

Appendix A. Code

```

package r3bot

import boofcv.alg.distort.brown.LensDistortionBrown
import boofcv.alg.geo.PerspectiveOps
import boofcv.struct.calib.{CameraModel, CameraPinholeBrown}
import georegression.struct.point.{Point2D_F64, Point3D_F64}
import r3bot.CameraInTheWorld.{AverageEstimation, RobustEstimation}
import r3bot.QRVision1.{cameraModel, logger, streamCamera}
import scalismo.registration.LandmarkRegistration
import scalismo.sampling._
import scalismo.sampling.proposals._
import scalismo.sampling.evaluators._
import scalismo.sampling.algorithms._
import scalismo.sampling.loggers.AcceptRejectLogger
import scalismo.sampling.proposals.MixtureProposal.implicit._
import scalismo.utils.Random

import scala.collection.{immutable, mutable}

object UncertaintyVision {

  implicit val random: Random = Random(56L)

  case class CameraTProposal(proposalTSdev: Double) (implicit rnd: Random)
    extends ProposalGenerator[PoseWithFocalLength] with
    SymmetricTransitionRatio[PoseWithFocalLength] {
    override def propose(current: PoseWithFocalLength):
    PoseWithFocalLength = {
      // given `current` -> create a new camera proposal
      val x = current.location.x + proposalTSdev *
      rnd.scalaRandom.nextGaussian()
      val y = current.location.y + proposalTSdev *
      rnd.scalaRandom.nextGaussian()
      val z = current.location.z + proposalTSdev *
      rnd.scalaRandom.nextGaussian()
      PoseWithFocalLength(Location(x, y, z), current.orientation,
      current.focallengthX, current.focallengthY)
    }
  }

  case class CameraRProposal(proposalRSdev: Double) (implicit rnd: Random)
    extends ProposalGenerator[PoseWithFocalLength] with
    SymmetricTransitionRatio[PoseWithFocalLength] {
    override def propose(current: PoseWithFocalLength):
    PoseWithFocalLength = {
      val xAngle = current.orientation.xAngle + proposalRSdev *
      rnd.scalaRandom.nextGaussian()
      val yAngle = current.orientation.yAngle + proposalRSdev *
      rnd.scalaRandom.nextGaussian()
      val zAngle = current.orientation.zAngle + proposalRSdev *
      rnd.scalaRandom.nextGaussian()
      // new angles are range limited, so that angles > than 360 deg
      reshape to the deg from 0
    }
  }
}

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        PoseWithFocalLength(
            current.location,
            Orientation(xAngle % (2 * math.Pi), yAngle % (2 * math.Pi),
zAngle % (2 * math.Pi)),
            current.focallengthX, current.focallengthY
        )
    }
}

case class IntrinsicProposal(focallengthProposalSdev: Double)(implicit
rnd: Random)
    extends ProposalGenerator[PoseWithFocalLength] with
SymmetricTransitionRatio[PoseWithFocalLength] {
    override def propose(current: PoseWithFocalLength):
PoseWithFocalLength = {
        val focallengthX = current.focallengthX + focallengthProposalSdev *
rnd.scalaRandom.nextGaussian()
        val focallengthY = current.focallengthY + focallengthProposalSdev *
rnd.scalaRandom.nextGaussian()
        PoseWithFocalLength(current.location, current.orientation,
focallengthX, focallengthY)
    }
}

/// project uncertainties in the world to the image plane
/// using the specified world-to-image transform and evaluate around the
anchor point
def projectWorldUncertaintyToImage(dx: Double, dy: Double, dz: Double,
worldToImage: Point3D_F64 =>
Point2D_F64,
                                anchorPoint: Point3D_F64): (Double,
Double) = {
    /// idea:
    /// build 8 points of 3d cube dx, dy, dz around anchorPoint
    /// project all to image plane
    /// take min/max in x and y direction
    val cubePoints: IndexedSeq[(Double, Double, Double)] = IndexedSeq(
        (0, 0, 0),
        (dx, 0, 0),
        (0, dy, 0),
        (0, 0, dz),
        (dx, dy, 0),
        (0, dy, dz),
        (dx, 0, dz),
        (dx, dy, dz)
    )
    /// place around anchor point
    val anchoredCube = cubePoints.map(case (x, y, z) =>
        new Point3D_F64(
            anchorPoint.x + x - dx/2,
            anchorPoint.y + y - dy/2,
            anchorPoint.z + z - dz/2)
    )
    /// project to image
    val imagePoints = anchoredCube.map(worldToImage)
    /// min / max
    val minY = imagePoints.map(_.y).min
    val maxY = imagePoints.map(_.y).max
    val minX = imagePoints.map(_.x).min

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    val maxX = imagePoints.map(_.x).max
    // lengths in each direction
    // minimal uncertainty in image is always half a pixel positioning
    error
    (math.max(0.5, maxX - minX), math.max(0.5, maxY - minY))
  }

  case class ImageFiducialLikelihood(measurementXSdev: Double,
    measurementYSdev: Double,
    measurementZSdev: Double,
    detectedFiducials:
IndexedSeq[Fiducial],
    cameraModel: CameraPinholeBrown)
  extends DistributionEvaluator[PoseWithFocalLength] {

    override def logValue(sample: PoseWithFocalLength): Double = {
      // get the camera from the pose we introduced
      val camera = PoseWithFocalLengthToCamera(sample, cameraModel)

      // total transform from world onto image plane, including
      perspective camera transform
      val worldToImage: Point3D_F64 => Point2D_F64 =
PerspectiveOps.createWorldToPixel(
        camera.cameraModel,
        camera.worldToCamera).transform

      val fidLikelihoods: IndexedSeq[Double] = detectedFiducials.map {
fid: Fiducial =>
        // fiducial 2d image position of all 4 corners
        val controlsInImage = fid.controls2D

        // fiducial 3d world positions of all 4 corners
        val worldPosition =
YuMiReferenceFiducials.referenceFiducialTransforms(fid.id)
        val refControls = fid.controlsInWorld(worldPosition)

        // evaluate distance
        val fiducialLH: Double = refControls.zip(controlsInImage).map {
          case (refPoint, observedPoint) =>
            // project reference into image
            val projectedRef: Point2D_F64 = worldToImage(refPoint)
            // pixel uncertainties of each corner in the image plane
            val (pixelXSdev, pixelYSdev) =
projectWorldUncertaintyToImage(
              measurementXSdev,
              measurementYSdev,
              measurementZSdev,
              worldToImage,
              refPoint)
            // evaluate difference to observed point with Gaussian model
            val logLHX = GaussianEvaluator(projectedRef.x,
pixelXSdev).logValue(observedPoint.x)
            val logLHY = GaussianEvaluator(projectedRef.y,
pixelYSdev).logValue(observedPoint.y)
            logLHX + logLHY
          }.sum
        // total likelihood of this fiducial (all corner cond.
        independent)

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        fiducialLH
    }
    // total likelihood of all fiducials (all fiducials cond.
independent)
    fidLikelihoods.sum
}
}

class ProposalLogger() extends AcceptRejectLogger[PoseWithFocalLength]
{
    var accepted: Int = 0
    var rejected: Int = 0

    var acceptedMap: mutable.Map[String, Int] = mutable.Map.empty[String,
Int].withDefaultValue(0)
    var rejectedMap: mutable.Map[String, Int] = mutable.Map.empty[String,
Int].withDefaultValue(0)

    override def accept(current: PoseWithFocalLength,
        sample: PoseWithFocalLength,
        generator:
ProposalGenerator[PoseWithFocalLength],
        evaluator:
DistributionEvaluator[PoseWithFocalLength]): Unit = {
        //println(f"Sample has been ACCEPTED: proposal=$sample (from
current=$current)")
        acceptedMap(generator.toString) += 1
        accepted += 1
    }

    override def reject(current: PoseWithFocalLength,
        sample: PoseWithFocalLength,
        generator:
ProposalGenerator[PoseWithFocalLength],
        evaluator:
DistributionEvaluator[PoseWithFocalLength]): Unit = {
        //println(f"Sample has been REJECTED: proposal=$sample (from
current=$current)")
        rejectedMap(generator.toString) += 1
        rejected += 1
    }
}

def SphericalGeomStats(angles: Seq[Double]): (Double, Double) = {
    // transform each angle to x, y
    def toXY(angle: Double): (Double, Double) = (math.cos(angle),
math.sin(angle))
    val XYAngles = angles.map(toXY)
    val xCoord = XYAngles.map(CartCoordinates => CartCoordinates._1)
    val yCoord = XYAngles.map(CartCoordinates => CartCoordinates._2)

    // average of [x, y]
    val xAnglesMean = xCoord.sum / xCoord.size
    val yAnglesMean = yCoord.sum / yCoord.size

    // standard deviation of [x, y]. Std Dev like sqrt(-2*ln(Module of
mean of samples))
    val AnglesStDev = math.sqrt(-math.log(math.pow(xAnglesMean, 2) +

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math.pow(yAnglesMean, 2))

    // transform [x, y] -> angle
    def toAngle(x: Double, y: Double): Double = {
        if (math.atan2(y, x) > 0) {
            math.atan2(y, x)
        }
        else {
            math.atan2(y, x) + 2 * math.Pi
        }
    }
    (toAngle(xAnglesMean, yAnglesMean), AnglesStDev)
}

def EuclideanGeomStats(values: Seq[Double]): (Double, Double) = {
    // average of [x, y]
    val Mean = values.sum / values.size

    // standard deviation of [x, y]. Variance like  $E[x^2]/N - M[x]^2$ 
    val valuesSq = values.map(value => math.pow(value, 2))
    val StDev = math.sqrt(valuesSq.sum / valuesSq.size - math.pow(Mean,
2))
    (Mean, StDev)
}

    // get camera with the focal length parameters from the new proposal
    def PoseWithFocalLengthToCamera(PoseWithFocalLength:
PoseWithFocalLength, cameraModel: CameraPinholeBrown): CameraInTheWorld =
    {
        val currentPose = Pose(PoseWithFocalLength.location,
PoseWithFocalLength.orientation)
        val newCameraModel: CameraPinholeBrown = new CameraPinholeBrown(
            PoseWithFocalLength.focallengthX, PoseWithFocalLength.focallengthY,
            cameraModel.skew, cameraModel.cx, cameraModel.cy,
cameraModel.width, cameraModel.height
        )
        newCameraModel.radial = cameraModel.radial.clone()
        newCameraModel.t1 = cameraModel.getT1
        newCameraModel.t2 = cameraModel.getT2

        CameraInTheWorld.fromCamera(currentPose.toSe3, newCameraModel)
    }

def main(args: Array[String]): Unit = {
    val image = streamCamera.nextGrayImage

    val proposalBroadTSdev: Double = 10
    val proposalBroadRSdev: Double = 0.1
    val proposalNarrowTSdev: Double = 1
    val proposalNarrowRSdev: Double = 0.01
    val FocalLengthProposalSdev: Double = 15
    val measurementXSdev: Double = 20
    val measurementYSdev: Double = 20
    val measurementZSdev: Double = 20

    // getting the IDs of reference fiducials
    val refIds = YuMiReferenceFiducials.referenceIds
    // detecting all fiducials that match the large size. In our case

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reference + objects are both large
    val fidSqBinaryLarge = Fiducial.findAllFiducials(image,
Fiducial.defaultSquareBinaryDetector(cameraModel, 47.0))
    // from all those fiducials, take only the ones that match the
reference IDs (to find the camera)
    val refFids = fidSqBinaryLarge.filter { fid =>
refIds.contains(fid.id) }
    // object fiducials set (no use for now)
    val objFids = fidSqBinaryLarge.filter { fid =>
!refIds.contains(fid.id) && fid.id != MagicWand.wandId }

    val likelihoodEvaluator: DistributionEvaluator[PoseWithFocalLength] =
ImageFiducialLikelihood(
    measurementXSdev, measurementYSdev, measurementZSdev,
    refFids,
    cameraModel
)

    // 4 proposals. A broad (bigger jumps) and a narrow (local
exporation) for T and R
    val proposalBroadT: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
CameraTProposal(proposalBroadTSdev)
    val proposalBroadR: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
CameraRProposal(proposalBroadRSdev)
    val proposalNarrowT: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
CameraTProposal(proposalNarrowTSdev)
    val proposalNarrowR: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
CameraRProposal(proposalNarrowRSdev)
    val proposalFocalLength: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
IntrinsicProposal(FocalLengthProposalsdev)

    val mixedProposal: ProposalGenerator[PoseWithFocalLength]
    with SymmetricTransitionRatio[PoseWithFocalLength] =
MixtureProposal(
    0.20 *: proposalBroadT + 0.20 *: proposalBroadR + 0.20 *:
proposalNarrowT + 0.20 *: proposalNarrowR + 0.20 *: proposalFocalLength
)

    val mh = Metropolis(mixedProposal, likelihoodEvaluator)

    // finding camera translation vector and rotation matrix
    val camera = CameraInTheWorld.findCameraPositionUsingManyFiducials(
    refFids,
    YuMiReferenceFiducials.referenceFiducialTransforms,
    cameraModel,
    //MostStableEstimation,
    AverageEstimation,
    //RobustEstimation(maxIteration = 100, inlierThreshold = 4.0)
)

    // obtain the CamParameters we need from the camera
camera match {
    case Some(camera: CameraInTheWorld) =>

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// translation
val x = camera.centerTranslation.getX
val y = camera.centerTranslation.getY
val z = camera.centerTranslation.getZ

// rotation
// transform of the matrix to a square matrix
val OrientationMatrix =
GeoUtils.boofToScalismo3D(camera.orientation)
// determination of Euler angles (phi, theta, psi) from the
retrieved rotation matrix
val Angles_3D =
LandmarkRegistration.rotMatrixToEulerAngles(OrientationMatrix.toBreezeMat
rix)

val xAngle = Angles_3D(2)
val yAngle = Angles_3D(1)
val zAngle = Angles_3D(0)

val focallengthX = cameraModel.fx
val focallengthY = cameraModel.fy

val initialCamEstimate = PoseWithFocalLength(x, y, z, xAngle,
yAngle, zAngle, focallengthX, focallengthY)
val logger = new ProposalLogger()

val samples = mh.iterator(initialCamEstimate,
logger).take(10000).toIndexedSeq

val posteriorSamples: IndexedSeq[PoseWithFocalLength] =
samples.drop(5000).take(5000)

// calculate the mean and standard deviation of the samples
// translation coordinates.
val xValues = posteriorSamples.map{case
PoseWithFocalLength(Location(x, _, _), Orientation(_, _, _),_,_) => x}
val yValues = posteriorSamples.map{case
PoseWithFocalLength(Location(_, y, _), Orientation(_, _, _),_,_) => y}
val zValues = posteriorSamples.map{case
PoseWithFocalLength(Location(_, _, z), Orientation(_, _, _),_,_) => z}
val (xMean, xStDev) = EuclideanGeomStats(xValues)
val (yMean, yStDev) = EuclideanGeomStats(yValues)
val (zMean, zStDev) = EuclideanGeomStats(zValues)

// rotation coordinates.
val xAngles = posteriorSamples.map(pose =>
pose.orientation.xAngle)
val yAngles = posteriorSamples.map(pose =>
pose.orientation.yAngle)
val zAngles = posteriorSamples.map(pose =>
pose.orientation.zAngle)
val (xAnglesMean, xAnglesStDev) = SphericalGeomStats(xAngles)
val (yAnglesMean, yAnglesStDev) = SphericalGeomStats(yAngles)
val (zAnglesMean, zAnglesStDev) = SphericalGeomStats(zAngles)

// focal length
val fx = posteriorSamples.map{case
PoseWithFocalLength(Location(_, _, _), Orientation(_, _, _),fx,_) => fx}

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    val fy = posteriorSamples.map{ case
PoseWithFocalLength(Location(_, _, _), Orientation(_, _, _), _, fy) => fy}
    val (fxMean, fxStDev) = EuclideanGeomStats(fx)
    val (fyMean, fyStDev) = EuclideanGeomStats(fy)

    println(f"accepted = ${logger.accepted} and rejected =
${logger.rejected}")
    println(f"acceptedMap = ${logger.acceptedMap}")
    println(f"rejectedMap = ${logger.rejectedMap}")

    println(f"Initial focal length: ${focallengthX},
${focallengthY}")
    println(f"All the results are shown in the order: x, y, z,
xAngle, yAngle, zAngle")
    println(f"The Camera is located in:")
    println(f"Means: ${xMean}, ${yMean}, ${zMean}, " +
    f"${xAnglesMean}, ${yAnglesMean}, ${zAnglesMean}")
    println(f"StdDev: ${xStDev}, ${yStDev}, ${zStDev}, " +
    f"${xAnglesStDev}, ${yAnglesStDev}, ${zAnglesStDev}")
    println(f"The Camera has the following focal length:")
    println(f"Mean: ${fxMean}, ${fyMean}")
    println(f"StdDev: ${fxStDev}, ${fyStDev}")

    // propagation of camera uncertainty to object uncertainty
    // get all cameras from the location samples sequence obtained
    val cameras: IndexedSeq[CameraInTheWorld] = samples.map(pose =>
PoseWithFocalLengthToCamera(pose, cameraModel))

    println("In process: estimation of the pose of the detected
fiducials/objects")
    val objectsForCam: IndexedSeq[IndexedSeq[VisualObject]] =
    // case focal length is constant. Initial cameraModel
maintained
    if (logger.acceptedMap(proposalFocalLength.toString) == 0) {
    // fiducials carry the camera model internally to provide an
estimated pose, and there's no need to change it now
    val objectsForCam: IndexedSeq[IndexedSeq[VisualObject]] =
cameras.map(
    (camera: CameraInTheWorld) =>
VisualObject.fromFiducials(objFids, camera)
    )
    objectsForCam
    }
    // case focal length independent to avoid calibration step
    else {
    // need to redetect and keep detector to reconfigure for each
intrinsic setting
    val fiducialDetector =
Fiducial.defaultSquareBinaryDetector(cameraModel, 47.0)
    fiducialDetector.detect(image)

    val objectsForCam: IndexedSeq[IndexedSeq[VisualObject]] = for
(camera <- cameras) yield {
    // need to update the camera model used by this fiducial
(internally)
    // reconfiguration of the detector with the new camera
model

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        val lensDistortion = new
LensDistortionBrown(camera.cameraModel)
        fiducialDetector.setLensDistortion(lensDistortion,
camera.cameraModel.width, camera.cameraModel.height)

        // re-estimation of fiducials pose
        val allFiducials =
Fiducial.fromAllDetections(fiducialDetector)
        val objFids = allFiducials.filter { fid =>
!refIds.contains(fid.id) && fid.id != MagicWand.wandId }

        // save all the visual object pose of all detected objects,
for this specific camera
        objFids.map(fid => VisualObject.fromFiducial(fid, camera))
    }
    objectsForCam
}
println(s"Calculation of object statistics")

    // 1: create long list of all objects (flatten: unpack inner
lists to 1 large list)
    val allObjectSamples: IndexedSeq[VisualObject] =
objectsForCam.flatten
    // 2: group by the object ids to get corresponding groups of
samples of the same object
    val objSamples = allObjectSamples.groupBy(obj => obj.id)
    // 3: for each id / object run the statistics analysis
    for (objId <- objSamples.keys) {
        // all samples belonging to this object id
        val visObjSamples = objSamples(objId)
        // mapping the Visual Object to a Pose (with Euler angles (phi,
theta, psi))
        val objectPoses = visObjSamples.map { case
VisualObject(location, orientation, _) =>
            val orientationMatrix =
GeoUtils.boofToScalismo3D(orientation)
            val Angles_3D =
LandmarkRegistration.rotMatrixToEulerAngles(orientationMatrix.toBreezeMat
rix)
            Pose(Location(location.x, location.y, location.z),
Orientation(Angles_3D(2), Angles_3D(1), Angles_3D(0)))
        }

        val posteriorObjPoses: IndexedSeq[Pose] =
objectPoses.drop(19).take(19)
        val fidXValues = posteriorObjPoses.map(pose => pose.location.x)
        val fidYValues = posteriorObjPoses.map(pose => pose.location.y)
        val fidZValues = posteriorObjPoses.map(pose => pose.location.z)
        val (fidXMean, fidXStDev) = EuclideanGeomStats(fidXValues)
        val (fidYMean, fidYStDev) = EuclideanGeomStats(fidYValues)
        val (fidZMean, fidZStDev) = EuclideanGeomStats(fidZValues)

        val fidXAngles = posteriorObjPoses.map(pose =>
pose.orientation.xAngle)
        val fidYAngles = posteriorObjPoses.map(pose =>
pose.orientation.yAngle)
        val fidZAngles = posteriorObjPoses.map(pose =>
pose.orientation.zAngle)

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        val (fidXAnglesMean, fidXAnglesStDev) =  
            SphericalGeomStats(fidXAngles)  
        val (fidYAnglesMean, fidYAnglesStDev) =  
            SphericalGeomStats(fidYAngles)  
        val (fidZAnglesMean, fidZAnglesStDev) =  
            SphericalGeomStats(fidZAngles)  
  
        println(f"Object fiducial with id: ${objId}")  
        println(f"has the location:")  
        println(f"${fidXMean} ${fidXStDev}")  
        println(f"${fidYMean} ${fidYStDev}")  
        println(f"${fidZMean} ${fidZStDev}")  
        println(f"${fidXAnglesMean} ${fidXAnglesStDev}")  
        println(f"${fidYAnglesMean} ${fidYAnglesStDev}")  
        println(f"${fidZAnglesMean} ${fidZAnglesStDev}")  
    }  
  
    case None =>  
        logger.debug("no origin found")  
    }  
}  
}
```

Appendix B. Improving the camera accuracy

Object	fiducial	with	id:	30#		
has	the	location:	0.92355	229.602	-8.5106	
with	this	uncertainty:	2.20928	3.00226	2.42432	
and	the	orientation:	6.23382	6.17671	3.12644	
with	this	uncertainty:	0.00412	0.0031	0.00654	
Object	fiducial	with	id:	944#		
has	the	location:	494.919	381.967	-35.88	
with	this	uncertainty:	2.28966	4.9687	2.8406	
and	the	orientation:	0.10424	0.10058	0.02039	
with	this	uncertainty:	0.00404	0.0032	0.00651	
Object	fiducial	with	id:	20#		
has	the	location:	244.763	-44.995	-28.618	
with	this	uncertainty:	3.03118	3.79848	2.565	
and	the	orientation:	0.0394	6.27264	2.30164	
with	this	uncertainty:	0.00464	0.00222	0.00617	
Object	fiducial	with	id:	5#		
has	the	location:	369.875	295.57	-36.377	
with	this	uncertainty:	2.2278	4.36358	2.53425	
and	the	orientation:	0.04777	6.25714	0.96565	
with	this	uncertainty:	0.00198	0.00475	0.00611	

Table B. 1. Object uncertainties in the case: Objects in $Z = 0 + 3 \text{ Ref Fid} + \text{Ref Fid } Z > 0 + \text{High Std Dev}$

Object	fiducial	with	id:	30#		
has	the	location:	1.59563	229.708	-6.017	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	6.2324	6.18237	3.12218	
with	this	uncertainty:	1.49E-08	2.11E-08	1.49E-08	
Object	fiducial	with	id:	944#		
has	the	location:	494.891	383.128	-34.489	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	0.10327	0.05112	0.01252	
with	this	uncertainty:	1.49E-08	1.49E-08	1.83E-08	
Object	fiducial	with	id:	20#		
has	the	location:	244.531	-44.754	-23.411	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	0.02609	6.27058	2.29863	
with	this	uncertainty:	0	1.83E-08	0	
Object	fiducial	with	id:	5#		
has	the	location:	370.101	295.591	-31.715	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	0.04141	6.26883	0.96606	
with	this	uncertainty:	2.11E-08	1.05E-08	0	

Table B. 2. Object uncertainties in the case: Objects in $Z = 0 + 3 \text{ Ref Fid} + \text{Ref Fid } Z > 0 + \text{Low Std Dev}$

Object	fiducial	with	id:	30#		
has	the	location:	-1.9207	235.056	-9.3583	
with	this	uncertainty:	2.14911	4.67679	5.54922	
and	the	orientation:	0.01316	6.2378	3.10821	
with	this	uncertainty:	0.00251	0.00237	0.00644	
Object	fiducial	with	id:	20#		
has	the	location:	238.642	-43.097	4.91044	
with	this	uncertainty:	3.28564	3.33607	4.66941	
and	the	orientation:	0.0271	0.0805	2.28619	
			4.82E-			
with	this	uncertainty:	04	0.00341	0.0065	
Object	fiducial	with	id:	5#		
has	the	location:	369.256	294.44	-15.081	
with	this	uncertainty:	2.02934	2.79574	5.02487	
and	the	orientation:	6.23944	0.01987	0.95445	
				9.12E-		
with	this	uncertainty:	0.00332	04	0.00646	
Object	fiducial	with	id:	22#		
has	the	location:	-25.855	-24.066	143.564	
with	this	uncertainty:	2.93029	4.45415	5.1188	
and	the	orientation:	6.18765	0.11013	0.01062	
with	this	uncertainty:	0.00262	0.00226	0.0063	

Table B. 3: Objects uncertainties in the case: Objects in Z = 0 + 3 Ref Fid + Ref Fid Z = 0 + High Std Dev

Object	fiducial	with	id:	30#		
has	the	location:	-1.6961	231.499	-1.1781	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	0.01503	6.23368	3.12367	
			3.49E-	1.49E-		
with	this	uncertainty:	08	08	0	
Object	fiducial	with	id:	20#		
has	the	location:	242.767	-43.473	11.8341	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	0.3686	6.11362	2.25139	
with	this	uncertainty:	0	0	0	
Object	fiducial	with	id:	5#		
has	the	location:	368.472	296.301	-10.555	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	6.25082	0.0058	0.967	
				1.05E-		
with	this	uncertainty:	0	08	0	
Object	fiducial	with	id:	22#		
has	the	location:	-21.859	-27.959	151.284	
with	this	uncertainty:	0.1781	0.40934	0.18743	
and	the	orientation:	6.18366	0.11315	0.02538	
				1.05E-	3.16E-	
with	this	uncertainty:	0	08	08	

Table B. 4: Objects uncertainties in the case: Objects in Z = 0 + 3 Ref Fid + Ref Fid Z = 0 + Low Std Dev

Object	fiducial	with	id:	22#		
has	the	location:	-24.762	-35.582	134.358	
with	this	uncertainty:	0.20757	0.3642	0.19634	
and	the	orientation:	6.24185	0.08732	0.03388	
with	this	uncertainty:	0	0	0	
Object	fiducial	with	id:	30#		
has	the	location:	4.68791	227.862	5.16268	
with	this	uncertainty:	0.20757	0.3642	0.19634	
and	the	orientation:	0.00623	0.02093	3.12648	
with	this	uncertainty:	0	1.49E-08	0	
Object	fiducial	with	id:	944#		
has	the	location:	495.205	382.297	10.3585	
with	this	uncertainty:	0.20757	0.3642	0.19634	
and	the	orientation:	0.04046	6.246	0.00251	
with	this	uncertainty:	0	0	2.58E-08	
Object	fiducial	with	id:	5#		
has	the	location:	370.151	295.385	0.45136	
with	this	uncertainty:	0.20757	0.3642	0.19634	
and	the	orientation:	6.23916	6.23069	0.96662	
with	this	uncertainty:	0	0	2.11E-08	
Object	fiducial	with	id:	20#		
has	the	location:	243.419	-45.223	1.6539	
with	this	uncertainty:	0.20757	0.3642	0.19634	
and	the	orientation:	6.23366	0.06898	2.29511	
with	this	uncertainty:	0	0	0	

Table B. 5: Objects uncertainties in the case: Objects in $Z = 0 + 2$ Ref Fid + Ref Fid $Z = 0 +$ Low Std Dev

Object	fiducial	with	id:	22#		
has	the	location:	-4.5016	3.25411	-15.944	
with	this	uncertainty:	3.19382	4.00857	5.21947	
and	the	orientation:	6.25431	0.04543	0.00593	
with	this	uncertainty:	0.0026	0.00227	0.00645	
Object	fiducial	with	id:	30#		
has	the	location:	2.74369	233.388	-9.5111	
with	this	uncertainty:	2.20763	3.96029	5.62994	
and	the	orientation:	6.26095	6.26352	3.1139	
with	this	uncertainty:	0.00252	0.00235	0.0065	
Object	fiducial	with	id:	944#		
has	the	location:	498.319	378.266	-7.4502	
with	this	uncertainty:	1.9692	2.51964	5.04907	
and	the	orientation:	0.02536	6.26101	6.27613	
with	this	uncertainty:	0.00257	0.0023	0.00658	
Object	fiducial	with	id:	5#		
has	the	location:	372.311	294.4	-16.882	
with	this	uncertainty:	2.06371	2.60801	5.10793	
and	the	orientation:	6.26949	6.25753	0.95811	
with	this	uncertainty:	0.00332	9.26E-04	0.00661	
Object	fiducial	with	id:	20#		
has	the	location:	241.337	-44.596	-4.6617	
with	this	uncertainty:	3.41936	2.91574	4.74621	
and	the	orientation:	0.02073	0.04914	2.28843	
with	this	uncertainty:	4.81E-04	0.00341	0.00652	

Table B. 6: Object uncertainties in the case: Objects in $Z = 0 + 2$ Ref Fid + Ref Fid $Z = 0 +$ High Std Dev

Object	fiducial	with	id:	911#		
has	the	location:	500.588	10.5468	-40.039	
with	this	uncertainty:	5.43005	4.94856	0.62981	
and	the	orientation:	6.23028	6.26897	0.03945	
with	this	uncertainty:	0.00436	0.00459	0.00623	
Object	fiducial	with	id:	30#		
has	the	location:	-1.6525	233.67	-21.975	
with	this	uncertainty:	6.63212	2.87473	2.83946	
and	the	orientation:	6.18424	6.22353	3.14025	
with	this	uncertainty:	0.00418	0.00476	0.00647	
Object	fiducial	with	id:	1234#		
has	the	location:	-4.0998	387.802	-13.345	
with	this	uncertainty:	7.49966	2.83976	3.48794	
and	the	orientation:	0.07028	0.04553	0.0128	
with	this	uncertainty:	0.00424	0.0047	0.00643	
Object	fiducial	with	id:	5#		
has	the	location:	367.685	304.483	-30.186	
with	this	uncertainty:	7.07528	4.24802	1.41603	
and	the	orientation:	0.03905	6.21722	0.98225	
				9.77E-		
with	this	uncertainty:	0.00627	04	0.00595	
Object	fiducial	with	id:	20#		
has	the	location:	246.319	-38.181	-34.163	
with	this	uncertainty:	5.13921	3.71373	0.63346	
and	the	orientation:	6.25457	0.00242	2.31175	
				8.67E-		
with	this	uncertainty:	04	0.00627	0.00629	

Table B. 7: Object uncertainties in the case: Objects in $Z = 0 + 2$ Ref Fid + Ref Fid $Z > 0$ + High Std Dev

Object	fiducial	with	id:	30#		
has	the	location:	2.35065	235.409	-4.6234	
with	this	uncertainty:	1.96174	2.30138	2.02108	
and	the	orientation:	6.25358	6.22379	3.12026	
with	this	uncertainty:	0.00105	0.00225	0.00458	
Object	fiducial	with	id:	20#		
has	the	location:	239.857	-41.016	-23.522	
with	this	uncertainty:	2.55899	1.68584	1.51554	
and	the	orientation:	6.27176	6.28128	2.28648	
with	this	uncertainty:	0.00116	0.0022	0.00463	
Object	fiducial	with	id:	5#		
has	the	location:	370.349	298.38	-19.735	
with	this	uncertainty:	1.93081	1.56877	1.46704	
and	the	orientation:	0.01758	6.24144	0.95575	
				7.40E-		
with	this	uncertainty:	0.00238	04	0.00473	

Table B. 8: Object uncertainties in the case: Objects in $Z = 0 + 4$ Ref Fid + Ref Fid $Z > 0$ + High Std Dev

Object	fiducial	with	id:	30#		
has	the	location:	2.01142	234.625	-7.2768	
with	this	uncertainty:	1.9978	2.14117	5.0594	
and	the	orientation:	0.01338	6.24291	3.11375	
			1.05E-			
with	this	uncertainty:	03	0.00226	0.00458	
Object	fiducial	with	id:	20#		
has	the	location:	239.26	-42.987	-11.218	
with	this	uncertainty:	2.85144	1.55852	4.38253	
and	the	orientation:	0.00394	0.05865	2.2847	
			1.16E-			
with	this	uncertainty:	03	0.0022	0.00455	
Object	fiducial	with	id:	5#		
has	the	location:	370.516	296.368	-20.29	
with	this	uncertainty:	1.88842	1.50249	4.49007	
and	the	orientation:	6.27235	6.26685	0.95273	
				7.38E-	4.65E-	
with	this	uncertainty:	0.00238	04	03	

Table B. 9: Object uncertainties in the case: Objects in Z = 0 + 4 Ref Fid + Ref Fid Z = 0 + High Std Dev

Object	fiducial	with	id:	30#		
has	the	location:	4.06566	228.871	4.94772	
with	this	uncertainty:	0.10781	0.09841	0.18964	
and	the	orientation:	0.0167	6.24815	3.12782	
				1.49E-		
with	this	uncertainty:	0	08	0	
Object	fiducial	with	id:	20#		
has	the	location:	244.412	-44.601	0.43258	
with	this	uncertainty:	0.10781	0.09841	0.18964	
and	the	orientation:	6.28202	0.06479	2.29861	
				2.79E-		
with	this	uncertainty:	08	0	0	
Object	fiducial	with	id:	5#		
has	the	location:	370.496	295.68	-10.024	
with	this	uncertainty:	0.10781	0.09841	0.18964	
and	the	orientation:	6.27103	6.2737	0.96712	
				1.83E-	1.05E-	
with	this	uncertainty:	0	08	08	

Table B. 10: Object uncertainties in the case: Objects in Z = 0 + 4 Ref Fid + Ref Fid Z = 0 + Low Std Dev

Object	fiducial	with	id:	30#	
has	the	location:	241.344	-28.034	-5.6227
with	this	uncertainty:	2.50102	1.61736	1.48714
and	the	orientation:	6.26331	0.0351	2.24303
with	this	uncertainty:	0.00125	0.00215	0.00461
Object	fiducial	with	id:	20#	
has	the	location:	263.913	199.203	153.952
with	this	uncertainty:	1.92917	1.49858	1.53496
and	the	orientation:	5.98133	6.2603	3.17661
with	this	uncertainty:	0.00115	0.0022	0.00463
Object	fiducial	with	id:	5#	
has	the	location:	259.595	274.494	153.483
with	this	uncertainty:	1.92812	1.50136	1.57358
and	the	orientation:	0.23016	6.11412	3.71323
with	this	uncertainty:	0.00206	0.00143	0.00431

Table B. 11: Object uncertainties in the case: Objects in $Z > 0 + 4$ Ref Fid + Ref Fid $Z > 0 +$ High Std Dev

Object	fiducial	with	id:	911#	
has	the	location:	502.204	7.46524	-14.832
with	this	uncertainty:	3.46091	3.3008	0.7409
and	the	orientation:	6.16667	0.04869	0.03775
with	this	uncertainty:	0.00359	0.00318	0.00202
Object	fiducial	with	id:	30#	
has	the	location:	249.228	-25.98	-15.575
with	this	uncertainty:	3.40172	2.85699	1.1954
and	the	orientation:	6.25252	6.26938	2.26323
with	this	uncertainty:	3.39E-04	0.00478	0.00184
Object	fiducial	with	id:	1234#	
has	the	location:	-0.4215	383.312	3.63118
with	this	uncertainty:	4.09051	2.36221	3.29135
and	the	orientation:	0.05504	0.02316	0.0127
with	this	uncertainty:	0.00351	0.00327	0.00193
Object	fiducial	with	id:	5#	
has	the	location:	263.172	268.094	157.77
with	this	uncertainty:	3.36956	2.31835	2.06476
and	the	orientation:	0.16295	6.11999	3.74796
with	this	uncertainty:	0.0048	7.48E-04	0.00263
Object	fiducial	with	id:	20#	
has	the	location:	267.517	194.468	146.096
with	this	uncertainty:	3.27287	2.36314	1.81335
and	the	orientation:	5.63519	0.04695	3.1874
with	this	uncertainty:	0.00362	0.00315	0.00168

Table B. 12: Object uncertainties in the case: Objects in $Z > 0 + 2$ Ref Fid + Ref Fid $Z > 0 +$ High Std Dev

Object	fiducial	with	id:	30#		
has	the	location:	247.937	-30.58	-12.069	
with	this	uncertainty:	2.88092	3.95016	2.53007	
and	the	orientation:	6.2675	6.26524	2.25294	
with	this	uncertainty:	0.00459	0.00232	0.00616	
Object	fiducial	with	id:	944#		
has	the	location:	495.875	380.497	-14.494	
with	this	uncertainty:	2.22603	5.112	2.91929	
and	the	orientation:	0.02185	0.00239	0.00267	
with	this	uncertainty:	0.00405	0.00315	0.00609	
Object	fiducial	with	id:	20#		
has	the	location:	272.44	195.15	153.621	
with	this	uncertainty:	1.91809	3.70817	2.42093	
and	the	orientation:	6.19834	6.24759	3.19219	
with	this	uncertainty:	0.00395	0.00329	0.00621	
Object	fiducial	with	id:	5#		
has	the	location:	265.5	271.635	146.617	
with	this	uncertainty:	1.94914	3.69518	2.45796	
and	the	orientation:	0.20741	6.08683	3.73167	
with	this	uncertainty:	0.00258	0.00447	0.00644	

Table B. 13: Object uncertainties in the case: Objects in $Z > 0 + 3 \text{ Ref Fid} + \text{Ref Fid } Z > 0 + \text{High Std Dev}$

Object	fiducial	with	id:	30#		
has	the	location:	238.8	-25.739	-1.5944	
with	this	uncertainty:	2.05401	2.90163	4.63761	
and	the	orientation:	6.2641	0.04792	2.23471	
with	this	uncertainty:	2.80E-04	0.00231	0.00162	
Object	fiducial	with	id:	20#		
has	the	location:	260.836	202.453	145.677	
with	this	uncertainty:	1.91336	2.64682	4.72	
and	the	orientation:	5.70208	0.05342	3.15851	
with	this	uncertainty:	0.00201	0.00119	0.00167	
Object	fiducial	with	id:	5#		
has	the	location:	259.04	275.883	156.31	
with	this	uncertainty:	1.90712	2.63028	4.75803	
and	the	orientation:	0.23096	6.10012	3.70662	
with	this	uncertainty:	1.06E-04	0.00237	0.00142	

Table B. 14: Object uncertainties in the case: Objects in $Z > 0 + 3 \text{ Ref Fid} + \text{Ref Fid } Z = 0 + \text{High Std Dev}$

Object	fiducial	with	id:	30#		
has	the	location:	240.076	-28.094	-9.1961	
with	this	uncertainty:	2.73874	1.51482	4.37142	
and	the	orientation:	6.26346	0.06497	2.24153	
with	this	uncertainty:	0.00124	0.00215	0.00457	
Object	fiducial	with	id:	20#		
has	the	location:	262.85	201.969	147.643	
with	this	uncertainty:	1.87838	1.41427	4.55176	
and	the	orientation:	5.88554	0.04769	3.16179	
with	this	uncertainty:	0.00113	0.00221	0.00469	
Object	fiducial	with	id:	5#		
has	the	location:	259.401	276.997	148.525	
with	this	uncertainty:	1.78044	1.4147	4.62938	
and	the	orientation:	0.23776	6.1165	3.71156	
with	this	uncertainty:	0.00206	0.00143	0.00431	

Table B. 15: Object uncertainties in the case: Objects in $Z > 0 + 4$ Ref Fid + Ref Fid $Z = 0 +$ High Std Dev

Object	fiducial	with	id:	22#		
has	the	location:	-3.9134	5.26125	-39.154	
with	this	uncertainty:	3.27459	3.84488	5.33299	
and	the	orientation:	0.06918	0.03837	6.27728	
with	this	uncertainty:	0.00257	0.0023	0.00635	
Object	fiducial	with	id:	30#		
has	the	location:	242.426	-28.33	-18.69	
with	this	uncertainty:	3.40457	2.85058	4.87159	
and	the	orientation:	0.26407	6.13221	2.2204	
with	this	uncertainty:	5.46E-04	0.0034	0.00649	
Object	fiducial	with	id:	944#		
has	the	location:	496.835	378.245	13.2805	
with	this	uncertainty:	1.97802	2.5953	5.15648	
and	the	orientation:	0.05684	6.24132	6.27621	
with	this	uncertainty:	0.00257	0.0023	0.0065	
Object	fiducial	with	id:	5#		
has	the	location:	252.89	265.194	155.917	
with	this	uncertainty:	1.99202	2.51637	5.36524	
and	the	orientation:	0.21235	6.18177	3.71246	
with	this	uncertainty:	7.01E-04	0.00339	0.00614	
Object	fiducial	with	id:	20#		
has	the	location:	256.098	191.601	146.145	
with	this	uncertainty:	2.17437	2.52559	5.22639	
and	the	orientation:	5.79264	0.10866	3.15647	
with	this	uncertainty:	0.00263	0.00225	0.00664	

Table B. 16: Object uncertainties in the case: Objects in $Z > 0 + 2$ Ref Fid + Ref Fid $Z = 0 +$ High Std Dev

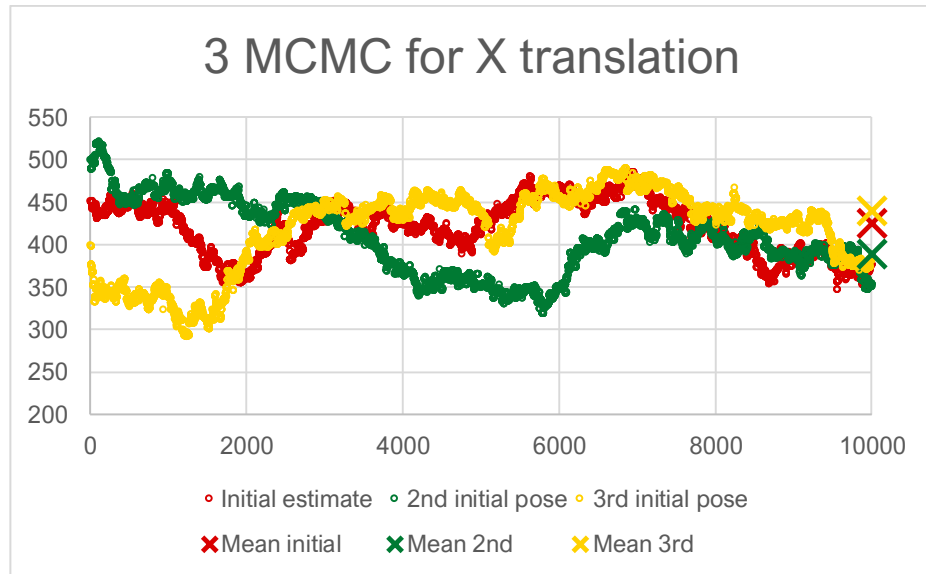


Figure B. 1: Evolution of the X translation coordinate of the camera, in 3 MCMC methods

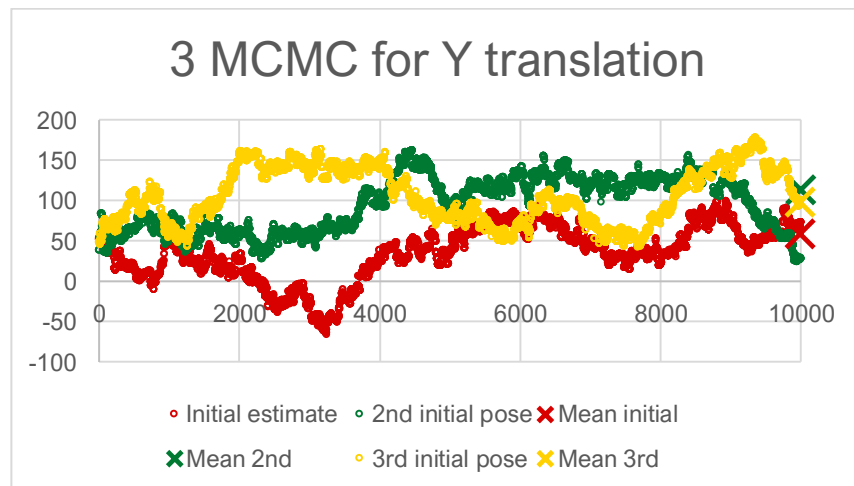


Figure B. 2: Evolution of the Y translation coordinate of the camera, in 3 MCMC methods

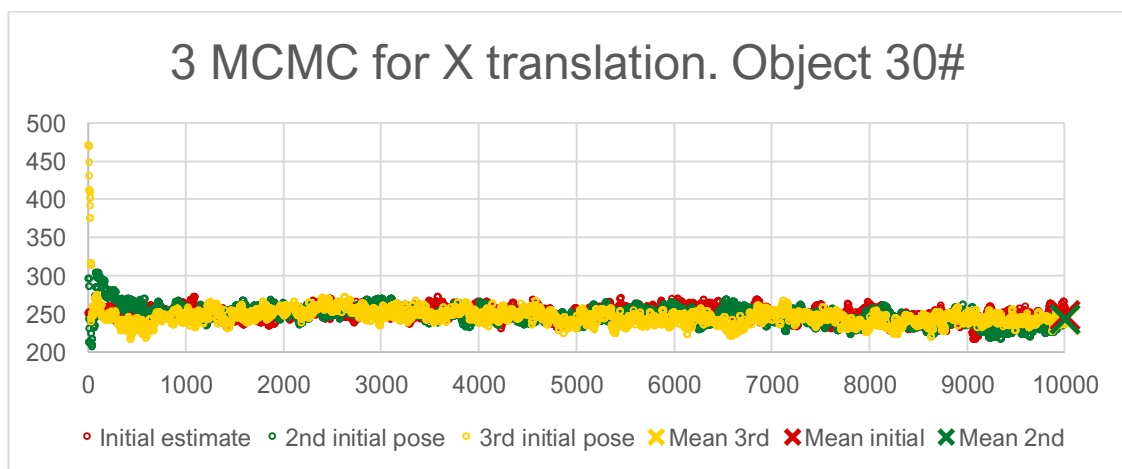


Figure B. 3: Evolution of the X translation coordinate of an object, in 3 MCMC methods

Appendix C. Intrinsic parameters impact

Object	fiducial	with	id:	30#
has	the	pose:		
Mean X	Std Dev X	249.4378931	0.898353736	
Mean Y	Std Dev Y	-62.85138487	0.186605302	
Mean Z	Std Dev Z	12.74248219	4.313840605	
Mean xAngle	Std Dev xA	6.278405299	0.017745785	
Mean yAngle	Std Dev yA	6.231667018	0.024583443	
Mean xAngle	Std Dev zA	5.443930226	0.001373991	
Object	fiducial	with	id:	20#
has	the	pose:		
Mean X	Std Dev X	-15.42570742	2.297913617	
Mean Y	Std Dev Y	-13.12542412	0.179546326	
Mean Z	Std Dev Z	275.7215594	2.82577714	
Mean xAngle	Std Dev xA	6.069678517	0.013958115	
Mean yAngle	Std Dev yA	0.244792475	0.0197343	
Mean xAngle	Std Dev zA	6.086226513	0.002012382	
Object	fiducial	with	id:	5#
has	the	pose:		
Mean X	Std Dev X	401.0619108	0.237139943	
Mean Y	Std Dev Y	297.6414632	1.766137725	
Mean Z	Std Dev Z	-9.19976052	1.982712984	
Mean xAngle	Std Dev xA	0.017250772	0.023654795	
Mean yAngle	Std Dev yA	0.017159622	0.010802006	
Mean xAngle	Std Dev zA	2.613154434	7.59E-04	

Table C. 1: Object uncertainties with 5 mm Std Dev + estimating focal length

Object	fiducial			30#
has	the	pose:		
Mean X	Std Dev X	249.4175762	0.178099791	
Mean Y	Std Dev Y	-61.0001765	0.409337281	
Mean Z	Std Dev Z	9.451857476	0.187427837	
Mean xAngle	Std Dev xA	6.265747386	0	
Mean yAngle	Std Dev yA	6.225588714	0	
Mean xAngle	Std Dev zA	5.444159811	1.49E-08	
Object	fiducial			20#
has	the	pose:		
Mean X	Std Dev X	-15.46400378	0.178099791	
Mean Y	Std Dev Y	-10.96631213	0.409337281	
Mean Z	Std Dev Z	272.9666053	0.187427836	
Mean xAngle	Std Dev xA	6.07857865	2.11E-08	
Mean yAngle	Std Dev yA	0.238238949	2.36E-08	
Mean xAngle	Std Dev zA	6.089765554	0	
Object	fiducial			5#
has	the	pose:		
Mean X	Std Dev X	401.2685094	0.178099791	
Mean Y	Std Dev Y	298.6748113	0.409337281	
Mean Z	Std Dev Z	-10.82710407	0.187427837	
Mean xAngle	Std Dev xA	0.02296895	0	
Mean yAngle	Std Dev yA	0.016001476	0	
Mean xAngle	Std Dev zA	2.616416262	0	

Table C. 2: Object uncertainties with 5 mm Std Dev + focal length constant

```

acceptedMap = Map(IntrinsicProposal(15.0) -> 1367 CameraRProposal(0.1) -> 21
CameraTPProposal(10.0) -> 983 CameraTPProposal(1.0) -> 1922 CameraRProposal(0.01) -> 995)
rejectedMap = Map(IntrinsicProposal(15.0) -> 683 CameraRProposal(0.1) -> 1935
CameraTPProposal(10.0) -> 1041 CameraTPProposal(1.0) -> 127 CameraRProposal(0.01) -> 925)
PoseWithFocalLength(Location(422.99933458317656 14.8901435285044 940.7073152121089)
Orientation(-3.1050971974640373 0.067655064596953 0.0020328821351304053)
580.720301286552 581.2618104860386)
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 295.174308164837 64.3635363604006 945.424275306897 3.12146070172515
6.22828449309208 0.0629799379196312
StdDev: 54.8958658659404 53.3494704311281 112.121733207559 0.0568481664824254
0.0489719665061318 0.0751904983166392
The Camera has the following focal length:
Mean:                    593.6213                510.8725
StdDev:                  98.47362                37.5573
Calculation              of                    object              statistics
Object                   fiducial              with                 id:
has                       the                   location:            11#
    12.94205             7.616208
    3.842521             1.391751
    21.78267             9.847711
    6.192256             0.006847
    6.232272             0.035977
    0.00361              4.87E-04
Object                   fiducial              with                 id:
has                       the                   location:            22#
    187.3476             4.454538
    -128.131             1.307066
    8.195664             9.545368
    6.03689              0.003881
    0.583946             0.046503
    6.216749             0.010734
Object                   fiducial              with                 id:
has                       the                   location:            30#
    255.2074             3.37904
    -57.1555             1.374863
    24.98802             10.01016
    0.082207             0.03533
    6.160532             0.040915
    5.444032             8.83E-04
Object                   fiducial              with                 id:
has                       the                   location:            5#
    166.1087             4.806502
    217.9509             1.584807
    181.9603             7.868248
    1.33E-04             0.032175
    6.113574             0.027241
    1.103111             0.004518
Object                   fiducial              with                 id:
has                       the                   location:            20#
    353.5479             1.180415
    283.9283             3.616539
    -2.5854              4.5465
    6.200796             0.034345
    0.110499             0.073799
    3.543301             0.014839

```

Table C. 3: Output when estimating the focal length with: Std Dev = 20 mm + 2 Ref Fid + All in Z = 0

accepted = 4660 and rejected = 5339
 acceptedMap = Map(IntrinsicProposal(15.0) -> 1173 CameraRProposal(0.1) -> 9 CameraTPProposal(10.0) -> 825 CameraTPProposal(1.0) -> 1792 CameraRProposal(0.01) -> 861)
 rejectedMap = Map(IntrinsicProposal(15.0) -> 877 CameraRProposal(0.1) -> 1967 CameraTPProposal(10.0) -> 1183 CameraTPProposal(1.0) -> 132 CameraRProposal(0.01) -> 1180)
 PoseWithFocalLength(Location(440.90440370577 54.1921132889342 946.2479341791751)
 Orientation(3.1373114662223394 0.0858343128047822 -0.002704213195178162) 580.720301286552
 581.2618104860386)
 All the results are shown in the order: x y z xAngle yAngle zAngle
 The Camera is located in:
 Means: 348.262095086735 294.293613548004 1112.45560756849 2.93084641705198 6.27977411156971
 6.25744242639689
 StdDev: 32.9114338928901 51.6134582145996 131.752753497808 0.026993791628002 0.0309173698571412
 0.0192909654605179
 The Camera has the following focal length:
 Mean: 635.689460851006 675.898014048142
 StdDev: 74.8251386338849 89.9315656191675
 In process: estimation of the pose of the detected fiducials/objects
 Calculation of object statistics

Object has	fiducial the	with location:	id:	11#
	-1.01578	7.066187		
	4.742659	1.579209		
	12.50307	9.58304		
	6.131422	0.010764		
	0.036334	0.039451		
	6.28293	0.002199		
Object has	fiducial the	with location:	id:	22#
	180.7668	3.664533		
	-125.973	1.481631		
	8.668105	8.328417		
	6.193071	0.025771		
	0.022557	0.059224		
	0.026512	0.011253		
Object has	fiducial the	with location:	id:	30#
	249.8437	2.67433		
	-55.6919	1.50696		
	14.86237	8.227244		
	6.246847	0.037726		
	6.254129	0.078949		
	5.442588	0.010067		
Object has	fiducial the	with location:	id:	5#
	161.2141	4.124127		
	224.8807	1.816971		
	167.8965	7.089414		
	0.057995	0.034034		
	6.221341	0.036231		
	1.11515	0.008497		
Object has	fiducial the	with location:	id:	20#
	352.6434	1.055188		
	278.4609	4.457284		
	-6.76299	7.543578		
	0.013402	0.047863		
	6.251164	0.039269		
	3.563337	0.003414		

Table C. 4: Output when estimating the focal length with: Std Dev = 20 mm + 3 Ref Fid + All in Z = 0

accepted = 4348 and rejected = 5651
 acceptedMap = Map(IntrinsicProposal(15.0) -> 1037 CameraRProposal(0.1) -> 11
 CameraTProposal(10.0) -> 721 CameraTProposal(1.0) -> 1788 CameraRProposal(0.01) ->
 791)
 rejectedMap = Map(IntrinsicProposal(15.0) -> 1017 CameraRProposal(0.1) -> 1944
 CameraTProposal(10.0) -> 1311 CameraTProposal(1.0) -> 150 CameraRProposal(0.01) ->
 1229)
 PoseWithFocalLength(Location(440.79476016937446 20.7902031889537
 934.5812332907284) Orientation(-3.1095690914029737 0.0859905581317112
 0.001460178053970035) 580.720301286552 581.2618104860386)
 All the results are shown in the order: x y z xAngle yAngle zAngle
 The Camera is located in:
 Means: 360.560782713528 126.392293045199 892.924189572582 3.05707329326355
 0.00438771457343922 6.278542452669
 StdDev: 32.7325441605378 27.1600358792926 51.6317106029282 0.0325648735890721
 0.0345225137419755 0.0163267814698297
 The Camera has the following focal length:
 Mean: 530.954097194911 524.319470113295
 StdDev: 34.7356789642552 35.8975563667176
 In process: estimation of the pose of the detected fiducials/objects
 Calculation of object statistics

Object has	fiducial the	with location:	id:	22#
184.3133		1.572837		
-126.901		1.528304		
4.280769		5.798324		
6.005265		0.002345		
0.563379		0.007606		
6.221056		0.002033		
Object has	fiducial the	with location:	id:	30#
252.8191		1.507692		
-56.4609		1.534132		
15.50112		5.815474		
0.020029		0.005877		
6.219329		0.007245		
5.445684		2.53E-04		
Object has	fiducial the	with location:	id:	5#
165.0884		1.563306		
218.8365		1.554089		
175.5144		4.944121		
0.052021		0.005048		
6.153179		0.004419		
1.106241		9.34E-04		
Object has	fiducial the	with location:	id:	20#
353.3293		1.385331		
281.7323		1.705933		
-5.56879		5.453797		
6.212614		0.01011		
6.275413		0.009125		
3.56226		0.001683		

Table C. 5: Output when estimating the focal length with: Std Dev = 20 mm + 4 Ref Fid + All in Z = 0


```

accepted = 3978 and rejected = 6021
acceptedMap = Map(IntrinsicProposal(15.0) -> 1151 CameraRProposal(0.1) -> 6
CameraTProposal(10.0) -> 574 CameraTProposal(1.0) -> 1801
CameraRProposal(0.01) -> 446)
rejectedMap = Map(IntrinsicProposal(15.0) -> 940 CameraRProposal(0.1) -> 1930
CameraTProposal(10.0) -> 1411 CameraTProposal(1.0) -> 193
CameraRProposal(0.01) -> 1547)
PoseWithFocalLength(Location(431.8985552723741 22.6133638468044
939.0624264563506) Orientation(-3.1133777424505915 0.0762599644188445 -
0.0016349352848254973) 580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 352.391440396219 36.9961738412862 1174.2972388422
3.15335829182291 6.2806309779396 6.28049146567241
StdDev: 21.4832703099365 36.6666501002907 46.1170673864375
0.0309191069955526 0.0176959251298537 0.012795267524646
The Camera has the following focal length:
Mean: 702.064745789644 709.377270161852
StdDev: 30.5167713221455 28.737895006025
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object          fiducial          with          id:          30#
has             the              location:
252.3932        1.472671
-57.1311        1.539852
19.25823        5.824545
0.025483        0.00581
6.210879        0.007456
5.446041        2.34E-04
Object          fiducial          with          id:          20#
has             the              location:
353.8142        1.356195
279.6841        1.675551
1.75966         5.452365
6.229351        0.007413
0.027472        0.010106
3.557469        0.001861
Object          fiducial          with          id:          5#
has             the              location:
164.704         1.538576
217.8506        1.559226
180.1423        4.934037
0.049645        0.00533
6.13304         0.004163
1.101887        7.90E-04

```

Table C. 6: Output when estimating the focal length with: Std Dev = 20 mm + 5 Ref Fid + All in Z = 0

```

accepted = 3861 and rejected = 6138
acceptedMap = Map(IntrinsicProposal(15.0) -> 1074 CameraRProposal(0.1) -> 1
CameraTProposal(10.0) -> 501 CameraTProposal(1.0) -> 1775
CameraRProposal(0.01) -> 510)
rejectedMap = Map(IntrinsicProposal(15.0) -> 978 CameraRProposal(0.1) -> 1973
CameraTProposal(10.0) -> 1481 CameraTProposal(1.0) -> 186
CameraRProposal(0.01) -> 1520)
PoseWithFocalLength(Location(429.49765191661646 26.2519754449971
940.3766572845713) Orientation(-3.1168005942766643 0.0737515115439825 -
0.0030381165125138955) 580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 453.127097166686 4.26615765090276 985.181780540424
3.18565210845514 0.0928098118055421 6.28086533176263
StdDev: 30.6149549630248 17.6309010660676 51.8833693887833
0.0149661423001495 0.0266119832769982 0.0139561512898468
The Camera has the following focal length:
Mean: 608.752620658664 604.048699167644
StdDev: 37.8234163416725 35.1707894875426
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object          fiducial          with          id:          30#
has             the              location:
                251.8202         1.463105
                -56.6633         1.545232
                20.23059         5.824454
                0.02576          0.005742
                6.206922         0.007394
                5.446023         2.38E-04
Object          fiducial          with          id:          20#
has             the              location:
                353.3749         1.347935
                280.7673         1.694743
                -0.80843         5.459835
                6.231181         0.008313
                0.018568         0.009956
                3.559277         0.001776
Object          fiducial          with          id:          5#
has             the              location:
                163.2985         1.53172
                219.4318         1.564296
                178.4507         4.944405
                0.044305         0.00526
                6.138422         0.004231
                1.101079         7.97E-04

```

Table C. 7: Output when estimating the focal length with: Std Dev = 20 mm + 6 Ref Fid + All in Z = 0

```

accepted = 5272 and rejected = 4727
acceptedMap = Map(IntrinsicProposal(15.0) -> 1258 CameraRProposal(0.1) -> 21
CameraTProposal(10.0) -> 1022 CameraTProposal(1.0) -> 1851 CameraRProposal(0.01) -> 1120)
rejectedMap = Map(IntrinsicProposal(15.0) -> 760 CameraRProposal(0.1) -> 1979
CameraTProposal(10.0) -> 1013 CameraTProposal(1.0) -> 109 CameraRProposal(0.01) -> 866)
PoseWithFocalLength(Location(473.59683257399513 13.6879950904812 923.4425790022817)
Orientation(-3.0989153928028994 0.119709438876544 0.009887490679135354)
580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 521.243905127163 311.21112642502 756.12116035131 2.74379898595863
0.174843368347535 6.16555089843833
StdDev: 47.6591670208004 38.4576977321971 57.5345820045342 0.0293745434903589
0.0524608685014522 0.0607959644979575
The Camera has the following focal length:
Mean: 579.236125579961 389.671814108533
StdDev: 63.5163505565109 44.3289666695236
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object      fiducial      with      id:      22#
has         the           location:
            181.307      4.352118
            -120.38      1.901228
            -35.1381     20.25691
            6.259479    0.08092
            0.065854    0.096149
            0.025619    0.013214
Object      fiducial      with      id:      30#
has         the           location:
            249.705     3.537863
            -51.0948    1.594537
            -31.175    20.00161
            6.244297 0.029741
            0.045461 0.082531
            5.440941 0.010299
Object      fiducial      with      id:      5#
has         the           location:
            165.1511  4.687257
            224.9231  0.670297
            141.4931  17.6566
            0.123741 0.030824
            6.208952 0.039315
            1.116074 0.012124
Object      fiducial      with      id:      20#
has         the           location:
            353.3835  1.726264
            281.5702  3.240716
            -32.3473 13.689
            6.2718   0.044007
            6.219083 0.03114
            3.569995  0.00326
    
```

Table C. 8: Output when estimating the focal length with: Std Dev = 20 mm + 2 Ref Fid + One in Z > 0



```

accepted = 4749 and rejected = 5250
acceptedMap = Map(IntrinsicProposal(15.0) -> 1172 CameraRProposal(0.1) -> 11
CameraTProposal(10.0) -> 845 CameraTProposal(1.0) -> 1846 CameraRProposal(0.01) -> 875)
rejectedMap = Map(IntrinsicProposal(15.0) -> 838 CameraRProposal(0.1) -> 1962
CameraTProposal(10.0) -> 1200 CameraTProposal(1.0) -> 132 CameraRProposal(0.01) ->
1118)
PoseWithFocalLength(Location(466.855955797015 10.3705676760992 921.3915513969216)
Orientation(-3.100789773774954 0.113129066109755 0.0013178826175287179)
580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 409.197375797343 176.540600982797 1041.73640839681 3.02400342050588
0.0551179549588737 0.0476945763739302
StdDev: 26.4575228047526 26.134161855672 50.5098652329661 0.025727230801744
0.0272715085132481 0.0272300942714535
The Camera has the following focal length:
Mean: 653.606372739736 576.097595024732
StdDev: 33.7349084901181 28.7745496580734
In process: estimation of the pose of the detected fiducials/objects
Object          fiducial          with          id:    22#
has             the               location:
                187.1424         3.409207
                -132.565         0.844991
                -14.2765         17.73056
                6.012438         0.015787
                0.52505          0.027162
                6.242908         0.009686
Object          fiducial          with          id:    30#
has             the               location:
                256.2051         2.880157
                -61.9678         0.690479
                -5.7292          17.48944
                6.242694         0.021125
                0.011355         0.041396
                5.448817         0.009319
Object          fiducial          with          id:    5#
has             the               location:
                169.3391         2.688825
                214.0183         0.843752
                162.2122         14.70614
                0.106283         0.017662
                6.181707         0.018641
                1.11353          0.007891
Object          fiducial          with          id:    20#
has             the               location:
                357.7531         2.165155
                273.3562         2.397527
                -11.7369         17.70033
                6.260307         0.022606
                6.248762         0.018733
                3.568837         0.006899

```

Table C. 9: Output when estimating the focal length with: Std Dev = 20 mm + 3 Ref Fid + One in Z > 0

```

accepted = 4339 and rejected = 5660
acceptedMap = Map(IntrinsicProposal(15.0) -> 1214 CameraRProposal(0.1) -> 3
CameraTProposal(10.0) -> 732 CameraTProposal(1.0) -> 1769 CameraRProposal(0.01) -> 621)
rejectedMap = Map(IntrinsicProposal(15.0) -> 840 CameraRProposal(0.1) -> 1912
CameraTProposal(10.0) -> 1344 CameraTProposal(1.0) -> 149 CameraRProposal(0.01) -> 1415)
PoseWithFocalLength(Location(446.4390689731874 3.20492126778201 924.8817513480653)
Orientation(-3.0922565348557334 0.0920615037754657 0.0043529332058538095)
580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.2618104860386
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 491.727698133061 72.6214747775872 1166.98116266333 3.12050813623768
0.10964565983331 0.00719888185993831
StdDev: 36.9142462451948 20.715424959943 40.093797724985 0.0182060964237596
0.027670809807116 0.0167224207999578
The Camera has the following focal length:
Mean: 731.233616562933 703.5602612476
StdDev: 30.7617955118425 32.7074961941577
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object      fiducial      with      id:      22#
has         the           location:
185.5319      2.814376
-131.616      0.748363
-13.6011      6.156243
6.26794       0.009718
0.072047      0.020137
0.03041       0.006984
Object      fiducial      with      id:      30#
has         the           location:
255.2419      2.297952
-61.0293      0.52064
-2.64421      6.3098
6.239892      0.012471
0.055874      0.027304
5.44516       0.007032
Object      fiducial      with      id:      5#
has         the           location:
163.4266      0.669578
212.1242      1.162503
164.6559      5.015055
0.11677       0.010799
6.198706      0.012514
1.125353      0.008585
Object      fiducial      with      id:      20#
has         the           location:
358.3169      0.882791
271.2651      1.633496
5.740185      9.139148
0.012313      0.013508
6.242682      0.010119
3.572771      0.007452
    
```

Table C. 10: Output when estimating the focal length with: Std Dev = 20 mm + 4 Ref Fid + One in Z > 0



accepted = 4058 and rejected = 5941
 acceptedMap = Map(IntrinsicProposal(15.0) -> 1154 CameraTProposal(10.0) -> 588
 CameraTProposal(1.0) -> 1794 CameraRProposal(0.01) -> 522)
 rejectedMap = Map(IntrinsicProposal(15.0) -> 850 CameraRProposal(0.1) -> 2012
 CameraTProposal(10.0) -> 1422 CameraTProposal(1.0) -> 184 CameraRProposal(0.01) ->
 1473)
 PoseWithFocalLength(Location(438.04971166677285 7.82115622154152
 932.9354223642625) Orientation(-3.0981919867093324 0.0816382635757172 -
 1.1815564530209846E-4) 580.720301286552 581.2618104860386)
 Initial focal length: 580.720301286552 581.261810486038
 All the results are shown in the order: x y z xAngle yAngle zAngle
 The Camera is located in:
 Means: 459.104188662242 -2.2164237484791 1093.31169542355 3.18763621696372
 0.0891409909278017 6.28160654031882
 StdDev: 19.5433164282963 51.9440207315169 31.3164740891756 0.0460161502813599
 0.0172272015399265 0.0136367116642992
 The Camera has the following focal length:
 Mean: 681.253885726074 673.510739109293
 StdDev: 25.98201091968 28.4873003622704
 In process: estimation of the pose of the detected fiducials/objects
 Calculation of object statistics

Object has	fiducial the	with location:	id:	30#
253.0103		0.471963		
-61.8566		0.512798		
10.32427		5.697445		
6.273238		0.011741		
0.031006		0.024752		
5.438364		0.002274		
Object has	fiducial the	with location:	id:	20#
357.9655		0.742803		
268.7921		1.48818		
10.97999		8.83702		
6.277547		0.013805		
6.271672		0.013075		
3.562276		0.001714		
Object has	fiducial the	with location:	id:	5#
162.579		0.928086		
210.1784		0.539244		
171.7976		4.700269		
0.113072		0.012668		
6.171869		0.011942		
1.110481		0.002874		

Table C. 11: Output when estimating the focal length with: Std Dev = 20 mm + 5 Ref Fid + One in Z > 0

```

accepted = 3827 and rejected = 6172
acceptedMap = Map(IntrinsicProposal(15.0) -> 980 CameraRProposal(0.1) -> 3
CameraTProposal(10.0) -> 497 CameraTProposal(1.0) -> 1739 CameraRProposal(0.01) ->
608)
rejectedMap = Map(IntrinsicProposal(15.0) -> 999 CameraRProposal(0.1) -> 1974
CameraTProposal(10.0) -> 1565 CameraTProposal(1.0) -> 197 CameraRProposal(0.01) ->
1437)
PoseWithFocalLength(Location(438.886221464816 4.29689946330672
929.3237164710116) Orientation(-3.0953035679662193 0.0850088063745822 -
6.307635845169578E-5) 580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 448.836765913808 -3.18294522751601 939.438556847308 3.19487561969737
0.0942331718917391 0.00172652094564046
StdDev: 28.2907180146605 34.4861991929635 37.6238822881778
0.0339885693268769 0.0269023399076068 0.0135713815994233
The Camera has the following focal length:
Mean: 583.625611148603 582.59235920435
StdDev: 28.6866287767627 28.866381925157
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object          fiducial          with          id:    30#
has             the               location:
                251.0598          0.454552
                -62.0514          0.523326
                0.808774          5.763837
                6.239819          0.014148
                0.030455          0.025991
                5.436524          0.002588
Object          fiducial          with          id:    20#
has             the               location:
                357.8619          0.794493
                269.3296          1.605097
                7.963592          9.133572
                0.015451          0.013789
                6.246764          0.010574
                3.563158          0.001042
Object          fiducial          with          id:    5#
has             the               location:
                161.9616          0.873135
                211.9182          0.563608
                167.7739          4.819343
                0.106879          0.011731
                6.181512          0.012142
                1.110723          0.00303

```

Table C. 12: Output when estimating the focal length with: Std Dev = 20 mm + 6 Ref Fid + One in Z > 0

Inlier size 24
 accepted = 3923 and rejected = 6076
 acceptedMap = Map(IntrinsicProposal(15.0) -> 1074 CameraRProposal(0.1) -> 4
 CameraTPProposal(10.0) -> 513 CameraTPProposal(1.0) -> 1804
 CameraRProposal(0.01) -> 528)
 rejectedMap = Map(IntrinsicProposal(15.0) -> 1023 CameraRProposal(0.1) -> 1882
 CameraTPProposal(10.0) -> 1563 CameraTPProposal(1.0) -> 182
 CameraRProposal(0.01) -> 1426)
 PoseWithFocalLength(Location(436.58953750650664 9.34985221371669
 933.6758069388287) Orientation(-3.0997473609584483 0.0811633692033235 -
 9.649076316729406E-4) 580.720301286552 581.2618104860386)
 Initial focal length: 580.720301286552 581.261810486038
 All the results are shown in the order: x y z xAngle yAngle zAngle
 The Camera is located in:
 Means: 382.979889616498 -5.75660274475765 1045.42295132669
 3.19507816000386 0.023683778748912 6.27990205611389
 StdDev: 23.7507155437271 29.4582206397028 51.8963787674677
 0.0289253749674521 0.0234470086352874 0.0157277664844321
 The Camera has the following focal length:
 Mean: 642.680388633629 641.525714689009
 StdDev: 35.3004597429518 31.6765828707343
 In process: estimation of the pose of the detected fiducials/objects
 Calculation of object statistics

Object has	fiducial the	with location:	id:	30#
251.8129	0.495452			
-61.8053	0.505901			
8.926377	5.879855			
6.260795	0.011906			
0.031312	0.025232			
5.435693	0.002347			
Object has	fiducial the	with location:	id:	20#
357.5297	0.759756			
269.084	1.5601			
10.44992	9.141013			
6.278347	0.015052			
6.251505	0.012012			
3.564684	0.001358			
Object has	fiducial the	with location:	id:	5#
162.2177	0.938526			
210.5004	0.531254			
172.505	4.665378			
0.10034	0.012461			
6.166193	0.011571			
1.109552	0.002658			

Table C. 13: Output when applying the robust estimation algorithm and estimating focal length with: Std Dev = 20 mm + 6 Ref Fid + One in Z > 0


```

accepted = 1678 and rejected = 8321
acceptedMap = Map(IntrinsicProposal(15.0) -> 231 CameraTProposal(10.0) ->
53 CameraTProposal(1.0) -> 1345 CameraRProposal(0.01) -> 49)
rejectedMap = Map(IntrinsicProposal(15.0) -> 1790 CameraRProposal(0.1) ->
1923 CameraTProposal(10.0) -> 2034 CameraTProposal(1.0) -> 624
CameraRProposal(0.01) -> 1950)
PoseWithFocalLength(Location(442.9260071766578 8.85140872776292
930.7340762421055) Orientation(-3.0974166146429196 0.0870119813125029
0.003115946352317159) 580.720301286552 581.2618104860386)
Initial focal length: 580.720301286552 581.261810486038
All the results are shown in the order: x y z xAngle yAngle zAngle
The Camera is located in:
Means: 448.089761597889 12.9813527758052 919.609527295389
3.18151983734245 0.0931720444253914 0.00396492488820101
StdDev: 4.26989001058385 3.42172994649986 10.3184945247022
0.00340305418676987 0.00406362364121468 0.0028216809086771
The Camera has the following focal length:
Mean: 574.041382125458 572.179123907099
StdDev: 7.52311162635733 7.5857621890317
In process: estimation of the pose of the detected fiducials/objects
Calculation of object statistics
Object      fiducial      with      id:      30#
has         the           location:
251.3852      0.867433
-62.9339      0.323266
0.339798      3.718054
0.005018      0.014657
6.271336      0.022482
5.443199      0.00141
Object      fiducial      with      id:      5#
has         the           location:
162.6721      1.395301
209.6169      0.201117
164.2153      3.064068
0.081191      0.013027
6.153359      0.011484
1.104844      0.002603
Object      fiducial      with      id:      20#
has         the           location:
354.029       0.373196
271.4355      1.346093
-6.91875      1.398382
6.236574      0.021295
6.260602      0.016392
3.566603      0.002414

```

Table C. 14: Output when estimating the focal length with: Std Dev = 5 mm + 4 Ref Fid + One in Z > 0