Sparse generalized polynomial chaos expansion for non-intrusive uncertainty quantification in aerodynamic computations

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ABSTRACT

Because of the high complexity of steady-state or transient fluid flow solvers, non-intrusive uncertainty propagation techniques have been developed in aerodynamic simulations for the consideration of random inputs such as the operational conditions or some geometrical data of the profile. Polynomial surrogate models based on dedicated collocation sets or generalized polynomial chaos (gPC) have usually been implemented [1,2], though kriging-based or radial basis function surrogates may also be envisaged [3]. Polynomial representations suffer from the so-called curse of dimensionality when the number of inputs increases since the evaluation of the expansion coefficients becomes intractable in this situation. Sparse quadrature rules may be achieved using the algorithm proposed by Smolyak (see *e.g.* [1] and references therein), but we envisage in this work to use the sparsity of the output signal, or quantity of interest, in trying to circumvent the dimensionality concern. Indeed, sparsity in the gPC basis expansion is expected to be enhanced in higher dimensions, where it is commonly observed that many cross-interactions between the input parameters are actually negligible. This yields only a small fraction of the polynomial coefficients to be significant, hence a sparse signal. In this context the number of samples needed for the synthesis is typically less than the one anticipated by the Shannon sampling theorem. We therefore expect to achieve a successful signal recovery by the techniques known under the terminology of compressed sensing [4], which are reported to be highly efficient for such sparse signals using incoherent random projections for their reconstruction. The procedure shall be illustrated on some basic examples of flow simulations about uncertain profiles, or fluid-structure interaction test cases with uncertain structural parameters.

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