NEAR-INTERFACE MODELING IN LARGE-EDDY SIMULATION OF TWO-PHASE TURBULENT FLOW

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Abstract. The smallest hydrodynamic length scales in interfacial turbulent flow are located at the interface between phases/fluids as a result of two-way coupling phenomena. These interface-generated scales are, typically, several times smaller than that of the dissipative scales in the surrounding bulk flow as characterized by Kolmogorov’s theory. Consequently, the cost of performing large-eddy simulations of this type of flow deviates significantly from its theoretical optimum toward values in the order of the direct numerical simulation of turbulence. Therefore, to maintain the cost of scale-resolving approaches linear with respect to the Reynolds number, this work investigates the modeling of the small-scale fluid motions in the vicinity of two-phase interfaces in turbulent flows. Given the resemblance between the flow structures in the near-interface regions and those found in the boundary layers of turbulent wall-bounded flow, the modeling methodology proposed is inspired by ideas developed for turbulent flows interacting with solid walls, but modified to capture slip-velocity effects between phases/fluids. The performance of the approach is \textit{a priori} assessed by utilizing data from direct numerical simulations of decaying isotropic turbulence laden with droplets of super-Kolmogorov size, demonstrating its feasibility and potential for reducing the cost of large-eddy simulation studies of two-phase turbulence.

1 INTRODUCTION, MOTIVATION AND OBJECTIVES

The interaction of turbulence with two-phase interfaces encompasses very complex multiscale flow phenomena of interest in a wide range of engineering and scientific applications, like for example rain formation in clouds [1], spray atomization in combustors [2, 3, 4], and microbubble generation in biomedical and processing technologies [5]. In such systems, the dispersed phase typically undergoes significant rates of deformation and break-up/coalescence events due to vortical and straining flow motions, and/or external forces, that greatly increase the surface area between phases. In these near-interface regions, the flow-topology of the turbulence is modulated as a result of two-way coupling effects [6], favoring low-enstrophy/high-dissipation motions with characteristic sizes significantly smaller than the dissipative scales in the bulk of the carrier fluid [7, 8].

The inherent nonlinearity of the flow mechanisms encountered, together with the numerous challenges associated with performing experiments, renders computational approaches an indispensable tool for the analysis, design and optimization of two-phase turbulence phenomena in industrial and natural problems. In this regard, direct numerical simulation (DNS) of turbulent two-phase flows [9], resolving all
length and time scales of turbulence interacting with interfaces, is now feasible for moderate Reynolds numbers or for reduced computational complexity; i.e., interfaces undergoing small deformation and/or limited number of droplets/bubbles. Selected examples of DNS studies in the past decade include mixing layers [10, 11, 12], bubbly flows [13, 14, 15], sprays and atomization processes [16, 17, 18], and multiphysics flows [19, 20, 21]. The flexibility and degree of detail of such high-fidelity (HF) calculations provides very insightful information for the characterization of the underlying flow physics. In addition, the results obtained can be used as reference data for the development of coarse-grained models useful in the design and optimization of engineering systems. However, the computational costs of these fine-grained simulations are extraordinarily high, requiring (typically) large allocations in powerful computing facilities. The necessity of high-performance computing (HPC) resources arises from the stringent spatio-temporal resolutions required to capture, for sufficient long periods of time, e.g., large-eddy turnover time in turbulence-dominated flows, the wide range of turbulent length and time scales [22] and the temporal evolution of interfaces [23, 24, 25, 26] as they change topology and break-up/coalesce.

The computational cost of studying turbulence can be significantly reduced by means of large-eddy simulation (LES) approaches [27, 28, 29], in which the small-scale motions in the bulk of the flow are modeled [30, 31] instead of resolved. Although much less explored than in single-phase turbulence, the extension of LES strategies to multiphase flow problems started in the 2000s marked by the gradual migration from Reynolds-Averaged Navier-Stokes (RANS) modeling to scale-resolving turbulence simulation including LES and its multiphase flow sub-variants [32]: dispersed-flow LES [referred to as large-eddy and structure simulation (LESS)], and interfacial-flow LES [referred to as large-eddy and interface simulation (LEIS)]. This transition was mainly driven by the limited predictive performance of statistical turbulence modeling in multiphase flows. The derivation of the LESS equations is detailed in the work by Lakehal et al. [33], and the resulting formulation has been applied, based on two-fluid approaches [34], to study, for instance, bubbly flow [35], sprays [36], particle-laden flow [37], and buoyant plumes [38]. While the LESS variant is best suited for a range of problems in which one of the phases is dispersed in the other, LEIS [39] provides superior accuracy, at expenses of higher computational costs, by directly resolving the interface dynamics and turbulent motions down to the grid resolution. LEIS has been applied, for example, to calculate turbulent gas-liquid flows involving sheared interfaces [40], clustering of bubbles in wall-bounded flows [41], and primary breakup in atomization processes [42].

Away from solid walls, LES has proven to be a computationally tractable approach to simulate single-phase, unsteady turbulent flows [43, 44] over the past decades. However, as analyzed in a recent DNS study of isotropic turbulence laden with finite-size droplets by Dodd & Jofre [8], the smallest hydrodynamic scales in turbulent two-phase flows are located at the interfacial regions, presenting characteristic sizes that are 2–3 times smaller than that of the dissipative Kolmogorov scales in the surrounding bulk flow. In particular, the flow topologies in these regions resemble those found in wall-bounded flows. Therefore, to bypass the near-interface stringent resolution requirements and make LES approaches cost-efficient for the study of turbulent two-phase flows, this work focuses on the derivation of the theoretical framework and a priori analyses of interfacial layer flow models based on ideas inspired from wall-modeled LES [45, 46] approaches. To that end, this paper is organized as follows. First, in Section 2, important dimensionless numbers and characteristic scales in two-phase turbulence are introduced. A description of the filtered LES equations is given in Section 3. Next, in Section 4, the derivation of the near-interface flow modeling approach is presented. Finally, in Section 5, the work is concluded and future directions are proposed.
Figure 1: Diagram of the energy cascade in terms of eddy sizes $\ell$ (logarithmic scale) at very high Reynolds number, showing the main turbulent scales and ranges. Illustration adapted from Pope [49].

2 DIMENSIONLESS NUMBERS AND CHARACTERISTIC SCALES

The dynamics of multiphase flows is governed by a variety of nondimensional numbers [47, 48]. Their importance depends on the physical mechanisms driving the flow. In turbulent flows, the ratio between inertial and viscous forces is characterized by the Reynolds number defined as

$$Re = \frac{\rho L U}{\mu} = \frac{LU}{\nu},$$

(1)

where $L$ and $U$ are characteristic length and velocity scales, and $\rho$, $\mu$ and $\nu = \mu/\rho$ are the density and dynamic and kinematic viscosities, respectively, of one of the phases. For inertia-dominated two-phase flows, pressure differences scale proportionally to $\rho U^2$, and as a result the normal stress condition at interfaces introduces the dimensionless Weber number, expressing the ratio of inertia to surface tension forces, given by

$$We = \frac{\rho L U^2}{\sigma},$$

(2)

with $\sigma$ the surface tension coefficient. Eliminating the velocity $U$ between the Reynolds and Weber numbers results in the Ohnesorge number, $Oh = \mu/\sqrt{\rho \sigma L}$, which compares viscous and capillary forces.

The Reynolds and Weber numbers define a two-dimensional (2-D) space where different turbulent two-phase regimes/phenomena can be distinguished. Associated to the different regimes are a set of characteristic scales related to the energy and break-up cascades. As presented in the paragraphs below, the flow mechanisms describing their behavior are described by the turbulent eddy sizes and bubble/droplet lengthscales, respectively.

As conceptualized in Richardson’s (statistically steady-state) view of the energy cascade [50] for single-phase flows, turbulence can be considered to be composed of eddies of different sizes $\ell$. The eddies in the largest size range are characterized by the lengthscale $\ell_0$, which is comparable to the integral flow scale $L \approx 6\ell_0$. These turbulent motions are relatively slow, but very energetic, dominated by inertial effects, and unstable. As a result, they break up, transferring their energy to somewhat smaller eddies. These smaller eddies undergo a similar break-up process, and transfer their energy to yet smaller eddies. This energy cascade continues down to the Kolmogorov scale $\eta$ [51], where molecular diffusion is effective in dissipating the kinetic energy and stabilizing the flow motions. As illustrated in Figure 1, the energy cascade can be separated in three distinct parts: (i) the energy-containing range where energy is
L. Jofre and M. S. Dodd

<table>
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<tr>
<th>Initial drop topology</th>
<th>Break-up morphology</th>
<th>Secondary droplets</th>
<th>Initial drop topology</th>
<th>Break-up morphology</th>
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<tr>
<td>No breakup</td>
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<td>Multimode</td>
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<td>$0 &lt; W_e &lt; 3$</td>
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<td>Sheet stripping</td>
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<td>$3 &lt; W_e &lt; 5$</td>
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<td>$100 &lt; W_e &lt; 350$</td>
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<td>Bag breakup</td>
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<td>$W_e &gt; 350$</td>
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Figure 2: Classification of break-up morphologies and secondary droplet formation as function of the Weber number for gas-liquid systems ($Oh < 0.1$). Graphics based on Pilch & Erdmann [52].

introduced (production rate $P$), (ii) the inertial subrange in which energy is transferred to smaller eddies (transfer $T(\ell)$), and (iii) the dissipation range where viscous effects become dominant (dissipation rate $\varepsilon$). The inertial subrange is delimited by the length scales $\ell_{EI} \approx 1/6 \ell_0$ and $\ell_{DI} \approx 60\eta$. The eddy size $\ell_{EI}$ demarcates the separation between the anisotropic large eddies ($\ell > \ell_{EI}$) and the isotropic small eddies ($\ell < \ell_{EI}$), whereas $\ell_{DI}$ splits the universal equilibrium range between the inertial subrange ($\ell_{EI} > \ell > \ell_{DI}$) and the dissipation range ($\ell < \ell_{DI}$). In connection to LES approaches, in which the filter and grid sizes $\Delta_{LES}$ are expected to be sufficiently fine to resolve (approximately) 80% of the kinetic energy in the bulk of the phases [49], the lengthscale $\ell_{EI}$ provides the theoretical maximum LES resolution, i.e., $\Delta_{LES} < \ell_{EI}$.

In the case of multiphase flows with small Ohnesorge numbers ($Oh < 0.1$), such as gas-liquid systems, the importance of viscous forces is relatively small and the Weber number becomes the principal parameter describing break-up behavior. For initially spherical drops, various break-up morphologies have been observed as a function of $We = \rho_c u^2 d / \sigma$, including vibrational, bag, bag-and-stamen, multimode, sheet-thinning and catastrophic modes, where $\rho_c$ is the density of the carrier phase, $d$ is the diameter of the dispersed phase, and $u$ is the relative velocity between phases. Following the work by Pilch & Erdmann [52], the break-up morphology classification of drops based on $We$, or liquid columns viewed from the top, and secondary droplet formation is depicted in Figure 2. The no break-up condition is defined by cases where the drop stays intact, while in vibrational mode, the drop breaks into few large droplets. Bag break-up is generally characterized by the occurrence of a single bag-like shape. If more than one bag is formed, the morphology is considered multimode. Finally, sheet stripping and catastrophic break-up phenomena occur when the edges of the drop are accelerated and separated from the main body before the volumetric core. As shown in the figure, the break-up morphology determines the distribution of the secondary droplets generated from vortical and straining fragmentation mechanisms. The droplet size at which the break-up cascade ends, due to the balance between fragmentation and surface tension forces, corresponds to the Hinze scale [53] defined as

$$R_H \sim \left( \frac{\sigma}{\rho_c} \right)^{3/5} \varepsilon^{-2/5}.$$  \hspace{1cm} (3)

As explained in this paragraph, the set of dimensionless numbers ($Re, We$) and characteristic scales ($\ell_{EI}$,
L. Jofre and M. S. Dodd

The strategy chosen in this work to perform simulations of two-phase turbulence is based on the LEIS approach. The particular methodology developed captures interfaces (topology deformations and break-up processes) on the computational mesh by selecting the grid resolution to be in the order of the Hinze scale, while filters the small scale turbulent fluctuations in the bulk of the phases. In addition, as thoroughly described in Section 4, the near-interface flow motions are modeled instead of resolved to maintain the LEIS approach cost-efficient. In contrast to LES strategies in which the flow and interfaces are filtered [54, 55], resulting in (potentially) large computational savings at expenses of significantly complex mathematical formulations and closure models, the approach presented in this work, although (presumably) more expensive by construction, is designed to be efficient in a large portion of the $Re$-$We$ diagram (yellow/green color region) by balancing computational cost, modeling complexity and accuracy. In detail, Figure 3 is constructed by considering homogeneous isotropic turbulence (HIT) laden with finite-size droplets/bubbles with characteristic diameters in the order of the Taylor microscale $\lambda_T$, and with relative velocities determined by the root-mean-square (rms) turbulent fluctuations $u_{rms}$. Under such assumptions, and knowing that $\eta/\ell_0 \sim Re^{-3/4}$ and $\varepsilon = 15 \nu u_{rms}^2/\lambda_T^2$, the $RH/\eta \sim Re^{18/40} We^{-3/5}$ and $RH/\ell_{EI} \sim Re^{-3/10} We^{-3/5}$ ratios are obtained. Therefore, the lines in the $Re$-$We$ diagram demarcate three different regions: (i) $RH > \ell_{EI}$, $We$ is small and the resulting interfacial structures could be captured with relatively coarse grids, but the condition $\Delta_{LES} < \ell_{EI}$ requires finer resolutions; (ii) $\eta < RH < \ell_{EI}$, portion of the diagram in which the strategy presented is envisioned to be efficient since the mesh required to capture interfaces corresponds to a LES resolution in the bulk of the phases; and (iii) $RH < \ell_{EI} & RH < \eta$, region in which $We$ is considerably larger than $Re$, and as a result the dynamics of the dispersed phase is dominated by break-up processes — methodologies based on point-particle assumptions and/or sub-Hinze-scale modeling should be considered for this regime.

3 LARGE-EDDY SIMULATION EQUATIONS

At isothermal conditions and with no phase change, the equations of fluid motion describing immiscible two-phase incompressible flow are the continuity and Navier-Stokes equations [47]

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot (\mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\mathbf{u} \mathbf{u}) + \nabla \cdot \mathbf{S} + \nabla \cdot \mathbf{f},$$

where $\mathbf{u}$ and $p$ are the velocity and pressure of the flow, $\rho$ and $\mu$ are the piecewise density and viscosity of the one-fluid system, $\mathbf{S}$ is the stress tensor, $n_\Gamma$ is the normal vector, and $\delta(x - x_\Gamma)$ is the Dirac delta function concentrated at the interface location $x_\Gamma$, which evolves according to the velocity field as

$$\frac{dx_\Gamma}{dt} = \mathbf{u}(x_\Gamma, t).$$

The derivation of this set of one-fluid conservation equations assumes that the interface is sharp with respect to the hydrodynamic flow scales, and consequently makes use of the following interfacial jump conditions

$$[\rho]|_\Gamma = \rho_2 - \rho_1 \quad \text{and} \quad [\mu]|_\Gamma = \mu_2 - \mu_1,$$

$$[\mathbf{u}] = 0 \quad \text{and} \quad [-p + n_\Gamma \cdot 2\mu \mathbf{S} \cdot n_\Gamma]|_\Gamma = -\nabla \cdot \mathbf{f}.$$
with $S = 1/2 [\nabla u + (\nabla u)^T]$ the rate-of-strain tensor, and subscripts 1, 2 and $\Gamma$ denoting phase/fluid 1 and 2 and the interface between them, respectively.

The LES equations are derived by applying a low-pass filter $G$ to the equations of fluid motion. The filter decomposes any flow variable $\phi(x,t)$ into large- $\bar{\phi}$ and small-scale $\phi'$ contributions, i.e., $\phi = \bar{\phi} + \phi'$. The filtered part is defined as

$$\bar{\phi}(x,t) = \int_{\Omega} G(x,x',\Delta) \phi(x',t) \, dx',$$  

(9)

with $x$ and $x'$ position vectors in the domain $\Omega$, and $\Delta$ the characteristic width of the filter. However, as introduced in Section 2, the methodology chosen in this work to perform scale-resolving studies of two-phase turbulence is based on a particular version of the LEIS approach; the small flow scales in the bulk of the phases are filtered, while interfaces are resolved. As a result, the filtered interface evolution equation takes the form

$$\frac{d x_{\Gamma}}{dt} = \mathbf{\bar{u}}(x_{\Gamma},t),$$  

(10)

and, since the interface topology is resolved and the right-hand sides involve no velocity terms, the filtered interfacial jump conditions become

$$[\rho]_{\Gamma} = \rho_2 - \rho_1 \quad \text{and} \quad [\mu]_{\Gamma} = \mu_2 - \mu_1,$$  

$$[\mathbf{u}]_{\Gamma} = 0 \quad \text{and} \quad [-p + \mathbf{n}_{\Gamma} \cdot 2 \mu \mathbf{S} \cdot \mathbf{n}_{\Gamma}]_{\Gamma} = -\sigma K,$$  

(11) (12)
Finally, assuming that differentiation and filtering commute [56, 57], the filtered continuity and Navier-Stokes equations result in

$$\nabla \cdot \mathbf{u} = 0,$$

(13)

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \nabla \cdot (\mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \sigma \mathcal{A} \delta (x - x_G) - \rho \nabla \cdot \tau,$$

(14)

where, following Leonard’s decomposition [58], $\tau = \mathbf{uu} - \mathbf{u}\mathbf{u}$ is the subfilter scale (SFS), or turbulent, stress tensor. The resolved scales of LES $\delta$ are characterized by the filter applied to the conservation equations. In a general context, the filtering and discretization operators are different [59]. However, in most cases the spatial discretization is chosen to be specifically the low-pass filter [27], and therefore $\tau$ is habitually referred to as the subgrid-scale (SGS) tensor. Based on the particularization of Sagaut & Germano [60] to incompressible two-phase flows with no phase change, $\tau$ is not altered due to the interfacial discontinuity since $[\mathbf{u}]_G = 0$, only (potentially) by the turbulence modulation resulting from the interaction between dispersed and carrier phases as discussed in Section 4.

### 4 NEAR-INTERFACE FLOW MODELING APPROACH

The Kolmogorov scale, as described in Section 2, characterizes the smallest flow motions in single-phase HIT. In the case of two-phase turbulence, however, the discrete phase introduces (potentially smaller) additional scales through two main mechanisms: (i) deformation and break-up processes, and (ii) velocity gradients and wakes in the vicinity of interfaces. For example, the formation of ligaments and films [61] with sizes determined by the Hinze scale, and boundary-layer-like flow structures [8] with thicknesses dependent on the material and flow properties of the different phases/liquids.

In this section, since the LEIS approach under study is designed to capture the evolution of interfaces down to the Hinze scale, the near-interface flow modeling is focused on the coarse-grained representation of the small-scale, sheared flow motions generated at the interfacial regions to largely improve the cost-effectiveness of the calculations. In particular, the recent work by Dodd & Jofre [8] quantified that the DNS of HIT laden with finite-size droplets requires an order of magnitude larger number of grid points than its single-phase counterpart. Therefore, similar to LES of wall-bounded flows in which models (or special boundary conditions) are required to enlarge the resolution near solid surfaces, the paragraphs below will explore the extension of wall modeling approaches [45, 46] to two-phase turbulent flows.

The first step is to connect boundary-layer theory [62] with the flow structure near interfaces. Following
Csanady’s “slip wall a la Millikan” approach [63], let us consider the mean velocity distribution in the vicinity of an approximately flat interface\(^1\) assuming fully-developed steady-state conditions, in the absence of inertial fluctuations, and with uniform shear stress in the interface-normal direction (i.e., constant relative velocity between phases). As sketched in Figure 4, let the interfacial boundary layer be conceptually divided into an inner and outer region, such that:

1.- In the inner region, the mean velocity profile \(U(\hat{n})\) and turbulence statistics are presumed to depend only on the distance from the interface \(\hat{n}\), the kinematic viscosity of the carrier phase \(\nu_c = \mu_c / \rho_c\), and the interfacial shear stress

\[
\tau_I = \sqrt{(t_1 \cdot 2\mu S \cdot n_I)^2 + (t_2 \cdot 2\mu S \cdot n_I)^2},
\]

where \(t_1\) and \(t_2\) are two orthogonal unit vectors that are tangent to the interface. Note that in the absence of surface tension gradients (no Marangoni stresses), the shear stress is continuous across the interface, i.e., \(\tau_I = \tau_{Ic} = \tau_{Id}\). In addition, \(\tau_I\) is utilized to characterize the friction velocity \(u_{tI} \equiv \sqrt{\tau_I / \rho_c}\) and viscous lengthscale \(\delta_{v,I} \equiv \nu_c / u_{tI}\), and nondimensionalize the variables \(U^+ \equiv U / u_{tI}\) and \(\hat{n}^+ \equiv \hat{n} u_{tI} / \nu_c\) on the carrier-phase side of the interface. Upon dimensional analysis it can be shown that the relative velocity \(U^+ - U_{I}^+\), with \(U_{I}^+ \equiv U_I / u_{tI}\) the interface velocity in viscous units, is of the form

\[
U^+ - U_{I}^+ = f(\hat{n}^+).
\]

This inner region is composed of: (i) the viscous sublayer \((0 < \hat{n}^+ < 5)\) where turbulent fluctuations are suppressed, molecular viscosity dominates momentum transport, and the relative velocity increases linearly with distance from the interface, i.e., \(U^+ - U_{I}^+ = \hat{n}^+\); (ii) the log-law region \((\hat{n}^+ > 30)\), which overlaps with the outer region, where momentum transport is dominated by turbulent fluctuations (Reynolds stresses); and (iii) a buffer layer \((5 < \hat{n}^+ < 30)\) transitional between (i) and (ii). The inner region extends from the interface to an approximate distance of \(\hat{n} \approx 0.15\delta\), where \(\delta\) is the thickness of the boundary layer defined by the location in which the flow velocity is 99\% of the free stream velocity \(U_\infty\), i.e., \(U(\delta) = 0.99U_\infty\).

2.- In the outer region, the mean velocity profile (and turbulence statistics) depends on the friction velocity \(u_{tI}\) and the boundary layer thickness \(\delta\), but not on the kinematic viscosity \(\nu_c\) (momentum transport is entirely dominated by Reynolds stresses), and therefore is function of

\[
U_{I}^+ - U^+ = g\left(\frac{\hat{n}}{\delta}\right).
\]

The outer region extends the region \(30 / Re_{\tau I} < \hat{n} / \delta < 1\), with the friction Reynolds number \(Re_{\tau I} \equiv u_{tI} \delta / \nu_c\) characterizing the ratio between the outer (\(\delta\)) and inner (\(\delta_{v,I}\)) region lengthscales.

3.- In the overlap region, where the inner and outer regions meet, an asymptotic matching argument based on Eqs. 16 and 17 (similar to Millikan’s analysis [64]) implies the existence of a logarithmic relationship between the mean velocity and distance from the interface expressed as

\[
U^+ - U_{I}^+ = \frac{1}{\kappa} \ln \hat{n}^+ + A^+ \quad \text{for} \quad \hat{n} / \delta_{v,I} \to \infty \quad \text{(inner region)},
\]

\[
U_{I}^+ - U^+ = \frac{1}{\kappa} \ln \frac{\hat{n}}{\delta} + B^+ \quad \text{for} \quad \hat{n} / \delta \to 0 \quad \text{(outer region)},
\]

\(^1\)In HIT laden with finite-size droplets, the ratio between cross-sectional perimeter and mean boundary-layer thickness is \(\pi D_0 / \delta \sim 20\) based on DNS studies by Dodd & Jofre [8].
where $\kappa \approx 0.41$ is the von Kármán constant, and $A^+$ and $B^+$ are parameters that depend on the details of the interface and the flow field, respectively. For example, in the case of solid boundaries, $A^+ \approx 5$ for smooth surfaces and $B^+ \approx 2.3$ for zero-pressure-gradient flows. Adding these two equations yields the slip law

$$U_\infty^+ - U_\Gamma^+ = \frac{1}{\kappa} \ln Re_{\tau}\Gamma + A^+ + B^+. \quad (20)$$

Once the classical boundary-layer theory has been adapted to interfacial flows, the final step is to propose a modeling strategy for the near-interface region. As introduced in the sections above, in this work we explore the utilization of approaches developed for wall-stress modeling. Many different methodologies have been proposed in the past decades — see, for example, the recent review by Bose & Park [65] for a detailed exposition — with the aim of representing various boundary-layer phenomena and with different balances between accuracy and computational cost. However, as a first analysis of this problem, we focus on a simple model in which the interfacial-shear stress is algebraically\(^2\) related to the velocity at some distance $\hat{n}^*$ from the interface. In the absence of pressure gradient effects on the boundary layer, the LES velocity profile can be assumed to satisfy a logarithmic law [66] in the form

$$\pi(\hat{n}^+) - u_{\Gamma} = u_{\tau}\Gamma \left[ \frac{1}{\kappa} \ln \left( \frac{\hat{n}^* u_{\tau}\Gamma}{\nu_c} \right) + C \right], \quad (21)$$

where $u_{\Gamma}$ and $u_{\tau}\Gamma$ are the (unknown) interface and friction velocities, respectively, and $C$ is an intercept coefficient particular to the problem. The modeling strategy, therefore, is to utilize the above equation to prescribe the velocity profile near the interface. The problem, however, is that we have 1 equation (Eq. 21) and 2 unknowns ($u_{\Gamma}$ and $u_{\tau}\Gamma$) rendering the system undetermined. To remediate this problem, it is assumed that $u_{\Gamma}$ is dominated by the LES resolved (large) scales, and consequently it can be approximated from the velocity of the grid point capturing the interface, i.e., $u_{\Gamma} \approx u_{\hat{n}=0}$; the implicit assumption is that the kinetic energy of the viscous + buffer layers is much smaller than that of the log-law region. Once $u_{\Gamma}$ is obtained, the system is determined, and the LES velocity at a matching location $\hat{n}^*$ in the range $30 < \hat{n}^+ < 0.15 Re_{\tau}\Gamma$ is utilized to iteratively calculate $u_{\tau}\Gamma$ from Eq. 21.

5 SUMMARY, CONCLUSIONS AND FUTURE WORK

The computational cost of studying turbulence in two-phase systems can be notably reduced by means of LES approaches, in which the large eddies are resolved while their interaction with the small-scale flow motions are modeled. Away from phase interfaces, LES has proven to be (over the past decades) an attractive strategy able to reduce the simulation expense, in terms of grid points per spatial dimension $N$, to a linear relation with the Reynolds number given as $N^3 \sim Re$; the cost of performing DNS is $N^3 \sim Re^{9/4}$. However, similar to the case of solid walls in single-phase turbulence, the performance of the methodology significantly decays as a result of the necessity to properly resolve with fine meshes the boundary layers generated at interfaces.

The near-interface flow modeling approach presented in this work, therefore, aims at keeping the cost of LES linear with respect to the Reynolds number when phase interfaces interact with turbulent flows. The initial step of the methodology is to connect boundary-layer theory with the flow structure near
interfaces by following Millikan’s theoretical analysis applied to the case of interfaces (conceptualized as slip walls). The final step consists in proposing models, inspired from wall-modeling ideas for example, to represent the boundary-layer flow phenomena near interfaces. As a first exploratory work, we have chosen to study simple models in which the interfacial-shear stress is algebraically related to the velocity as a function of distance to the interface, e.g., logarithmic law applied to interfacial flow.

Ongoing work is focused on a priori analyses to assess the potential of the approach presented to reduce the cost of LES while maintaining good accuracy in the context of high-fidelity simulations. The flow studied corresponds to DNS of HIT laden with finite-size droplets for different Weber numbers and ratios of carried and dispersed phase density and viscosity. Future work will consider the application of the modeling strategy to full cases involving the LES of interfacial problems, like for example turbulent two-phase jets, waves and bubbly flow.

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