OPTIMAL DESIGN OF LATTICE STRUCTURE CONSIDERING CONSTRAINTS THROUGH ADDITIVE MANUFACTURING PROCESS

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Abstract. In this research, an optimization design of the lattice structure is investigated, which is based on the ground structure method. In recent years, as a result of rapid development of additional manufacturing technology, it has become possible to manufacture complicated shapes including periodic lattice structure. However, in real metal lattice samples, geometric imperfections may exist in every unit because of the stochastic influence of feasible processing path. In this study, a new ground structure method which does not leave elements with small cross sectional area was proposed. In particular, the effect of geometric constraints caused by additive manufacturing techniques on optimized results are discussed based on the robust topology optimization combining perturbation methods for quantifying uncertainty. In the end, a robust topology optimization was also discussed as a problem to minimize expected value and standard deviation of compliance.

1 INTRODUCTION

Recently, due to the dramatic advance of additive manufacturing technology for metals using laser beams, more precise and complicated structures can be produced easily. For example, the micro-lattice structure which contains slender beams with a dimension of micrometre can be fabricated by using a metal 3D printer based on Selective Laser Melting (henceforth, depict as SLM) technology. As a result, many researches on topology optimization for lightweight structure produced by 3D printers have been published in many journals. However, the metal 3D printer cannot produce any shapes, and there are some geometrical constraints for producible shapes. If some constraints cannot be satisfied, large amount of geometrical imperfections that cannot meet the requirements and lack of mechanical response would be observed. Besides,
since the lattice structures are composed of slender beams, the optimized results obtained by ground structure method can be applied directly as a final product. Also, in the ground structure method, the numerical model is composed of 2D beams and trusses, the numerical algorithm becomes more simple than the conventional optimization methods for continuous model.

In our study, an optimization design of the lattice structure is investigated, which is based on the ground structure method, and the effects of structural robustness on the optimized results were discussed as a problem to minimize expected value and standard deviation of compliance.

2 ANALYTICAL METHOD

2.1 Conventional ground structure method

The ground structure method is one of the optimization method that finds the optimum structure excluding unnecessary elements in the frame structure in which each node is connected by a large number of beam elements. The conventional ground structure method sets the cross-sectional area of each member by using the following equation;

\[ A_e = \alpha_e A_{\text{max}}, \quad 0 \leq \alpha_e \leq 1 \quad (e = 1, 2, \ldots, N_e) \]  

(1)

In Eq.(1), \( A_e \) is the cross-sectional area of each beam element, \( A_{\text{max}} \) is the maximum cross-sectional area, \( \alpha_e \) is the design variable of each element of element number \( e \) changing between 0 and 1, and \( N_e \) is the number for all elements. In our optimization algorithm, the cross-sectional area of each element was optimized by changing the design variable \( \alpha = \{ \alpha_1, \alpha_2, \ldots, \alpha_{N_e} \} \).

The optimization problem dealt with in this research is formulated as the following problem which minimizes the compliance \( C \) of the whole structure under a volume constraint \( W \);

\[
\text{minimize} \quad C(\alpha) = d^T K d
\]

subject to \( W(\alpha) = \sum_{e=1}^{N_e} W(\alpha_e) \leq W \)

\[
0 \leq \alpha_e \leq 1 \quad (e = 1, 2, \ldots, N_e)
\]

(2)

In Eq.(2), \( K \) is the overall stiffness matrix and \( d \) is the nodal displacement vector. \( W(\alpha) \) is the total volume of the member, and \( W \) is the constraint value of the total volume.

2.2 Ground structure method applying three-phase material model

As described in Section 2.1, the ground structure calculation is a method for finding the optimum structure by regarding the cross-sectional area of each member as a design object. However, this method does not necessarily provide a simple shape consisting of thick elements only, and some thin elements would remain. Therefore, in order to eliminate such thin whose design variable \( \alpha_e \) has a value between 0 and 1 effectively, Eq.(3) is generally used to calculate the cross section as;

\[ A_e = \alpha_e^p A_{\text{max}}, \quad 0 \leq \alpha_e \leq 1 \quad (e = 1, 2, \ldots, N_e) \]  

(3)
As compared with Eq.(1), Eq.(3) has an exponent $p$ for $\alpha_e$. Equation (3) is simple and easy to be programmed, but there is an unavoidable problem for fabricating the optimized shape, since there are no restriction of the minimum size for $A_e$.

On the other hand, as for the cross-sectional area $A_e$ for struts in a lattice fabricated by SLM, it can be designed freely within the range $A_{\min} \leq A_e \leq A_{\max}$. However, in the method for calculating the cross-sectional area of each beam element by using Eq.(3), the cross-sectional area would converge to 0 or $A_{\max}$. Based on this fact, a new formula for controlling the cross-sectional area is used as follows:

$$A_e = \beta_e p \{ (1 - \alpha_e)A_{\min} + \alpha_e A_{\max} \} \quad (4)$$

This equation is based on the theory of three-phase material model proposed by Kato et al.[1] Since it is not restricted that the cross-sectional area of the beam element is between $A_{\min}$ and $A_{\max}$, no penalties is set for the design variable $\alpha_e$. On the other hand, since the design variable $\beta_e$ has a penalty, it is unlikely that there is an element whose cross-sectional area is less than $A_{\min}$.

In this paper, the minimum producible diameter is set to 0.2 mm, so the minimum cross-sectional area can be calculated as $A_{\min} = 0.031416 \text{mm}^2$. In addition, the maximum producable diameter is 0.4 mm, and the maximum cross-sectional area is set to $A_{\min} = 0.125665 \text{mm}^2$.

### 2.3 Robust optimization method

In a real metal lattice specimen, it contains some geometric uncertainty and does not always have the rigidity as designed. So, in this research, we performed robust optimization, assuming that Young’s modulus $E$ of each beam element contains uncertainty. The Young’s modulus $E$ of the beam element is modeled by an uncorrelated normal distribution such that the standard deviation $\sigma$ is 10% of the averaged Young’s modulus.

In our robust optimization, Eq.(5) is used instead of the objective function in Eq.(2) as;

$$\text{minimize } \hat{C} = \frac{w_E}{\mu^*}E[C] + \frac{1 - w_E}{\sigma^*}\text{Std}[C] \quad (5)$$

Here, $E[C]$ and $\text{Std}[C]$ represent the expected value of compliance and the standard deviation of compliance, respectively. Considering that the Young’s modulus of each element is uncorrelated, $E[C]$ and $\text{Std}[C]$ are respectively obtained by Eq.(6) as;

$$E[C] = f^T d_0 + \sigma^2 \sum_i d_i^T K_0 d_i$$

$$\text{Std}[C] = \sqrt{f^T \left( \sigma^2 \sum_i d_i d_i^T \right) f} \quad (6)$$

Here, $\mu^*$ is the value of $E[C]$ for $(w_E/\mu^*, (1 - w_E)/\sigma^*) = (1, 0)$, and $\sigma^*$ is the value of $\text{Std}[C]$ for $(w_E/\mu^*, (1 - w_E)/\sigma^*) = (0, 1)$. Also, the parameter $w_E$ is a constant arbitrarily defined between 0 and 1 and adjusts the weight of the expected value and standard deviation of compliance in
the objective function. In this paper, \( w_E \) is varied between 0 and 1, and the influence of the difference in the objective function on the optimized result is examined.

The sensitivity to the design variable \( \alpha_e \) in the objective function defined by Eq.(5) can be calculated from Eq.(7):

\[
\frac{\partial \hat{C}}{\partial \alpha_e} = -\frac{w_E}{\mu^h} \left\{ d_e^T \frac{\partial K_{e0}}{\partial \alpha_e} d_{e0} + \zeta^T e \frac{\partial K_{e0}}{\partial \alpha_e} d_{e0} + \sigma^2 \sum_i \left( d_{ei}^T \frac{\partial K_{ei}}{\partial \alpha_e} d_{e0} + 2 d_{ei}^T \frac{\partial K_{ei}}{\partial \alpha_e} d_{ei} \right) \right\}
- \frac{1 - w_E}{\sigma^2 \text{Std}[C]} \left\{ \sigma^2 \sum_i f^T d_i \left( d_{ei}^T \frac{\partial K_{ei}}{\partial \alpha_e} d_{e0} + 2 d_{ei}^T \frac{\partial K_{ei}}{\partial \alpha_e} d_{ei} \right) \right\}
\]

(7)

Here, parameters \( d_i, K_i \) and \( \zeta \) in Eqs.(5) and (6) can be found in reference[2]. The sensitivity to the design variable \( \beta_e \) is also determined in the same method as \( \alpha_e \).

Robust optimization can be summarized in the form of the following algorithm:

1. Set initial model data (shape, material characteristics, boundary conditions). Determine the initial value of the element design variables in vectors \( \alpha \) and \( \beta \).

2. Assemble the stiffness matrix \( K_0 \) without uncertainty.

3. Structural analysis is performed using \( K_0 \) to solve the displacement vector \( d_0 \).

4. Assemble \( K_i \) and solve for \( d_i \) and \( \zeta \).

5. Evaluate the expected value and standard deviation of compliance using Eq.(6).

6. Compute the sensitivity of the objective function using Eq.(6).

7. Update the element design variables in vectors \( \alpha \) and \( \beta \) using OC method.

8. Check convergence; if not converged go to step 2.

2.4 Boundary conditions and physical property values

In this study, an optimization design is conducted or the core part (shown in pink) in a sandwich structure subjected to three-point bending load as shown in Fig. 1(a). As a boundary condition, a 1/4 model as shown in Fig. 1(b) was used by considering a geometrical symmetry. A sandwich structure with a core of BCC lattice structure as shown in Fig. 2 is used as the base model. In this study, the volume constraint is set to 208.96 mm\(^3\) according to the volume of the core part when the diameter of all beam elements in the BCC lattice structure shown in Fig. 2 is 0.2 mm. In addition, the dimensions and physical properties of each part are shown in Table 1 below.
Figure 1: Schematic of three-point bending condition

(a) Schematic of three-point bending condition (b) Boundary condition applied in FEM

Figure 1: Analysis model

Figure 2: Layout of BCC lattice structure before optimization

Table 1: Values of geometrical and material parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>The length of between indenters $l$</td>
<td>64 mm</td>
</tr>
<tr>
<td>The length of the sandwich beam $L$</td>
<td>80 mm</td>
</tr>
<tr>
<td>The width of the sandwich beam $w$</td>
<td>10 mm</td>
</tr>
<tr>
<td>The thickness of core $t_c$</td>
<td>7.5 mm</td>
</tr>
<tr>
<td>The thickness of facesheet $t_f$</td>
<td>1 mm</td>
</tr>
<tr>
<td>External force $P$</td>
<td>100 N</td>
</tr>
<tr>
<td>Young’s modulus of the core and facesheet $E$</td>
<td>180 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio of the core and facesheet $\nu$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

3 ANALYSIS RESULTS

3.1 Effect of proposed equation on cross-sectional area range

Figure 3 shows the optimization results of the existing method by using Eq.(3) and the proposed method using Eq.(4). In addition, Fig. 4 shows a histogram of the number of beam elements in each section area.

The optimization discussed in this section does not use the robust optimization method according to Eq.(5), but uses the objective function shown in Eq.(2).

The optimized structure obtained by the conventional method with $p = 1$ has beam elements with small diameter. Some of the struts are not satisfied the minimum condition $A_{\text{min}} \leq A_e$, so they cannot be fabricated by SLM. The optimized structure obtained by the existing method with $p = 2$ has a convergence in cross section of almost all elements to 0 or the maximum, and the
compliance $C$ increase in comparison with the case of $p = 1$. On the other hand, the optimum structure obtained by the proposed method has elements with various cross-sectional areas, and there are no elements whose area is less than $A_{\text{min}}$. In addition, the compliance $C$ is almost the same as the conventional method with $p = 1$.

(a) Conventional method with $p = 1$ ($C = 3.786$)  
(b) Conventional method with $p = 2$ ($C = 4.138$)

(c) Proposed method ($C = 3.799$)

**Figure 3:** Comparison of results in each method

(a) Conventional method with $p = 1$  
(b) Conventional method with $p = 2$

(c) Proposed method

**Figure 4:** Histogram of the number of beam elements in each section area
3.2 Robust optimization results

The effects of the weights of the objective function on the optimal shape are compared in Fig. 5. Figure 5(b) shows the optimum structure obtained by \( w_{E} = 1.0 \), namely, the analysis that minimizes the expected value of compliance. It is almost the same as the optimal structure by deterministic optimization shown in Fig. 5(a). In addition, by comparing results shown in Figs. 5(b)–(d), it can be confirmed that as the value of \( w_{E} \) decreases, the number of beam elements with large cross-sectional area would decrease and the total number of beam elements increases.

Figure 6 shows the effect of the weighting factor \( w_{E} \) on the objective function. As shown in Fig. 6, the expected value of the compliance decreases and the standard deviation of compliance increased as \( w_{E} \) increases. Therefore, the expected value of the compliance and the standard deviation are in a trade-off relationship, and the balance can be arbitrarily determined by designers by changing the factor \( w_{E} \).

![Figure 5: Examination of the influence on the result by weight coefficient \( w_{E} \)](image)

![Figure 6: Relationship between weighting factor \( w_{E} \) and expected value and standard deviation of compliance](image)
4 FABRICATION AND EXPERIMENT

4.1 Verification of molding accuracy

In our study, the initial and optimized shape of BCC structures are fabricated using the metal 3D printer (3D Systems, ProX 300). A photograph of the fabricated test piece is shown in Fig. 7. In order to measure the beam diameter of the fabricated product, a test piece was photographed using a scanning electron microscope (SEM) JCM-6000. One of the images taken is shown in Fig. 8.

Figure 7: Photograph of test pieces molded by metal 3D printer

Figure 8: Picture of beam element taken by SEM

The diameter of ten beam elements a–j (see Fig. 9) was measured from the pictures. The design value of the diameter by optimization and the average value of the diameter obtained by measuring from the SEM image are compared in Tables 2 and 3 below.
According to Table 2 and 3, although there are some errors, the diameter of the designed beam element is almost equal to the measured value. Therefore, it is concluded that the lattice structure composed of struts with various area $A_e$ can be fabricated appropriately, which is based on the optimized result derived from using our ground structure method.

### 4.2 Experimental result

A three-point bending test was conducted for the fabricated test pieces. The load-displacement diagram obtained by the experiment is shown in Fig. 10. In addition, Table 4 shows the results of calculating compliance from the graph.

![Load-displacement diagram obtained by experiment](image-url)

**Figure 10:** Load-displacement diagram obtained by experiment
According to Fig. 10, compared to the test piece before optimization, the test piece after optimization is higher in rigidity and the maximum load is also larger. On the other hand, there are still large errors between the analysis value and the experimental value of compliance, which is due to the inferior quality of metal additive manufacturing on molding. As shown in Fig. 8, the fabricated test pieces have a roughness in the strut surface, and the diameter contributing to the mechanical properties would be smaller than the diameter obtained by measurement.

## 5 CONCLUSION

In this study, optimization was carried out using the new ground structure method in consideration of restriction due to additive manufacturing, and the following findings were obtained by performing the fabrication and three-point bending experiment.

1. In our method, the cross-sectional area $A_e$ is controlled between $A_{\text{min}}$ and $A_{\text{max}}$, which is based in the three-phase material model proposed by Kato et al.\cite{1}. The optimized shape can be fabricated by metal 3D printer.

2. In our calculation, the structural robustness is considered and the effect of designed performance (to minimize the expected value or the standard deviation for the compliance) on the optimized shape is discussed.

3. The optimum shape obtained in this research can be fabricated by using the metal 3D printer. The product quality is strongly dependent on the fabrication condition. In the next work, the structural optimal design in consideration with the appropriate fabrication condition and its robustness will be studied.

## REFERENCES


<table>
<thead>
<tr>
<th>Constituion</th>
<th>Analysis values</th>
<th>Experimental values</th>
<th>Relative error [%]</th>
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<td></td>
<td></td>
<td>Specimen ① Specimen ② Specimen ③ Average</td>
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<td>11.690 12.812 13.351 12.617</td>
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<td>5.335 6.392 6.098 5.907</td>
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<td>Fig. 7(c)</td>
<td>4.104</td>
<td>6.850 7.500 6.476 6.917</td>
<td>68.55</td>
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