Space-Time Decomposition of A Posteriori Error Estimate for Linear and Nonlinear Parabolic Equations with Semi-Implicit Schemes

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ABSTRACT

In recent years there has been a growing interest in a particular class of nonlinear parabolic equations known as phase-field models, which is characterised by evolving diffuse interfaces instead of sharp interfaces. These models are energy dissipative and mass conservative. Moreover, these models exhibit alternating fast and slow variations in time and impose severe demands on the resolution of the interface. There are many semi-implicit finite difference time-stepping schemes which inherit these properties at the discrete level [1]. Obviously, to efficiently and accurately simulate these problems, there is a need to employ a posteriori error estimates to drive adaptive mesh refinement and adaptive time-step selection for fully discrete schemes.

In this contribution, we introduce a general space-time adaptive framework for linear and nonlinear parabolic equations, based on a duality-based two-level error estimator [2]. The framework employs semi-implicit time-discretizations combined with Galerkin finite elements in space. According to Verfurth's decomposition [3], we split the global space-time error estimate into a spatial part and a temporal part, and develop an efficient space-time adaptive refinement methodology. A blockwise adaptive approach is one of the key strategies, which has been discussed and developed by Carey et al [4]. Following this idea, we divide the time domain into blocks, and apply the adaptive algorithm for each block. Several numerical experiments are presented to demonstrate the proposed adaptive algorithm.

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