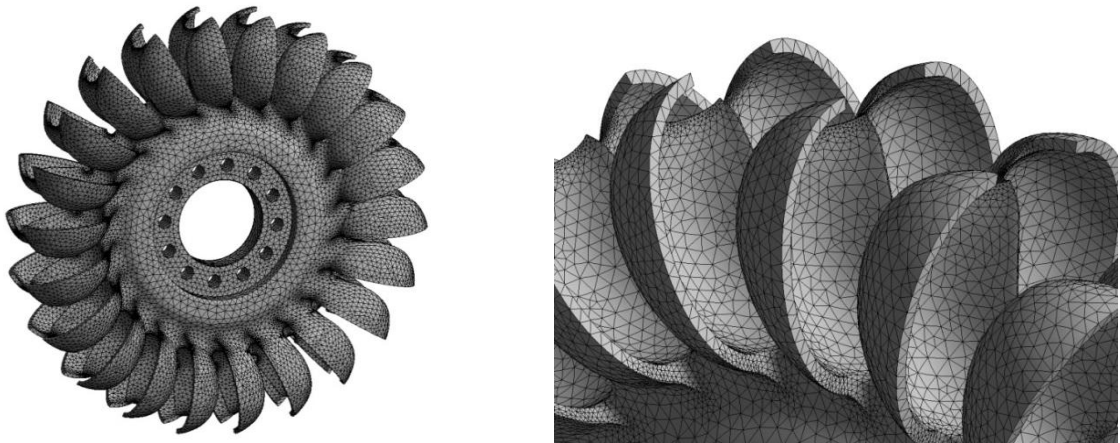
223

<b>Conditions</b>	Free body
<b>Material</b>	Stainless steel
<b>Density</b>	7500 kg/m <sup>3</sup>
<b>Young's modulus</b>	2x10 <sup>11</sup>
<b>Type of mesh elements</b>	Tetrahedral
<b>Number of mesh elements</b>	400000

224

*Table 4. Information about the simulation of the runner*

225



*Figure 6. Left: Mesh of the whole runner, right: detailed mesh of the buckets*

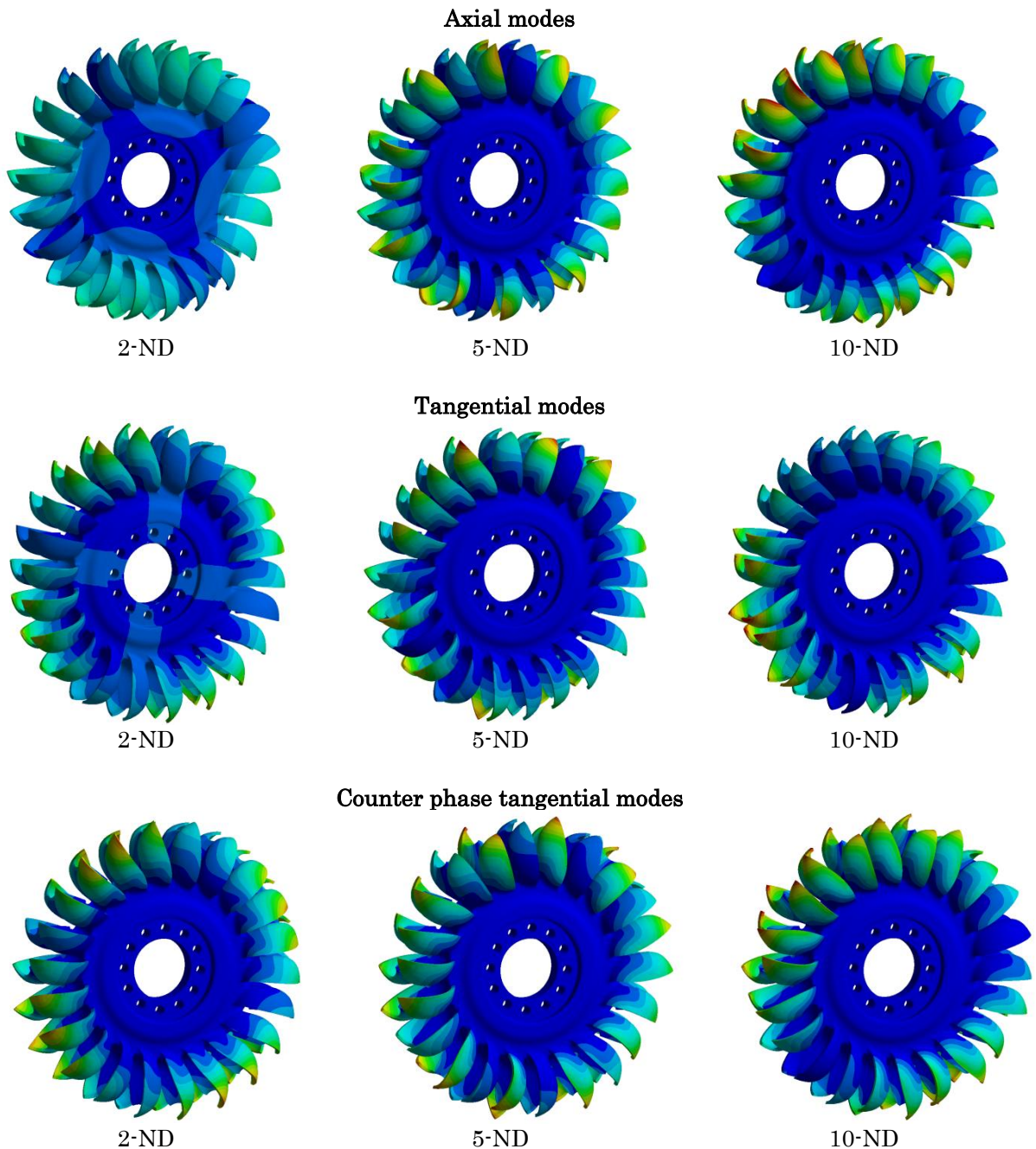
226

#### 227 4.2. Mode shapes description

228 The simulation shows that the runner has several variants of every bucket mode shape,  
 229 in which both the wheel and the buckets deform. Every variant is featured by a certain  
 230 number of nodal diameters of the wheel. These nodal lines delimit the runner areas that  
 231 vibrate in opposite phases, and between them the buckets and the disk have maximum  
 232 deformation. The buckets located on the nodal diameters do not deform. It is worth  
 233 noting that as the number of nodal diameters increase, so does the vibrational frequency  
 234 of the structure. There is a total amount of 22 modal shapes, which reach a maximum  
 235 number of 11 nodal diameters.

236 The axial, tangential and counter phase tangential modal shapes of the Pelton runner  
 237 with two (2-ND), five (5-ND) and ten (10-ND) nodal diameters are displayed in Figure 7.  
 238 The order of appearance of the mode shapes is the same as in the pure bucket modes. In  
 239 the first place, one finds the axial modes, where wheel and buckets are deforming in the  
 240 axial direction. When the number of nodes increases, the deformation of the disk is  
 241 minimized and only a deformation in the area of the buckets is noticed. In that case the

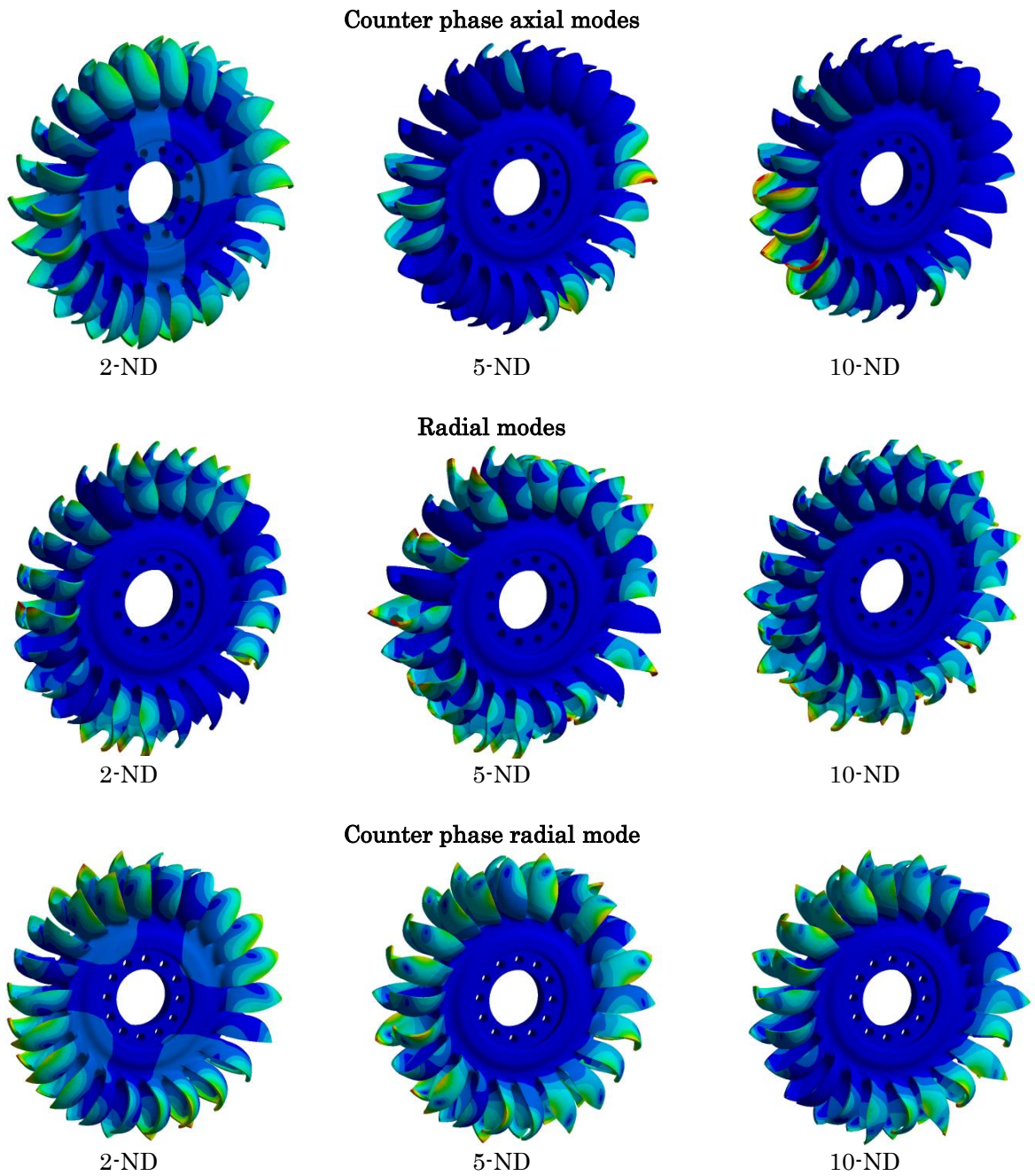
242 maximum deformation, which is found at the tip of the buckets, is increased with respect  
 243 to lower ND modes. The second group corresponds to the tangential modes. The disk  
 244 deforms in the radial direction for the first ND modes, while for higher ND only the  
 245 bending of the buckets is significant, just like in the axial case. In the third group, the  
 246 counter phased tangential mode involves the movement of the bucket tips in opposite  
 247 phases. In this case there is no deformation of the disk in the 2-ND, and the maximum  
 248 deformation of the tips of the buckets is kept almost constant for different ND, unlike  
 249 the previous cases.



*Figure 7. Runner modes*

250 In Figure 8 the next four modes of the Pelton runner are displayed also for 2-ND, 5-ND  
 251 and 10-ND. One can clearly identify the 2-ND runner deformation in all of them, except  
 252 in the radial mode, in which the deformation in the disk is much lower than in the

253 buckets. In the counter phase axial modes, the deformation of the disk is in the radial  
 254 direction, while in the counter phase radial mode it is in the axial direction. In all of  
 255 them, the deformation of the tip/rim of the buckets increases with the number of nodal  
 256 diameters.



*Figure 8. Runner modes 2*

257 *4.3. Experimental study*

258 The numerical results were validated by performing an Experimental Modal Analysis  
 259 (EMA) in the prototype. The first experimental test was addressed to studying the pure  
 260 mode shapes and frequencies of the Pelton runner, without the influence of the rotor.  
 261 Therefore, the runner was suspended with a rope and placed in an accessible location of  
 262 the power station to carry out impact tests. This was equivalent to considering the



263 runner as a free body (without any constraint) due to the small influence of the rope on  
264 the vibration of the runner. The procedure consisted in doing impacts with an  
265 instrumented hammer on different locations of the structure and in recording the  
266 vibrational response with different sensors. The ones used in this experiment were  
267 industrial K-Shear® accelerometers from KISTLER type 8752A50. Their sensitivity was  
268 100 mV/g and the acceleration range  $\pm 50$  g. They were mounted on clean and flat  
269 locations of the runner to ensure reliable and accurate measurements. The hammer had  
270 a sensitivity of 223  $\mu$ V/N.

271 To perform the EMA the location of the sensors was chosen accurately to represent the  
272 main mode shapes of the runner. Four accelerometers were placed on the surface of the  
273 disk (axial direction) with approximately 90 degrees between them and the same radial  
274 position. Moreover, four accelerometers were placed on different locations of the buckets  
275 to detect the axial, tangential and radial modes. The axial accelerometers were adhered  
276 on the outer surface of the bucket rim, approximately in the middle section, and the  
277 tangential and radial accelerometers were placed on the tip of the bucket, on  
278 perpendicular surfaces. Since the vibration of a runner is complex, different series of  
279 impacts were carried out. In each series the accelerometers were relocated in different  
280 parts of the buckets. One distribution to detect tangential modes of the runner is  
281 displayed in Figure 9.



282

283

*Figure 9. Accelerometers disposition on the hanged runner*

284 The vibration time signal was transmitted to a recording module LAN-XI Data  
285 Acquisition Hardware from Brüel & Kjær. The recorded time signal was later converted  
286 into an acceleration-frequency signal (spectrum) by performing a Fast Fourier  
287 Transformation (FFT). The time-force signal from the instrumented hammer was also  
288 transmitted to the acquisition module and was later used to perform a Frequency  
289 Response Function (FRF) with the signal from the accelerometer. The FRF divides the  
290 accelerometer signal by the hammer signal thus representing the excitability of the  
291 structure, regardless of the force magnitude. An Operational Deflection Shape (ODS)  
292 was created to identify the modal shapes of the runner for every natural frequency (see  
293 Figure 10 left). The frequencies were obtained by peak picking from the FRF of different  
294 sensors. In Figure 10 right the FRF of one accelerometer can be seen. The lower chart is  
295 the amplitude of the acceleration/force ratio and the upper chart is the phase difference  
296 between the accelerometer and hammer signal. Two representations of tangential mode  
297 shapes are displayed in the graph; 1-ND on the left and 4-ND on the right.

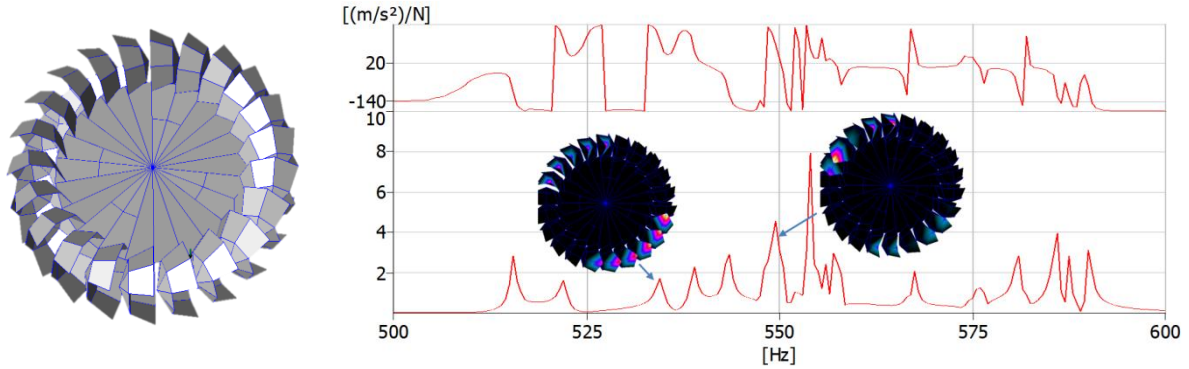


Figure 10. Left, ODS model of the Pelton runner; right, FRF and ODS mode shapes of experimental impacts

298 In Table 5, the axial natural frequencies obtained from the experimental tests are  
 299 compared to the frequencies obtained with the numerical model. In Table 6, the same  
 300 comparison is shown for the tangential frequencies. It can be seen that the mode shapes  
 301 of the real runner correspond to the ones of the numerical model and for all the  
 302 frequencies the error is less than 2%. With this, it is proved that the numerical model  
 303 created is validated because it represents well the real prototype behavior.

	2-ND	3-ND	4-ND	5-ND	6-ND	7-ND
<b>Exp. [Hz]</b>	195,5	356	424	449,5	461	466,5
<b>Num. [Hz]</b>	194,3	353,9	423,1	449,4	461,1	467,1
<b>Rel. error [%]</b>	< 1%	< 1%	< 1%	< 1%	< 1%	< 1%

304 Table 5. Comparison of runner axial frequencies between simulation and experiment

	2-ND	3-ND	4-ND	5-ND	6-ND	7-ND
<b>Exp. [Hz]</b>	517	541,75	547,5	549	550,7	552,25
<b>Num. [Hz]</b>	519,3	545,6	551,97	554,87	556,32	557,09
<b>Rel. error [%]</b>	< 1%	< 1%	< 1%	1,07%	1,02%	< 1%

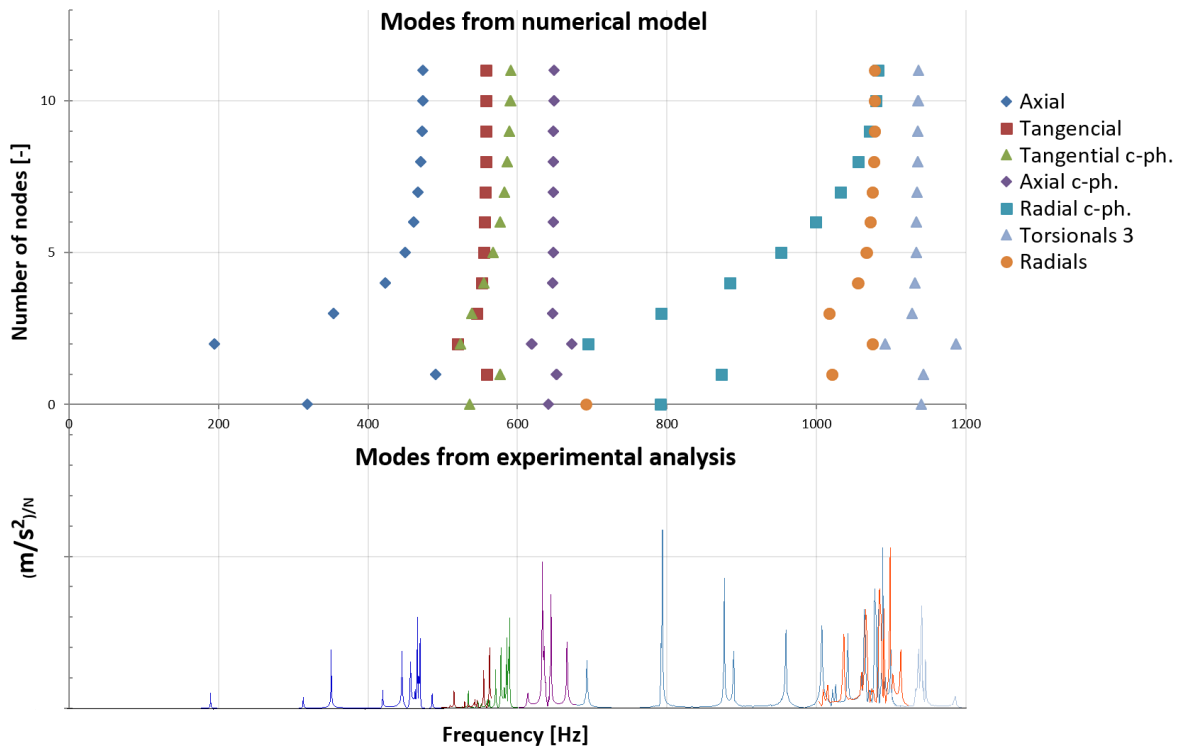
305 Table 6. Comparison of runner tangential frequencies between simulation and experiment

306

## 307 5. Results and discussion

308 This section attempts to analyze the modes of a Pelton runner by comparing the  
 309 vibrational modes of the wheel with those of a single bucket. The interaction between  
 310 both elements will be studied in order to have a better understanding of the modal  
 311 behavior of the turbine. In Figure 11, the frequencies obtained from the numerical model  
 312 (upper chart) can be compared with the experimental results (lower chart). Every type of  
 313 bucket mode shape is attributed a color to relate more easily the peaks from the  
 314 experimental tests with the numerical modes. The modes studied are located in a range  
 315 between 0 and 1200Hz. The first modes found are the axial modes between 195 Hz and  
 316 473 Hz. They are followed by the tangential modes (in phase and counter phase), which  
 317 appear between 519 and 591Hz and overlap each other. Next the counter phase axial  
 318 modes are found in a very narrow frequency range between 618 and 648 Hz. The counter  
 319 phase radial modes and the radial modes are found in the next frequency band, being the  
 320 former ones spread over a wide range of frequencies and the latter concentrated in a

321 narrow band (1017 to 1077 Hz). Similar to the counter phase axial modes, the counter  
 322 phase radial-axial modes have small differences between them.



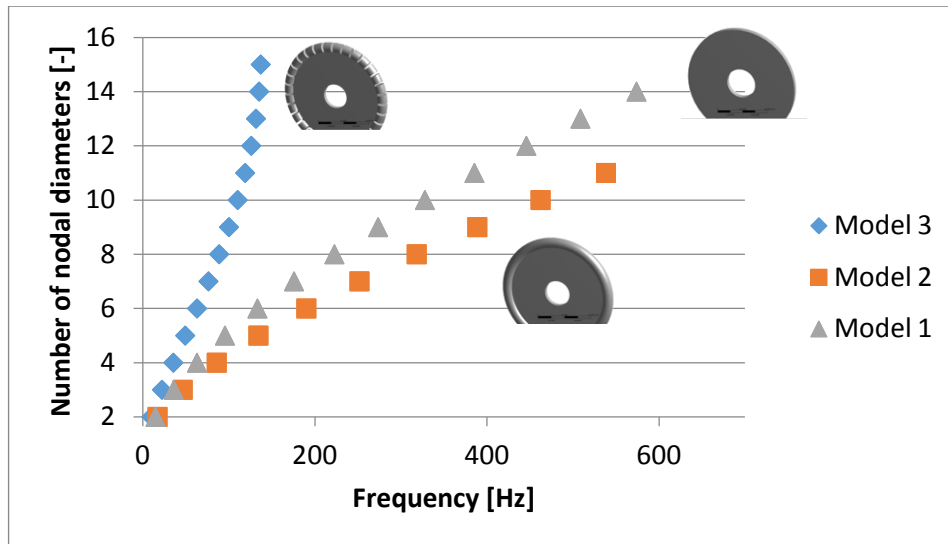
323

324 *Figure 11. Numerical and experimental modes of a Pelton runner: a) Numerical results, b)*  
 325 *Response spectrum after the impacts*

326 *5.1. Analysis of the coupling between the disk and the buckets*

327 A noticeable fact is the nonlinear increase of the frequencies, which are distributed in a  
 328 similar way to an asymptote. The modes in lower frequencies are those with a fewer  
 329 number of runner nodal diameters. As the number of nodal lines increases, so does the  
 330 frequency, but at a decreasing rate. For instance, the axial mode with 2 ND has a  
 331 frequency of 194,3Hz and the one with 3 ND a frequency of 353,9Hz, which is about 82%  
 332 of increase. However, the 4-ND mode is found at 423,1Hz, which is less than 20%  
 333 increase with respect to the 3-ND mode. After 5-ND, frequencies converge in a small  
 334 range, with a frequency variation lower than 3%. This effect is caused by the interaction  
 335 between the modal shapes of the disk and the modal shapes of the buckets.

336 To appreciate the contribution of every component to the global modal shapes of the  
 337 turbine, three simplified models have been studied: a disk (model 1), a disk with a single  
 338 peripheral mass (model 2) and a disk with separated peripheral masses (model 3). The  
 339 distribution of frequencies can be seen in Figure 12. In model 1, the frequencies increase  
 340 steadily with the number of nodal diameters. This is attributed to the fact that the mass  
 341 oscillating between the nodal diameters becomes more restricted, thus having an  
 342 increase in the stiffness. Attaching a peripheral mass to the disk (model 2) increases the  
 343 overall stiffness of the modes. However, when it comes to the disk with attached masses  
 344 (model 3), the frequencies change their distribution. The maximum frequency is lowered.  
 345 We can say that for a fewer number of nodal diameters the modes are governed by the  
 346 disk (the frequencies are very similar for both models) while for an increased number of  
 347 nodal diameters, the modes become more dominated by the masses.



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Figure 12. Frequencies of a disk and of a disk with masses

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To see the progression from a disk-dominated mode to a bucket-dominated mode the deformation of the outer diameter of the disk in the axial direction in model 1 and model 3 has been represented in Figure 13. It can be seen that in model 1 the maximum displacement is kept almost constant. This fits the behavior observed in the distribution of frequencies: the modal mass is kept almost constant, and the frequency increases steadily due to the stiffness increase. However, in model 3 the deformation of the disk decreases largely between 2 ND and 4 ND. From 5 ND the deformation barely shows a reduction. Some conclusions can thus be extracted from the chart. First, the increase in disk nodal diameters from 2 ND to 4 ND happens to reduce largely the vibrating mass of the disk, which explains the large difference between the first frequencies. Second, for modes higher than 5 ND the deformation of the disk is very small and shows almost no variation, even if the number of nodal diameters is increased. It can be said then that in the higher modes almost all the vibration is performed by the buckets, and that their base is similarly restricted, what help explaining why all the frequencies are so similar. Therefore, when studying the runner, the mode in the highest frequency, which corresponds to 11 ND, has the most resemblance to a pure bucket mode. The base of the bucket has the smallest angle between nodal diameters, being the most rigid one.

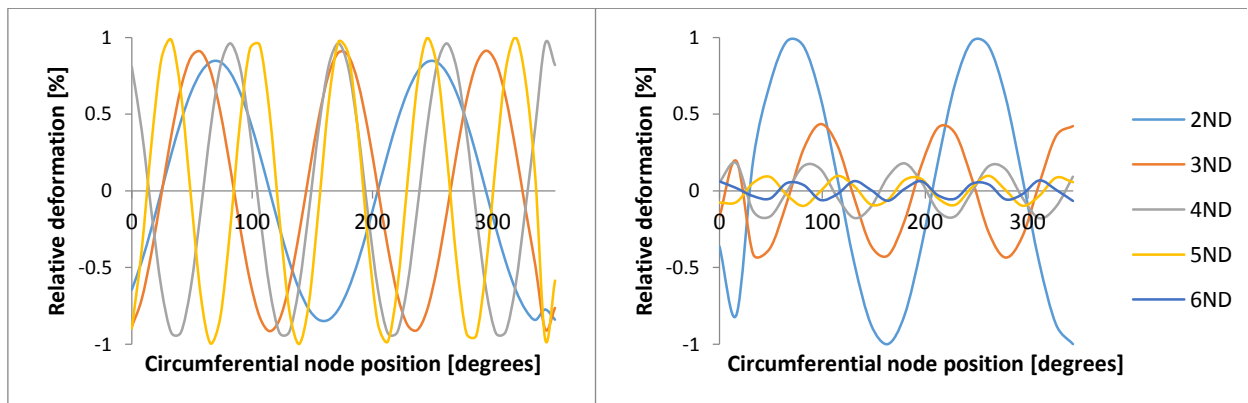


Figure 13. Relative deformation of the outer periphery modes in: left, the disk and right, the disk with masses



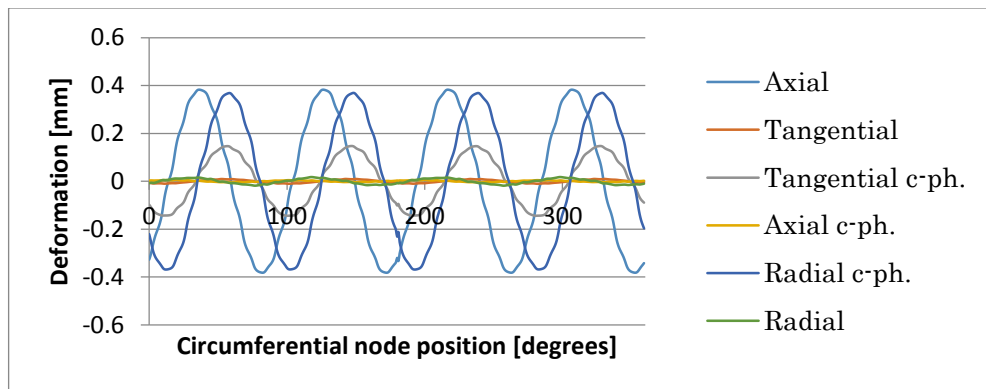
368 To check the similarity of the 11-ND mode to a simple bucket vibration, another analysis  
 369 has been performed. The natural frequencies of a single bucket and the natural  
 370 frequencies obtained in the 11-ND runner mode are indicated in Table 7. The frequencies  
 371 are similar, with less than 4% of difference. It is worth noting, though, that this  
 372 variation differs from one bucket mode shape to another. The axial and the counter  
 373 phase radial mode shapes are the most affected by the change in the boundary  
 374 conditions of the disk.

	Single bucket [Hz]	Runner mode 11ND [Hz]	Rel. difference [%]
Axial	487,8	473,7	2,98%
Tangential	564,9	558,2	1,20%
Tangential c-ph.	590,9	591,2	0,05%
Axial c-ph.	635,0	648,6	2,10%
Radial c-ph.	1120,4	1082,3	3,52%
Radial	1091,9	1077,9	1,30%

375 *Table 7. Variation in the bucket frequencies for different modes*

376 *5.2. Effect of the bucket mode shapes*

377 As indicated above, the effect of the coupling to the disk affects differently on every  
 378 bucket mode. Some of them converge in a small range, like tangential and counter phase  
 379 axial modes. Such behavior can be attributed to the nature of the deformation of the  
 380 bucket and its interaction with the disk. Some modes imply a low contribution of the  
 381 disk, for example the counter phase axial modes, which are very local and only affect the  
 382 rim of the buckets. For these types of modes, a change in the stiffness of the coupling to  
 383 the disk doesn't affect much the vibration of the buckets. On the other hand, it is also  
 384 important to mention that the increase in the number of nodal diameters affects  
 385 basically the stiffness in the axial direction. In circumferential and radial direction, the  
 386 disk is very stiff, regardless of the number of nodal diameters. Therefore, the modes with  
 387 a large axial deformation of the disk (see axial and counter phase radial in Figure 7 and  
 388 Figure 8) are more sensitive to the disk stiffness variation. Tangential and counter phase  
 389 axial modes have a significant deformation of the disk but in the radial direction. In  
 390 Figure 14 the deformation of the runner of 2-ND mode shapes at the base of the buckets  
 391 in the axial direction has been plotted. It is clear that the axial and counter phase  
 392 radials are the mode shapes most influenced by the axial stiffness of the disk.



393  
 394 *Figure 14. 2-ND axial deformation of the base of the buckets for every mode shape*

## 396 **6. Modal behavior of other Pelton turbines**

397 The main geometrical features of a Pelton turbine are related to its head  $H$ , discharge  $Q$   
 398 and number of nozzles. The specific speed is defined as

$$n_s = \frac{nQ^{0.5}}{E^{0.75}} \quad \text{Eq. (1)}$$

399 where  $n$  is the rotational speed of the turbine,  $Q$  the discharge and  $E$  the specific energy  
 400 and is used to classify hydraulic machines. The specific speed  $n_s$  is related to the head of  
 401 the turbine and, for Pelton, this is connected to the ratio between the runner diameter  
 402 and the jet diameter  $D/d_j$ . In like manner, the discharge of the turbine is related to the  
 403 dimensions of the bucket. Thus the dimensions of a Pelton runner (from the hydraulic  
 404 point of view) are characterized by the specific speed  $n_s$ . However, even for the same  $n_s$ ,  
 405 runners may bear some differences, especially in the structural design, depending on the  
 406 manufacturer and on the year of construction.

407 So far, the modal behavior of Pelton turbines has only been studied for a single case. To  
 408 determine to which extent the results are representative of other turbines, a runner with  
 409 the same basic dimensions and characteristics as the one studied previously is analyzed  
 410 in order to determine how the modal behavior may vary between designs with the same  
 411 hydrodynamic features. Afterwards, other runners with different specific speeds will be  
 412 analyzed and compared.

### 413 *6.1. Influence of the structural design for runners with the same $n_s$*

414 The runner studied in this section is located in the same power plant and has the same  
 415 features of runner 1-A (listed in Table 2). The runner is connected to the rest of the  
 416 machine. However, the mechanical design is different. This prototype will be referred to  
 417 as 1-B. Figure 15 left shows a view of the turbine 1-B with the casing disassembled.



Figure 15. Left, runner 1-B attached to the machine and left, accelerometers placed on one bucket

418 An experimental modal analysis was carried out to determine the modal shapes and  
 419 frequencies of the runner. The procedure and the instrumentation were the same  
 420 described in the experiment of prototype 1-A. Impacts in the axial, radial and tangential

421 direction were performed on all the accessible buckets and the natural frequencies and  
 422 corresponding mode shapes were identified. A picture of the accelerometers during the  
 423 impact tests is shown in Figure 15 right.

424 The experimental results of runner 1-B are presented in Figure 16. It is seen that the  
 425 runner mode shapes appear in the same order as the runner 1-A. The modes with fewer  
 426 nodal diameters (dominated by the deformation of the disk) have lower frequencies than  
 427 the modes with more nodal diameters (dominated by the bucket). As seen in the previous  
 428 results, the axial and counter-phase radial modes are also the most affected by the  
 429 stiffness of the disk, because of the difference between the frequencies of 2-ND and 11-  
 430 ND. The distribution is asymptotic as well. However, the natural frequencies of runner  
 431 1-B are higher than 1-A.

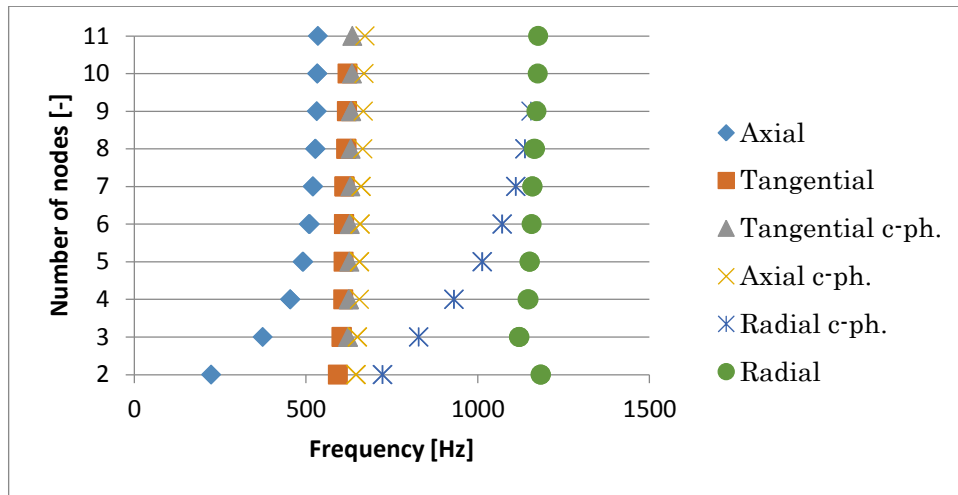


Figure 16. Left, natural frequencies of an old runner with buckets with ribs and right, view of the runner

432 In Table 8, the axial frequencies of runners 1-B have been compared to those of runner 1-  
 433 A. The frequencies dominated by the deformation of the disk have a difference of less  
 434 than 2% between both designs. Nevertheless, as the vibration becomes more located in  
 435 the buckets the difference between 1-B and 1-A increases. The frequencies of the bucket-  
 436 dominated modes of design 1-B are around 10% higher than 1-A. In Table 9, the  
 437 difference in tangential frequencies between both designs is also presented. All the  
 438 modes are affected similarly by the variation in the mechanical design, being the  
 439 frequencies of runner 1-B around 9% higher than the runner 1-A.

	3ND	4ND	5ND	6ND	7ND	8ND	11ND
Runner 1-B [Hz]	373,5	453,5	489,8	508	519,3	526,3	534,3
Runner 1-A [Hz]	369,8	426,9	450,3	461,3	467,2	470,5	473,7
Rel. diff. [%]	1,0	5,8	8,1	9,2	10,0	10,6	11,0

Table 8. Axial frequencies in the old runner and the new runner

440  
 441  
 442

	3ND	4ND	5ND	6ND	7ND	8ND	11ND
Runner 1-B [Hz]	603,3	608,1	609,1	610,1	611,4	617	620,6
Runner 1-A [Hz]	550,8	553,1	555,5	556,5	557,2	557,6	558,2
Rel. diff. [%]	8,7	9,0	8,8	8,8	8,9	9,6	10,1

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*Table 9. Tangential frequencies in the old runner and the new runner*

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From the previous analysis, it can be deduced that the mechanical design mainly affects the frequencies dominated by the vibration of the buckets. The difference between runners of the same hydraulic design can be up to 10%. This variation is mainly due to the shape of the buckets. The buckets of runner 1-A and runner 1-B are represented in Figure 17. The design is different, especially in the back area. The buckets of the older runner design (1-B) are supported by ribs in the rear side in order to increase the stiffness and the resistance to the impacts of the jet. The new runner design (1-A), though, does not have these supports and has a more polished shape. This design variation has a large effect on the natural frequencies. It can be thus concluded that for prototypes with the same hydrodynamic characteristics, only the disk dominated modes can be expected to be similar. The bucket-dominated modes change due to a variation of the rear design, and with the less supported buckets of new designs, the bucket-dominated modes can be significantly reduced.

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*Figure 17. Left, buckets of runner 1-A and, right, buckets of runner 1-B*

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### *6.2. Influence of the structural design for runners with different $n_s$*

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In the previous section, the modal behavior of two Pelton runners with the same dimensions and hydraulic design has been studied. To see if this can be extrapolated to other machines, the modal behavior of a prototype Pelton turbine with a different hydraulic design has been studied experimentally (see Figure 18). The information regarding the turbine and the runner, which will be referred to as runner 2, is listed in Table 10 and Table 11 respectively. The procedure followed to identify the natural frequencies and the modal shapes was similar to the one described for the first studied turbine.



Figure 18. View of the Pelton unit 2

<b>Rated head</b>	556 m	<b>Nozzles</b>	2
<b>Rated output</b>	12 MW	<b>Jet diameter <math>d_j</math></b>	135 mm
<b>Speed</b>	600 min <sup>-1</sup>	<b>Specific speed <math>n_s</math></b>	17
<b>Runners</b>	1	<b>Groups</b>	3

Table 10. Characteristics of Pelton turbine 2

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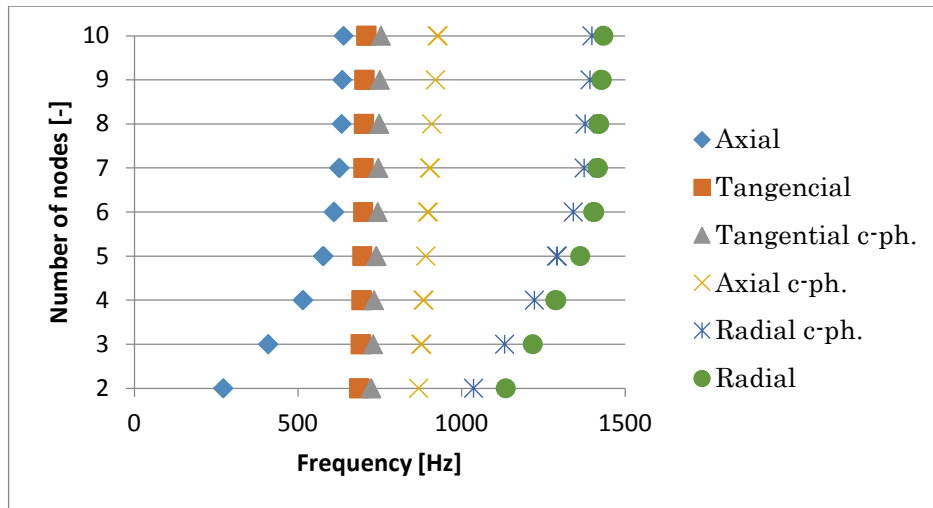
<b>Pitch diameter <math>D_I</math></b>	1500 mm
<b>Jet diameter <math>d_j</math></b>	135 mm
<b>Bucket width <math>b</math></b>	427 mm
<b>No. of buckets</b>	21
<b>Ratio <math>b/D_I</math></b>	0,285
<b>Material</b>	Stainless steel Cr/Ni 6/4

469

Table 11. Characteristics of runner 2

470 The results obtained after the impact tests are represented in Figure 19. The natural  
 471 frequencies of this prototype are higher compared to the prototype 1-A and 1-B because  
 472 of the reduced size of the runner. Nevertheless, the order of appearance of the runner  
 473 mode shapes is the same, even though the dimensions are different. The lowest  
 474 frequencies correspond to the axial mode shapes of the runner, and these are followed by  
 475 the tangential and axial counter-phase modes. Regarding the distribution of frequencies,  
 476 the axial and the radial counter-phase modes are the most affected by the stiffness of the  
 477 disk and show an asymptotic transition from disk-dominated modes to bucket-dominated  
 478 modes. In this case, though, also the radial in-phase modes interact with the disk, while  
 479 in the other prototypes this mode shape wasn't affected by the stiffness of the base. Since  
 480 the mode shapes of the bucket become more complex at higher frequencies, the similarity  
 481 between different prototypes is reduced due to the variation in the design. However, the  
 482 simplest vibration modes of the disk and the buckets (axial and tangential modes) show  
 483 a similar pattern.





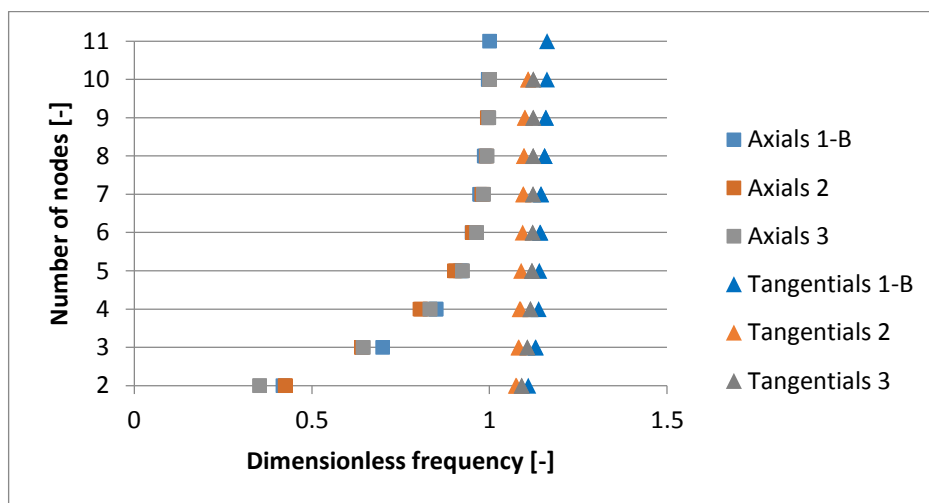
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Figure 19. Frequency distribution for Pelton turbine 2

486 Even though the prototypes studied have similar modal characteristics, they have  
 487 different dimensions ( $b/D_1$ ) and sizes. To be able to compare the results between runners  
 488 with different features, the frequencies have to be represented in a non-dimensional  
 489 way. In Figure 20, the axial and tangential frequencies of three studied runners have  
 490 been represented. All the frequencies have been referred to the maximum frequency of  
 491 the axial modes, the one corresponding to 11-ND, to see if the non-dimensional  
 492 distribution of the modes and the distance between axial and tangential modes follow  
 493 the same pattern between different runners. The prototypes represented in the chart are  
 494 runner 1-B and 2. Moreover, the results obtained from a turbine with a rated head of 800  
 495 m ( $n_s=18$ ), which will be referred to as runner 3, have also been included. The data  
 496 regarding runner 3 is not disclosed due to confidentiality.

497 Observing the results in Figure 20, it can be seen that for the three studied runners the  
 498 relative distribution of natural frequencies is comparable. Regarding the axial modes,  
 499 the transition from disk-dominated to bucket-dominated modes follows the same  
 500 asymptotic pattern. The first disk-dominated axial frequency (2-ND) is 30-45% lower  
 501 than the highest axial frequency (11-ND). Tangential frequencies are around 15% higher  
 502 with respect to the bucket-dominated axial frequencies in the three cases.



503

504

Figure 20. Axial and tangential natural frequencies of several runners in a non-dimensional form

505 With this information, it can be concluded that for the studied Pelton runners, the modal  
506 behavior has very similar characteristics. The simplest vibration modes of the buckets  
507 can be compared to the bending of a cantilever beam. Bending in the circumferential  
508 direction of the disk is always more difficult for the bucket than bending in the axial  
509 direction, so the axial modes are always at lower frequencies than tangential modes. In  
510 addition, since the main dimensions of the buckets of a Pelton runner are always relative  
511 to their width  $b$  (with small differences depending on the manufacturer, year of  
512 construction and so on), it is reasonable to expect tangential frequencies to appear near  
513 the axial frequencies. Future studies should be aimed at gathering more experimental  
514 data from prototypes and at determining how this behavior varies depending on the  
515 design, manufacturing process, material and so on.

516

## 517 *7. Conclusions*

518 In this paper a detailed study of the modal response of Pelton turbines is presented.  
519 Prototypes from different power plants have been analyzed numerically and  
520 experimentally for this purpose.

521 The first part of the study consisted in analyzing in-depth the modal behavior of a Pelton  
522 runner without constraints (without being connected to the shaft). The prototype was  
523 hanged by a rope in the power plant and fully accessible to perform experimental tests. A  
524 3D scanning of the complete structure was performed to obtain the CAD geometry of the  
525 runner. The analysis was carried out following a systematic approach. First, the mode  
526 shapes of the bucket were determined numerically. Then, the whole runner structure  
527 was analyzed numerically and experimentally. After that, the effects of the interaction  
528 between the buckets and the disk were discussed.

529 The typical modes of a Pelton bucket were classified according to the direction of the  
530 deformation and to the number of transversal nodes. In this way, the bucket modes were  
531 identified as axial, tangential or radial, and as phase or counter-phase, depending on the  
532 phase between the bucket halves.

533 The analysis of the runner was performed with a numerical model and with a thorough  
534 experimental investigation. The results showed that for every kind of bucket mode, the  
535 runner has several variants, which are coupled to the modes of the disk ( $N$  nodal  
536 diameters). For each group of bucket modes, the frequencies increase with the number of  
537 nodal diameters of the disk. It was noticed that in lower frequencies the vibration is  
538 global to all the runner (behaves like a disk) and in higher frequencies the vibration is  
539 more restricted and is dominated by the vibration of the buckets. In addition, some  
540 bucket modes are more affected by the stiffness of the disk than others and this affects  
541 the distribution of frequencies. For example, tangential modes gather in a small  
542 frequency range, while axial modes are more spread. The natural frequencies of the  
543 whole runner are lower than the ones obtained with a fixed single bucket.

544 The second part of the study consisted in determining the effect of structural design in  
545 turbines with the same hydrodynamic characteristics. For that purpose, an experimental  
546 modal analysis was performed in another prototype of the same power plant. The main  
547 dimensions and specific speed were the same. The results showed that the mode shapes  
548 appear in the same order and that the frequencies are similarly distributed. The

549 frequencies dominated by the vibration of the disk are similar in both runners. However,  
550 the bucket-dominated frequencies vary up to 10%, because of the different bucket design  
551 and especially the use of rear supports in the older runner.

552 In the third part, a prototype with different hydraulic and mechanical characteristics  
553 was studied experimentally. The results indicated that the mode-shapes, as well as their  
554 order of appearance, are the same. The natural frequencies were distributed in a similar  
555 way, except the radial counter-phase modes.

556 Finally, to overcome the difficulty of comparing turbines with different sizes and  
557 dimensions, the axial and tangential frequencies of three runners with different designs  
558 were represented in a non-dimensional way. The results showed that the distribution is  
559 similar and that tangential frequencies are 15% higher than axial bucket-dominated  
560 frequencies. This is reasonable due to the standardized dimensions of the buckets with  
561 respect to its width.

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